Energies and widths of $T=\frac{3}{2}$ states in $A=11$ nuclei

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We have calculated energies and widths of the three lowest $T=\frac{3}{2}$ states in ^{11}B, ^{11}C, and ^{11}N. Comparison with data on known levels suggests that the $\frac{1}{2}^+$, $T=\frac{3}{2}$ states in ^{11}B and ^{11}C have been misidentified. Our calculations for ^{11}N are in agreement with measurements.

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I. INTRODUCTION

Interest in ^{11}N has led us to reexamine the supposed $T=\frac{3}{2}$ states in the $T_z=\pm\frac{1}{2}$ nuclei ^{11}B and ^{11}C, which are analogs and double analogs, respectively, of the levels in ^{11}Be. The lowest three levels of ^{11}Be, at excitation energies of 0.0, 0.32, and 1.78 MeV, have J^π of $\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{5}{2}^+$, respectively. Their configurations are dominantly $^{10}\text{Be}\otimes n$, with n in $2s\frac{1}{2}$, $1p\frac{1}{2}$, and $1d\frac{5}{2}$ orbitals. The assignments are confusing for the corresponding $T=\frac{3}{2}$ states in the $T_z=\pm\frac{1}{2}$ nuclei.

^{11}B *and* ^{11}C. A casual inspection suggests either large isospin mixing or incorrect identication of levels. We discuss the situation in detail below.

In ^{11}Be and ^{11}N, the parentage of these $T=\frac{3}{2}$ states is unique, viz., $^{10}\text{Be}\otimes n$ and $^{10}\text{C}\otimes p$, respectively; but in ^{11}B and ^{11}C, two parentages contribute to each.

For example, in ^{11}B

$$^{11}\text{B}\left(T=\frac{3}{2}\right)=\frac{1}{3}{}^{10}\text{Be}\otimes p+\frac{2}{3}{}^{10}\text{B}^*\otimes n \tag{1}$$

and in ^{11}C

$$^{11}\text{C}\left(T=\frac{3}{2}\right)=\frac{2}{3}{}^{10}\text{B}^*\otimes p+\frac{1}{3}{}^{10}\text{C}\otimes n,$$

where ^{10}B* denotes $T=1$ levels of ^{10}B.

The parent states ^{10}Be, ^{10}B*, and ^{10}C are predominantly the lowest $(0^+,1)$ and $(2^+,1)$ levels. We will use these below to compute expected energies and widths.

There have been at least six experiments attempting to populate the $\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{5}{2}^+$ levels of ^{11}N, the mirrors of the corresponding levels in ^{11}Be. The most recent one by Markenroth *et al.* [1] used radioactive beams of ^{10}C at GANIL and MSU to study the $^{10}\text{C}+p$ elastic resonance scattering, a reasonable but difficult experiment, both in execution and in analysis. Others use exotic reactions with ^{14}N and ^{12}N, such as the experiment of Oliveira *et al.* [2] (not listed in [1]), using the reaction $^{10}\text{B}(^{14}\text{N},^{13}\text{B})^{11}\text{N}$. The results of these experiments are listed in Table I [3–6]. The energies and widths vary appreciably for the $\frac{1}{2}^+$ level and widely for the widths of the other two.

II. ISOBARIC MULTIPLET MASS EQUATION

If the reported energies [7] of the $T=\frac{3}{2}$ levels of ^{11}Be, ^{11}B, and ^{11}C are correct, the levels of ^{11}N should be given by the Isoboric Multiplet Mass Equation (IMME):

$$M(T_z)=a+bT_z+cT_z^2,$$

which yields

$$^{11}\text{N}={}^{11}\text{Be}+3(^{11}\text{C}-{}^{11}\text{B}), \tag{2}$$

where each term is the mass of the state with specific J^π. The results are shown in Table II, which lists the excitation energies of the ^{11}Be, ^{11}B, ^{11}C levels [7] followed by the IMME results for ^{11}N and the experimental values from Table I. The ^{11}N values are relative to $(^{10}(\text{C})+p)$.

We see that the agreement is excellent for the $\frac{1}{2}^-$ and $\frac{5}{2}^+$ levels of ^{11}N, but that the IMME prediction for the $\frac{1}{2}^+$ level is off by 0.31 to 0.67 MeV. If the ^{11}B and ^{11}C energies for the $(\frac{1}{2}^+,\frac{3}{2})$ levels are correct, this discrepancy might be the result of a T_z^3 term in the IMME. Its coefficient d is given by

$$d=\frac{1}{2}[^{11}\text{C}-{}^{11}\text{B}]-\frac{1}{6}[^{11}\text{N}-{}^{11}\text{Be}], \tag{3}$$

yielding $d=52$ to 112 keV for the $\frac{1}{2}^+$ state.

Antony *et al.* [8] list 27 quartets from $A=7$ to $A=41$. Only $A=9$ has a nonzero d coefficient (5.2 ± 1.7) keV. A value of 10–20 times that for $A=11$ would appear outrageous. No reasonable amount of isospin mixing could result in such a large d coefficient.

A simpler explanation might be that the true $(\frac{1}{2}^+,\frac{3}{2})$ levels have not been correctly identified experimentally.

III. POTENTIAL-MODEL CALCULATIONS

We explore this possibility further by computing energies and widths in a simple potential model. With good isospin, the spectroscopic factor S should be the same for all four

TABLE I. Experimental levels of ^{11}N. Energies [relative to (^{10}C+p)] and widths are in MeV.

Reaction	$\frac{1}{2}^+$		$\frac{1}{2}^-$		$\frac{5}{2}^+$	
	E	Γ	E	Γ	E	Γ
^{14}N(^{3}He,^{6}He)[a]			2.24(10)	0.74(10)		
^{12}C(^{14}N,^{15}C)[b]			2.18(5)	0.44(8)	3.63(5)	0.40(8)
^{9}Be(^{12}N,^{11}N)[c]	1.45(40)	>0.4	2.24(10)	0.74(10)		
^{10}B(^{14}N,^{13}B)[d]	1.63(5)	0.4(1)	2.16(5)	0.25(8)	3.61(5)	0.50(8)
(^{10}C+p)[e]	$1.27^{(+18)}_{(-5)}$	1.44(20)	$2.01^{+(15)}_{-(5)}$	0.84(20)	3.75(5)	0.60(5)
^{12}O decay[f]	<1.45					

[a]Reference [3].
[b]Reference [4].
[c]Reference [5].
[d]Reference [2].
[e]Reference [1].
[f]Reference [6].

members of the $T=\frac{3}{2}$ quartet. This quantity S can be determined experimentally in a single-nucleon transfer reaction or, for unbound states, from the experimental width Γ_{sp} of the state compared to a calculated single-particle width Γ_{sp}: $C^2S=\Gamma_{exp}/\Gamma_{sp}$, where C^2 values are the coefficients in Eq. (1).

Information on known spectroscopic factors in ^{11}Be is listed in Table III [9–14]. For the ground state (g.s.), S ranges from 0.73 to 0.84—i.e., ^{11}Be (g.s.) is dominantly ^{10}Be(g.s.)$\otimes 2s\frac{1}{2}$. The remaining ~20% of the wave function is almost certainly dominated by ^{10}Be$(2^+)\otimes 1d\frac{5}{2}$. Because Coulomb energies for $2s\frac{1}{2}$ and $1d\frac{5}{2}$ behave differently as a function of Z, the computed energies in ^{11}B, ^{11}C, and ^{11}N will depend somewhat on the precise value of this admixture. However, the range is reasonably small in Table III, and we use $0.80(0^+\otimes 2s\frac{1}{2})+0.20(2^+\otimes 1d\frac{5}{2})$ for the $\frac{1}{2}^+$ state of ^{11}Be.

The value of S for the $\frac{1}{2}^-$ state of ^{11}Be is less certain, but here the particle to be coupled to the core is in the 1p orbital for all reasonable configurations, and hence, the computed energies will be very insensitive to the value of S.

The value of S for the $\frac{5}{2}^+$ state of ^{11}Be is near 0.50, with a large uncertainty. For the present work we use the measured width $\Gamma_{exp}=100\pm 20$ keV, together with our calculated $\Gamma_{sp}=175$ keV to get $S=0.57\pm 0.11$.

To calculate energies and decay widths in the other nuclei we use the Woods-Saxon model. As in Ref. [15] we use $r_0=1.25$ fm and $a=0.65$ fm, and assume the resonances are those of $d\theta/dE$ where θ is the scattering phase at proton energy E.

TABLE II. IMME vs Experimental values for the levels of ^{11}N (E_x in MeV for ^{11}Be, ^{11}B, ^{11}C; E relative to ^{10}C+p for ^{11}N).

J	^{11}Be[a]	^{11}B[a]	^{11}C[a]	^{11}N$_{IMME}$	^{11}N$_{exp}$[b]
$\frac{1}{2}^+$	0	12.557(16)	12.16(4)	1.94(13)	1.27-1.63
$\frac{1}{2}^-$	0.320	12.916(12)	12.51(3)	2.24(10)	2.01-2.24
$\frac{5}{2}^+$	1.778	14.34(2)	13.90(2)	3.59(9)	3.61-3.75

[a]From Ref. [7].
[b]Range of values from Table I.

This approach has difficulties as the resonance energy approaches the top of the Coulomb plus centrifugal barrier. In those cases, we obtain resonance energies by extrapolation and single-particle widths by matching smoothly to asymptotic penetrabilities.

We assume that the $T=1$ levels of the $A=10$ cores, which are the parents of the $A=11$ levels are the $(0^+,1)$ and $(2^+,1)$ levels. The S factors of the $(0^+,1)$ cores are taken from Table V, which we discuss shortly and the remainder is ascribed to the $(2^+,1)$ cores.

Our results for the energies are displayed in Table IV. For $\frac{1}{2}^-$ and $\frac{5}{2}^+$ the results are excellent (deviations of -20 to $+50$ keV in ^{11}B and ^{11}C). In ^{11}N the experimental uncertainty is too large for $\frac{1}{2}^-$ for a valid test, but the $\frac{5}{2}^+$ result is quite satisfactory.

However, for $\frac{1}{2}^+$ the deviations in ^{11}B and ^{11}C are much larger: -113 keV and $+320$ keV, respectively. We suspect misidentification of these levels—which we explain further below with S comparisons.

Calculated widths for single-particle decay to the appropriate 0^+ core are listed in Table V. (Decays to the 2^+ core are energetically forbidden, or nearly so.) We then compute S from known widths: $S=\Gamma_{exp}/C^2\Gamma_{sp}$. For the $\frac{5}{2}^+$ level in ^{11}B and ^{11}C, both proton and neutron decay are possible in isospin allowed channels and we sum $C^2\Gamma_{sp}$ for those in calculating S.

For the $\frac{1}{2}^-$ and $\frac{5}{2}^+$ levels, we note reasonable agreement, within the uncertainties, for S in ^{11}B, ^{11}C, and ^{11}N. However, for the $\frac{1}{2}^+$ state, the spectroscopic factors in ^{11}B and ^{11}C are only about 30% of the expected value, i.e., the states

TABLE III. Experimental spectroscopic factors for ^{11}Be.

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{5}{2}^+$
0.73[a]	0.63[a]	0.57[f]
0.77[b]	0.96[b]	0.50[b]
0.84[c]		
0.80[d]		
0.84[e]		

[a]Reference [9].
[b]Reference [10].
[c]Reference [11].
[d]Reference [13].
[e]Reference [14].
[f]Reference [12] and Table IV.

TABLE IV. Comparison of the $T=\frac{3}{2}$ quartets of $A=11$.

J	config	^{11}Be	^{11}B	^{11}C	^{11}N	$S\Gamma(^{11}\mathrm{N})$
$\frac{1}{2}^+$	$(0^+\otimes 2s)$	0	12.371	11.679	1.081	
	$(2^+\otimes 1d)$	0	12.734	12.503	2.441	
	$0.80(0^+)+0.20(2^+)$[a]	0	12.444	11.844	1.353	0.87
	exp	0	12.557(16)	12.16(4)	1.22-1.63[b]	0.4-1.6[b]
	calc−exp	0	−0.113(16)	−0.32	−0.28 to +0.08	
$\frac{1}{2}^-$	$0^+\otimes 1p$	0.320	12.834	12.359	2.040	
	$2^+\otimes 1p$	0.320	13.049	12.807	2.723	
	$0.74(0^+)+0.16(2^+)$[c]	0.320	12.890	12.476	2.218	0.71
	exp	0.320	12.916(12)	12.51(3)	2.01-2.24[b]	0.25-0.84
	calc−exp	0	−0.017(12)	−0.02(3)	−0.02 to +0.21	
$\frac{5}{2}^+$	$0^+\otimes 1d$	1.778	14.366	13.985	3.770	
	$2^+\otimes 2s$	1.778	14.365	13.903	3.478	
	$0.57(0^+)+0.43(3^+)$[a]	1.778	14.366	13.949	3.645	0.37
	exp	1.778	14.34(2)	13.90(2)	3.66(6)[b]	0.50(10)
	calc−exp	0	+0.03(2)	+0.05(2)	−0.01(6)	

[a]Table III.
[b]Table I.
[c]Tables III and V.

that have been suggested as $\frac{1}{2}^+$, $T=\frac{3}{2}$ in these nuclei have widths that are only about $\frac{1}{3}$ of the expected values. In ^{11}N the exact g.s. width is uncertain, but is consistent with expectation. We suggest that the $\frac{1}{2}^+$, $T=\frac{3}{2}$ levels in ^{11}B and ^{11}C remain to be observed. Perhaps the reactions ^{10}Be(d,n), ^{9}Be$(^{3}$He$,p)$, and ^{9}Be$(^{3}$He$,n)$ should be investigated. In ^{9}Be$(t,p)^{11}$Be [12] the cross-section ratio $\sigma(\frac{1}{2}^+)/\sigma(\frac{1}{2}^-)$ is about 0.24, whereas in an earlier ^{9}Be$(^{3}$He$,p)^{11}$B$(T=\frac{3}{2})$ measurement [16] the ratio is $\sigma(\frac{1}{2}^+)/\sigma(\frac{1}{2}^-)=1.1$. Of course, this is further evidence that the $\frac{1}{2}^+$ state has been misidentified. For $T=\frac{3}{2}$ levels, $\sigma(\frac{1}{2}^+)/\sigma(\frac{1}{2}^-)$ should be equal for (t,p), $(^{3}$He$,p)$ and $(^{3}$He$,n)$ under equivalent kinematic conditions.

It might be possible to investigate ^{11}C states in a ^{10}C(d,p) reaction with a radioactive ^{10}C beam.

IV. CONCLUSION

In ^{11}B, ^{11}C, and ^{11}N the measured energies of the lowest $\frac{1}{2}^-$ and $\frac{5}{2}^+$ states are in agreement both with the IMME and

TABLE V. Computed spectroscopic factors for the resonance energies of the $A=11$ quartets. (Energies and widths in MeV.)

J	Nucleus	E_{exp}	Γ_{sp}	Γ_{exp} [a]	$S=\Gamma_{exp}/C^2\Gamma_{sp}$
$\frac{1}{2}^+$	^{11}Be				0.80[b]
	^{11}B	1.33	2.40	0.21(2)	0.26
	^{11}C	1.73	2.40	0.27(5)	0.26
	^{11}N	1.45[c]	1.28	>0.7[c]	>0.55
$\frac{1}{2}^-$	^{11}Be				0.80(16)[b]
	^{11}B	1.69	0.87	0.20(3)	0.69(10)
	^{11}C	2.08	1.10	0.49(4)	0.67(6)
	^{11}N	2.24[d]	1.02	0.74(10)[d]	0.73(10)
$\frac{5}{2}^+$	^{11}Be	1.275	0.175	0.100(20)	0.57(11)
	^{11}B[e]	3.11, 1.15	0.59, 0.15	0.25(2)	0.84(7)
	^{11}C[e]	3.49, 0.79	0.74, 0.06	0.2(1)	0.4(2)
	^{11}N	3.66[c]	0.65	0.50(5)[c]	0.77(8)

[a]Reference [7].
[b]Average for Table III.
[c]Average for Table I.
[d]Reference [3].
[e]Both proton neutron energy and Γ_{sp} listed.

simple potential-model calculations. Measured S factors also agree across the quartet for these two states.

For $\frac{1}{2}^+$, however, energies disagree with the IMME. In ^{11}B and ^{11}C they disagree with potential-model calculations. Furthermore, the S factors in ^{11}B and ^{11}C are only one-third their expected values. We suggest the $\frac{1}{2}^+ T=\frac{3}{2}$ levels in these two nuclei have yet been located and we mention possible reactions to use in searching for them. The $\frac{1}{2}^+$ energy of ^{11}N is in reasonable agreement with calculations, but a better experimental value is desirable. One possible reaction is $d(^{10}\mathrm{C}^{11}\mathrm{N})n$.

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