SU(4) symmetry and Wigner energy in the infinite nuclear matter mass model

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The measures for SU(4) symmetry and the Wigner energy, in terms of double binding energy differences, for even-even and odd-odd nuclei in $A=56$–100 mass region are studied using the infinite nuclear matter (INM) mass model of atomic nuclei. The INM model predicts that the SU(4) symmetry is broken in even-even nuclei but remnants of this symmetry should be present in $N=Z$ odd-odd nuclei in this region. Similarly, the estimates of the Wigner energy indicate that the $T=0$ states in these nuclei should start appearing around 0.5 MeV above the ground states that are $T=1$ in nature.

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With the advent of radioactive ion beam facilities, in the last few years considerable interest has been generated in investigating structure of heavy nuclei near the proton drip line and, in particular, the $N=Z$ nuclei in the mass range $A=56$–100 [1,2]. These nuclei are expected to give new insights into neutron-proton (*np*) correlations that are hitherto unknown. Towards this end, for example, using nuclear masses the questions studied so far in literature are, (i) signatures for Wigner's spin-isospin SU(4) symmetry [3]; (ii) relationship between $T=0$ pairing and the so-called Wigner energy [4]; (iii) effects of spin-orbit force on the relative positions of $T=0$ and $T=1$ states in odd-odd nuclei [5]; (iv) possible formation of a condensate of $T=0$ *np* Cooper pairs [6]; (v) pairing vibrations (phonons) in the isospin channels [7], etc. All the results in Ref. [3–7] are limited by the known experimental masses, especially those of the $N=Z$ odd-odd nuclei with $A>60$; see [8] for experimental data. Therefore, a natural question is whether there exists a good model for predicting nuclear masses. Then using such a model it is possible to go beyond $A=60$ and study the issues involved in (i)–(v). One such model is the infinite nuclear matter (INM) model introduced recently [9,10]. In this Brief Report the focus is on (i) and (ii), i.e., on signatures of SU(4) symmetry and Wigner energy in $A=56$–100 nuclei.

The INM model [9,10] is based on the Hugen Holtz-Van Hove (HVH) theorem [11] of many-body theory. For asymmetric nuclear matter, the theorem primarily connects the neutron and proton Fermi energies $\epsilon_n=(\partial E/\partial N)_Z$ and $\epsilon_p=(\partial E/\partial Z)_N$ and the mean energy per particle E/A as $E/A=[(1+\beta)\epsilon_n+(1-\beta)\epsilon_p]/2$, where β is the asymmetry parameter $(N-Z)/(N+Z)$. Extending the HVH theorem to finite nucleus it is shown that the total energy could be written as sum of three distinct parts, namely, the nuclear matter part E, a global part f, and a local part η. The first two parts governing the global behavior of all nuclei, such as the volume, surface, symmetry, coulomb, and pairing energies are characterized by five parameters and these are determined once for all using known masses. The local energies $\eta(N,Z)$ satisfying linear difference equations are determined by using an ensemble averaged procedure as described in detail elsewhere [10]. The model, as such, is found, to be quite successful both for mass predictions [10] as well as for the determination of nuclear saturation properties including the nuclear incompressibility [9]. More recently [12] it is shown that the INM model, via the propagation of $\eta(N,Z)$ into unknown regions, predicts quenching of magic numbers near the drip lines, as implied by the astrophysical *r*-process nuclidic abundances [13]. Encouraged by this success, INM masses are employed in this Brief Report in calculating the measures for SU(4) symmetry and the Wigner energy, in terms of double binding energy differences, for even-even and odd-odd nuclei in the $A=56$–100 mass region.

Van Isacker *et al.* [3] suggested recently that the double binding energy differences, defining average *np* interaction δV_{np} as

$$\delta V_{np}^{ee}(N,Z)=\frac{1}{4}[B(N,Z)+B(N-2,Z-2)-B(N-2,Z)-B(N,Z-2)]$$

$$\delta V_{np}^{oo}(N,Z)=[B(N,Z)+B(N-1,Z-1)-B(N-1,Z)-B(N,Z-1)] \qquad (1)$$

for even-even (ee) and odd-odd (oo) nuclei, respectively, carry the signatures of Wigner's spin-isospin SU(4) symmetry. In Eq. (1) $B(N,Z)$ represents the negative binding energy of a given nucleus. In order to obtain the binding energy differences in Eq. (1) in the SU(4) limit, one starts with the valence nucleons (m in number) in an oscillator shell. Then, the $U(4)$ irreducible representations (irreps) for this system are denoted by $\{F_1,F_2,F_3,F_4\}$, where $\Sigma_i F_i=m$ and $F_1\geqslant F_2\geqslant F_3\geqslant F_4\geqslant 0$. The irreps of the corresponding SU(4) group are $\{f_1,f_2,f_3\}=\{F_1-F_4,F_2-F_4,F_3-F_4\}$. It is often convenient to use the O(6) group that is isomorphic to the SU(4) group [14]. The irreps of O(6) are $[\omega_1,\omega_2,\omega_3]=[(f_1+f_2-f_3)/2,(f_1-f_2+f_3)/2,(f_1-f_2-f_3)/2]$. Identifying the O(6) [or SU(4)] irreps for the ground states and assuming that the binding energies are linear in the quadratic Casimir invariant (C_2) of SU(4) [or O(6)], the δV_{np} in Eq. (1) can be evaluated in the SU(4) limit. The eigenvalues of $C_2(SU(4))$ in a given SU(4) irrep are $\langle C_2(SU(4))\rangle^{\{f_1,f_2,f_3\}}=4\langle C_2(O(6))\rangle^{[\omega_1,\omega_2,\omega_3]}=4(\omega_1(\omega_1+4)+\omega_2(\omega_2+2)+\omega_3^2)$. For even-even nuclei

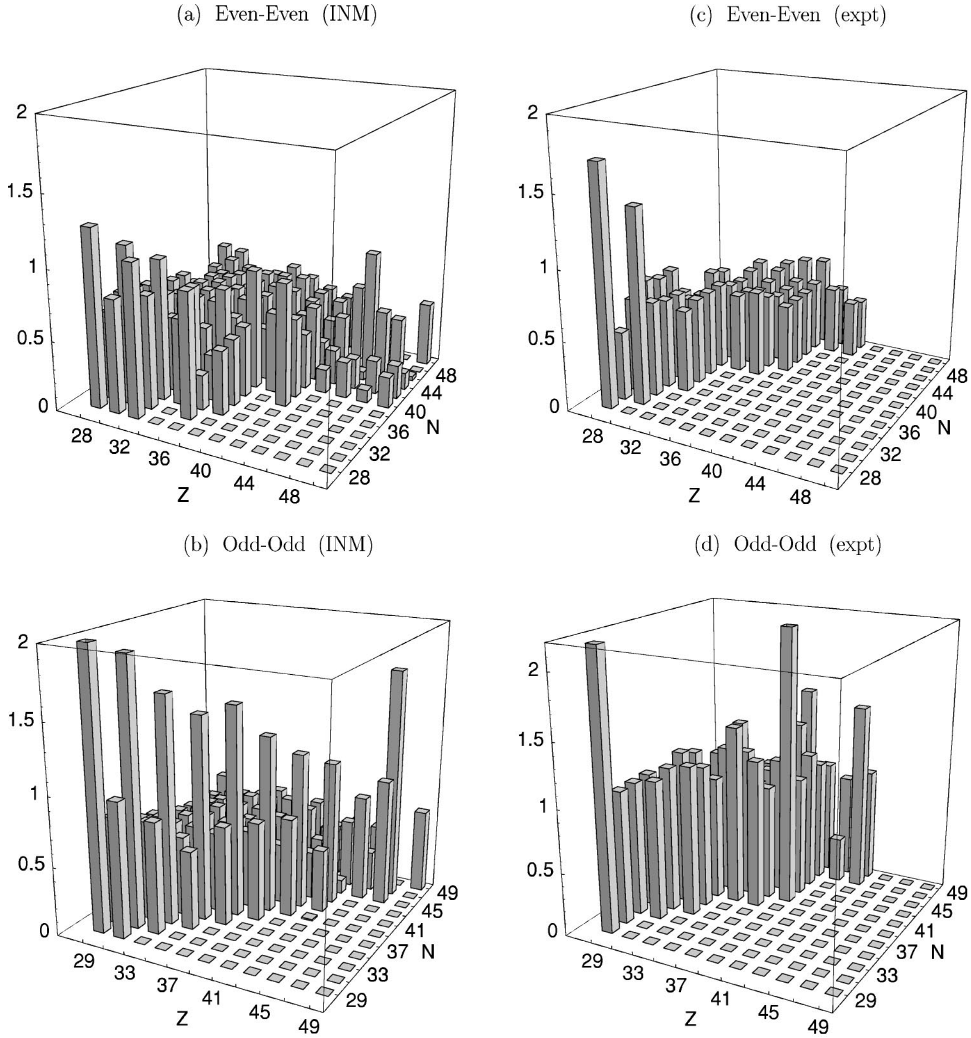

FIG. 1. Double binding energy differences $|\delta V_{\mathrm{np}}|$ defined by Eq. (1) for even-even and odd-odd nuclei for various (N,Z) values are shown as a bar chart. (a) Results from INM masses for even-even nuclei. (b) Same as (a) but for odd-odd nuclei. (c),(d) Same as (a),(b) but from experimental masses.

the ground state O(6) irreps are $[T=|N-Z|/2]$. However, for odd-odd $N=Z$ nuclei the ground state is $[1]$ and for $N\neq Z$ nuclei it is $[T,1]$. Similarly for odd-A nuclei with $N=Z\pm1$ the irreps are $[\frac{1}{2},\frac{1}{2},\pm\frac{1}{2}]$ and in other cases the irreps are $[T\pm\frac{1}{2},\frac{1}{2},\frac{1}{2}]$. Using these, as pointed out in [3], with $B(N,Z)=a+bC_2[SU(4)]$ the $|\delta V_{np}/b|$ takes value 10 for $N=Z$ nuclei and 2 for $N\neq Z$ nuclei (this result is valid for both even-even and odd-odd nuclei). Van Isacker *et al.* [3] showed, by using the measures defined by Eq. (1), that the $(2s1d)$ shell nuclei exhibit the presence of SU(4) symmetry remarkably. Further, they speculate that SU(4), in the form of pseudo-SU(4), may be present in the $A=56$–80 mass region.

In order to test SU(4) signatures in the $A=60$–100 mass region, the INM masses [10] are used in calculating the double binding energy differences given by Eq. (1). The results are shown in Figs. 1(a) and 1(b). For comparison, the results of Eq. (1) with the available experimentally determined masses are shown in Figs. 1(c) and 1(d). As pointed out in the beginning the experimental data is limited and especially the important domain of $N=Z$ region is largely missing. Therefore it is not possible to infer about SU(4) symmetry from Figs. 1(c) and 1(d). However, from Figs. 1(a) and 1(b) it is seen that the INM model predicts interesting structures. As seen from Fig. 1, the INM model predicts enhancements in $|\delta V_{pn}|$ along the $N=Z$ line for both even-even and odd-odd nuclei. However the enhancements in odd-odd nuclei are somewhat larger. These enhancements are typically by a factor of 2 in odd-odd and about 1.5 in even-even nuclei. Note that in the exact symmetry limit, the enhancement should be by a factor of 5; for the $(2s1d)$ shell nuclei they are typically 3–3.5. For example, for $N=Z=38$, $|\delta V_{pn}|$ is 0.86 MeV while for the neighboring $N\neq Z$ nuclei the values are 0.38, 0.51, 0.49, 0.58 MeV. Similarly for $N=Z=37$ the value is 1.53 MeV while for the neighboring $N\neq Z$ nuclei the values are 0.57, 0.72, 0.58, 0.72 MeV. In order to reemphasize this observation, the results in Figs. 1(a) and 1(b) are presented in a different form in Fig. 2. It is clearly seen that the difference in δV_{np} between the $N=Z$ and $N\neq Z$ nuclei is rather small in even-even nuclei and it is negligible beyond $Z=40$. On the other hand, for odd-odd nuclei the differences are much larger. Thus one can conclude that SU(4) is broken in the $A=56$–100 mass region but relatively SU(4) should be a better symmetry in the $N=Z$ odd-odd nuclei as compared to even-even nuclei. Finally, as seen from Figs. 1(c) and 1(d) the experimental data for $N=Z=28,29,30$ also show the presence of SU(4) symmetry to the same degree as indicated by the INM results.

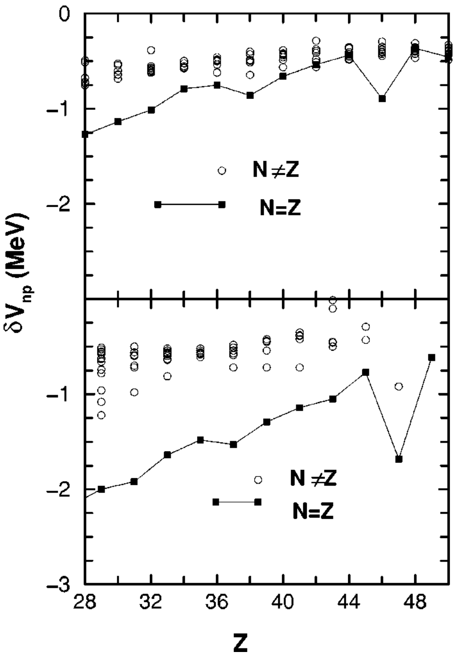

FIG. 2. δV_{np} vs Z from INM masses, for even-even and odd-odd nuclei.

Now we will consider the Wigner energy [item (ii) mentioned in the beginning] in terms of double binding energy differences.

Experimental masses of even-even and odd-odd nuclei exhibit cusps at $N=Z$ indicating additional binding in these nuclei with protons and neutrons occupying the same shell model orbitals. In order to account for this feature, following the SU(4) symmetry considerations, an additional term called Wigner energy (E_w) is added in nuclear mass formulas [4]

$$E_w = W(A)|N-Z| + d(A)\pi_{np}\delta_{NZ}, \tag{2}$$

where $\pi_{np}=(1-\pi_p)(1-\pi_n)/4$ and $\pi_n=(-1)^N$ and $\pi_p=(-1)^Z$ being nucleon number parities. The second term is nonzero only for $N=Z$ odd-odd nuclei. Note that by combining the first term of Eq. (2) and the standard symmetry energy term $(N-Z)^2/A$ gives $|N-Z|(|N-Z|+\alpha)=T(T+\alpha)$ term; $\alpha=4$ corresponds to SU(4) and $\alpha=1$ in the jj coupling shell model. For further discussion on the $T(T+\alpha)$ term, see [15]. A recent significant development is the introduction of indicators, in terms of double binding energy differences, for determining $W(A)$ and $d(A)$ in Eq. (2). Applying the $\delta V^{ee}_{np}(N,Z)$ in Eq. (1) for any (N,Z) nucleus (hereafter called $\delta V_{np}(N,Z)$), the $W(A)$ parameter for even-even nuclei with $N=Z=A/2$ and odd-odd nuclei with $N=Z=(A/2)-1$ are given by

$$W^{ee}(A)=\delta V_{np}\left(\frac{A}{2},\frac{A}{2}\right)-\frac{1}{2}\left[\delta V_{np}\left(\frac{A}{2},\frac{A}{2}-2\right)+\delta V_{np}\left(\frac{A}{2}+2,\frac{A}{2}\right)\right] \tag{3}$$

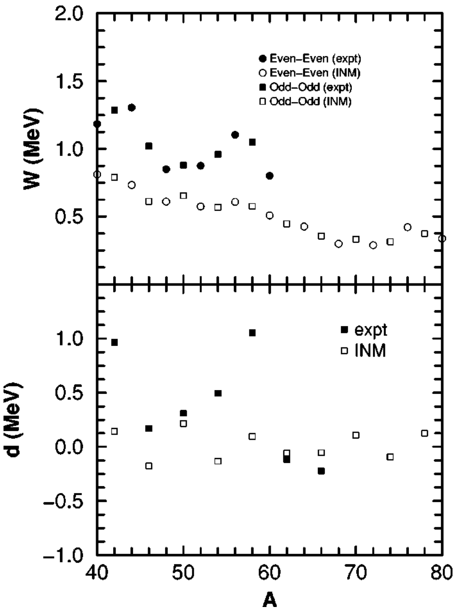

FIG. 3. Wigner energy parameters $W(A)$ and $d(A)$ vs A. Shown are the results of the INM model and those deduced from experimentally known masses.

$$W^{oo}(A-2)=-\delta V_{np}\left(\frac{A}{2},\frac{A}{2}-2\right)+\frac{1}{2}\left[\delta V_{np}\left(\frac{A}{2}-2,\frac{A}{2}-2\right)+\delta V_{np}\left(\frac{A}{2},\frac{A}{2}\right)\right].$$

Similarly $d(A)$ in odd-odd nuclei with $N=Z=A/2$ is given by

$$d(A)=-4\,\delta V_{np}\left(\frac{A}{2}+1,\frac{A}{2}-1\right)+2\left[\delta V_{np}\left(\frac{A}{2},\frac{A}{2}-2\right)+\delta V_{np}\left(\frac{A}{2}+2,\frac{A}{2}\right)\right]. \tag{4}$$

Note that the $W(A)$ and $d(A)$ formulas in Eqs. (3) and (4) are based on the assumption that they behave as $A^{-\alpha}$ with $\alpha\sim 1$. Employing measured nuclear masses with $A<62$, Satula *et al.* [4] showed that $W(A)\simeq 47/A^{0.95}$ MeV. More importantly they showed that $d(A)/W(A)\sim 1$ when the excitation energy of the lowest known $T=0$ state is added to the binding energies of odd-odd nuclei with $T=1$ as the ground state. Without this correction, for $A\geqslant 50$ the $d(A)\sim 0$. Thus the $T=0$ states in odd-odd nuclei with $A\sim 60$ are expected to start appearing at about an energy that is equal to $W(A)$ MeV from the ground states. These results indicate that the origin of the Wigner energy is in the $T=0$ pairing of the nuclear interaction. In order to go beyond $A=62$, the INM model masses are used in Eqs. (3) and (4) to obtain $W(A)$ and $d(A)$. The results are shown in Fig. 3. It is clear that the estimate $W(A)\sim 47/A^{0.95}$ MeV extends well up to $A=80$. For example, Fig. 3 shows that $W(A)\sim 0.4$ MeV for $A\sim 70$. Moreover, in this domain it is seen that $d(A)\sim 0$ as expected. Thus in the $A=60$–80 region, one can expect that the $T=0$ states start appearing around 0.5 MeV above the ground states that are $T=1$ in nature. For example it is known that for ^{74}Rb the $T=0$ states start appearing from 1 MeV above the ground state [2]. In fact this can be taken as an indication of the success of the INM model. The origin of $d(A)\sim 0$ in the $A=60$–100 region is probably related to the known fact that for $N=Z$ nuclei near the proton drip line the Fermi energies for proton and neutron are far from being degenerate unlike in the β-stable region. For example, from the neutron and proton separation energies given in [12], it is seen that the difference between them is ~ 4 MeV in ^{24}Mg while it is ~ 10 MeV in ^{70}Br.

In conclusion, in the $A=56$–100 region, the INM model predicts that the SU(4) symmetry is a broken symmetry but relatively SU(4) should be a better symmetry in the $N=Z$ odd-odd nuclei as compared to the even-even nuclei. Similarly, the Wigner energy parameter $W(A)$ is estimated by the model to be ~ 0.5 MeV and the parameter $d(A)\sim 0$ MeV indicating that the $T=0$ states in odd-odd $N=Z$ nuclei in this domain will start appearing at ~ 0.5 MeV excitation from the ground state. The issues involved in items (iii)–(v) mentioned in the beginning will be addressed elsewhere using the INM model.

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