p_T fluctuations in high-energy p-p and A-A collisions

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The event-by-event p_T fluctuations in proton-proton and central Pb-Pb collisions, which have been experimentally studied by means of the so-called Φ measure, are analyzed. The contribution due to the correlation that couples the average p_T to the event multiplicity is computed. The correlation appears to be far too weak to explain the preliminary experimental value of $\Phi(p_T)$ in p-p interactions. The significance of the result is discussed.

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I. INTRODUCTION

The transverse momentum fluctuations in proton-proton and central Pb-Pb collisions at 158 GeV per nucleon have been recently measured [1] on event-by-event basis. To eliminate trivial "geometrical" fluctuations due to an impact parameter variation the so-called Φ measure [2] has been used. Φ is constructed in such a way that it is exactly the same for nucleon-nucleon (N-N) and nucleus-nucleus (A-A) collisions if the A-A collision is a simple superposition of N-N interactions. Consequently, Φ is independent of the centrality of A-A collision in such a case. On the other hand, Φ equals zero when the interparticle correlations are entirely absent. A critical analysis of the Φ measure can be found in [3,4]. For the central Pb-Pb collisions, value of $\Phi(p_T)$ measured in the laboratory rapidity window (4.0, 5.5) equals 4.6 ± 1.5 MeV [1]. Preliminary result for proton-proton interactions in the same acceptance is 5 ± 1 MeV [1]. Although the two values are close to each other the mechanisms behind them seem to be very different. It has been shown [1] that the correlations, which are of a short range in the momentum space as those due to the Bose-Einstein statistics, are responsible for the positive value of $\Phi(p_T)$ in the central Pb-Pb collisions. When the short range correlations are excluded $\Phi(p_T)$ is reduced to 0.6 ± 1 MeV [1]. Our calculations have indeed demonstrated [5,6] that the effect of Bose statistics of pions modified by the hadron resonances fully explains the observed $\Phi(p_T)$ in the central Pb-Pb collisions. On the other hand, the short range correlations have been experimentally shown [1] to provide a negligible contribution to the p_T fluctuations in the p-p interactions. Thus, the data suggest that the dynamical long range correlations are reduced in the central Pb-Pb collisions (when compared to p-p) with the short range due to the Bose statistics being amplified. The former feature is a natural consequence of the system evolution towards the thermodynamic equilibrium. The amplification of the quantum statistics effect results from an increased particle population in the final state phase space. Since various dynamical correlations contribute to $\Phi(p_T)$ a question emerges, what is the dynamical correlation in the nucleon-nucleon interactions that appear to be washed out in the central nucleus-nucleus collisions. The aim of this paper is to discuss the question.

The average transverse momentum $\langle p_T\rangle$, which is measured at a given multiplicity N, is known to depend on N in proton-proton collisions [7]. The correlation is negative for the collision energies below, say, $\sqrt{s}=50$ GeV and positive for the higher energies. At a beam energy of 205 GeV ($\sqrt{s}=19.7$ GeV), which is very close to that of the NA49 measurement [1], $\langle p_T\rangle$ significantly decreases with N [8]. The data are shown in Fig. 1, where we have merged the results for positive and negative particles. As already discussed in Ref. [2], the correlation that couples $\langle p_T\rangle$ to N leads to

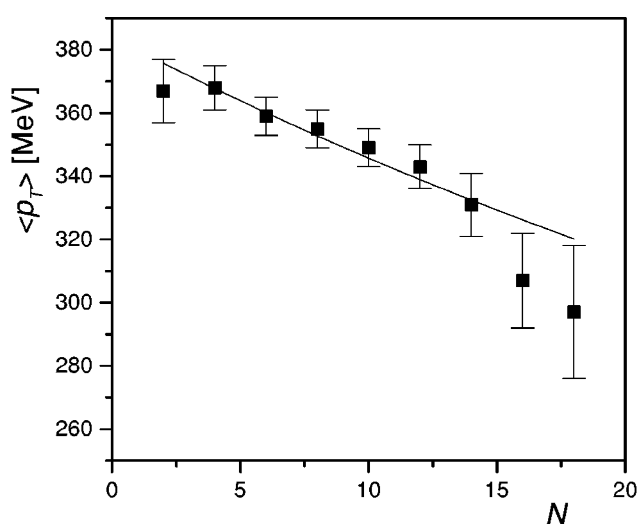

FIG. 1. Average transverse momentum as a function of charged particle multiplicity in nondiffractive p-p collisions at 205 GeV. The data are taken from Ref. [8]. The line corresponds to $\langle N\rangle=6.56$, $T=167$ MeV and $\Delta T=1.25$ MeV, see the text for the parameter's meaning.

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$\Phi(p_T)>0$. Here, we compute $\Phi(p_T)$ as a function of the correlation strength. Analytical and numerical results are presented. The effect of the finite acceptance is studied and comparison with the experimental data is performed.

II. ANALYTICAL CALCULATION

Let us first introduce the Φ measure. One defines a single-particle variable $z \stackrel{\text{def}}{=} x-\overline{x}$ with the overline denoting averaging over a single particle inclusive distribution. Here, we identify x with the particle transverse momentum. The event variable Z, which is a multiparticle analog of z, is defined as $Z \stackrel{\text{def}}{=} \sum_{i=1}^{N}(x_i-\overline{x})$, where the summation runs over particles from a given event. By construction, $\langle Z\rangle=0$ where $\langle\cdots\rangle$ represents averaging over events. Finally, the Φ measure is defined in the following way:

$$\Phi \stackrel{\text{def}}{=} \sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}}-\sqrt{\overline{z^2}} \quad .$$

The correlation $\langle p_T\rangle$ vs N is introduced to our model calculations through the multiplicity dependent temperature or slope parameter of p_T distribution. Specifically, the single particle transverse momentum distribution in the events of multiplicity N is chosen in the form suggested by the thermal model, i.e.,

$$P_{(N)}(p_T)\sim p_T \exp\left[-\frac{\sqrt{m^2+p_T^2}}{T_N}\right] \quad , \tag{1}$$

where m is the particle mass while T_N is the multiplicity dependent temperature defined as

$$T_N=T+\Delta T(\langle N\rangle-N)$$

with ΔT controlling the correlation strength. For $N=\langle N\rangle$ one gets $T_N=T$. The inclusive transverse momentum distribution, which determines $\overline{z^2}=\overline{p_T^2}-(\overline{p}_T)^2$, reads

$$P_{\text{incl}}(p_T)=\frac{1}{\langle N\rangle}\sum_N \mathcal{P}_N N P_{(N)}(p_T) \quad ,$$

where $\mathcal{P}_N$ is the multiplicity distribution.

The N-particle transverse momentum distribution in the events of multiplicity N is assumed to be the N product of $P_{(N)}(p_T)$. Therefore, all interparticle correlations different than $\langle p_T\rangle$ vs N are entirely neglected. Then, one easily finds

$$\langle Z^2\rangle=\sum_N \mathcal{P}_N\int_0^\infty dp_T^1\cdots\int_0^\infty dp_T^N(p_T^1+\cdots+p_T^N-N\overline{p}_T)^2 \times P_{(N)}(p_T^1)\cdots P_{(N)}(p_T^N).$$

Assuming that the particles are massless and the correlation is weak, i.e., $T\gg\Delta T(\langle N^2\rangle-\langle N\rangle^2)^{1/2}$ the calculation of Φ can be performed analytically. The results read

$$\frac{\langle Z^2\rangle}{\langle N\rangle}=2T^2-4\frac{T\Delta T}{\langle N\rangle}(\langle N^2\rangle-\langle N\rangle^2)+2\frac{\Delta T^2}{\langle N\rangle^3}(2\langle N^4\rangle\langle N\rangle^2 -4\langle N^3\rangle\langle N^2\rangle\langle N\rangle+\langle N^3\rangle\langle N\rangle^2-2\langle N^2\rangle\langle N\rangle^3+2\langle N^2\rangle^3 +\langle N\rangle^5),$$

$$\overline{z^2}=2T^2-4\frac{T\Delta T}{\langle N\rangle}(\langle N^2\rangle-\langle N\rangle^2)+2\frac{\Delta T^2}{\langle N\rangle^2}(3\langle N^3\rangle\langle N\rangle-2\langle N^2\rangle \times\langle N\rangle^2+\langle N\rangle^4-2\langle N^2\rangle^2),$$

$$\Phi(p_T)=\sqrt{2}\frac{\Delta T^2}{T\langle N\rangle^3}(\langle N^4\rangle\langle N\rangle^2-2\langle N^3\rangle\langle N^2\rangle\langle N\rangle-\langle N^3\rangle\langle N\rangle^2 +\langle N^2\rangle^3+\langle N^2\rangle^2\langle N\rangle), \tag{2}$$

where terms of the third and higher powers of ΔT have been neglected. One observes that the lowest nonvanishing contribution to Φ is of the second order in ΔT. The above formulas are much simplified for the Poisson multiplicity distribution. Then, one finds

$$\frac{\langle Z^2\rangle}{\langle N\rangle}=2T^2-4T\Delta T+2\Delta T^2(2\langle N\rangle^2+5\langle N\rangle+1),$$

$$\overline{z^2}=2T^2-4T\Delta T+2\Delta T^2(3\langle N\rangle+1),$$

$$\Phi(p_T)=\sqrt{2}\,\frac{\Delta T^2}{T}(\langle N\rangle^2+\langle N\rangle).$$

The multiplicity distribution of charged particles produced in high energy proton-proton collisions is, of course, not Poissonian. First of all, the number of charged particles is always even due to the charge conservation. The multiplicity distribution of positive (or negative) particles is also not Poissonian—the dispersion does not grow as $\sqrt{\langle N\rangle}$ but it follows the so-called Wróblewski formula [10], i.e., the dispersion is the linear function of $\langle N\rangle$. However, for the average multiplicities as low as those discussed here the Poisson distribution provides a reasonable approximation. Therefore, the multiplicity distribution, which is further used in our calculation, is Poissonian for negative pions with the number of positive pions being exactly equal to that of negative charge. Thus, the charge conservation is satisfied in every event. For such a multiplicity distribution $\Phi(p_T)$, which is given by approximate Eq. (2), reads

$$\Phi(p_T)=2\sqrt{2}\frac{\Delta T^2}{T}(\langle N\rangle^2+3\langle N\rangle). \tag{3}$$

III. NUMERICAL SIMULATION

The results from the preceding section are instructive but the adopted approximations are very rough. So, let us present more realistic Monte Carlo calculations that can be confronted with the experimental data [1]. The proton-proton collisions are simulated event by event in the following way. For every event we first generate the multiplicity of negative particles from the Poisson distribution and then add the equal

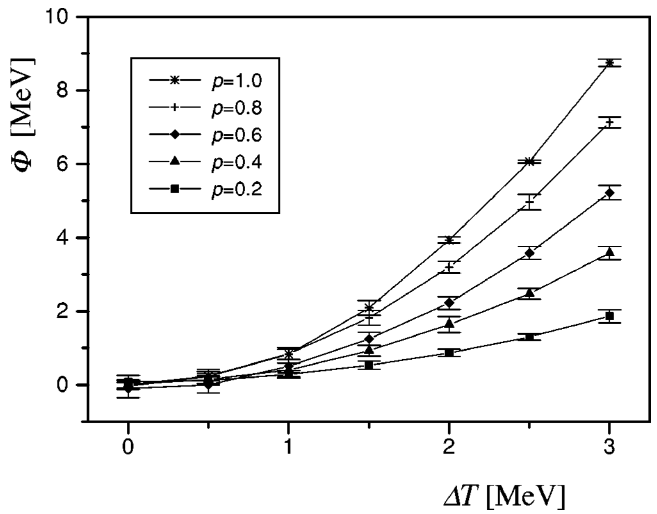

FIG. 2. $\Phi(p_T)$ as a function of the correlation strength ΔT for several values of the acceptance probability p.

number of positive particles. The average multiplicity of negative particles has been taken as $\langle N^- \rangle = 3.28$ that is the experimentally observed negative multiplicity in nondiffractive proton-proton interactions at 205 GeV [9]. This is the collision energy corresponding to the data from Fig. 1. Further, we attribute the transverse momentum from the distribution (1) to each particle assuming that all particles are pions. The numerical values of the temperature and correlation strength have been found fitting the data [8] shown in Fig. 1. We have got $T = 167 \pm 1.5$ MeV and $\Delta T = 1.25 \pm 0.25$ for charged particle multiplicity $\langle N \rangle = 2\langle N^- \rangle = 6.56$.

Due to the particle registration inefficiency and finite detector coverage of the final state phase space, only a fraction of particles produced in high-energy collisions is usually observed in the experimental studies. As an input to our model calculations we have taken the $\langle p_T \rangle - N$ correlation observed in the full phase space. The rapidity, which determines the acceptance domain, is not considered at all. Therefore, there is no difference whether a particle is lost due to the limited acceptance or due to the tracking inefficiency. Our Monte Carlo simulation takes into account the two effects in such a way that each generated particle—positive or negative pion—is registered with the probability p and rejected with $(1-p)$. Below, we further discuss the procedure in the context of NA49 data [1]. The results of our simulation are collected in Fig. 2, where $\Phi(p_T)$ as a function of ΔT for several values of p is shown. We have checked that our simulation fully reproduces the results from [2], where only negative particles in the full acceptance have been studied.

As seen in Fig. 2, $\Phi(p_T)$ grows quadratically with ΔT in agreement with the approximate Eq. (3). The increase of $\Phi(p_T)$ with p also follows from Eq. (3). Indeed, when a particle is detected with the probability p the temperature dispersion effectively decreases and the multiplicity $\langle N \rangle$ should be replaced by $p\langle N \rangle$ in Eq. (3). Consequently, $\Phi(p_T)$ grows quadratically with the observed particle multiplicity and the correlation, which is easily observable in the full phase space, is hardly seen in an acceptance as small as 20%. This behavior is very different than that found in Refs. [5,6] where $\Phi(p_T)$ due to the Bose-Einstein correlations has been computed. Then, $\Phi(p_T)$ is independent of the particle multiplicity.

The NA49 measurement of $\Phi(p_T)$ in proton-proton collisions has been performed in the transverse momentum and pion rapidity intervals (0.005, 1.5) GeV and (4.0, 5.5), respectively [1]. Only about 20% of all produced particles have been observed. Our simulation gives $\Phi(p_T) = 0.41 \pm 0.07$ MeV for the values of $\Delta T = 1.25$ MeV and $p = 0.2$ that are adequate for the NA49 measurement. Thus, the theoretical result significantly underestimates the preliminary experimental one that is, as already noted, $\Phi(p_T) = 5 \pm 1$ MeV [1].

One wonders whether the discrepancy is not caused by our highly simplified procedure of taking into account the effect of finite acceptance. We first note that the rapidity coverage of the NA49 measurement that is (4.0, 5.5) in the laboratory translates (1.1, 2.6) in the center-of-mass frame. Therefore, the observed pions are not far from the very central rapidity region. Since most of pions originate from the domain the average characteristics of all pions and that of the central ones are expected to be similar to each other. Indeed, the data from [8] show that $\langle p_T \rangle$ for all pions and those from the central region are essentially the same. Therefore, the correlation strength parameter $\Delta T = 1.25$ MeV, which corresponds to the $\langle p_T \rangle - N$ correlation averaged over full phase space, seems to be applicable not only to all pions but to the central ones as well. One further notes that $\langle p_T \rangle$ shown in [8] changes with the rapidity similarly for different N. Consequently, the correlation $\langle p_T \rangle$ vs N only weakly varies with y and it is hard to expect that the correlation strength observed in the NA49 acceptance domain is significantly larger than that averaged over the whole phase space. The data [8] suggests rather the opposite effect. Therefore, we conclude that our simplified procedure cannot distort the results dramatically and that the correlation $\langle p_T \rangle$ vs N does not explain the NA49 p-p preliminary data.

As already mentioned, Φ is constructed is such a way that it is exactly the same for nucleon-nucleon and nucleus-nucleus collisions if the latter is a simple superposition of former ones. Therefore, $\Phi(p_T)$ shown in Fig. 2 holds for nucleus-nucleus when all secondary interactions are neglected. Since our model neglects the Bose-Einstein correlations it can be compared with the NA49 data for the central Pb-Pb collisions when the short range correlations are excluded. In such a case, our result $\Phi(p_T) = 0.41 \pm 0.07$ MeV for $\Delta T = 1.25$ MeV and $p = 0.2$ appears to be compatible with the experimental value $\Phi(p_T) = 0.6 \pm 1$ MeV [1]. The smallness of $\Phi(p_T)$ reported by NA49 collaboration is then not surprising at all. As follows from Fig. 2 it is caused by the limited acceptance. However, it is premature to draw a conclusion about p_T correlations in Pb-Pb collisions until the origin of $\Phi(p_T)$ p-p interactions is not explained.

IV. CONCLUDING REMARKS

We have studied how a correlation, which couples the average p_T to the event multiplicity, influences the transverse momentum fluctuations observed by means of the Φ measure. An approximate analytical formula has been derived and then a numerical simulation has been performed. The effect of the finite detector acceptance has been taken into account. The procedure is highly simplified but it seems to be adequate for the NA49 data. It has been shown that the effect of the correlation $\langle p_T \rangle$ vs N is very weak if the particles from the small acceptance region are studied. Consequently, the correlation is far too weak to explain the preliminary experimental value of $\Phi(p_T)$ in proton-proton collisions [1]. If the preliminary data is confirmed by the final analysis one should look for other sources of dynamical p_T fluctuations. The effect of the conservation laws presumably plays no role in the acceptance as small as 20%. The decays of hadron resonances that strongly correlate the momenta of decay products might be important. However, the hadron resonances are present in proton-proton and nucleus-nucleus collisions as well. Consequently, the reported reduction of long range dynamical correlations in the central Pb-Pb collisions when compared to *p-p* [1] remains unexplained. The situation is much simpler if preliminary data on $\Phi(p_T)$ in proton-proton collision [1] overestimates the real value. Then, $\Phi(p_T)$ from *p-p* and central Pb-Pb are close to each other. There is also no conflict between our calculations and the experimental data. However, one cannot conclude that the long range correlations present in the proton-proton interactions are washed out in the central heavy-ion collisions. The problem obviously needs further experimental and theoretical studies. In particular, the data on proton-proton and nucleus-nucleus collisions from an enlarged acceptance are needed.

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