Neutron structure function $F_2^n(x)$ from deep inelastic electron scattering off few-nucleon systems

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The possibility of a reliable extraction of the neutron deep inelastic structure function, $F_2^n(x)$, for $x<0.85$ from joint measurements of deep inelastic structure functions of deuteron, ^{3}He, and ^{3}H is investigated. The model dependence in this extraction, linked to the possible different interactions between nucleons in nuclei, is shown to be weak, if the nuclear structure effects are properly taken into account. A combined analysis of the deep inelastic structure functions of these nuclei is proposed to study effects beyond the impulse approximation.

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I. INTRODUCTION

The knowledge of both proton and neutron deep inelastic structure functions (DISF's) at large values of the Bjorken variable $x=Q^2/(2M\nu)$ could give access to the valence u and d quark distributions in the nucleon [1] [$Q^2=-q^2$, with $q\equiv(\nu,\vec{q})$ the four-momentum transfer]. Usually, deuteron data have been employed to gain information on the neutron unpolarized DISF, $F_2^n(x)$, but uncertainties remain, linked to the EMC effect in the deuteron [2,3]. Medium off-shell effects, different from the binding effects and related to a nucleon structure in nuclei different from the free one, have been often advocated (see, e.g., Refs. [4,5]). Although these effects have been found to be small, it was argued [6] that the standard treatment of deuteron data [7] could be unfair. Indeed, such a treatment employes convolution formulas, neglecting medium effects beyond the impulse approximation (IA), and leads to the value 1/4 for the ratio $r(x)=F_2^n(x)/F_2^p(x)$, when $x\to 1$. At variance, an analysis which includes medium effects [6] moves such a value towards that of 3/7, suggested by PQCD arguments [8].

Recently, the possible use of an unpolarized ^{3}H target has been discussed [9]. In particular, an experiment has been proposed [10,11], aimed at determining $F_2^n(x)$ at large x from the measurement of the ratio $E^{HeT}(x)=F_2^{He}(x)/F_2^T(x)$ between the unpolarized structure functions of ^{3}He, $F_2^{He}(x)$, and ^{3}H, $F_2^T(x)$. Indeed, using this ratio one is expected to reduce the effects of systematic errors in the measurements, as well as the effects of theoretical model dependences and, in particular, of contributions beyond the impulse approximation. As far as the latter are concerned, the differences between the EMC effect in ^{3}He [12] and in ^{3}H are expected to be small, because of isospin symmetry [10].

In Ref. [13] a reliable recurrence procedure has been proposed, within the impulse approximation [14,15], to extract $F_2^n(x)$ in the range $0<x\leqslant 0.9$ from the experimental ratio $E^{HeT}(x)$. It has been shown that, at high x, nuclear structure effects, i.e., Fermi motion and nuclear binding, are relevant and cannot be overlooked.

In this paper, using the same approach of Ref. [13], we will show that the extraction of $F_2^n(x)$ from the ratio $E^{HeT}(x)$, up to $x<0.85$, is weakly dependent upon the different possible interactions between nucleons in nuclei. Furthermore, we suggest a method to check the role of effects beyond the impulse approximation and the reliability of the many different expressions proposed for the description of the DISF's of nuclei (see, e.g., Refs. [2,3,14,16,17]). Our approach is based on a joint analysis of the experimental ratios of (i) deuteron to proton, $E^{Dp}(x)=F_2^D(x)/F_2^p(x)$, (ii) ^{3}He to deuteron, $E^{HeD}(x)=F_2^{He}(x)/F_2^D(x)$, and (iii) ^{3}He to ^{3}H, $E^{HeT}(x)=F_2^{He}(x)/F_2^T(x)$, DISF's. We accurately take care of nucleon motion and nucleon binding in the two- and three-nucleon systems and explicitly consider the Coulomb interaction in the evaluation of the ^{3}He spectral functions. To this end, we take advantage of the very accurate wave functions of ^{3}He and ^{3}H systems, which can be calculated for realistic interactions within the correlated hyperspherical harmonics (CHH) approach of Ref. [18]. For an easy presentation, only the case of infinite momentum transfer in the Bjorken limit is considered, but it is straightforward to generalize our approach to the realistic case of finite momentum transfer values [19].

The paper is organized as follows. In Sec. II, the general formalism for the DISF's is presented; in Sec. III and IV recurrence relations for the extraction of $F_2^n(x)$ from DISF's of few-nucleon systems are proposed and the sensitivity to the interaction between nucleons is investigated; conclusions are drawn in Sec. V.

II. GENERAL FORMALISM

Our analysis is based on IA, which is usually employed for the calculation of nuclear structure functions at intermediate values of x, i.e., when the very small-x and the very large-x regions are excluded [20]. In IA the nucleon structure

is assumed to be the same as for free nucleons, and the DISF's for the deuteron, $F_2^D(x)$, for ^{3}He, $F_2^{He}(x)$, and for ^{3}H, $F_2^T(x)$ can be written, in the Bjorken limit, as follows:

$$F_2^D(x)=\int_x^{M_D/M}[F_2^p(x/z)+F_2^n(x/z)]f^D(z)dz, \qquad (1)$$

$$F_2^{He}(x)=2\int_x^{M_{He}/M}F_2^p(x/z)f_p^{He}(z)dz + \int_x^{M_{He}/M}F_2^n(x/z)f_n^{He}(z)dz, \qquad (2)$$

$$F_2^T(x)=\int_x^{M_T/M}F_2^p(x/z)f_p^T(z)dz+2\int_x^{M_T/M}F_2^n(x/z)f_n^T(z)dz, \qquad (3)$$

where M, M_D, M_{He}, M_T are the masses of nucleon, deuteron, ^{3}He, and ^{3}H, respectively. Different expressions have been proposed, see, e.g., Refs. [3,17], for the distributions $f^D(z)$ and $f_{p(n)}^{He(T)}(z)$, which describe the structure of the deuteron and of the three-nucleon systems. In this paper, we consider the following ones [14]:

$$f^D(z)=\int d\vec{p}\, n^D(|\vec{p}|)\,\delta\left(z-\frac{pq}{M\nu}\right)zC, \qquad (4)$$

$$f_{p(n)}^{He(T)}(z)=\int dE\int d\vec{p}\, P_{p(n)}^{He(T)}(|\vec{p}|,E)\,\delta\left(z-\frac{pq}{M\nu}\right)zC'. \qquad (5)$$

In Eqs. (4) and (5), $n^D(|\vec{p}|)$ is the nucleon momentum distribution in deuteron, the functions $P_p^{He(T)}(|\vec{p}|,E)$ and $P_n^{He(T)}(|\vec{p}|,E)$ are the proton and neutron spectral functions in ^{3}He (^{3}H), respectively, [21] $\vec{p}$ and E the nucleon three-momentum and removal energy, C and C' normalization factors. The Coulomb interaction is explicitly taken into account in the evaluation of the ^{3}He spectral function, unless otherwise explicitly specified. In Eqs. (4) and (5), to ensure the 4-momentum conservation at the virtual photon-nucleon vertex, the nucleon is assumed to be off-mass shell, i.e., $p\equiv(p^0,\vec{p})$ with $p^0=M_{D(He,T)}-\sqrt{(E+M_{D(He,T)}-M)^2+|\vec{p}|^2}$.

It has to be noted that the definitions (4) and (5) of the distributions $f^D(z)$ and $f_{p(n)}^{He(T)}(z)$, because of the off-mass-shell nucleon energy, p^0, already include the off-shell effects considered in the x-rescaling model of Ref. [22] (and, therefore, also the effects related to the derivative of the nucleon structure functions studied in Ref. [23]).

III. EXTRACTION OF $F_2^n(x)$ FROM ^{3}He AND ^{3}H DISF's

To perform our study we use the proton and neutron spectral functions for ^{3}He and ^{3}H that were obtained in Ref. [24] with the *RSC* [25], *AV*14 [26], and *AV*14 + Brazil three-body force (TBF) [27] interactions (in the last case the Brazil three-body force was properly tuned in Ref. [18] to obtain the experimental binding energy of ^{3}H). Furthermore, we have specifically evaluated, along the same lines of Ref. [24], the spectral functions for the *AV*18 interaction [28], for the *AV*18+*UrbanaIX* (*UIX*) TBF interaction [29] and for the *AV*18+*TM′* TBF interaction, which is a new version of the original *Tucson-Melbourne* (*TM*) TBF [30], from the corresponding CHH wave functions. Note that the *UIX* TBF was specifically proposed in Ref. [29] to get, together with the *AV*18 two-body interaction, the experimental binding energy of light nuclei and reproduces the binding energies of both ^{3}He and ^{3}H. The *TM′* TBF was properly modified in Ref. [31] to ensure consistency with chiral symmetry. The values of the strength and cutoff parameters of the *TM′* TBF are taken from Ref. [32]: with these values the *AV*18 + *TM′* interaction describes the $A=3$ ground state energies.

Let us define the super-ratio, $R^{HeT}(x)$ [10],

$$R^{HeT}(x)=\frac{F_2^{He}(x)/[2F_2^p(x)+F_2^n(x)]}{F_2^T(x)/[2F_2^n(x)+F_2^p(x)]}=E^{HeT}(x)\frac{2r(x)+1}{2+r(x)}. \qquad (6)$$

In IA the super-ratio is a functional of $r(x)$ ($R^{HeT}(x)=R^{HeT}[x,r(x)]$). Indeed from Eqs. (2),(3), and (6) one has

$$R^{HeT}[x,r(x)]=\frac{2r(x)+1}{2+r(x)}\times\frac{\int_x^{M_{He}/M}F_2^p(x/z)[2f_p^{He}(z)+r(x/z)f_n^{He}(z)]dz}{\int_x^{M_T/M}F_2^p(x/z)[f_p^T(z)+2r(x/z)f_n^T(z)]dz}. \qquad (7)$$

The extraction of $r(x)$ from the experimental ratio $E^{HeT}(x)$ can proceed, through Eq. (6), with the help of theoretical estimates of $R^{HeT}(x)$. Actually, from Eq. (6) one can immediately obtain the following equation for the ratio $r(x)$:

$$r(x)=\frac{E^{HeT}(x)-2R^{HeT}[x,r(x)]}{R^{HeT}[x,r(x)]-2E^{HeT}(x)}. \qquad (8)$$

In IA, Eq. (8) is a self-consistent equation, which allows one to determine $r(x)$. If the distributions $f_{p(n)}^{He(T)}(z)$ are represented by a Dirac δ function, $f_{p(n)}^{He(T)}(z)=\delta(z-1)$, then $R^{HeT}(x)=1$ and Eq. (8) becomes trivial. This hypothesis works reasonably well at small x, but is not a good approximation at $x>0.75$, so that $R^{HeT}(x)\neq 1$, as shown in Fig. 1. As a consequence, if (i) the experimental quantity $E^{HeT}(x)$ is simulated by its theoretical estimate, evaluated in IA through Eqs. (2) and (3) with some model for $r(x)$; and (ii) the approximation $R^{HeT}(x)=1$ is used to calculate $r(x)$ by Eq. (8), then one obtains a function which differs $\sim$10% at $x=0.85$ from the model for $r(x)$ used to simulate $E^{HeT}(x)$. Therefore, at high x one cannot approximate $R^{HeT}(x)$ by 1, if a good accuracy is required.

Fortunately, as illustrated in Fig. 1(a), the model dependence of $R^{HeT}(x)$, due to the different, possible two-body and three-body interactions between nucleons in ^{3}He and ^{3}H, is very weak for any x. Indeed, there is a substantial

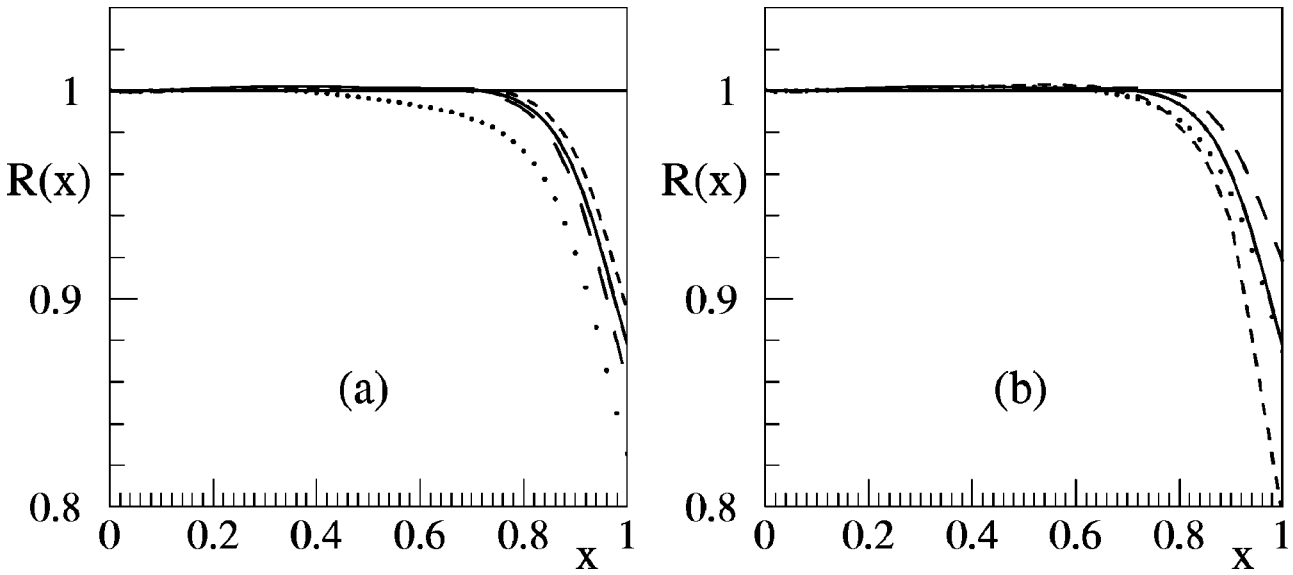

FIG. 1. (a) The super-ratio $R^{HeT}(x)$ [Eq. (7)] with $F_2^{n(p)}(x)$ from Ref. [33] for different nuclear interactions. Solid and long-dashed lines correspond to the $AV18 + UIX$ TBF and RSC interactions, respectively (the results for $AV18$, $AV18 + TM'$ TBF, $AV14$, and $AV14 +$ Brazil TBF are essentially identical to the ones for $AV18 + UIX$ TBF and are not shown). The short-dashed line corresponds to the $AV14$ interaction without the Coulomb interaction for ^{3}He. The dotted line is obtained as the solid one, but using the nucleon momentum distributions for the $AV18+UIX$ interaction, instead of the nucleon spectral functions. (b) The super-ratio $R^{HeT}(x)$ for the $AV18+UIX$ interaction. Dashed, dotted, and solid lines correspond to the models of Refs. [34], [35], and [33] for $F_2^{n(p)}(x)$, respectively (the model of Ref. [36] gives almost identical results of the model of Ref. [35] and is not shown); long-dashed line: as the solid one with $F_2^n(x)$ multiplied by $(1+0.5x^2)$ (see text).

cancellation of interaction effects in the numerator and in the denominator. In particular, the introduction of a three-body force yields very small effects in $R^{HeT}(x)$ at $x \leqslant 0.90$. Only if the Coulomb interaction is neglected in the evaluation of the ^{3}He spectral functions, one obtains relevant effects, since this interaction acts exclusively in the numerator. However, sensible differences in $R^{HeT}(x)$ are obtained for $x \geqslant 0.5$, if the nucleon spectral functions are replaced by the corre-

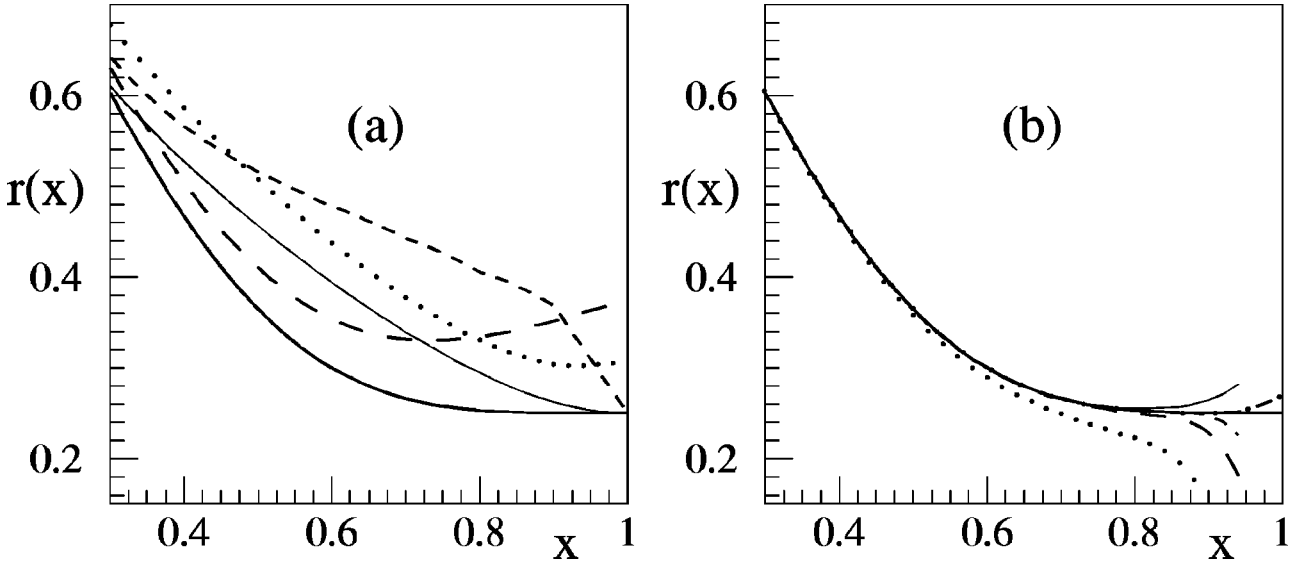

FIG. 2. (a) The ratio $r(x)$ for different parametrizations of nucleon DISF's. Thick-solid, dashed, dotted, and thin-solid lines correspond to the models of Refs. [33], [34], [35], and [36] for the nucleon DISF's, respectively. The long-dashed line corresponds to the model of Ref. [33], multiplied by $(1+0.5x^2)$. (b) $r^{(n)}(x)$, obtained by the recurrence relation (9) for $n=20$, using the nucleon DISF's of Ref. [33] and the $AV18+UIX$ spectral function for $E^{HeT}(x)$. Different spectral functions are used for $R^{HeT}(x)$: dot-dashed, thin-solid, dashed, and long-dashed lines correspond to the $AV18+UIX$ TBF, $AV18$, $AV14$, and RSC spectral functions, respectively (the results for $AV14$ + Brazil TBF and for $AV18+TM'$ TBF are almost indistinguishable up to $x=0.9$ from the ones for $AV18+UIX$ TBF and are not shown). The dotted line is $r^{(20)}(x)$, obtained using the nucleon momentum distributions for the $AV18+UIX$ TBF interaction in the evaluation of $R^{HeT}(x)$, instead of the spectral functions. The thick solid line is the ratio $r(x)$ for the nucleon DISF's of Ref. [33].

sponding nucleon momentum distributions, since this approximation yields much larger effects in $f_{p(n)}^{He(T)}(z)$ than the different interactions (see also Ref. [14] for the relevance of the spectral function).

Since several models for the nucleon DISF's, to be used in Eq. (7), are available, the sensitivity of the super-ratio to the different parametrizations has also to be checked. A scale of $Q^2=10\ (\mathrm{GeV}/c)^2$ has been chosen for the evaluation of the nucleon DISF's. Such a scale is low enough to allow the use of many of the available models and, at the same time, relevant differences are not expected between the results obtained at $Q^2=10\ (\mathrm{GeV}/c)^2$ and the ones corresponding to the Bjorken limit. The super-ratio Eq. (7), evaluated by using the DISF's given in [33–36], is shown in Fig. 1(b).

In Ref. [13] we showed that, within IA, Eq. (8) can be solved by recurrence

$$r^{(n+1)}(x)=\frac{E^{HeT}(x)-2\,R^{HeT}[x,r^{(n)}(x)]}{R^{HeT}[x,r^{(n)}(x)]-2\,E^{HeT}(x)} \tag{9}$$

starting from a reasonable zero-order approximation, $r^{(0)}(x)$. Since no data are presently available for $F_2^T(x)$, we simulated the experimental ratio $E^{HeT}(x)$ by a theoretical IA estimate. Both $E^{HeT}(x)$ and $R^{HeT}(x)$ were evaluated with the same nucleon spectral functions. The nucleon DISF's of Ref. [33] were used in the calculation of $E^{HeT}(x)$, while, to generate the zero-order approximation $r^{(0)}(x)$ to be used in $R^{HeT}(x)$, the neutron one was arbitrarily modified by the factor $(1+0.5x^2)$ to change its behavior at high x. Using the nucleon spectral functions obtained from the $AV18+UIX$ TBF interaction, a sequence which rapidly converges to $r(x)$ of Ref. [33] is obtained in the range $0\leqslant x\leqslant 0.9$. In particular, up to $x=0.85$ an accuracy better than 1% is obtained with only ten iterations. Starting from very different zero-order approximations $r^{(0)}(x)$, for instance the ratios corresponding to the nucleon DISF's of Refs. [34], [35] or [36] [see Fig. 2(a)], while still evaluating $E^{HeT}(x)$ from the nucleon DISF's of Ref. [33], convergences of a similar quality and to the same $r(x)$ of Ref. [33] have been obtained. Therefore, one can conclude that, up to $x=0.85$, the recurrence relation converges to the correct result, almost independently of the starting point $r^{(0)}(x)$ [see the dot-dashed line in Fig. 2(b)]. A convergence of the same quality is obtained if the spectral functions used for the calculation of $E^{HeT}(x)$ and $R^{HeT}(x)$ correspond to another interaction, e.g., the RSC interaction [13]. The convergence of the recurrence relation to the correct result can be related to the similarity between the distributions $f_{p(n)}^{He(T)}(z)$ and $\delta(z-1)$. Near $x\sim 1$, where $f_{p(n)}^{He(T)}(z)$ no more acts as a Dirac δ function in Eqs. (2) and (3), the recurrence relation is unable to solve Eq. (8).

In order to check the model dependence of our approach, due to the different assumptions for the interaction between nucleons in nuclei, we repeat the whole procedure of Ref. [13], but using for the evaluation of the super-ratio $R^{HeT}(x)$ spectral functions corresponding to different interactions than the $AV18+UIX$ one, employed for the calculation of our simulated "experimental" ratio $E^{HeT}(x)$. The spectral functions corresponding to RSC, $AV14$, $AV14$ + Brazil TBF, $AV18$ and $AV18$ + TM' TBF interactions are considered. In the range $0\leqslant x\leqslant 0.85$ the ratio $r(x)$ extracted by the recurrence relation after twenty iterations differs from the one used for $E^{HeT}(x)$ less than 3%, for any of the considered interactions [see Fig. 2(b)]. Actually, if only interactions able to give the experimental value for the binding energy of ^{3}H are considered (i.e., $AV14$ + Brazil, $AV18$ + TM' and $AV18$ + UIX), the model dependence in the extraction of $r(x)$ is at most 1% in the range $0\leqslant x\leqslant 0.85$. Furthermore, these results are essentially independent of the model for the ratio $r(x)$, which is used in the evaluation of $E^{HeT}(x)$. Let us stress that the recurrence procedure yields somewhat larger

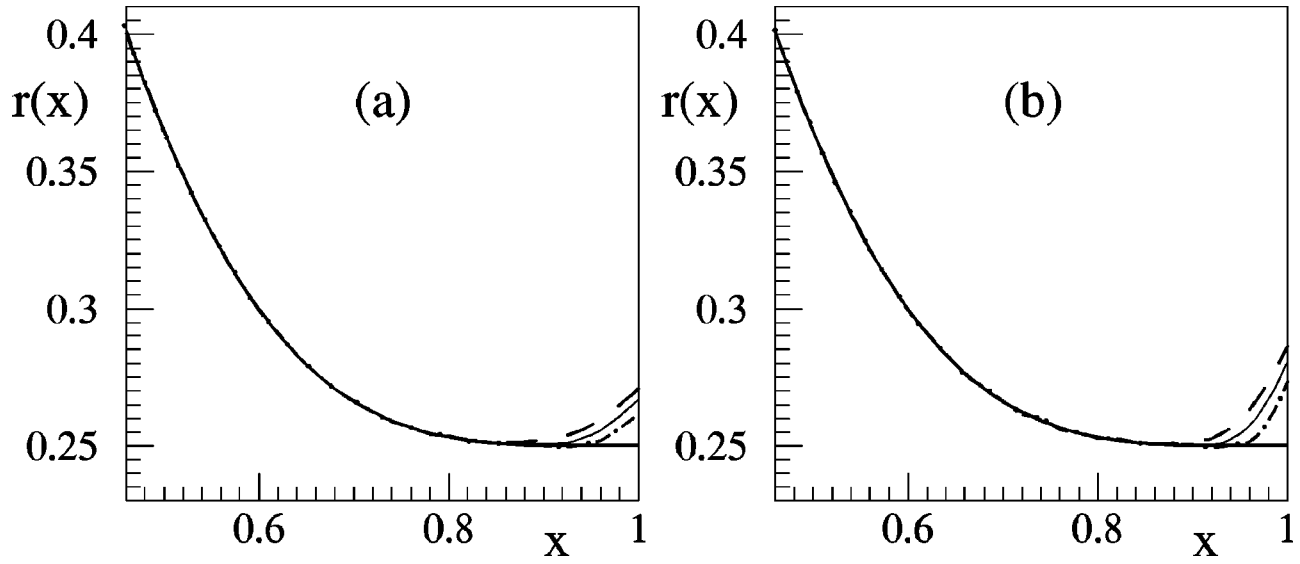

FIG. 3. (a) The ratio $r(x)$ obtained by the recurrence relation (10), using the $AV18$ interaction both for $E^{Dp}(x)$ and $R^{Dp}(x)$. Long-dashed, thin-solid, and dot-dashed lines are $r^{(n)}(x)$ for $n=3,6,20$ iterations, respectively. The thick solid line is the ratio $r(x)$ for the nucleon DISF's of Ref. [33], used to evaluate $E^{Dp}(x)$. (b) The same as in (a), but for the recurrence relation (11), concerning the ratio of ^{3}He to deuteron DISF. The $AV18$ and the $AV18+UIX$ TBF interactions have been used for the ^{2}H and ^{3}He DISF's, respectively.

differences with respect to the input $r(x)$ ($\sim 4\%$ at $x=0.85$), if the Coulomb interaction is neglected in the ^{3}He spectral functions considered for the evaluation of $R^{HeT}(x)$. However, these differences are not to be included in the model dependences, since the Coulomb interaction can be exactly taken into account, e.g., within the CHH approach.

Let us note that, in order to apply the recurrence relation (9), the knowledge of the function $E^{HeT}(x)$ is needed on the whole range $0<x<1$, even if one is interested in $r(x)$ for $x<0.85$ only. However, in the near future $E^{HeT}(x)$ will not be experimentally accessible for $x \geqslant 0.85$. To investigate the possible effects on the extraction of $r(x)$ due to this problem, we change the ''experimental'' ratio $E^{HeT}(x)$ by an arbitrary factor $(1+0.5x^{20})$, which modifies only the large x region, and repeat the recurrence extraction procedure. The ratio $E^{HeT}(x)$ is essentially unchanged by the factor $(1+0.5x^{20})$ up to $x=0.8$, is modified by 2% at $x=0.85$ and by 50% at $x=1$. Then, after twenty iterations one obtains convergence to the same $r(x)$ up to $x=0.8$ and only a 5% difference at $x=0.85$.

Therefore, within IA, the proposed procedure is able to yield reliable information on $F_2^n(x)$ in the x range accessible at TJLAB [10], whenever nucleon binding in nuclei and the Coulomb interaction in the ^{3}He spectral function are correctly taken into account. On the contrary, if the momentum distribution is used for the evaluation of $R^{HeT}(x)$, instead of the spectral function, the iterative procedure converges to a function $r(x)$, which differs from the correct one more than 13% for $x \geqslant 0.8$ [see Fig. 2(b)].

Let us note that our results hold unchanged if, instead of Eq. (5), a different expression (see, e.g., [3,16,17]) is used to evaluate both $E^{HeT}(x)$ and $R^{HeT}(x)$.

IV. EXTRACTION OF $F_2^n(x)$ FROM A JOINT ANALYSIS OF ^{2}H, ^{3}He AND ^{3}H DISF's

Many different expressions have been proposed to describe the DISF's of nuclei and to explain the EMC effect (see, e.g., Refs. [2,3,14,16,17] and references quoted therein), which are based on convolution formulas or involve medium effects beyond IA. The different models can clearly affect the extraction of $F_2^n(x)$. For instance, in Ref. [6] it was shown that, at large x, medium effects beyond IA can considerably modify the neutron DISF extracted from the experimental deuteron DISF. Again in the case of the deuteron, in Ref. [37] it was shown that sizable effects are obtained in the extraction of neutron DISF if, instead of the model given by Eqs. (1) and (4), one adopts a convolution model developed within the front-form Hamiltonian dynamics with a Poincaré-covariant current operator. Although the effects of the different expressions proposed for the DISF's of nuclei are present both in the numerator and in the denominator of $R^{HeT}(x)$ and they should at least partially compensate in the ratio, their relevance in the extraction of $F_2^n(x)$ has to be carefully investigated. A possible check of the correctness of the different theoretical expressions could be performed by comparing the neutron DISF, independently extracted from the experimental ratios $E^{Dp}(x)$, $E^{HeD}(x)$ and $E^{HeT}(x)$, using a coherent framework for the evaluation of the deuteron, ^{3}He and ^{3}H DISF's. Indeed, the theoretical super-ratios corresponding to $E^{Dp}(x)$ and $E^{HeD}(x)$ will be much more affected by the model used for the evaluation of the structure functions than in the case of $R^{HeT}(x)$. Then, one can take advantage of this model dependence for a test of the theoretical models: the proper expressions for the DISF's of nuclei should lead to the same results for the neutron DISF extracted from any of the above mentioned experimental ratios.

This analysis, performed with actual estimates of medium correction terms or using different convolution formulas, is outside the scope of the present work. Here we only wish to show that $F_2^n(x)$ can be extracted from the ratios $E^{Dp}(x)$ and $E^{HeD}(x)$ through the following recurrence relations, based on IA and analogous to the one of Eq. (9),

$$r^{(n+1)}(x)=\frac{E^{Dp}(x)}{R^{Dp}[x,r^{(n)}(x)]}-1 =\frac{F_2^{Dexp}(x)[1+r^{(n)}(x)]}{\int_x^{M_D/M}[1+r^{(n)}(x/z)]F_2^p(x/z)f^D(z)dz}-1, \tag{10}$$

$$r^{(n+1)}(x)=\frac{E^{HeD}(x)-2\,R^{HeD}[x,r^{(n)}(x)]}{R^{HeD}[x,r^{(n)}(x)]-E^{HeD}(x)}, \tag{11}$$

with natural definitions for the super-ratios $R^{Dp}(x)=F_2^D(x)/[F_2^p(x)+F_2^n(x)]$ and $R^{HeD}(x)=F_2^{He}(x)[F_2^p(x)+F_2^n(x)]/\{F_2^D(x)[2F_2^p(x)+F_2^n(x)]\}$.

As we did before for Eq. (9), we simulate the experimental ratios $E^{Dp}(x)$ and $E^{HeD}(x)$ by theoretical estimates using Eqs. (1), (2), (4), and (5) with a given momentum distribution or given spectral functions for the deuteron and for ^{3}He, respectively. Then, we evaluate $R^{Dp}(x)$ and $R^{HeD}(x)$ in IA with the same nucleon momentum distribution or spectral functions, and assume a function $r^{(0)}(x)$ as the zero-order approximation. As shown in Fig. 3, the convergence of the recurrence relations (10) and (11) to the input model for the ratio $r(x)$, used for the calculation of the simulated ''experimental'' quantities, is very fast. Furthermore, as in the case of Eq. (9), the extracted $r(x)$ is essentially independent of the function $r^{(0)}(x)$, assumed as the zero-order approximation.

The evaluation of the model dependence due to the nuclear interaction in the extraction of $r(x)$ by the recurrence relations (10) and (11) deserves a separate analysis for each one of these two equations. For the deuteron-proton case [Eq. (10)] we evaluate $R^{Dp}(x)$ by means of different nucleon momentum distributions than the one corresponding to the $AV18$ interaction used to simulate $E^{Dp}(x)$. Using any of the already mentioned two-body interactions, the function $r^{(n)}(x)$ obtained after twenty iterations differs less than 4% up to $x \leqslant 0.80$ and by 8% at $x=0.85$ from the function $r(x)$ used for $E^{Dp}(x)$. For the ^{3}He-deuteron case [Eq. (11)], we simulate $E^{HeD}(x)$ through Eqs. (1) and (2), using the $AV18$ interaction for the deuteron and the $AV18+UIX$ TBF interaction for the ^{3}He, respectively. Then, using any of the men-

tioned two-body and three-body interactions to evaluate $R^{HeD}(x)$, differences as high as 12% at $x=0.8$ and 25% at $x=0.85$ are found between $r^{(20)}(x)$ and the parametrization for $r(x)$ used to simulate $E^{HeD}(x)$. However, if the model dependence in the extraction of $r(x)$ due to the nuclear interaction is estimated, as it has to be, considering only the differences generated by interactions able to correctly reproduce the experimental binding energy of ^{3}H, then the effects of the possible different interactions is reduced to 1% at most, up to $x=0.85$. Therefore, both for $E^{Dp}(x)$ and $E^{HeD}(x)$, the effects of the different nuclear interactions on the extraction of $F_2^n(x)$ are well under control.

V. CONCLUSIONS

In this paper, recurrence relations for the extraction of $F_2^n(x)$ for $x<0.85$ from DISF's of deuteron, ^{3}He and ^{3}H have been proposed within IA. These recurrence relations, which require a zero-order approximation for the neutron structure function $F_2^n(x)$, have been shown to be rapidly convergent and essentially insensitive to the zero-order approximation. Moreover, they are only very weakly dependent on the interaction between nucleons in nuclei, whenever the $A=3$ binding energies are correctly reproduced. In the case of the three-nucleon systems, the relevance of accurate calculations which take into account the nuclear structure by means of the spectral function was stressed. In particular, we have investigated the role played by the Coulomb interaction in ^{3}He, for a good accuracy in the extraction of $F_2^n(x)$ at high values of x.

Summarizing, we suggest to take advantage of the very well known nuclear structure of few-nucleon systems to extract $F_2^n(x)$ from a joint analysis of deuteron, ^{3}He and ^{3}H DISF's. Our approach can be easily extended to include the analysis of ^{4}He DISF. We stress that, while waiting for the ^{3}H experiments in order to perform a more complete investigation, a simultaneous analysis of the experimental ratios $E^{Dp}(x)$ and $E^{HeD}(x)$ in a wide range of x should be carried out. In these cases, the model dependences in the evaluation of the structure functions will be bigger than in the $E^{HeT}(x)$ case, but the comparison of the results obtained from the recurrence relations (10) and (11), including possible contributions beyond IA in the evaluation of the super-ratios $R^{Dp}(x)$ and $R^{HeD}(x)$, could already give useful information on the role of medium effects and consequently allow a more reliable extraction of $F_2^n(x)$. In the case of $E^{HeD}(x)$, one should accurately take care of three-body forces which give the experimental ^{3}H binding energy. Indeed considerable differences are obtained if interactions which do not reproduce ^{3}H binding energy are used in the evaluation of $R^{HeD}(x)$. At variance, because of isospin symmetry, these effects largely cancel out in the ratio of ^{3}He to ^{3}H DISF. This fact supports the usefulness of measurements of the ratio $E^{HeT}(x)$ for the extraction of the neutron deep inelastic structure function.

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