

Nucleation versus spinodal decomposition in a first order quark hadron phase transition

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We investigate the scenario of homogeneous nucleation for a first order quark-hadron phase transition in a rapidly expanding background of quark gluon plasma. Using an improved preexponential factor for homogeneous nucleation rate, we solve a set of coupled equations to study the hadronization and the hydrodynamical evolution of the matter. It is found that significant supercooling is possible before hadronization begins. This study also suggests that spinodal decomposition competes with nucleation and may provide an alternative mechanism for phase conversion particularly if the transition is strong enough and the medium is nonviscous. For weak enough transition, the phase conversion may still proceed via homogeneous nucleation.

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I. INTRODUCTION

The hadronization of quark gluon plasma (QGP) possibly produced in the early universe or expected to be formed in relativistic heavy-ion collisions [1] has been the focus of much attention during the past few years. The quark gluon plasma (QGP), if formed, would expand hydrodynamically and would cool down until it reaches a critical temperature T_C where a phase transition from the quark matter to the hadron matter begins. Although the plasma has to hadronize, the mechanism of hadronization still remains an open question. The percolation model calculations are used in the case of a second order phase transition. In a first order scenario, the dynamics of the phase transition has been modeled in several ways. In the most idealized picture, the temperature of the plasma is held fixed at $T=T_C$ until the phase conversion is completely over. Assuming an isoentropic expansion in $(1+1)$ dimension, the hadronization in the above picture gets completed at $\tau_h=r\tau_C$ where r is the ratio of the degrees of freedom of QGP and hadronic phases and τ_C is the proper time at which the QGP cools down to the temperature T_C . In reality, a first order phase transition is characterized by a large nucleation barrier that separates the two phases at $T=T_C$ and the hadronization will not begin unless the matter supercools below T_C . Alternatively, the theory of homogeneous nucleation has been invoked to study the first order phase transition which is more realistic than the above idealized adiabatic scenario and has been in use for quite some time in the cosmological context [2]. In this picture, the transition is initiated by the nucleation of critical-size hadron bubbles from a supercooled metastable QGP phase. These hadron bubbles can grow against surface tension, converting the QGP phase into the hadron phase as the temperature drops below the critical temperature, T_C . For strong enough transition, the large amplitude fluctuations are suppressed so that the nucleation begins from a (nearly) homogeneous background of supercooled metastable phase. This has been the basis of homogeneous nucleation theory [3] based on which the QCD phase transition has been studied extensively [4–9]. However, for a weak enough transition, the matter may not remain in a pure homogeneous state even at $T=T_C$ due to the transitions that may occur above T_C . Evidence of such pretransitional phenomena are quite common

in several condensed matter studies particularly in the case of isotropic to nematic transition in liquid crystals. The possibility of such a transition had also been investigated during the cosmological electroweak phase transition in the early universe [10,11]. Due to high rate of thermal fluctuations and slow cooling of the universe, a strong phase mixing is expected even at temperatures above T_C where the new phase is highly metastable. In such situations, the phase transition either proceeds through percolation [11], if not, the dynamics of the first order transition is going to be quite different than the standard theory of homogeneous nucleation [12]. Similar phenomena is also expected in the case of a quark hadron phase transition both in the early universe as well as in the plasma expected to be formed during the relativistic heavy ion collisions [13,14]. An ideal quark gluon plasma in $(1+1)$ dimension expands as per the Bjorken scaling where $T^3\tau$ is constant [15]. This scaling would mean that the rate of change of temperature is higher for the plasma at earlier time as compared to the plasma at later time. In the case of early universe, due to high initial temperature, the rate of cooling becomes quite slow by the time it approaches T_C . However, the QGP produced during the heavy ion collisions will have smaller initial temperature and will cool much more rapidly as compared to the early universe. Another difference as compared to the early universe is that the QGP produced at RHIC and LHC energies may attain kinetic equilibrium in a very short time $\approx 0.3-0.7$ fm/c but will remain far off from chemical equilibrium [16,17]. Such a plasma will be more gluon rich and many more quark and anti-quark pairs will be needed before the plasma achieves chemical equilibrium. The chemically unsaturated plasma will cool still at a faster rate since additional energy will be consumed in approaching the chemical equilibration [17,18]. Recently, we have investigated the effect of thermal fluctuations leading to phase mixing by modeling them as subcritical hadron bubbles [19]. Although the equilibrium density distribution of these subcritical bubbles can be quite large, their equilibration time scale is larger than the cooling time scale for the QGP. As a consequence, for RHIC and LHC energies, they will not build up to a level capable of modifying the predictions from homogeneous nucleation theory: the QGP has to supercool and the hadronization may still proceed through the nucleation of critical bubbles within a nearly homogeneous back-

ground of quark-gluon plasma.

For the QGP formed at RHIC and LHC energies, the question to be asked is how long the plasma would continue to supercool. It is expected that the plasma will supercool up to some temperature T_m until the density of nucleated hadron bubbles become sufficient to heat up the medium due to the release of latent heat [6]. Since the medium is heated up, the nucleation is again switched off at some point and the hadronization continues due to the growth of previously nucleated hadron bubbles. Related to this another crucial aspect which needs investigation is that how fast the barrier between the metastable QGP phase and the stable hadron phase decreases as the system supercools. For strong enough supercooling, the barrier between the stable and metastable minima completely vanishes leading to a point of inflection at $T=T_S$ known as spinodal instability. Thus, the rapidly quenched system leaves the region of metastability and enters the highly unstable spinodal region before a substantial amount of nucleation begins. The spinodal decomposition has also been suggested as a possible mechanism of phase conversion for a rapidly expanding system of quarks and gluons [20]. The second scenario of phase transition may lead to coherent pion emission due to the formation of disoriented chiral condensates [21].

For the case of QGP produced in heavy ion collision, the route to hadronization either through nucleation or through spinodal decomposition depends sensitively on several factors like nucleation rate, effective potential used to model the dynamics of the phase transition and also on the hydrodynamical evolution that decides the expansion rate of the matter. The homogeneous nucleation rate is estimated from the factor $I=I_0 \exp(-\Delta F_C/T)$ where I_0 is the product of dynamical and statistical prefactors and ΔF_C is the minimum energy needed to create a critical hadron bubble. The exponential factor dominates the rate at temperature close to T_C whereas the prefactor I_0 plays the significant role away from T_C . The most commonly used expression for the prefactor is $I_0 \sim T^4$ [22] which may be alright for the case of early universe but not for the heavy ion collisions where we expect a large amount of supercooling. The theory of homogeneous nucleation assumed significance for the study of phase transition in QGP produced in the relativistic heavy ion collision after Csernai and Kapusta [4] derived an improved prefactor using coarse grained field theory. An important aspect of their formalism is the presence of viscosity which is essential for the removal of latent heat away from the hadron bubble. The prefactor vanishes for an ideal plasma. This would suggest that a dissipative dynamics should be followed for the phase transition to be completed through nucleation [9]. In a subsequent work [5], Ruggeri and Friedman, on the other hand, derived a prefactor which does not vanish in the limit of zero viscosity. Recently, we have also derived a prefactor which is more general and also reproduces the Csernai-Kapusta and Ruggeri-Friedman results in the limiting cases [23]. The prefactor I_0 in all these estimates is found to be less as compared to T^4 . In this work, we estimate the nucleation rate using an improved prefactor as derived in Ref. [23]. The argument $\Delta F_C/T$ that appears in the exponential depends on the phenomenological potential used to describe

a first order phase transition. Since the lattice predictions [24,25] are not yet conclusive about the nature and the strength of the transition, we use an effective potential that follows the bag equation of state and also covers a wide range from very weak to very strong first order phase transitions. Finally, we study the amount of supercooling and rate of hadronization by solving self consistently the nucleation rate along with energy momentum conserving hydrodynamic equation both for dissipative and nondissipative plasma. The dissipative plasma makes the evolution slow and also generates extra entropy. From this study we show that spinodal decomposition may compete with homogeneous nucleation if the plasma is nondissipative and the transition is relatively stronger. Since the amount of supercooling is less in case of a dissipative plasma, the phase conversion may still proceed through the nucleation of critical size hadron bubbles from a homogeneous background of supercooled quark gluon plasma.

The paper is organized as follows. In Sec. II, we briefly review the bag equation of state and an effective potential used to model the phase transition. The nucleation rate with various prefactors are discussed in Sec. III. Finally, we present our results in Sec. IV followed by the conclusion in Sec. V.

II. THE EFFECTIVE POTENTIAL AND THE EQUATION OF STATE

The spinodal instability corresponds to the inflection point in the effective potential (below T_C) used to study the dynamics of a phase transition. Obviously, the temperature T_S at which the instability occurs will depend on the parameters of the effective potential. Since the order as well as strength of the quark hadron phase transition is still an unsettled issue, we consider a more generic form of the potential which covers a wide range from very strong to very weak first order phase transition. The parameters of the effective potential are determined from the physical observables like the surface tension σ , the correlation length ξ and also from the requirement that the potential difference between the two minima should correspond to the pressure difference between the quark and hadron phases. We follow the bag equation of state to estimate various thermodynamical observables. For example, the pressure can be estimated from the relation $P = T \sum \ln Z_i / V - B$ where the partition function $\ln Z_i$ for a single fermion or boson is given by [27]

$$\ln Z_i = \pm \frac{gV}{2\pi^2} \int_0^\infty dk k^2 [\ln(1 \pm e^{-\sqrt{k^2 + m_i^2}/T})], \quad (1)$$

where the + and - stand for fermions and bosons, respectively. In the above, the chemical potential μ has been set to zero. Assuming, a baryon free quark gluon plasma which consists of u , d , and s quarks and gluons, the total pressure can be written as

$$\begin{aligned}
 p_q &= \frac{16\pi^2}{90} T^4 + \sum_i^{n_f} \frac{12T}{2\pi^2} \int_0^\infty dk k^2 \ln(1 + e^{-\sqrt{k^2 + m_i^2}/T}) - B, \\
 &= \frac{16\pi^2}{90} T^4 + \frac{7\pi^2}{60} T^4 \sum_i^{n_f} f(m_i/T) - B,
 \end{aligned} \quad (2)$$

where n_f is the number of flavors and m_i ($i = u, d, s$) are their masses. The function $f(m_i/T) = (360/7\pi^4) \int_0^\infty du u^2 \ln(1 + e^{-\sqrt{u^2 + (m_i/T)^2}})$ and B is the bag constant. In Eq. (2), the first term is due to massless gluons while the second term needs to be evaluated for different physical masses of the quarks. If the quarks are taken as massless, we get

$$p_q = a_q T^4 - B,$$

where

$$a_q = \frac{19\pi^2}{36}. \quad (3)$$

If the physical masses of quarks are used as $m_u \approx m_d \approx 8$ MeV and $m_s \approx 160$ MeV, then at the critical temperature ($T_C = 160$ MeV) we get $f(m_u/T) \approx 1$ and $f(m_s/T) \approx 0.8$. Thus

$$a_q = \frac{37\pi^2}{90} + 0.8 \frac{7\pi^2}{60}, \quad (4)$$

which is not much different from the one for the massless gas. Similarly, the total pressure in hadron phase consisting of n different types of particles and resonances is given by

$$\begin{aligned}
 p_h &= -T \sum_{i=1}^n \frac{g_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 - e^{-\sqrt{k^2 + m_i^2}/T}), \\
 &= \frac{\pi^2}{90} T^4 \sum_{i=1}^n g_i h(m_i/T).
 \end{aligned} \quad (5)$$

Here, $h(m_i/T) = -(45/\pi^4) \int_0^\infty du u^2 \ln(1 - e^{-\sqrt{u^2 + (m_i/T)^2}})$. If we consider only a massless pion gas, the above summation contributes only a factor of 3. Assuming the hadron gas consisting of π , K , η , ρ , and ω with appropriate mass and degeneracy [$m_\pi = 137.0$ MeV ($g = 3$), $m_K = 496.0$ MeV ($g = 4$), $m_\eta = 547.45$ MeV ($g = 1$), $m_\rho = 768.50$ MeV ($g = 9$), $m_\omega = 781.94$ MeV ($g = 3$)] we get

$$p_h \approx a_h T^4,$$

where

$$a_h = 4.5 \frac{\pi^2}{90} \quad \text{at } T = 160 \text{ MeV}. \quad (6)$$

In addition to the above equation of state, we use the following phenomenological potential to describe the phase transition [26]:

$$V(\phi, T) = a(T) \phi^2 - bT \phi^3 + c \phi^4, \quad (7)$$

where b and c are positive constants. Although, the exact knowledge of the order parameter ϕ is not necessary for the present purpose, it can be related to entropy or energy density [4] or to the sigma and pion field [20]. The above potential has two minima, one at $\phi_q = 0$ and the other one at

$$\phi_h = (3bT + \sqrt{9b^2T^2 - 32ac})/8c, \quad (8)$$

which in the present case will represent quark and hadron phases, respectively. These phases are separated by a maximum that occurs at ϕ_m given by

$$\phi_m = (3bT - \sqrt{9b^2T^2 - 32ac})/8c. \quad (9)$$

In the thin wall approximation [22], b and c can be expressed in terms of surface tension σ and correlation length ξ as [14]

$$b = \frac{1}{\sqrt{6\sigma\xi^5 T_C^2}}, \quad c = \frac{1}{12\xi^3 \sigma}. \quad (10)$$

The height of the degenerate barrier at $T = T_C$ or at $a(T_C) = b^2 T_C^2 / 4c$ is given by

$$V_b(T_C) = \frac{3\sigma}{16\xi(T_C)}. \quad (11)$$

Thus the potential can describe first-order phase transitions, with varying strength by either changing σ or ξ or both. The latent heat is also a measure of the strength of the first order phase transition which in the bag model is given by $4B$. The requirement that at all the temperatures, the difference between the two minima should be equal to the pressure difference between the two phases [14] fixes the third parameter $a(T)$ as

$$\begin{aligned}
 \Delta p &= p_h - p_q = -[V(\phi_h) - V(0)] \\
 &= -[a(T) - bT\phi_h + c\phi_h^2] \phi_h^2.
 \end{aligned} \quad (12)$$

Scavenius and Dumitru [20] have used a linear sigma model (LSM) potential and fitted the potential of the form given by Eq. (7) to fix the parameters a , b , and c . With this form of potential they obtain surface tension σ and correlation length ξ . We directly use surface tension (σ) and correlation length (ξ) to fix the parameters b and c . For any value of b and c the nonperturbative vacuum effect is taken care of by bag constant B . At $T = T_C$, $\phi_h = bT_C/2c = \sqrt{6\sigma\xi}$. For $\sigma = 10$ MeV/fm² and $\xi = 0.7$ fm (typical hadron size) we get $\phi_h = 91$ MeV which is nothing but the pion decay constant.

Figure 1 shows the plot of $V(\phi)$ as a function of ϕ at four different temperatures for typical values of $\sigma = 30$ MeV/fm² and $\xi(T_C) = 0.7$ fm. At $T = T_C$, the potential is degenerate with $V(\phi_q) = V(\phi_h) = 0$ being separated by a barrier at $\phi = \phi_m$. As the matter supercools, the hadron phase has lower free energy as compared to the QGP phase which is held fixed at $\phi_q = 0$ and also $V(\phi_q) = 0$. Thus, below T_C , the ϕ_q phase is metastable with respect to the stable hadron phase at $\phi = \phi_h$. At $T = T_S$, the potential develops an inflection point where ϕ_q and ϕ_m coincide. The condition

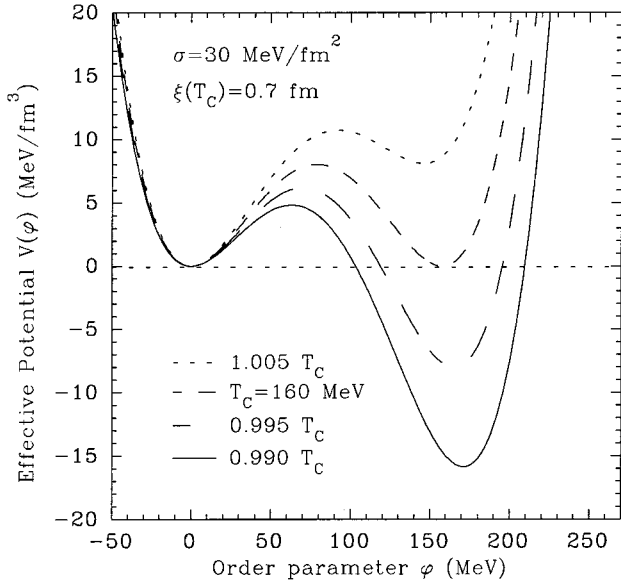


FIG. 1. The free energy density $V(\phi)$ as a function of ϕ at four different temperatures.

$\phi_q = \phi_m = 0$ leads to $a(T_S) = 0$ and $\phi_h = 3bT_S/4c$. The spinodal temperature T_S can be obtained analytically by solving Eq. (12) as

$$T_S = \left[\frac{B}{B + 27V_b(T_C)} \right]^{1/4} T_C. \quad (13)$$

At $T = T_S$ there exists only one minimum corresponding to the hadron phase. If the QGP supercools up to this point it will become unstable and may go to hadron phase by spinodal decomposition. It is worth noting that as $\sigma \rightarrow 0$ and $\xi \rightarrow \infty$, $T_S \rightarrow T_C$. The spinodal temperature depends on the strength of the transition. For a strong enough transition, σ is large and T_S is lower as compared to the case when the transition is weak. We are interested to know whether the system cools down to the temperature T_S . For comparison we denote the minimum temperature reached during the supercooling as T_m . It is the temperature in the supercooling region at which the system starts reheating due to the release of latent heat. The rate of nucleation will be suppressed for a stronger first order phase transition due to large barrier resulting in higher supercooling (i.e., smaller value of T_m). Thus, both T_S and T_m depend on the strength of the transition and need to be evaluated properly. While T_S can be estimated directly from Eq. (13), T_m requires a self-consistent solution of a set of equations involving the nucleation rate and energy momentum conserving hydrodynamical equations.

III. NUCLEATION RATE

The homogeneous nucleation theory assumes the formation of nucleating clusters within the initially homogeneous metastable state. One can use the Langer formalism [3], to calculate the probability of nucleation of hadron bubbles per unit time per unit volume from a homogeneous background

of quark gluon plasma, given by

$$I = I_0 \exp\left(-\frac{\Delta F_C}{T}\right). \quad (14)$$

Here, ΔF_C is the minimum energy needed to form a critical hadron bubble and I_0 is the prefactor which can be written as $I_0 = \kappa \Omega / 2\pi$. The statistical factor Ω is a measure of both the available phase space as the system goes over the saddle and of the statistical fluctuations at the saddle relative to the equilibrium states. The dynamical prefactor κ gives the exponential growth rate of the bubble or droplet sitting on the saddle.

A. Statistical prefactor

To understand the meaning of the statistical prefactor, consider a classical system with N degrees of freedom described by a set of N collective coordinates $\eta_i, i = 1, \dots, N$. The coarse-grained free energy functional $F\{\eta\}$ of the system has local minima $F\{\eta_i\}$ in the $\{\eta\}$ -space, corresponding to the metastable $\{\eta^0\}$ and stable states, separated by the energy barrier. The point of minimal energy along the barrier is the so-called saddle point $\{\eta^S\}$. The rate of the decay of the metastable state is determined by the steady-state current across the saddle point from the metastable to the stable minimum of $F\{\eta\}$. According to [3,28,29], the statistical prefactor can be written as

$$\Omega = \left(\frac{8\pi\sigma}{3|\lambda_1|} \right)^{3/2} \left(\frac{2\pi T}{|\lambda_1|} \right)^{1/2} \left[\frac{\det(M_0/2\pi T)}{\det(M'/2\pi T)} \right], \quad (15)$$

where λ_1 is the negative eigenvalue of the matrix $M_{ij} = \partial^2 F / \partial \eta_i \partial \eta_j$, evaluated at the saddle point. The subscript 0 denotes the metastable state and the prime indicates that the negative eigenvalue λ_1 as well as the zero eigenvalues of the matrix M_{ij} is omitted. The calculation of the fluctuation determinant in Eq. (15) is extremely difficult and adds to the uncertainty of the calculation of Ω . In Ref. [30], the above prefactor has been estimated under harmonic approximation using the product of eigenvalues of the mobility matrix M , evaluated at the saddle point and the metastable points. The final expression that accounts for fluctuation correction reads

$$\Omega_1 = \frac{2}{3\sqrt{3}} \left(\frac{\sigma}{T} \right)^{3/2} \left(\frac{R_C}{\xi} \right)^4, \quad (16)$$

where σ is the surface tension, ξ is the correlation length and R_C is the size of the critical droplet. Since R_C is infinite at $T = T_C$, Ω_1 diverges as $T \rightarrow T_C$. Alternatively, one can follow the earlier approach of Langer [3] where the fluctuation corrections are absorbed into the free energies of the metastable (F_0) and saddle point region (F_S) and the activation energy of the critical droplet is simply $\Delta F_C = F_S - F_0$. Therefore, omitting the fluctuation determinant, the expression for Ω is given by

$$\Omega_2 = \frac{32\pi^2 T^{1/2}}{|\lambda_1|^2} \left(\frac{\sigma}{3} \right)^{3/2}. \quad (17)$$

In the above, σ is interpreted everywhere as the true surface energy that includes fluctuation corrections. This approach has been adopted in Refs. [7,8] to estimate the statistical prefactor. In order to evaluate λ_1 , we will approximate the potential barrier between the metastable and stable states by the excess of the free energy ΔF corresponding to the formation of a spherical bubble of radius R . In the thin wall approximation [22], it can be written as the sum of the bulk and the surface energies as

$$\Delta F(R, T) = -\frac{4\pi}{3}R^3\Delta p + 4\pi R^2\sigma, \quad (18)$$

where Δp is the pressure difference between the hadron and QGP phases. Minimization of ΔF with respect to the radius R yields the free energy of the critical bubble

$$\Delta F_C = \frac{4}{3}\pi R_C^2\sigma, \quad R_C = \frac{2\sigma}{\Delta p}. \quad (19)$$

It is convenient to introduce a similarity number $\lambda_z = R_C\sqrt{4\pi\sigma/T}$ and the reduced radius $r = R/R_C$ [7,8,31]. Two functions $\Delta F_1(R)$ and $\Delta F_2(R)$ are similar if they have same similarity number. In the harmonic approximation, Eq. (18) can be expanded around the critical radius as

$$\begin{aligned} \frac{\Delta F}{T} &= \left(\frac{\Delta F}{T}\right)_{R=R_C} + \frac{1}{2T} \left(\frac{\partial^2 \Delta F}{\partial R^2}\right)_{R=R_C} (R-R_C)^2 \\ &= \frac{\Delta F_C}{T} - \lambda_z^2 (r-1)^2 \end{aligned} \quad (20)$$

and we get finally for the negative eigenvalue λ_1

$$\lambda_1 = -2T\lambda_z^2. \quad (21)$$

Following [7,8], we estimate Ω_2 using Eq. (17) and Eq. (21).

B. Dynamical prefactor

Csernai and Kapusta [4] have derived a dynamical prefactor κ by generalizing the coarse-grained effective field theory of Langer to a relativistic case. For a baryon-free plasma, where the thermal conductivity vanishes, the dynamical prefactor is found to depend on the viscosity coefficients and is given by

$$\kappa_{CK} = \frac{4\sigma(4\eta_q/3 + \zeta_q)}{(\Delta\omega)^2 R_C^3}. \quad (22)$$

Here η_q and ζ_q are, respectively, the shear and bulk viscosity coefficients of the quark phase and the $\Delta\omega$ is the difference between the enthalpy densities of the QGP and the hadron phases, $\omega = e + p$. According to Csernai and Kapusta (CK), there will be no bubble growth in the case of an ideal plasma with zero viscosity. Since viscosity is an essential ingredient in the above expression, for consistency, dissipative hydrodynamics should also be used to describe the space time evolution of the matter [9]. The CK approach has also been

extended to include thermal dissipation in addition to viscous damping for the case of baryon rich quark gluon plasma [32]. On the contrary, Ruggeri and Friedman [5] argued that for relativistic hydrodynamics the energy flow does not vanish in the absence of any heat conduction or viscous damping. Since the change of the energy density e in time is given in the low velocity limit by the conservation equation [4], $\partial e/\partial t = -\nabla \cdot (\omega \mathbf{v})$ which implies that the energy flow $\propto \omega \mathbf{v}$ is always present. Therefore, following a different approach, Ruggeri and Friedman derived an expression for κ , given by (for zero viscosity)

$$\kappa_{RF} = \left(\frac{2\sigma\omega_q}{R_C^3(\Delta\omega)^2}\right)^{1/2}. \quad (23)$$

The above result is in contradiction with the expression given by Eq. (22). Recently, following the Langer's procedure, we have solved the relativistic hydrodynamic in the hadron, quark, and the interfacial regions to obtain the dynamical prefactor. The κ as obtained in Ref. [23], is given by

$$\kappa = \kappa_0 + \kappa_v, \quad \kappa_0 = c_s \sqrt{\frac{\xi}{3R_C^3}}, \quad \kappa_v = \frac{\xi}{6R_C^3} \frac{1}{\omega_q} \left(\frac{4\eta_q}{3} + \zeta_q\right). \quad (24)$$

Here c_s is the velocity of sound in the massless gas. Under certain assumption for c_s , the above prefactor can also be written as [23]

$$\kappa \approx \left(\frac{2\sigma\omega_q}{R_C^3(\Delta\omega)^2}\right)^{1/2} + \frac{1}{c_s^2} \frac{\sigma(4\eta_q/3 + \zeta_q)}{(\Delta\omega)^2 R_C^3}. \quad (25)$$

As can be seen, the first term in the above equation is the same as obtained by Ruggeri and Friedman corresponding to the case of nonviscous plasma [see Eq. (23)]. The second term is similar to the result obtained by Csernai and Kapusta. Two important aspects of our result are (a) the prefactor κ can be written as a linear sum of a nonviscous (κ_0) and a viscous (κ_v) components and (b) the nonviscous component (κ_0) which depends on two parameters R and ξ is finite in the limit of zero viscosity. Further, it has been argued in Ref. [23] that the nonviscous part of the prefactor basically arises due to nonuniform pressure across the interface. In the present work, we will use Eq. (24) to calculate the dynamical prefactor.

IV. NUCLEATION AND THE EXPANSION DYNAMICS

To calculate the fraction of space converted to hadron phase using the nucleation rate $I(T)$, one requires a kinetic equation. The simplest of such equations is given in Ref. [6]. According to it, if the QGP cools to T_C at a proper time τ_C , then at some later time τ the fraction h of space which has been converted to hadronic gas is given by

$$h(\tau) = \int_{\tau_C}^{\tau} d\tau' I(T(\tau')) [1 - h(\tau')] V(\tau', \tau). \quad (26)$$

Here $V(\tau', \tau)$ is the volume of a bubble at time τ which had been nucleated at an earlier time τ' ; this takes into account the bubble growth. The factor $[1-h(\tau')]$ is the available space for new bubbles to nucleate. The model for bubble growth can be taken as [6,33]

$$V(\tau', \tau) = \frac{4\pi}{3} \left(R(T(\tau')) + \int_{\tau'}^{\tau} d\tau'' v(T(\tau'')) \right)^3, \quad (27)$$

where $v(T)$ is the velocity of the bubble growth at temperature T and is taken to be $v(T) = 3[1 - T/T_C]^{3/2}$ [6,34]. This expression is intended to apply only when $T > \frac{2}{3}T_C$ so that the growth velocity stays below the speed of sound of a massless gas, $c/\sqrt{3}$. At the critical temperature, both the nucleation rate and growth rate vanish. One can also write v in terms of pressure difference between the two phases as used in Ref. [35].

The evolution of the energy density is given by [15,36,37]

$$\frac{de}{d\tau} + \frac{D\omega}{\tau} = \frac{4\eta/3 + \zeta}{\tau^2} \equiv \frac{\mu}{\tau^2}, \quad (28)$$

where $D=1$ for the expansion in (1+1) dimension. The factors η and ζ are the shear and the bulk viscosity of the medium. For nonviscous plasma (for zero viscosity), the above equation follows the Bjorken's scaling solution where $T^3\tau$ is a constant. The energy momentum equation needs to be solved numerically for expansion in (3+1) dimensions. However, retaining the simplicity, we can still use Eq. (28) for spherical expansion [38,39] with the choice of $D=3$, although the scaling $u^\mu = x^\mu/\tau$ (where u^μ is the four velocity of the fluid) may not be valid if viscosity is included.

In the transition region, the energy density at a time τ can be written in terms of hadronic fraction $h(\tau)$ as

$$e(\tau) = e_q(T) + [e_h(T) - e_q(T)]h(\tau). \quad (29)$$

Here $e_q = 3p_q + 4B$ and $e_h = 3p_h$ are energy densities in the QGP and hadron phase, respectively. The viscosity at any time τ can also be written as [40]

$$\mu(\tau) = \mu_q(T) + [\mu_h(T) - \mu_q(T)]h(\tau), \quad (30)$$

where $\mu_q = 4\eta_q/3 + \zeta_q$ and $\mu_h = 4\eta_h/3 + \zeta_h$. The temperature dependence of the viscosities are taken as $\sim T^3$ [36].

V. RESULTS AND DISCUSSIONS

We compare the nucleation rates using different dynamical prefactors κ as given by

$$I_1 = \frac{\Omega_2 \kappa_{CK}}{2\pi} \exp\left(-\frac{\Delta F_C}{T}\right), \quad (31)$$

$$I_2 = \frac{\Omega_2 \kappa_0}{2\pi} \exp\left(-\frac{\Delta F_C}{T}\right),$$

$$I_3 = \frac{\Omega_2 \kappa_v}{2\pi} \exp\left(-\frac{\Delta F_C}{T}\right),$$

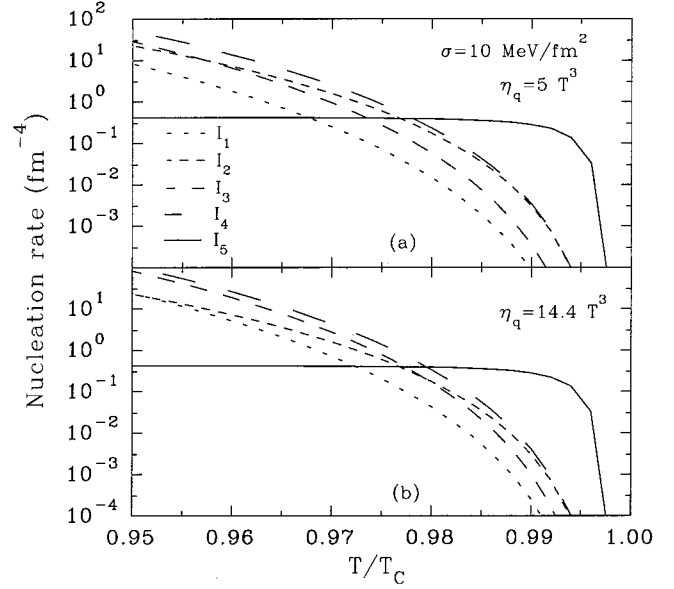


FIG. 2. Nucleation rates obtained using different prefactors for two viscosities using surface tension $\sigma = 10$ MeV/fm². The expressions for I_1 , etc., are given in the text.

$$I_4 = \frac{\Omega_2(\kappa_0 + \kappa_v)}{2\pi} \exp\left(-\frac{\Delta F_C}{T}\right),$$

$$I_5 = T^4 \exp\left(-\frac{\Delta F_C}{T}\right).$$

In all the cases, we use the same statistical prefactor Ω_2 which is more physical except for I_5 where the total prefactor is of the order of $\sim T^4$ as commonly used in the literature [22]. Further, we choose $\xi = 0.7$ fm and use Eq. (19) to estimate $\Delta F_C/T$. The nucleation rates I_2 and I_5 correspond to the case of a nondissipative plasma whereas viscosity enters as an essential ingredient for the evaluation of I_1 , I_3 , and I_4 . Figures 2 and 3 show the plot of above nucleation rates as a function of T/T_C for $\sigma = 10$ MeV/fm² and $\sigma = 30$ MeV/fm², respectively, each at two typical values of viscosity coefficient η_q . We consider only the shear viscosity η_q since the bulk viscosity ζ_q that provides resistance to expansion (or contraction) does not exist in the case of an incompressible fluid. Further, the viscosity coefficients η_h and ζ_h in hadron phase are neglected since the amount of supercooling is affected by viscosity of quark phase only. Notice that the most commonly used prefactor $\sim T^4$ overpredicts the nucleation rate (I_5) over a wide range of temperature particularly for small amount of supercooling and also when the transition is relatively stronger (see Fig. 3). Further notice, that both for strong and weak transitions but for moderately viscous medium, the nucleation rates are dominated by the nonviscous components [since I_4 is nearly the same as I_2 , see Figs. 2(a) and 3(a) for $\eta_q = 5T^3$]. The rates I_1 and I_3 which contain viscosity have only a second order effect unless the medium is highly dissipative. At $\eta_q = 14.4T^3$, the viscous component I_3 competes with the nonviscous compo-

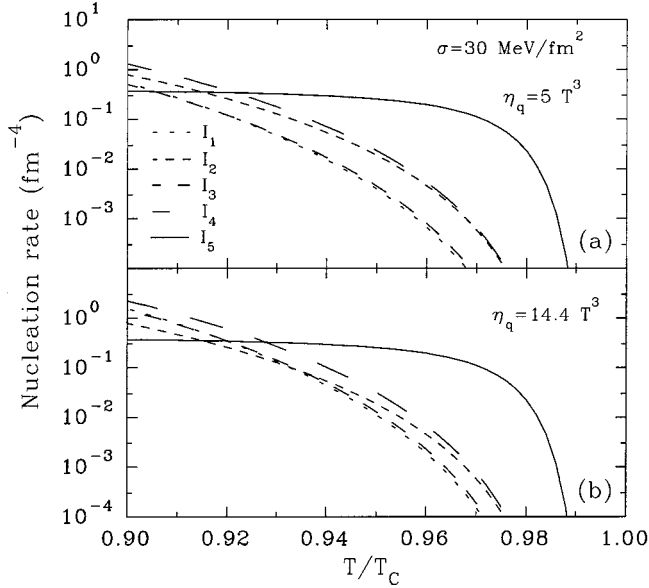


FIG. 3. Same as Fig. 2 but with $\sigma=30$ MeV/fm².

ment I_2 although I_1 is still lower [see Figs. 2(b) and 3(b)]. From the above studies we may conclude that the choice of T^4 overestimates whereas the rate (I_1) with the Csernai-Kapusta prefactor underestimates the nucleation rates in the region of interest as compared to I_4 . Since the nucleation rate I_4 has both dissipative and nondissipative components, we will use it to estimate the nucleation rate and supercooling in the subsequent study.

Next, we solve the coupled equations Eq. (26) and Eq. (28) to study the space time evolution of the matter. The critical temperature is fixed at $T_C=160$ MeV which gives $B^{1/4}=236$ MeV. The strength of the transition depends on σ and ξ or more explicitly on the ratio σ/ξ . Recent lattice calculations [25] predict σ between 2 to 10 MeV/fm². These calculations are without any dynamical quarks and thus are only indicative. Therefore σ is treated as a free parameter in the present study and $\xi=0.7$ fm seems the most reasonable value for the correlation length.

Another important parameter that affects the evolution is the time τ_C that the plasma takes to cool down to $T=T_C$. Obviously, τ_C will depend on the initial temperature (T_i), the formation time (τ_i) as well as on the expansion dynamics of the plasma. In the Bjorken scenario [15], $\tau_C=(T_i/T_C)^3\tau_i$. It is quite reasonable to assume that the plasma will expand in (1+1) dimension between τ_i and τ_C until the longitudinal dimension becomes comparable to the size of the colliding nuclei. Assuming $T_i=320$ MeV and $\tau_i=1$ fm/c, the Bjorken scaling predicts $\tau_C=8$ fm/c for expansion in (1+1) dimension. As mentioned before, τ_C is a crucial parameter that affect the solution of the coupled equations which in turn depends on the initial conditions as well as on the expansion scenario of the plasma. Since τ_C is not known exactly, we assume it of the order of 6 to 8 fm/c. From τ_C onwards, we consider the spherical expansion. This expansion scenario corresponds to the fastest cooling rate resulting in maximum supercooling. Therefore the choice of

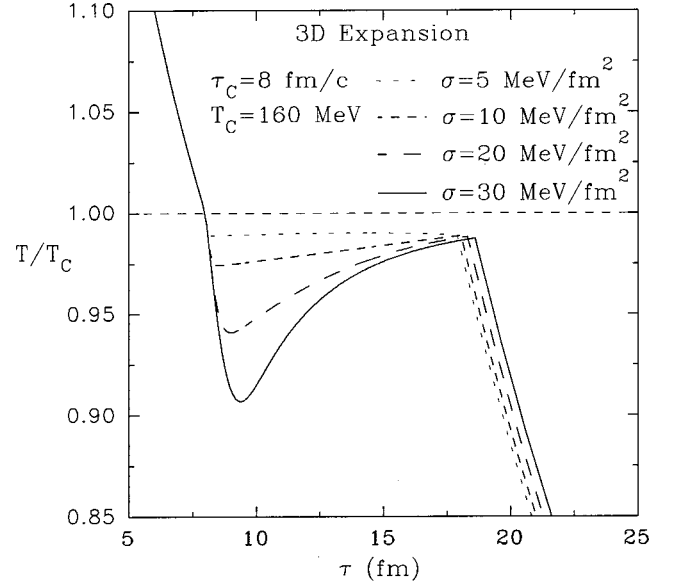


FIG. 4. The temperature evolution obtained with the nucleation rate I_4 for a spherically expanding system for different surface tensions.

$D=3$ provides a lower bound on T_m which we compare with the spinodal temperature T_S .

Figure 4 shows the plot of T/T_C as a function of τ for a nonviscous plasma at a typical value of $\tau_C=8$ fm/c. Unlike the ideal Maxwell construction (where the temperature is held fixed at $T=T_C$ until phase conversion is over), the system supercools up to T_m . At T_m , the number density of the nucleated hadron bubbles is sufficient to raise the temperature again due to the release of the latent heat. The amount of supercooling will obviously depend on the strength of the transition being more for a stronger transition (large value of σ) as compared to that for the weaker one (small value of σ). A faster expansion also reduces the number density of the nucleated hadron bubbles resulting in a larger supercooling. Although shown for $\tau_C=8$ fm/c, the amount of supercooling will increase further if τ_C is reduced due to faster expansion. We have also done the calculations for $\tau_C=6$ fm/c which are summarized in Fig. 6. For τ_C still smaller the expansion in (1+1) should be used.

Next, we consider the dissipative hydrodynamics. As mentioned before, the viscous contribution to the nucleation rate is not very significant if the transition is strong enough and also the medium is moderately viscous. However, the presence of a small amount of viscosity will affect the hydrodynamic evolution of the plasma. The temperature of the plasma will fall at a slower rate due to viscous heating of the medium. Figure 5 shows a plot of T/T_C as a function of τ for $\eta_q=0, 5T^3$, and $14.4T^3$. Here we have considered the case of a relatively stronger transition ($\sigma=30$ MeV/fm²) where the viscosity contribution to the nucleation rate is not very significant. However, significant effect can be seen on T_m which increases (less supercooling) with increasing viscosity as expected [9]. Figures 6 and 7 show the plot of T_m/T_C as a function of σ/ξ_q at $\tau_C=6$ fm/c and 8 fm/c, respectively, both for ideal as well as for dissipative plasma. The spinodal

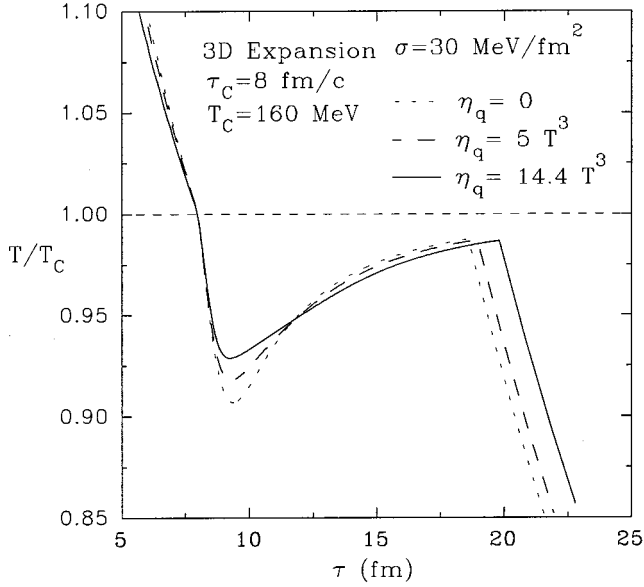


FIG. 5. The temperature evolution obtained with the nucleation rate I_4 for a spherically expanding system for different viscosity coefficients.

temperature T_S which depends on the ratio σ/ξ has also been plotted in the same figures for comparison. First consider the case of nondissipative plasma with zero viscosity. The curves T_m and T_S show a crossover point depending on the choice of τ_C . The nucleation rate is suppressed for a strong enough transition due to large nucleation barrier resulting in a higher amount of supercooling. On the other hand, for weak enough transition, the amount of supercooling is smaller and also well above the point of spinodal decomposition. Since T_m depends on expansion rate of the medium, the crossover point will sensitively depend on τ_C ; moving towards left for

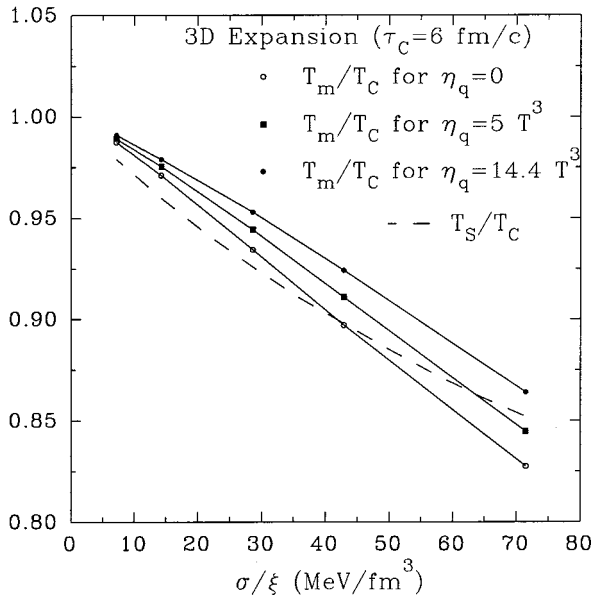


FIG. 6. The spinodal temperature T_S/T_C and minimum temperature T_m/T_C reached during supercooling as a function of σ/ξ using $\tau_C=6$ fm/c.

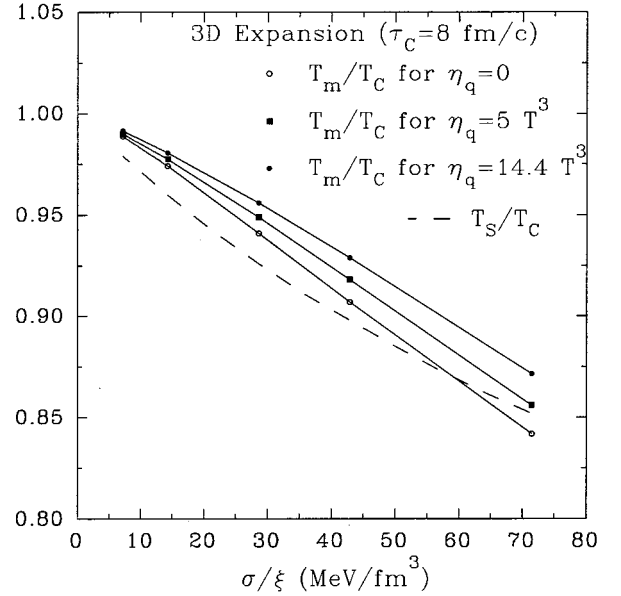


FIG. 7. Same as Fig. 6 but with $\tau_C=8$ fm/c.

the faster expansion. Qualitatively it can be concluded here that the homogeneous nucleation is still the dominant mechanism of phase conversion if the transition is weak enough. For stronger transition and fast enough cooling, the system may reach T_S and the phase conversion may proceed through the spinodal decomposition. If the plasma is viscous, the amount of supercooling is reduced further due to slow evolution of the medium. Even the crossover point also shifts towards right showing that the nucleation is the dominant mechanism over a wide range of σ/ξ , ratios. Due to uncertainties in τ_C , σ/ξ and η_q , it is difficult to say what is the exact scenario at RHIC and LHC energies. Assuming a weakly first order transition for σ in the range of 2 to 5 MeV/fm², $\xi \sim 0.7$ fm [25], $\eta_q \sim 2T^3$ (lower limit [41]), and $\tau_C \sim 4$ to 8 fm/c [18], the nucleation still seems to be the dominant mechanism of phase conversion as opposed to the spinodal decomposition.

It may be mentioned here that we have considered a dynamical growth rate which is exponential in nature. This may not be true at the later stage of the expansion. The fusion of bubbles may also play a role at the later stage when the density of bubbles is sufficiently high [38]. However, in the study of supercooling, we are very much in the domain of exponential region where the effect of bubble fusion can be ignored.

VI. CONCLUSIONS

We have investigated the mechanism of phase conversion from quark gluon plasma phase to hadron phase via two routes: the standard homogeneous nucleation and spinodal decomposition. The point of spinodal decomposition depends on the strength of the transition or more precisely on the ratio σ/ξ where σ and ξ are the surface tension and correlation length, respectively, at $T=T_C$. The nucleation

and supercooling, on the other hand, depend on the strength of the transition as well as on the expansion dynamics of the medium. We have solved a set of coupled equations to estimate the amount of maximum supercooling under spherical expansion scenario. Which way the hadronization will proceed depends sensitively on the nucleation and expansion dynamics since the nucleation and expansion time scales are comparable. Qualitatively we can describe the results as follows: For strong enough transition with zero or very small amount of viscosity, the system reaches the spinodal instability before the amount of nucleated hadron bubbles become

significant to begin phase conversion. The phase conversion in such a case will proceed via spinodal decomposition. If the medium is viscous or the transition is weak enough or both, the supercooling is much less. The phase conversion may still proceed through homogeneous nucleation. However, depending on the range of the parameters, there could be a competition between the homogeneous nucleation and the spinodal decomposition. A definite answer can be provided only when the parameters such as surface tension, τ_C , viscosity and also the expansion scenario are known precisely.

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