

Pion electroproduction amplitude

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(Received 15 May 2001; published 19 October 2001)

We analyze amplitudes for the pion electroproduction on proton derived from Lagrangians based on the local chiral $SU(2) \times SU(2)$ symmetries. We show that such amplitudes do contain information on the nucleon weak axial form factor F_A in both soft and hard pion regimes. This result invalidates Habermehl's recent claim that the pion electroproduction at threshold cannot be used to extract any information regarding F_A .

DOI: 10.1103/PhysRevC.64.055501

PACS number(s): 11.40.Ha, 13.10.+q, 12.39.Fe, 25.30.-c

I. INTRODUCTION

The low energy theorems for the pion production by an external electroweak interaction were formulated after the development of the current algebra and partial conservation of the axial current (PCAC) [1,2]. According to this development, the amplitude $M_\lambda^{nj}(q,k)$ for the production of a soft pion $\pi^n(q)$ off the nucleon by a vector-isovector current $J_\lambda^j(k)$,

$$\hat{J}_\lambda^j(k) + N(p) \rightarrow \pi^n(q) + N(p'), \quad (1)$$

given by the matrix element

$$M_\lambda^{nj}(q,k) = \langle N(p') \pi^n(q) | \hat{J}_\lambda^j(0) | N(p) \rangle, \quad (2)$$

is written as

$$f_\pi M_\lambda^{nj}(q,k) \xrightarrow{q \rightarrow 0} i q_\mu \langle p' | \int d^4 y e^{-iqy} T \times [\hat{J}_{5\mu}^n(y) \hat{J}_\lambda^j(0)] | p \rangle + \varepsilon^{njm} \langle p' | \hat{J}_{5\lambda}^m(0) | p \rangle. \quad (3)$$

Here the matrix element of the nucleon weak axial current is [3]

$$\langle p' | \hat{J}_{5\lambda}^m(0) | p \rangle = i \bar{u}(p') [g_A F_A(q_1^2) \gamma_\lambda \gamma_5 - 2igf_\pi \Delta_F^\pi(q_1^2) q_{1\lambda} \gamma_5] \frac{\tau^m}{2} u(p), \quad (4)$$

$q_1 = p' - p = k - q$ and $g_A = 1.267$.

Starting from Eq. (3), a "master formula" for the amplitude $M_\lambda^{nj}(q,k)$ can be derived [4]. For this purpose, the contribution to the divergence due to the coupling of the axial current to the external nucleon lines [Figs. 1(a) and 1(b)] can be extracted from the current-current amplitudes. The resulting equation is

$$\begin{aligned} f_\pi M_\lambda^{nj}(q,k) &= \frac{gf_\pi}{2M} \bar{u}(p') [\not{q} \tau^n S_F(P) \hat{J}_\lambda^j(k) \\ &+ \hat{J}_\lambda^j(k) S_F(Q) \not{q} \tau^n] u(p) + \varepsilon^{njm} i \bar{u}(p') \\ &\times [g_A F_A(q_1^2) \gamma_\lambda \gamma_5 - 2igf_\pi \Delta_F^\pi(q_1^2) q_{1\lambda} \gamma_5] \frac{\tau^m}{2} u(p). \end{aligned} \quad (5)$$

The vector-isovector current $\hat{J}_\lambda^j(k)$ is defined as

$$\hat{J}_\lambda^j(k) = i \left[F_1^V(k) \gamma_\lambda - \frac{\kappa^V}{2M} F_2^V(k) \sigma_{\lambda\eta} k_\eta \right] \frac{\tau^j}{2}. \quad (6)$$

It is clear that besides the nucleon Born terms, the soft pion amplitude, Eq. (5), contains the Kroll-Ruderman (contact) term including the nucleon weak axial form factor, and almost the pion pole production term, which is expected to contribute for both the pion photoproduction and electroproduction. To have this term present, Adler [4] changed by hand $q_1 \rightarrow q_1 - q$ in the induced pseudoscalar term [the last term on the right-hand side of Eq. (5)], thus adding an $O(q)$ piece. It was shown [5] that the evaluation of the contribution to the divergence of the current-current amplitudes using the current algebra and PCAC due to the process of Fig. 1(c) with $B = \pi$ yields two pieces [6]: one is exactly the pion pole production term and the other one cancels the induced pseudoscalar term. Then the soft pion production amplitude corresponds to Fig. 2 and it is

$$\begin{aligned} M_\lambda^{nj}(q,k) &= \frac{g}{2M} \bar{u}(p') [\not{q} \tau^n S_F(P) \hat{J}_\lambda^j(k) \\ &+ \hat{J}_\lambda^j(k) S_F(Q) \not{q} \tau^n] u(p) \\ &+ \frac{(k-2q)_\lambda}{(k-q)^2 + m_\pi^2} F_\pi(k^2) \varepsilon^{njm} g \bar{u}(p') \gamma_5 \tau^m u(p) \\ &+ i \frac{g_A}{2f_\pi} F_A[(k-q)^2] \varepsilon^{njm} \bar{u}(p') \gamma_\lambda \gamma_5 \tau^m u(p). \end{aligned} \quad (7)$$

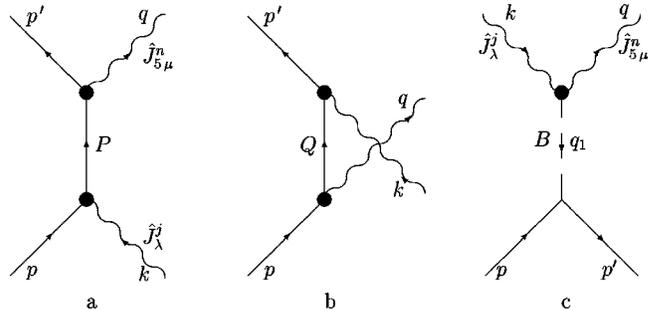


FIG. 1. The current-current amplitudes contributing to the first term on the right-hand side of Eq. (3). (a),(b) The nucleon Born terms; (c) the pole terms of the $B = \pi$ or a_1 range.

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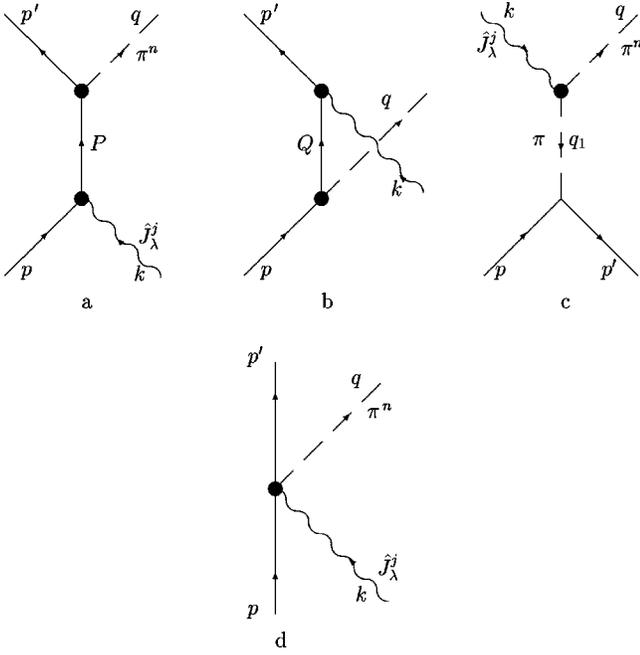


FIG. 2. The soft pion production amplitude according to Eq. (7). (a),(b) The nucleon Born terms; (c) the pion pole term; (d) the Kroll-Ruderman term.

The q dependence in the form factor $F_A[(k-q)^2]$ can be neglected.

Elimination of the induced pseudoscalar term from the pion electroproduction amplitude and the restoration of the pion pole production term was discussed also in Refs. [7,8]. Nevertheless, the amplitude with the induced pseudoscalar term was applied to extract the axial and pseudoscalar form factors from the pion electroproduction at threshold in Ref. [9]. The same flaw is present also in the earlier study of the pion electroproduction amplitude [10].

Low energy theorems for the amplitude $M_\lambda^{nj}(q,k)$ were derived [1,2,5,11] using the current conservation and PCAC. These theorems have found copious applications (see Refs. [12–15] and references therein). The relevance of the nucleon weak axial form factor to the pion electroproduction off the nucleon has been revisited recently by Haberzettl [12]. The pion production amplitude [16,17] was rederived [12] using an analog of Eq. (3) valid beyond the soft pion limit. Neglecting possible Schwinger terms, the amplitude is

$$M_\lambda^{nj}(q,k) = \frac{q^2 + m_\pi^2}{f_\pi m_\pi^2} \left[i q_\mu \langle p' | \int d^4 y e^{-iqy} T \right. \\ \left. \times [\hat{J}_{5\mu}^n(y) \hat{J}_\lambda^j(0)] | p \rangle + \varepsilon^{njm} \langle p' | \hat{J}_{5\lambda}^m(0) | p \rangle \right]. \quad (8)$$

The current-current amplitudes were constructed [12] by inserting photon lines in all possible places in the nucleon weak axial current graph of Fig. 1. After calculating the divergence of the current-current amplitudes, Haberzettl [12] obtained Eq. (19), which differs from our Eq. (8) by revers-

ing the left- and right-hand sides and by an additional term, $\bar{u}(p') \mathcal{W}_\lambda u(p)$, accompanying the pion production amplitude in the right-hand side of Eq. (19). According to the discussion [12] after Eq. (20), in order for Eq. (19) to be consistent with Eq. (8), this term should vanish:

$$\bar{u}(p') \mathcal{W}_\lambda u(p) = 0. \quad (9)$$

The derivation of Eq. (19) inspired Haberzettl to argue that the entire F_A dependence of its left-hand side is retained solely in \mathcal{W}_λ on the right-hand side, since the pion production amplitude $M_\lambda^{nj}(q,k)$ itself does not depend on F_A . Then the conclusion [12] that the pion electroproduction processes at threshold cannot be used to extract any information regarding the nucleon weak axial form factor.

This claim has been criticized [18,19], but the critics have been promptly refuted [20]. Actually, the only merit of the criticism made in Ref. [18] is that it attracted attention to the problem. Otherwise, the argument that the contribution to the first term on the right-hand side of Eq. (3) in the soft pion limit is only from the diagrams 1(a) and 1(b), is incorrect. The point is that if one of the currents is axial, then it can couple to the pion or a_1 meson lines that emerge from the vector-current- B -boson vertex, and also by direct contact to this vertex. The processes due to the pion emission contribute to the B -pole current-current amplitude by a portion

$$\Delta T_{\mu\lambda}^{\pi,nj}(B) = i f_\pi q_\mu \Delta_F^\pi(q^2) \Delta M_\lambda^{nj}(B), \quad (10)$$

while the divergence of the current-current amplitudes due to the other two processes yields

$$i q_\mu \Delta T_{\mu\lambda}^{c,nj}(B) = f_\pi \Delta M_\lambda^{nj}(B) + C_\lambda^{nj}(B), \quad (11)$$

where $\Delta M_\lambda^{nj}(B)$ is the associated pion production amplitude and a term $C_\lambda^{nj}(B)$ appears, since the chiral group is non-Abelian. Then the sum of the two contributions to the divergence of the B -pole current-current amplitude in Eq. (3) is

$$i q_\mu [\Delta T_{\mu\lambda}^{\pi,nj}(B) + \Delta T_{\mu\lambda}^{c,nj}(B)] = f_\pi \Delta M_\lambda^{nj}(B) + C_\lambda^{nj}(B), \quad (12)$$

and a new contribution to the pion production amplitude by the vector-isovector current appears even in the soft pion limit. As we have already discussed above for the case $B = \pi$ and as we shall see later also for $B = a_1$, this picture is valid, indeed.

In our opinion, Guichon's argument [18] against the applied formalism is also irrelevant. It is true that the formalism developed in Ref. [12] for the treatment of the interaction of the axial and vector currents is not standard, but it is formally correct. After making our own study of the problem reported here and comparing it with the earlier results [12], we came to the conclusion that the problem is not in the formalism, but is rather with a misinterpretation of the results by Haberzettl [12].

We shall first study the structure of the pion electroproduction amplitude derived from chiral Lagrangians based on the local chiral symmetry $SU(2) \times SU(2)$ [3,21–25]. These models have recently been used [26] to construct the one-

boson axial exchange currents for the Bethe-Salpeter equation and the transition amplitude for the radiative muon capture by proton [3]. For the chiral Lagrangians [3,23], we shall derive the pion electroproduction amplitude in the same way as it was done in Ref. [12]: we shall construct the current-current amplitudes and then we shall calculate the divergence. It will become clear that this way of constructing the pion production amplitude cannot change its content. Once the amplitude is built independently within the same concept (which is the case of the method developed in [12,16,17] and in our approach, too), then to calculate it using the divergence of the current-current amplitudes is a mere exercise in calculating the diagrams, since one should get the identity. Our chiral models provide the contact term of the pion production amplitude with the nucleon weak axial form factor $F_A(k^2)$ in the soft pion limit, as it should be, since the chiral invariance restricts them in such a way that they reproduce in the tree approximation predictions of the current algebra and PCAC. On the other hand, the method of Refs. [12,16,17], avoiding chiral invariance and based on the gauge invariance only, can produce in the pion electroproduction amplitude the contact term retaining only the form factor $F_1^V(k^2)$ in the soft pion limit.

Our pion production amplitude is valid for both soft and hard pions. We use first a minimal Lagrangian [21], built in

terms of the Yang-Mills gauge fields, for the direct construction of the pion electroproduction amplitude. For the Lagrangian [23] reflecting the hidden local $SU(2) \times SU(2)$ symmetry [24,25], we first construct the current-current amplitudes and then we calculate the divergence. We show that besides the amplitudes of Figs. 1(a) and 1(b) both exchanges, π and a_1 , contribute to the amplitude of Fig. 1. As a result, we obtain the pion production amplitude of our model minus the last term on the right-hand side of Eq. (3), as it should be, if the calculations are consistent. On the other hand, the divergence of the pion pole current-current amplitude provides the standard pion pole production amplitude, and the divergence of the a_1 meson pole current-current amplitude produces the new contact term, containing again the nucleon weak axial form factor $F_A(k^2)$ and other terms of the order at least $O(q)$ in the soft pion limit.

In Sec. II, we introduce the necessary formalism, in Sec. III we construct the pion production amplitude. Our results and conclusions are summarized in Sec. IV.

II. FORMALISM

Direct construction of the amplitude $M_\lambda^{nj}(q,k)$ will be carried out using the minimal Lagrangian [21,22]. Here we write only the necessary vertices

$$\begin{aligned} \Delta \mathcal{L}_{N\pi\rho a_1}^{YM} = & -\bar{N}\gamma_\mu\partial_\mu N - M\bar{N}N - i\frac{g}{2M}\bar{N}\hat{b}(\vec{\Pi}\cdot\vec{\tau})\gamma_5 N + ig_\rho\frac{g_A}{2f_\pi}\bar{N}\gamma_\mu\gamma_5(\vec{\tau}\cdot\vec{\rho}_\mu\times\vec{\Pi})N - ig_A g_\rho\bar{N}\gamma_\mu\gamma_5(\vec{\tau}\cdot\vec{a}_\mu)N - i\frac{g_\rho}{2}\bar{N} \\ & \times \left(\gamma_\mu\vec{\rho}_\mu - i\frac{\kappa_V}{4M}\sigma_{\mu\nu}\vec{\rho}_{\mu\nu} \right) \cdot \vec{\tau}N + g_\rho\vec{\rho}_\mu\cdot\vec{\Pi}\times\partial_\mu\vec{\Pi} - \frac{1}{f_\pi}\left(\vec{\rho}_{\mu\nu}\cdot\vec{a}_\mu\times\partial_\nu\vec{\Pi} + \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\Pi}\times\vec{a}_{\mu\nu} \right), \end{aligned} \quad (13)$$

where

$$\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu. \quad (14)$$

From the associated one-body currents we present only the vector-isovector current

$$J_{YM,\lambda}^j = -\frac{m_\rho^2}{g_\rho}\rho_\lambda^j. \quad (15)$$

We use the Lagrangian model [3,23], reflecting the hidden local symmetry to derive the amplitude $M_\lambda^{nj}(q,k)$ employing Eq. (8). It reads in the needed approximation

$$\begin{aligned} \mathcal{L}_{N\pi\rho a_1}^{HLS} = & -\bar{N}\gamma_\mu\partial_\mu N - M\bar{N}N + ig\bar{N}\gamma_5(\vec{\Pi}\cdot\vec{\tau})N - ig_\rho\frac{g_A}{2f_\pi}\bar{N}\gamma_\mu\gamma_5(\vec{\tau}\cdot\vec{\rho}_\mu\times\vec{\Pi})N - i\frac{g_\rho g_A^2}{f_\pi}\bar{N}\gamma_\mu(\vec{\tau}\cdot\vec{a}_\mu\times\vec{\Pi})N \\ & - ig_A g_\rho\bar{N}\gamma_\mu\gamma_5(\vec{\tau}\cdot\vec{a}_\mu)N - i\frac{g_\rho}{2}\bar{N}\left(\gamma_\mu\vec{\rho}_\mu - i\frac{\kappa_V}{4M}\sigma_{\mu\nu}\vec{\rho}_{\mu\nu} \right) \cdot \vec{\tau}N + i\frac{g_\rho g_A}{4f_\pi}\frac{\kappa_V}{2M}\bar{N}\gamma_5\sigma_{\mu\nu}(\vec{\Pi}\cdot\vec{\rho}_{\mu\nu})N \\ & + g_\rho\vec{\rho}_\mu\cdot\vec{\Pi}\partial_\mu\vec{\Pi} - g_\rho\partial_\nu\vec{\rho}_\mu\cdot\vec{\rho}_\mu\times\vec{\rho}_\nu + g_\rho(\vec{\rho}_\mu\times\vec{a}_\nu - \vec{\rho}_\nu\times\vec{a}_\mu)\cdot\partial_\mu\vec{a}_\nu + \frac{1}{f_\pi}\left(\vec{\rho}_{\mu\nu}\cdot\vec{a}_\mu\times\partial_\nu\vec{\Pi} + \frac{1}{2}\vec{\rho}_\mu\cdot\partial_\nu\vec{\Pi}\times\vec{a}_{\mu\nu} \right). \end{aligned} \quad (16)$$

The associated one-body currents are

$$J_{HLS,\lambda}^j = -\frac{m^2}{g_\rho} \rho_\lambda^j - 2f_\pi g_\rho (\vec{a}_\lambda \times \vec{\Pi})^j + O(|\vec{\Pi}|^2), \quad (17)$$

$$J_{HLS,5\mu}^n = -\frac{m_\rho^2}{g_\rho} a_\mu^n + f_\pi \partial_\mu \Pi^n - 2f_\pi g_\rho (\vec{\rho}_\mu \times \vec{\Pi})^n + \frac{1}{g_\rho} \left[\left(\frac{1}{2f_\pi} \partial_\nu \vec{\Pi} - g_\rho \vec{a}_\nu + e \vec{A}_\nu \right) \times \vec{\rho}_{\mu\nu} \right]^n. \quad (18)$$

III. PION ELECTROPRODUCTION AMPLITUDE

This section is devoted to the construction of the pion production amplitude starting from the chiral Lagrangians and currents of the preceding section and using the standard Feynman graph technique.

A. Pion electroproduction amplitude from the minimal chiral Lagrangian

Generally, the amplitude is graphically presented as in Fig. 2. The Born terms, Figs. 2(a) and 2(b), are of the same form as the first two terms on the right-hand side of Eq. (5), but only the current $\hat{J}_\lambda^j(k)$ is given [3] by the vector dominance model

$$\hat{J}_\lambda^j(k) = im_\rho^2 \Delta_{\lambda\zeta}^\rho(k) \left(\gamma_\zeta - \frac{\kappa_V}{2M} \sigma_{\zeta\delta} k_\delta \right) \frac{\tau^j}{2}. \quad (19)$$

Then the Born term is

$$M_{B,\lambda}^{nj} = \frac{g}{2M} \bar{u}(p') [\hat{q} \tau^n S_F(P) \hat{J}_\lambda^j(k) + \hat{J}_\lambda^j(k) S_F(Q) \hat{q} \tau^n] u(p). \quad (20)$$

Also the pion pole production term, Fig. 2(c), is of the standard form with the pion form factor provided again by the vector dominance model

$$M_{pp,\lambda}^{nj} = m_\rho^2 \Delta_{\lambda\zeta}^\rho(k) (q_{1\zeta} - q_\zeta) \Delta_F^\pi(q_1^2) \varepsilon^{njm} g \Gamma_{5\zeta}^m(p', p), \quad (21)$$

where

$$\Gamma_{5\zeta}^m(p', p) = \bar{u}(p') \gamma_\zeta \gamma_5 \tau^m u(p). \quad (22)$$

However, the contact term, Fig. 2(d), deserves more attention. Visually, it is given by two graphs of Figs. 3(a) and 3(b).

The amplitude corresponding to the graph Fig. 3(a) is

$$M_{c_1,\lambda}^{nj} = i \frac{g_A}{2f_\pi} m_\rho^2 \Delta_{\lambda\zeta}^\rho(k) \varepsilon^{njm} \Gamma_{5\zeta}^m(p', p) \rightarrow i \frac{g_A}{2f_\pi} F_1^V(k^2) \varepsilon^{njm} \Gamma_{5\lambda}^m(p', p), \quad (23)$$

where

$$\Gamma_{5\lambda}^m(p', p) = \bar{u}(p') \gamma_\lambda \gamma_5 \tau^m u(p). \quad (24)$$

This is the Kroll-Ruderman term in the pion production amplitude, which can be obtained by the minimal substitution [27]

$$\partial_\lambda \rightarrow \partial_\lambda \pm ie F_1^V A_\lambda$$

in the pseudovector πNN interaction [the third term on the right-hand side of Eq. (13)]. Let us now present the contact term, Fig. 3(b), arising from the last two vertices of Eq. (13):

$$M_{c_2,\lambda}^{nj} = i \frac{g_A}{f_\pi} \varepsilon^{njm} m_\rho^2 [k_\nu \Delta_{\lambda\eta}^\rho(k) - k_\eta \Delta_{\lambda\nu}^\rho(k)] \times (q_\nu + \frac{1}{2} q_{1\nu}) \Delta_{\eta\zeta}^{a_1}(q_1) \Gamma_{5\zeta}^m(p', p). \quad (25)$$

This amplitude is transverse by itself. Taking into account that $q_1 = k - q$, we can write

$$M_{c_2,\lambda}^{nj} = i \frac{g_A}{2f_\pi} m_\rho^2 \Delta_F^\rho(k^2) \Delta_F^{a_1}(k^2) k^2 \varepsilon^{njm} \Gamma_{5\lambda}^m(p', p) + \Delta M_{c_2,\lambda}^{nj}, \quad (26)$$

where in the soft pion limit

$$\Delta M_{c_2,\lambda}^{nj} = O(q, k^2). \quad (27)$$

Summing up the amplitudes given in Eq. (23) and (26), we get the total contact term $M_{c,\lambda}^{nj}$ of this model in the soft pion limit

$$M_{c,\lambda}^{nj} = i \frac{g_A}{2f_\pi} F_A(k^2) \varepsilon^{njm} \Gamma_{5\lambda}^m(p', p) + O(q, k^2). \quad (28)$$

In deriving Eqs. (23)–(28), we used the Weinberg relation $m_{a_1}^2 = 2m_\rho^2$ [28] and the vector dominance model form of the form factors $F_1^V(k^2)$ and $F_A(k^2)$, which is incorporated in the model Lagrangian, Eq. (13).

The contact term, Eq. (28), coincides in form with the last term of the soft pion amplitude Eq. (7). Let us stress that the nucleon weak axial form factor $F_A(k^2)$ appears in the contact term in order to satisfy the local chiral invariance. The

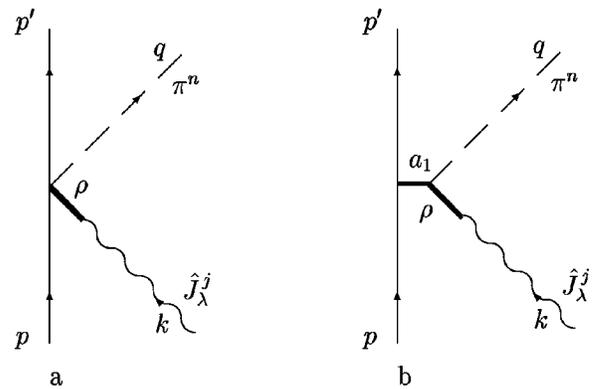


FIG. 3. The contact terms arising from the chiral Lagrangian, Eq. (13). (a) corresponds to the fourth term; (b) is constructed from the last two terms.

gauge condition for the pion electroproduction amplitude of this model, given by Eqs. (20)–(25), is in agreement with the Ward-Takahashi relation [29]

$$\begin{aligned} & k_\lambda [M_{B,\lambda}^{nj} + M_{pp,\lambda}^{nj} + M_{c_1,\lambda}^{nj} + M_{c_2,\lambda}^{nj}] \\ &= -\frac{m_\pi^2 + q^2}{m_\pi^2 + q_1^2} \varepsilon^{njm} g \Gamma_5^m(p', p). \end{aligned} \quad (29)$$

B. Pion electroproduction amplitude from the hidden local symmetry Lagrangian

In this section, the pion electroproduction amplitude will be obtained via Eq. (8), and for constructing the current-current amplitudes we use the chiral Lagrangian, Eq. (16), and the associated currents, Eqs. (17) and (18). This formalism has been employed in Ref. [3] for the derivation of the current-current amplitudes for the radiative muon capture by protons. Those amplitudes and the ones we need to calculate here are related by time reversal. The minimal amplitudes, direct analogs of the current-current amplitudes of Fig. 3 of Ref. [12], were derived for the radiative muon capture [30] almost 40 years ago and they were used directly in Refs. [31,32] for calculating the transition rate of the reaction

$$\mu + p \rightarrow n + \nu_\mu + \gamma. \quad (30)$$

The case of the pion electroproduction is simpler, since only the vector- and the axial-isovector currents contribute.

1. Nucleon Born terms

Let us start with the nucleon Born terms. According to our Lagrangian equation (16), we use the chiral model with the pseudoscalar πNN coupling, and in addition to the graphs Figs. 4(a) and 4(b), the contact term Fig. 4(c) should be calculated. There are three nucleon Born current-current amplitudes:

$$\begin{aligned} T_{\mu\lambda}^{B,nj}(1) &= -\bar{u}(p') [\hat{J}_{5\mu}^n(q) S_F(Q) \hat{J}_\lambda^j(k) \\ &+ \hat{J}_\lambda^j(k) S_F(P) \hat{J}_{5\mu}^n(q)] u(p), \end{aligned} \quad (31)$$

$$\begin{aligned} T_{\mu\lambda}^{B,nj}(2) &= -\frac{g_A}{2} q_\mu \Delta_F^\pi(q^2) m_\rho^2 \Delta_{\lambda\zeta}^\rho(k) \varepsilon^{njm} \Gamma_{5\zeta}^m(p', p) \\ &\equiv i f_\pi q_\mu \Delta_F^\pi(q^2) M_{B,\lambda}^{nj}(2), \end{aligned} \quad (32)$$

$$\begin{aligned} T_{\mu\lambda}^{B,nj}(3) &= i \frac{g_A}{2} q_\mu \Delta_F^\pi(q^2) \frac{\kappa_V}{2M} m_\rho^2 \Delta_{\lambda\eta}^\rho(k) k_\zeta \\ &\times \delta_{jn} \bar{u}(p') \gamma_5 \sigma_{\zeta\eta} u(p) \\ &\equiv i f_\pi q_\mu \Delta_F^\pi(q^2) M_{B,\lambda}^{nj}(3). \end{aligned} \quad (33)$$

The contribution of the induced pseudoscalar part of the axial current $\hat{J}_{5\mu}^n(q)$ to the amplitude $T_{\mu\lambda}^{B,nj}(1)$ is

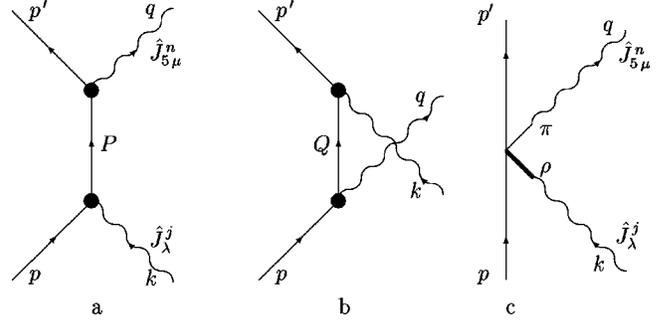


FIG. 4. The nucleon Born current-current terms given by the Lagrangian equation (16) and the currents, Eqs. (17) and (18).

$$\begin{aligned} T_{\mu\lambda}^{B,nj}(\pi, 1) &= f_\pi \bar{u}(p') [g \gamma_5 \tau^n S_F(Q) \hat{J}_\lambda^j(k) \\ &+ \hat{J}_\lambda^j(k) S_F(P) g \gamma_5 \tau^n] u(p) \\ &\equiv i f_\pi q_\mu \Delta_F^\pi(q^2) M_{B,\lambda}^{nj}(1). \end{aligned} \quad (34)$$

Calculation of the divergence of the amplitude $T_{\mu\lambda}^{B,nj}(1)$, Eq. (31), yields

$$i q_\mu T_{\mu\lambda}^{B,nj}(1) = f_\pi m_\pi^2 \Delta_F^\pi(q^2) M_{B,\lambda}^{nj}(1) + f_\pi \sum_{i=2}^3 M_{B,\lambda}^{nj}(i). \quad (35)$$

Then it follows from Eqs. (32), (33), and (35) that

$$\begin{aligned} i q_\mu \sum_{i=1}^3 T_{\mu\lambda}^{B,nj}(i) &= f_\pi m_\pi^2 \Delta_F^\pi(q^2) \sum_{i=1}^3 M_{B,\lambda}^{nj}(i) \\ &\equiv f_\pi m_\pi^2 \Delta_F^\pi(q^2) M_{B,\lambda}^{nj}, \end{aligned} \quad (36)$$

where $M_{B,\lambda}^{nj}$ is the nucleon Born pion electroproduction amplitude for the pseudovector πNN coupling, Eq. (20). The amplitudes $M_{B,\lambda}^{nj}(2)$ and $M_{B,\lambda}^{nj}(3)$ ensure the PCAC constraint.

2. Pion pole terms

The current-current amplitudes due to the pion exchange in Fig. 1(c) are presented in Figs. 5(a)–5(d). They are

$$\begin{aligned} T_{\mu\lambda}^{pp,nj}(a) &= i f_\pi q_\mu \Delta_F^\pi(q^2) m_\rho^2 \Delta_{\eta\lambda}^\rho(k) \\ &\times (q_{1\eta} - q_\eta) \Delta_F^\pi(q_1^2) \varepsilon^{njm} g \Gamma_5^m(p', p) \\ &\equiv i f_\pi q_\mu \Delta_F^\pi(q^2) M_{pp,\lambda}^{nj}, \end{aligned} \quad (37)$$

$$\begin{aligned} T_{\mu\lambda}^{pp,nj}(b) &= 2 i f_\pi m_\rho^2 \Delta_{\mu\eta}^{a_1}(q) \Delta_F^\rho(k^2) (q_1 \cdot k \delta_{\eta\lambda} \\ &- k_\eta q_{1\lambda}) \Delta_F^\pi(q_1^2) \varepsilon^{njm} g \Gamma_5^m(p', p) \\ &+ i f_\pi m_\rho^2 \Delta_F^{a_1}(q^2) \Delta_{\eta\lambda}^\rho(k) (q \cdot q_1 \delta_{\eta\mu} \\ &- q_\eta q_{1\mu}) \Delta_F^\pi(q_1^2) \varepsilon^{njm} g \Gamma_5^m(p', p), \end{aligned} \quad (38)$$

$$T_{\mu\lambda}^{pp,nj}(c) = -2 i f_\pi m_\rho^2 \Delta_{\mu\lambda}^{a_1}(q) \Delta_F^\pi(q_1^2) \varepsilon^{njm} g \Gamma_5^m(p', p), \quad (39)$$

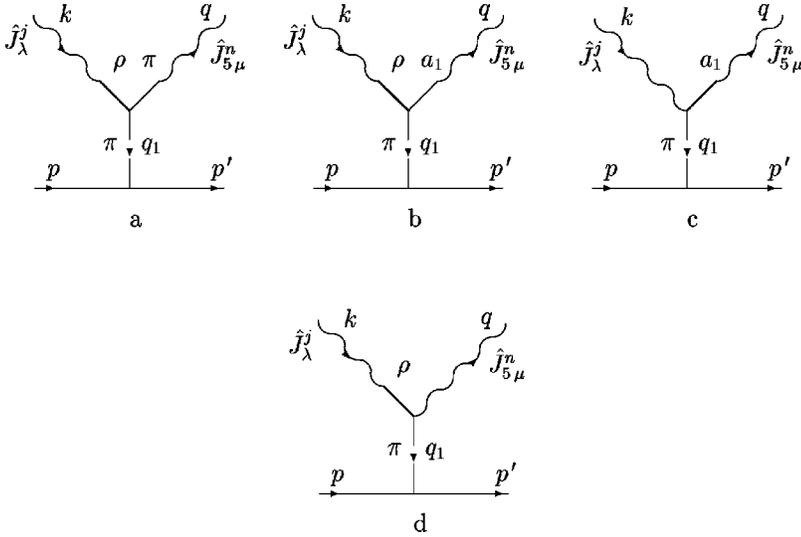


FIG. 5. The pion pole current-current amplitudes given by the Lagrangian equation (16) and the currents, Eqs. (17) and (18).

$$\begin{aligned}
T_{\mu\lambda}^{pp,nj}(d) &= 2if_{\pi}m_{\rho}^2\Delta_{\mu\lambda}^{\rho}(k)\Delta_{F}^{\pi}(q_1^2)\varepsilon^{njm}g\Gamma_5^m(p',p) \\
&\quad + if_{\pi}\Delta_{F}^{\rho}(k^2)(k_{\mu}q_{1\lambda} - q_1 \cdot k\delta_{\mu\lambda})\Delta_{F}^{\pi} \\
&\quad \times (q_1^2)\varepsilon^{njm}g\Gamma_5^m(p',p). \quad (40)
\end{aligned}$$

The process of Fig. 5(a) is the only one where the axial current is attached to the pion emitted from the vector-current- B -boson vertex. The associated pion production amplitude $M_{pp,\lambda}^{nj}$ is given by the same graph without the axial current wavy line.

Let us first calculate the divergence of the amplitudes, Eqs. (38)–(40). It is easy to find that the divergence of the first part of the amplitude, Eq. (38), and of the second part of the amplitude, Eq. (40), cancels. The rest provides

$$\begin{aligned}
iq_{\mu} \sum_{x=b,c,d} T_{\mu\lambda}^{pp,nj}(x) \\
= f_{\pi}M_{pp,\lambda}^{nj} - f_{\pi}q_{1\lambda}\Delta_{F}^{\pi}(q_1^2)\varepsilon^{njm}g\Gamma_5^m(p',p), \quad (41)
\end{aligned}$$

where $M_{pp,\lambda}^{nj}$ is the pion pole production amplitude defined in Eq. (37), and it coincides with the amplitude, Eq. (21), of Sec. III A. Let us note that Eq. (41) is a particular form of the general result given in Eq. (11).

Then the divergence of the pion pole current-current amplitudes, Eqs. (37)–(40), is

$$\begin{aligned}
iq_{\mu} \sum_{x=a,b,c,d} T_{\mu\lambda}^{pp,nj}(x) &= f_{\pi}m_{\pi}^2\Delta_{F}^{\pi}(q^2)M_{pp,\lambda}^{nj} \\
&\quad - f_{\pi}q_{1\lambda}\Delta_{F}^{\pi}(q_1^2)\varepsilon^{njm}g\Gamma_5^m(p',p). \quad (42)
\end{aligned}$$

Using this result and Eq. (4) for the matrix element of the nucleon weak axial current, we immediately see from Eq. (8) that the induced pseudoscalar disappears from the pion electroproduction amplitude, and the pion pole production am-

plitude $M_{pp,\lambda}^{nj}$, Eq. (21), appears instead, as already discussed above. Evidently, this statement is independent of the pion being soft or hard.

3. a_1 meson pole terms

The contributions to the current-current amplitude from the a_1 meson pole term, Fig. 1(c), are given in Figs. 6(a)–6(d). They have the form

$$\begin{aligned}
T_{\mu\lambda}^{a_1p,nj}(a) &= g_A m_{\rho}^2 q_{\mu} \Delta_{F}^{\pi}(q^2) \{ \Delta_{\nu\zeta}^{a_1}(q_1) \Delta_{F}^{\rho}(k^2) [(k \cdot q) \delta_{\nu\lambda} \\
&\quad - k_{\nu} q_{\lambda}] + \frac{1}{2} \Delta_{\nu\lambda}^{\rho}(k) \Delta_{F}^{a_1}(q_1^2) [(q \cdot q_1) \delta_{\zeta\nu} \\
&\quad - q_{\zeta} q_{1\eta}] \} \varepsilon^{njm} \Gamma_{5\zeta}^m(p',p) \\
&\equiv if_{\pi} q_{\mu} \Delta_{F}^{\pi}(q^2) M_{a_1p,\lambda}^{nj}(a), \quad (43)
\end{aligned}$$

$$\begin{aligned}
T_{\mu\lambda}^{a_1p,nj}(b) &= -g_A m_{\rho}^2 q_{\mu} \Delta_{F}^{\pi}(q^2) \Delta_{\lambda\zeta}^{a_1}(q_1) \varepsilon^{njm} \Gamma_{5\zeta}^m(p',p) \\
&\equiv if_{\pi} q_{\mu} \Delta_{F}^{\pi}(q^2) M_{a_1p,\lambda}^{nj}(b), \quad (44)
\end{aligned}$$

$$\begin{aligned}
T_{\mu\lambda}^{a_1p,nj}(c) &= g_A m_{\rho}^4 \Delta_{\lambda\eta}^{\rho}(k) [(q_{1\eta} - q_{\eta}) \Delta_{\mu\nu}^{a_1}(q) \Delta_{\nu\zeta}^{a_1}(q_1) \\
&\quad + q_{\nu} \Delta_{\mu\eta}^{a_1}(q) \Delta_{\nu\zeta}^{a_1}(q_1) - q_{1\nu} \Delta_{\mu\nu}^{a_1}(q) \Delta_{\zeta\eta}^{a_1}(q_1)] \\
&\quad \times \varepsilon^{njm} \Gamma_{5\zeta}^m(p',p), \quad (45)
\end{aligned}$$

$$\begin{aligned}
T_{\mu\lambda}^{a_1p,nj}(d) &= g_A m_{\rho}^2 \Delta_{F}^{\rho}(k^2) [k_{\eta} \delta_{\mu\lambda} \\
&\quad - k_{\mu} \delta_{\lambda\eta}] \Delta_{\eta\zeta}^{a_1}(q_1) \varepsilon^{njm} \Gamma_{5\zeta}^m(p',p). \quad (46)
\end{aligned}$$

The two processes, Figs. 6(a) and 6(b), contribute to the pion production amplitude providing the terms $M_{a_1p,\lambda}^{nj}(a)$ and $M_{a_1p,\lambda}^{nj}(b)$, which are given in Eq. (43) and (44), respectively.

Let us calculate the divergence of the sum of the amplitude equations (45) and (46). The result is

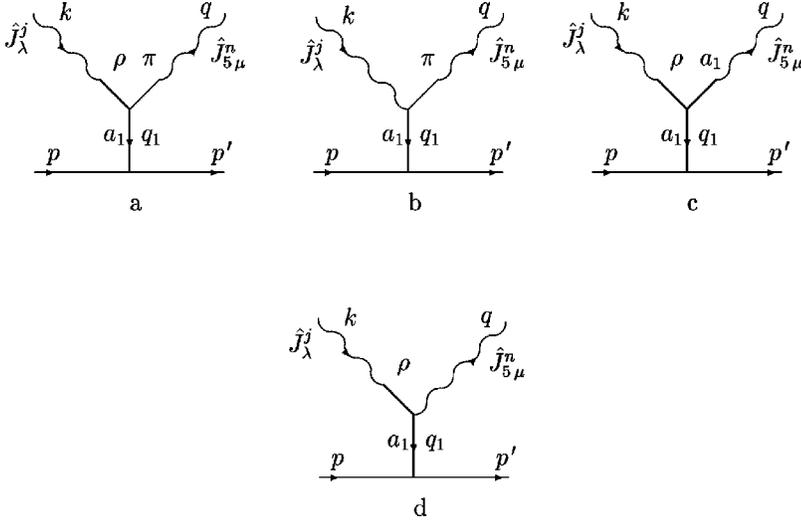


FIG. 6. The a_1 meson pole current-current amplitudes given by the Lagrangian equation (16) and the current equations (17) and (18).

$$i q_\mu \sum_{x=c,d} T_{\mu\lambda}^{a_1 p, nj}(x) = f_\pi \sum_{x=a,b} M_{a_1 p, \lambda}^{nj}(x) - i g_A m_\rho^2 \Delta_{\lambda\zeta}^{a_1}(q_1) \varepsilon^{njm} \Gamma_{5\zeta}^m(p', p). \quad (47)$$

Then it is clear that the divergence of the sum of all the a_1 meson pole current-current amplitudes, given by Eqs. (43)–(46), is

$$i q_\mu \sum_{x=a,b,c,d} T_{\mu\lambda}^{a_1 p, nj}(x) = f_\pi m_\pi^2 \Delta_F^\pi(q^2) \sum_{x=a,b} M_{a_1 p, \lambda}^{nj}(x) - i g_A m_\rho^2 \Delta_{\lambda\zeta}^{a_1}(q_1) \varepsilon^{njm} \Gamma_{5\zeta}^m(p', p). \quad (48)$$

Substituting this result into Eq. (8) we find that the first term in the matrix element of the axial current, Eq. (4), is cancelled. However, it does not mean that the dependence of the pion electroproduction amplitude on the nucleon weak axial form factor is eliminated. This will become clear in the next section, where we present the resulting pion production amplitude.

4. Resulting pion electroproduction amplitude

Now we present the pion electroproduction amplitude obtained in Secs. III B 1–III B 3 from the Lagrangian model, Eq. (16). For this purpose, we insert the results for the divergence of the current-current amplitudes, Eqs. (36), (42), and (48), into Eq. (8). The resulting pion electroproduction amplitude is

$$M_\lambda^{nj}(q, k) = M_{B, \lambda}^{nj} + M_{pp, \lambda}^{nj} + \sum_{x=a,b} M_{a_1 p, \lambda}^{nj}(x). \quad (49)$$

It is clear that the matrix element of the axial current is cancelled. The amplitudes $M_{B, \lambda}^{nj}$ and $M_{pp, \lambda}^{nj}$ are the well-known nucleon Born and pion pole terms, respectively. So only the contribution of the a_1 meson pole to the divergence of the current-current amplitudes remains to be considered.

Let us write the last two terms on the right-hand side of Eq. (49) explicitly. From the definitions in Eqs. (43) and (44) we have

$$M_{a_1 p, \lambda}^{nj}(a) = i \frac{g_A}{2f_\pi} m_{a_1}^2 \{ \Delta_{\nu\zeta}^{a_1}(q_1) \Delta_F^\rho(k^2) [k_\nu q_\lambda - (k \cdot q) \delta_{\nu\lambda}] + \frac{1}{2} \Delta_{\nu\lambda}^\rho(k) \Delta_F^{a_1}(q_1^2) [q_\zeta q_{1\eta} - (q \cdot q_1) \delta_{\zeta\nu}] \} \varepsilon^{njm} \Gamma_{5\zeta}^m(p', p), \quad (50)$$

$$M_{a_1 p, \lambda}^{nj}(b) = i \frac{g_A}{2f_\pi} m_{a_1}^2 \Delta_{\lambda\zeta}^{a_1}(q_1) \varepsilon^{njm} \Gamma_{5\zeta}^m(p', p). \quad (51)$$

In the soft pion limit, the amplitude $M_{a_1 p, \lambda}^{nj}(a) \sim O(kq)$, while the amplitude $M_{a_1 p, \lambda}^{nj}(b)$ restores the contact term retaining the nucleon weak axial form factor F_A . For its construction, the presence of the contact current [the second term on the right-hand side of Eq. (17)] is decisive. So this model is also consistent with predictions of the current algebra and PCAC. Let us emphasize that the amplitude $M_\lambda^{nj}(q, k)$, Eq. (49), satisfies the gauge condition, expressed in Eq. (29).

Of course, the amplitude $M_\lambda^{nj}(q, k)$ obtained from the Lagrangian equation (16) directly, does coincide with the one obtained from Eq. (49). However, the construction of this section allows us to see how the pion and a_1 meson poles contribute to the divergence of the current-current amplitudes.

IV. DISCUSSION AND SUMMARY

We have studied the structure of the pion electroproduction amplitude obtained from the Lagrangians incorporating the local chiral symmetry. From the minimal Lagrangian equation (13), constructed in terms of the Yang-Mills heavy meson fields, and using the current equation (15), we obtained the pion production amplitude directly. Actually, the model provides two contact terms. One of them is prescribed by the gauge invariance and it contains the form factor F_1^V .

Another one is due to the gauge chiral invariance. This term is transverse by itself. Combining these contact terms results in another contact term containing the nucleon weak axial form factor and a piece of the order $O(qk^2)$ in the soft pion limit.

The Lagrangian equation (16), reflecting the hidden local symmetry, helped us to construct the current-current amplitudes of Fig. 1. Subsequent calculation of the divergence of these amplitudes and the use of Eq. (8) led us to observe that the pion and a_1 exchanges in Fig. 1(c) do contribute non-negligibly even in the soft pion limit. The contribution proceeds in such a way that the matrix element of the axial current [the second term on the right-hand side of Eq. (8)] is eliminated and the pion pole and a_1 pole pion production terms appear. Moreover, one of the a_1 pole pion production amplitudes is nothing but the Kroll-Ruderman term containing again the nucleon weak axial form factor $F_A(k^2)$ in the soft pion limit. It was noted many years ago in Ref. [4] that this contact term originated from the a_1 meson exchange.

It is to be noted that the derivation of the pion production amplitude via Eq. (8) does not supply any additional dynamical input that is not already present in the considered model and we get the pion production amplitude, Eq. (49), which is the same as the one obtained from the Lagrangian equation (16) by the direct construction.

It follows from our results that the pion electroproduction amplitude does not contain the induced pseudoscalar part of the nucleon weak axial current either for soft or hard pions, which is at variance with Ref. [18]. It also means that the measurement of this quantity [9] is a misconception.

Let us now compare our results with those of Ref. [12]. In this paper, the formalism of the vector-current-axial-current interaction is formulated in such a way that the divergence of the current-current amplitudes, where the axial current is attached to the pion line, provides the whole pion production amplitude. As discussed [12] in connection with Eq. (19) for the pion electroproduction amplitude, Eq. (9) should hold. It was checked [12] that Eq. (9) is satisfied for the nucleon without the electroweak structure. Actually, Eq. (9) conforms to what we have, with the difference that in our model, one observes explicitly the validity of an analogous condition [the right-hand side of our Eq. (8) contains only the pion production amplitude] also for the nucleons having electroweak structure. In order to get sensible results beyond the soft pion limit, the same condition should hold also in the approach [12]. Otherwise, the pion production amplitude, derived directly, would differ from the one obtained via the

current-current amplitudes. This would be a dubious result, since it would not be clear which amplitude is correct. In our opinion, absence of a transparent proof of Eq. (9) [12] for the nucleons with the electroweak structure makes the whole Haberzettl calculation doubtful. Let us note that such a proof can be done only if this structure is introduced not phenomenologically, but at a microscopic level. This is achieved here within the concept of the hidden local symmetry and respecting the local chiral $SU(2)_R \times SU(2)_L$ invariance.

Moreover, it is not true that the right-hand side of Eq. (19) [12] depends on F_A only via \mathcal{W}_λ , as stated in the paragraph after Eq. (21). In other words, fulfilling Eq. (9) does not mean the elimination of the dependence of the pion electroproduction amplitude on F_A . The point is that it is the dynamical content of the model that dictates the form factor of the Kroll-Ruderman term. As we have seen in our study, if the model respects the local chiral symmetry, then this form factor is $F_A(k^2)$ in the soft pion limit, as it should be, in order to be in accord with the current algebra and PCAC. Actually, this result should be valid for any model possessing the local chiral symmetry. On the other hand, models, respecting the gauge invariance only, will provide the Kroll-Ruderman term retaining the form factor $F_1^V(k^2)$. So the claim [12] that the pion electroproduction processes at threshold cannot be used to extract any information regarding the nucleon weak axial form factor should be considered as precipitous.

On the contrary, the measurement of the nucleon weak axial form factor $F_A(k^2)$ in the electroproduction of charged pions on the proton at threshold makes a good sense. In the recent measurement [33] of this form factor by the $p(e, e' \pi^+)n$ reaction, the pion electroproduction amplitude Eq. (7) with the added Δ excitation terms was used to analyze the data. The value $M_A = 1.077 \pm 0.039$ GeV was found for the axial mass entering $F_A(k^2)$, which is consistent with the value of M_A known from neutrino scattering experiments.

Let us note that the study of the electroproduction of charged pions on the proton at threshold can also provide the information on the pion charge radius [15,34,35].

ACKNOWLEDGMENTS

This work was supported by Grant No. GA ČR 202/00/1669. We thank Professor F. C. Khanna for a critical reading of the manuscript and Mgr. J. Smejkal for discussions.

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- [1] S.L. Adler and R. Dashen, *Current Algebras* (Benjamin, New York, 1968).
 [2] V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics* (North-Holland/American Elsevier, Amsterdam, 1973).
 [3] J. Smejkal, E. Truhlík, and F.C. Khanna, *Few-Body Syst.* **26**, 175 (1999).
 [4] S.L. Adler, in *Proceedings of the 6th Hawaii Topical Confer-*

- ence in Particle Physics*, edited by D.N. Dobson *et al.* (University of Hawaii, Manoa, 1975), p. 5.
 [5] E.A. Ivanov and E. Truhlík, *Fiz. Elem. Chastits At. Yadra* **12**, 492 (1981) [*Sov. J. Part. Nucl.* **12**, 198 (1981)].
 [6] D.Yu. Bardin and E.A. Ivanov, *Fiz. Elem. Chastits At. Yadra* **7**, 726 (1976) [*Sov. J. Part. Nucl.* **7**, 286 (1976)].
 [7] N. Dombey and B.J. Read, *Nucl. Phys.* **B60**, 65 (1973).
 [8] D. Drechsler and L. Tiator, *J. Phys. G* **18**, 449 (1992).

- [9] S. Choi *et al.*, Phys. Rev. Lett. **71**, 3927 (1993).
- [10] G. Benfatto, F. Nicolo, and G.C. Rossi, Nucl. Phys. **B50**, 205 (1972).
- [11] A.I. Vainstein and V.I. Zakharov, Nucl. Phys. **B36**, 589 (1972).
- [12] H. Haberzettl, Phys. Rev. Lett. **85**, 3576 (2000).
- [13] S.L. Adler, Ann. Phys. (N.Y.) **50**, 189 (1968).
- [14] V. Bernard, N. Kaiser, and U.-G. Meißner, Nucl. Phys. **B383**, 442 (1992).
- [15] V. Bernard, N. Kaiser, and U.-G. Meißner, Phys. Rev. C **62**, 028201 (2000).
- [16] H. Haberzettl, Phys. Rev. C **56**, 2041 (1977).
- [17] H. Haberzettl, Phys. Rev. C **62**, 034605 (2000).
- [18] P.A.M. Guichon, hep-ph/0012126.
- [19] V. Bernard, N. Kaiser, and Ulf-G. Meißner, hep-ph/0101062.
- [20] H. Haberzettl, hep-ph/0103347.
- [21] E. Ivanov and E. Truhlík, Nucl. Phys. **A316**, 437 (1979).
- [22] E. Truhlík, Czech. J. Phys. **43**, 467 (1993).
- [23] J. Smejkal, E. Truhlík, and H. Göller, Nucl. Phys. **A624**, 655 (1997).
- [24] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988).
- [25] U.-G. Meissner, Phys. Rep. **161**, 213 (1988).
- [26] F.C. Khanna and E. Truhlík, Nucl. Phys. **A673**, 455 (2000).
- [27] J.L. Friar, Ann. Phys. (N.Y.) **104**, 380 (1977).
- [28] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).
- [29] P. Christillin and S. Servadio, Nuovo Cimento A **42**, 165 (1977).
- [30] H.P.C. Rood and H.A. Tolhoek, Nucl. Phys. **A70**, 658 (1965).
- [31] H.W. Fearing, Phys. Rev. C **21**, 1951 (1980).
- [32] D.S. Beder and H.W. Fearing, Phys. Rev. D **35**, 2130 (1987); **39**, 3493 (1989).
- [33] A. Liesenfeld *et al.*, Phys. Lett. B **468**, 20 (1999).
- [34] B. De Tollis and F. Nicolo, Nuovo Cimento A **48**, 281 (1967).
- [35] A.S. Esaulov, A.M. Pilipenko, and Yu.I. Titov, Nucl. Phys. **B136**, 511 (1978).