Isospin dependent Pauli blocking and nucleon mean free path in isospin-asymmetric nuclear matter

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The isospin, density, and energy dependence of the nucleon mean free path in isospin-asymmetric nuclear matter is obtained through geometrical considerations of isospin dependent Pauli blocking and Fermi motion. The nuclear medium isospin effects on nucleon mean free path are studied systematically. It is found that the isospin-dependent nuclear medium effects are important for mean free path of a nucleon in isospin-asymmetric nuclear matter. Meanwhile, the importance of isospin-dependent Pauli blocking in isospin-asymmetric nuclear matter is also discussed.

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The mean free path of a nucleon in nuclear matter is a crucial quantity in theoretical calculations of heavy-ion collisions [1] and also in the predictions of the nuclear transparency [2] measured in (e, e'p) reactions [3]. At low energy it is related to the question of one-body versus two-body dissipation and at medium to high energy to the question of validity of nuclear hydrodynamics or of hot spots. Meanwhile, it is directly connected to the microscopic optical potential, elastic scattering, and total reaction cross section in heavy-ion collisions [4,5]. Theoretically, it has been extensively investigated based on the phenomenological opticalmodel potential or the microscopic many-body theory [2,6-14]. In some previous studies [15,16,7], the nuclear medium effects on nucleon mean free path have been corrected through considering the Pauli blocking and Fermi motion while the isospin effects of nuclear medium were not included, which is, therefore, correct just for the isospinsymmetric nuclear medium. The recent advance in radioactive nuclear beam (RNB) physics provides people a unique opportunity to investigate nuclear physics at extreme isospin degrees of freedom [5,17–23]. Therefore, it is very important and interesting to explore the variation of nucleon mean free path as a function of the nuclear medium isospin degree of freedom.

In this paper, we report results of the first theoretical study on the nuclear medium isospin effects on nucleon mean free path in isospin-asymmetric nuclear matter by considering the isospin-dependent Pauli blocking and Fermi motion. Under reasonable approximation, particularly, we can give the analytical expression for nucleon mean free path in isospinasymmetric nuclear matter. It is found that the nuclear medium isospin effects are important in strongly isospinasymmetric nuclear matter. Meanwhile, the importance of isospin-dependent Pauli blocking in isospin-asymmetric nuclear matter is also discussed.

As in the previous studies [15,16,7,4], we calculate the medium corrections of the nucleon-nucleon (*N*-*N*) cross section by considering the Pauli blocking and Fermi motion. Then, the effective *N*-*N* total cross section $\overline{\sigma}_{NN}$ for a *N*-*N* collision in nuclear matter is given by

$$\bar{\sigma}_{NN} = \frac{1}{(4\pi k_{F_2}^3/3)} \int d\mathbf{k}_2 \frac{2k}{k_1} \int d\Omega \cdot \frac{d\sigma(\mathbf{k}, \mathbf{k}')}{d\Omega} \qquad (1)$$

where \mathbf{k}_1 and \mathbf{k}_2 are momenta of the incident and target nucleons, respectively, and $2\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ is the relative momentum and meanwhile we assume the momenta of the incident and target nucleons to be \mathbf{k}_1' and \mathbf{k}_2' , respectively, and the relative momentum to be $2\mathbf{k}'$ after a single collision. $[d\sigma(\mathbf{k},\mathbf{k}')]/d\Omega$ is the differential cross section that depends on the relative momenta $2\mathbf{k}$ and $2\mathbf{k}'$. k_{F_2} is the fermi momentum of the target nucleon in nuclear matter. It is well known that the main medium corrections for in-medium effective N-N cross sections, are due to three main factors, i.e., Pauli blocking of the intermediate states in nucleon-nucleon scattering, the energy-momentum dispersion relations because of the nuclear mean field, and Pauli blocking of the final states [24-26]. In the present work, our main interest is the nuclear medium isospin effects on the nucleon mean free path in isospin-asymmetric nuclear matter and the former two factors have been neglected and we use the free-space N-N cross section. Meanwhile the angular distribution for the free-space N-N cross section σ_{NN}^{free} is assumed to be isotropic, then Eq. (1) is reduced to

$$\bar{\sigma}_{NN} = \frac{1}{(4\pi k_{F_2}^3/3)} \int d\mathbf{k}_2 \frac{2k}{k_1} \sigma_{NN}^{free}(k) \frac{\Omega_{Pauli}}{4\pi}, \qquad (2)$$

with Ω_{Pauli} the solid angle allowed by the Pauli principle. If the incident nucleon resembles the target nucleon, for example, for neutron-neutron (n-n) or proton-proton (p-p)collision, then the Pauli principle requires:

$$|\mathbf{k}_1'| > k_{F_2}$$
 and $|\mathbf{k}_2'| > k_{F_2}$. (3)

If the incident nucleon does not resemble the target nucleon (n-p or p-n) and we assume the like-incident-particle Fermi

momentum in the nuclear matter to be k_{F_1} , then the Pauli principle requires

$$|\mathbf{k}_1'| > k_{F_1} \quad \text{and} \quad |\mathbf{k}_2'| > k_{F_2}. \tag{4}$$

From the energy and momentum conservation in the collision, the relative momentum $2\mathbf{k}'$ is a vector whose two ends can only locate on the surface of a sphere with center at \mathbf{k} and radius $|\mathbf{k}|$. These constraints yield an allowed scattering solid angle given by

$$\Omega_{Pauli} = \frac{2\pi (k_1^2 + k_2^2 - k_{F_1}^2 - k_{F_2}^2)}{k|\mathbf{k}_1 + \mathbf{k}_2|}.$$
 (5)

The requirement $\Omega_{Pauli} \ge 0$ determines the integration limit for \mathbf{k}_2 as

$$k_2^2 \ge k_{F_1}^2 + k_{F_2}^2 - k_1^2, \tag{6}$$

which gives us the range of integration, according to the magnitude of \mathbf{k}_1 , as

$$0 < k_2^2 < k_{F_2}^2 \quad \text{for} \quad k_1^2 \ge k_{F_1}^2 + k_{F_2}^2, \tag{7}$$

$$k_{F_1}^2 + k_{F_2}^2 - k_1^2 \le k_2^2 \le k_{F_2}^2$$
 for $k_1^2 \le k_{F_1}^2 + k_{F_2}^2$. (8)

Then, the effective total cross section $\bar{\sigma}_{\rm NN}$ can be written in the form

$$\bar{\sigma}_{NN} = \frac{1}{k_1 (4\pi k_{F_2}^3/3)} \int d\mathbf{k}_2 \frac{k_1^2 + k_2^2 - k_{F_1}^2 - k_{F_2}^2}{|\mathbf{k}_1 + \mathbf{k}_2|} \sigma_{NN}^{free}(k).$$
(9)

If we assume that the free-space *N*-*N* cross section $\sigma_{NN}^{free}(k)$ is a constant in the integral region with respect to \mathbf{k}_2 , then the $\sigma_{NN}^{free}(k)$ can be taken outside the integral in Eq. (9), and the integral has the following analytical expression:

$$\bar{\sigma}_{NN} = \begin{cases} \sigma_{NN}^{free}(k) \left(1 - \frac{k_{F_1}^2}{k_1^2} - \frac{2}{5} \frac{k_{F_2}^2}{k_1^2} \right), & \text{for } k_1^2 \ge k_{F_1}^2 + k_{F_2}^2, \\ \sigma_{NN}^{free}(k) \left[1 - \frac{k_{F_1}^2}{k_1^2} - \frac{2}{5} \frac{k_{F_2}^2}{k_1^2} + \frac{2}{5} \frac{(k_{F_1}^2 + k_{F_2}^2)^{5/2}}{k_1^2 k_{F_2}^3} \left(1 - \frac{k_1^2}{k_{F_1}^2 + k_{F_2}^2} \right)^{5/2} \right] & \text{for } k_1^2 \le k_{F_1}^2 + k_{F_2}^2. \end{cases}$$

$$(10)$$

For isospin symmetric nuclear matter, we obtain $k_{F_1} = k_{F_2} = k_F$ and Eq. (10) is reduced to the usual expression [15,16,7,4]

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$$\bar{\sigma}_{NN} = \begin{cases} \sigma_{NN}^{free}(k) \left(1 - \frac{7}{5} \frac{k_F^2}{k_1^2} \right), & \text{for } k_1^2 \ge 2k_F^2, \\ \sigma_{NN}^{free}(k) \left[1 - \frac{7}{5} \frac{k_F^2}{k_1^2} + \frac{2}{5} \frac{k_F^2}{k_1^2} \left(2 - \frac{k_1^2}{k_F^2} \right)^{5/2} \right] & \text{for } k_1^2 \le 2k_F^2, \end{cases}$$

$$(11)$$

which has been used extensively in literature, even for reactions induced by nuclei far from β -stability line [4,5,27–30]. From the difference between Eqs. (10) and (11), however, we expect that the isospin-dependent Pauli blocking would result in a larger deviation for strongly isospin-asymmetric nuclear matter that has been observed experimentally in exotic nuclei [31–33]. We will discuss this point later.

In asymmetric nuclear matter with neutron density ρ_n , proton density ρ_p , and total density ρ , it is convenient to introduce the isospin-average effective nucleon cross section

$$\bar{\sigma}_{N} = (\bar{\sigma}_{Nn}\rho_{n} + \bar{\sigma}_{Np}\rho_{p})/\rho = [\bar{\sigma}_{Nn}(1+\delta) + \bar{\sigma}_{Np}(1-\delta)]/2, N$$
$$= n \text{ and } p, \qquad (12)$$

with neutron excess $\delta = (\rho_n - \rho_p)/\rho$. The mean free path for

a nucleon traveling in asymmetric nuclear matter is then expressed as

$$\lambda_N = \frac{1}{\rho \bar{\sigma}_N}, N = n \text{ and } p.$$
 (13)

In the energy range of 10 MeV $\leq E_{\text{lab}} \leq 1000$ MeV, the experimental data of free-space *N*-*N* cross section σ_{NN}^{free} can be parametrized by [34]

$$\sigma_{np}^{free} = -70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta (\text{mb}),$$
(14)

$$\sigma_{pp(nn)}^{free} = 13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^4 \quad (\text{mb}),$$
(15)



FIG. 1. Energy dependence of *n*-*n* (a), *p*-*p* (b), *n*-*p* (c), and *p*-*n* (d) effective total cross sections at normal nuclear matter density (ρ =0.17 fm⁻³) and different neutron excesses δ =0.0, 0.3, 0.6, and 0.9. The corresponding free-space *N*-*N* cross sections are also included for comparison.

where β is the ratio of projectile nucleon velocity to light velocity. At normal nuclear matter density ($\rho = 0.17 \text{ fm}^{-3}$) and different neutron excesses $\delta = 0.0, 0.3, 0.6, \text{ and } 0.9$, Fig. 1 displays the energy dependence of *n*-*n*, *p*-*p*, *n*-*p*, and *p*-*n* effective total cross sections by calculating numerically Eq. (9). For comparison the corresponding free-space N-N cross section σ_{NN}^{free} is also included in the figure. It is indicated that the nuclear medium isospin effects on the N-N effective total cross sections are very obvious for the incident nucleon energy lower than about 300 MeV. The n-n and n-p effective total cross sections decrease with the increment of neutron excesses δ while the *p*-*p* and *p*-*n* effective total cross sections increase appreciably. With the increment of the energy, the nuclear medium effects as well as its isospin effects disappear gradually and the in-medium N-N effective total cross sections approach their free-space values eventually. The energy dependence of the neutron and proton mean free paths at normal nuclear matter density ($\rho = 0.17 \text{ fm}^{-3}$) and different neutron excesses $\delta = 0.0, 0.3, 0.6, \text{ and } 0.9$ are illustrated in Fig. 2, which also exhibits strong nuclear medium isospin effects for the nucleon energy lower than about 300 MeV, namely, the neutron mean free path increases with the increment of neutron excesses δ while the proton mean free path decreases strongly. In addition, the mean free path from the free-space N-N cross section σ_{NN}^{free} is also included for comparison. Again we find that the nuclear medium effects as well as its isospin effects disappear at higher energy and the in-medium nucleon mean free paths approach their values from the free-space N-N cross section. These phenomena are easy to understand since the Pauli blocking becomes weaker and weaker with the increment of the energy that results in the disappearance of the medium effects at higher energy. On the other hand, the larger neutron excesses δ in neutron-rich nuclear matter lead to the larger radius of Fermi sphere of the neutron in the nuclear matter, which results in the stronger reduction of the total cross section of neutron



FIG. 2. Energy dependence of neutron and proton mean free paths at normal nuclear matter density ($\rho = 0.17 \text{ fm}^{-3}$) and different neutron excesses $\delta = 0.0$, 0.3, 0.6, and 0.9. The result from the free-space *N*-*N* cross section is also plotted as a thick curve for comparison.

and thus the stronger enhancement of the mean free path of neutron. The situation of the proton is opposite to that of the neutron.

In Fig. 3 the neutron and proton mean free paths at E = 100 MeV and different neutron excesses $\delta = 0.0$ and 0.3 are plotted in a domain of nucleon density $\rho = 0.0$ to 0.4 fm⁻³, which are of interest in heavy-ion collisions induced by neutron-rich nuclei at intermediate energies. For comparison the mean free paths from the experimental free-space *N*-*N* cross sections are also included in Fig. 3. One can find from Fig. 3, that in the limit of very dilute nuclear matter the medium-corrected results approaches the free values, which results from the fact that the Pauli blocking disappears at very low densities. In addition, one can see that the medium correction and its isospin effects are more and more



FIG. 3. The neutron and proton mean free paths as a function of nuclear matter density ρ at E = 100 MeV and neutron excesses $\delta = 0.0$ and 0.3. For comparison, the mean free paths from the experimental free-space *N*-*N* cross sections are also included.



FIG. 4. The inverse mean free path $1/\lambda$ for neutron and proton as a function of the neutron excess δ at a fixed energy E=100 MeV and different densities ρ =0.085 (a), 0.17 (b), and 0.34 (c) fm⁻³. The solid lines correspond to the results using the isospindependent Pauli blocking Eq. (4) while the dashed lines correspond to the results using the isospin-independent Pauli blocking, namely, in the integral of Eq. (9) $k_{F_1} = k_{F_2} = k_F$ has been assumed. The dash-dotted lines represent the values at δ =0.0.

important with increment of nuclear matter density. Therefore, the usual ansatz to approximate the in-medium correction of *N*-*N* cross sections by a constant rescaling of the *N*-*N* cross sections or a linear ρ dependence [35,36] is not enough in the transport-model simulations of intermediate energy heavy-ion collisions induced by nuclei near drip line.

The inverse mean free path $1/\lambda$ for neutron and proton as a function of the neutron excess δ is plotted in Fig. 4 at a fixed energy E=100 MeV and different densities $\rho = 0.085$, 0.17, and 0.34 fm⁻³. The solid lines correspond to the results using the isospin-dependent Pauli blocking Eq. (4) while the dashed lines correspond to the results using the isospin-independent Pauli blocking, namely, in the integral of Eq. (9) we have assumed $k_{F_1} = k_{F_2} = k_F$ but Eq. (12) remains unchanged. From Fig. 4 one can see clearly that the isospin-dependent Pauli blocking effect is very important for larger neutron excesses and this effect become more and more important with increasing nuclear matter densities and neutron excesses. In fact, the importance of isospindependent Pauli blocking for the final states of collision nucleon pair has also been observed in the time evolution of isospin degrees of freedom and nuclear collective flow phenomenon in intermediate-energy heavy-ion collisions [37,38]. In addition, one can find from Fig. 4, that except for very small neutron excesses the shift of neutron and proton inverse mean free paths is not symmetric with respect to their common value at $\delta = 0.0$ (symbolized by dash-dotted lines) when the isospin-dependent Pauli blocking is used. The most striking effect of the isospin asymmetry is the sizable enhancement of the neutron mean free path at large neutron excesses, which implies that the nuclear surface would become more transparent to neutrons than protons in nucleoninduced reactions on nuclei near the neutron drip line. This effect would be especially more pronounced for the results at higher density as shown in Fig. 4. In calculations, we also find that these effects become much stronger at energy of a few tens MeV. These features imply that it seems to be dangerous, at least for lower energies, to use Eq. (11) in



FIG. 5. The energy dependence of *n*-*p* total cross sections for $\rho = 0.0$ and 0.18 fm⁻³ at neutron excess $\delta = 0.3$. The in-medium *n*-*p* differential cross sections from the Dirac-Brueckner approach based on Bonn A [24] are used.

nucleon-induced reactions on nuclei near drip line where the neutron excess δ deviates strongly from zero.

In the calculations above, only Pauli blocking of the final states has been considered as the medium effect on the N-N cross section and meanwhile σ_{NN}^{free} has been assumed to be isotropic. Using the in-medium n-p differential cross sections from the Dirac-Brueckner approach based on Bonn A [24], which include the medium corrections from Pauli blocking of intermediate state as well as mean field, we display in Fig. 5 the energy dependence of n-p total cross sections for $\rho = 0.0$ and 0.18 fm⁻³ at neutron excess $\delta = 0.3$. The thick and thin solid lines are in-medium n-p total cross sections for $\rho = 0.0$ and 0.18 fm⁻³, respectively, from Ref. [24]. The thin dashed line is the result by calculating numerically Eq. (1) for $\rho = 0.18$ fm⁻³ while the thin dotted line represents the result assuming n-p cross section to be isotropic. The thick dashed line is the result for $\rho = 0.0 \text{ fm}^{-3}$ assuming n-p cross section to be isotropic and the thick dotted line is the result from Eq. (10) for $\rho = 0.0$ fm⁻³. From Fig. 5 one can find that the isotropic assumption for N-N cross section results in a small error and the constant assumption for N-N total cross section in Eq. (10) is a better approximation at higher energies (higher than about 100 MeV/nucleon). Meanwhile, it is indicated in Fig. 5 that the medium effects from Pauli blocking of intermediate state as well as mean field are important for in-medium effective n-p total cross sections. Fortunately, here our main interest is the nuclear medium isospin effects on the nucleon mean free path and the previous conclusion remains qualitatively unchanged.

In summary, we presented a microscopic derivation of the isospin, density, and energy dependence of the nucleon mean free path in isospin-asymmetric nuclear matter through considering isospin-dependent Pauli blocking and Fermi motion. In this scenario, one can get very simple, even analytical, expressions for effective *N*-*N* cross sections or nucleon mean

free path in isospin-asymmetric nuclear matter, which is physically more transparent and easier to be applied. The calculated results show that the isospin-dependent nuclear medium effects are important for mean free path of a nucleon in strongly isospin-asymmetric nuclear matter at energies lower than about 300 MeV. Meanwhile, the isospindependent Pauli blocking in isospin-asymmetric nuclear matter is found to be important, at least at lower energies, for effective *N-N* cross sections or nucleon mean free path in strongly isospin-asymmetric nuclear matter that could be formed in heavy-ion collisions induced by nuclei far from

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 β -stability line or exist in exotic nuclei observed experimentally.

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