ϕ mesons from a hadronic fireball

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Production of ϕ mesons is considered in the course of heavy-ion collisions at SPS energies. We investigate the possible difference in momentum distributions of ϕ mesons measured via their leptonic $(\mu^+\mu^-)$ and hadronic (K^+K^-) decays. Rescattering of secondary kaons in the dense hadron gas together with the influence of in-medium kaon potential can lead to a relative decrease of the ϕ yield observed in the hadronic channel. We analyze how the in-medium modifications of meson properties affect apparent, reconstructed momentum distributions of ϕ mesons. Quantitative results are presented for central Pb+Pb collisions at $E_{\text{beam}} = 158$ GeV/nucleon.

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I. INTRODUCTION

The growing body of experimental information on fixed-target nucleus-nucleus collisions from AGS and CERN-SPS accelerators provides a reliable basis for the systematic investigation of strongly interacting hadronic matter under extreme conditions. Particularly, production of particles containing strange quarks is expected to reflect the reaction dynamics at the early stage of collisions. In this context, ϕ mesons, particles consisting mainly of $s\bar{s}$ pairs, are of a special interest. Since the interaction of ϕ mesons with non-strange hadronic matter is suppressed according to the Okubo-Zweig-Iizuka (OZI) rule, ϕ mesons are expected to decouple easily from the hadronic fireball.

Several calculations have been done for ϕ meson properties in dense hadronic environment (mainly at higher baryon densities) [1–7]. It has been found that modifications of the ϕ width and mass are sensitive to the strangeness content of the surrounding medium.

Recently, new experimental data from CERN-SPS on ϕ meson production in central Pb+Pb collisions at 158 GeV/nucleon beam energy became available. Advantage of the rich experimental program at CERN is that it allows to study the ϕ production via different ϕ decay channels. Results of the NA49 collaboration [8] are based on the ϕ meson identification via its hadronic decay $\phi \to K^+K^-$, while the NA50 collaboration has recently reported [9] about preliminary analysis of ϕ mesons identified via the dileptonic decay channel $\phi \to \mu^+\mu^-$.

The purpose of this paper is to show that the ϕ meson production spectra reconstructed via K^+K^- and $\mu^+\mu^-$ decay channels can be significantly different up to the level of present experimental observations.

After introductory considerations in Sec. II, we derive in Sec. III expressions for momentum distributions of ϕ mesons detected via KK and $\mu\mu$ channels. We take into account rescattering of decay kaons and in-medium modification of ϕ meson properties discussed in Sec. IV. Numerical results are presented in Secs. V and VI and conclusions are drawn in Sec. VII.

II. ϕ MESONS IN HADRON GAS

For our estimates of the observed ϕ meson spectra we assume a simple two-stage picture of central heavy-ion collisions, which is close to those considered within cascade-transport [10–14], hydrodynamical [15–17], and thermodynamical approaches [18–21]. The initial stage of a collision is characterized by a temperature close to the QCD phase transition $T \gtrsim T_c \approx 170 \pm 10$ MeV. Then the system expands up to the point when numbers of different kinds of particles freeze in—a chemical freeze-out. Thermodynamical parameters of this stage could be obtained by fitting the final total hadron multiplicities. Typical temperature is found to be $T_{\text{chem}} \sim 160 \pm 10$ MeV. During the second stage of the expansion, elastic scatterings change momentum distributions of hadrons until they cease and distributions freeze in. The freeze-out temperature can be extracted from a simultaneous fit to the single-particle m_T spectra of different particles supplemented by the analysis of particle correlation data. According to Refs. [19–22] one has $T = T_{\text{therm}} \sim 110 \pm 30$ MeV.

In our considerations we assume that the fireball created in heavy-ion collisions consists mainly of pions, kaons, and excited mesonic resonances in the central rapidity region.

Mean free path λ_ϕ of ϕ mesons in a hadron gas is estimated in Ref. [4]. Comparison with mean free paths of pions and kaons, $\lambda_{\pi,K}$, from Refs. [25,26] gives $\lambda_\pi \lesssim \lambda_K < \lambda_\phi$ for temperatures $T_{\text{therm}} < T < T_{\text{chem}}$. Hence, one can expect that ϕ mesons decouple from the pion-kaon subsystem at some earlier stage between the chemical and thermal freeze-out. At this stage, ϕ mesons stream freely out from the fireball. Pions and kaons, on the other hand, may still participate in mutual secondary interactions up to the stage of total thermal freeze-out. Therefore, if a ϕ meson decays inside a fireball via hadronic channel we have to take into account a possible interaction of its decay products with surrounding hadronic environment.

As a result of secondary interactions, the reconstructed invariant mass of a given pair of ϕ daughter kaons falls out from the original ϕ meson peak into the region identified as a combinatorial background. Therefore, the ϕ mesons decaying in medium can be partially unrecognized in experimental analysis of K^+K^- pairs. Together with negligible final state interaction of secondary dimuons originating from $\phi \to \mu^+\mu^-$ decays, this behavior results in the relative suppression of ϕ meson yield observed via the hadronic KK channel. Such a mechanism has been quantitatively studied in Ref. [27] where suppression at the level 40–60 % has been obtained from simulation using the RQMD code.

In this paper we discuss effects that may enhance the suppression of observed ϕ mesons identified via kaon channel. Possible increase of a ϕ meson width in medium will enlarge the probability of ϕ decay inside a fireball, enhancing thus consequences of the mechanism studied in Ref. [27]. Alternatively, we consider a possibility that the kaon decay channel of a ϕ meson becomes kinematically quenched in the medium. Additionally we argue that substantial relative difference in properties of K^+ and K^- meson in the hadronic environment caused by the isospin asymmetry and/or by the large baryonic admixture, would also prevent the reconstruction of ϕ mesons decaying into kaon pairs inside the medium.

III. DISTRIBUTION OF ϕ DECAY PRODUCTS

Let us denote the phase-space distribution of ϕ mesons in the center-of-mass system of two colliding nuclei at freeze-out as $f_\phi(\vec{x},\vec{p})$. Then the primary momentum distribution of ϕ mesons is given by

$$\eta_0(p)=\int_\Sigma d^3\sigma^\mu p_\mu f_\phi(\vec{x}_\phi,\vec{p}), \tag{3.1}$$

where integration goes over the fireball volume within a freeze-out hypersurface Σ (surface normal vector $d^3\sigma^\mu$ contracted with a ϕ meson momentum p^μ) [23]. Here we assume the case of a timelike freeze-out hypersurface, $d^3\sigma^\mu p_\mu>0$, which is relevant for applications given below (for discussions of alternative cases, see [24]). In the absence of any in-medium modifications of decay products and final state rescattering, the shape of *observed* momentum distributions $\eta_0(p)\Gamma^0_{\mu\mu}/\Gamma_{\rm tot}$ of muon pairs $(\mu^+\mu^-)_\phi$ and $\eta_0(p)\Gamma^0_{KK}/\Gamma_{\rm tot}$ of kaon pairs $(K^+K^-)_\phi$ would be the same. Here $\Gamma_{\rm tot}$ is the ϕ meson total width and Γ_{KK}, $\Gamma_{\mu\mu}$ are the ϕ partial decay widths in kaon and muon decay channels. Accordingly, in the experimental analysis, ϕ meson distribution is reconstructed from momentum distribution of decay products multiplied by the corresponding inverse branching ratio.

In this section we derive expressions for the apparent momentum distributions of ϕ mesons reconstructed via kaon and dimuon decay channels taking into account possible modifications of meson properties in medium and consequences of kaon rescattering.

Let us assume that the ϕ meson suffered the last interaction at position $\vec{x}_\phi$ inside the hadronic fireball. The probability that ϕ meson lives for a time t is

$$\mathrm{D}_\phi(t)=\exp\left[-\int_0^t \tilde{\Gamma}^*_{\rm tot}(t')dt'\right], \tag{3.2}$$

where $\tilde{\Gamma}_{\rm tot}=\Gamma_{\rm tot}m_\phi/E_\phi$ is the total width of a moving meson with energy $E_\phi=(m_\phi^2+p^2)^{1/2}$. The asterisk denotes in-medium values of the quantities.

After traveling for a given time t with velocity $\vec{v}_\phi=\vec{p}/E_\phi$, the ϕ meson decays into two kaons at the position $\vec{x}+\vec{v}_\phi t$ with a probability $\Gamma^*_{KK}(t)/\Gamma^*_{\rm tot}(t)$. In-medium values of widths Γ^* are determined by the current local temperature and density of the system. Daughter kaons from ϕ meson decay have momenta $\pm p_{KK}\vec{n}_K$ in the rest frame of the ϕ meson. The value of p_{KK} follows from the equation $m_\phi=\omega^*_K(p_{KK})+\omega^*_{\bar{K}}(p_{KK})$, where $\omega^*_K(p)$ and $\omega^*_{\bar{K}}(p)$ are in-medium spectra of kaons and antikaons. In the center-of-mass system (c.m.s.) of two colliding nuclei the kaon momenta are equal to

$$\vec{p}^{\,\pm}_K=\frac{\vec{p}_\phi}{2}\pm\delta\vec{p},$$
$$\delta\vec{p}=p_{KK}\{\vec{n}_K+\vec{n}_\phi(\gamma_\phi-1)(\vec{n}_K\cdot\vec{n}_\phi)\}, \tag{3.3}$$

where $\gamma_\phi=(1-v_\phi^2)^{-1/2}$. The unit vector $\vec{n}_K$ is uniformly distributed in the ϕ rest frame, the direction of $\vec{n}_\phi$ is determined by the ϕ meson momentum $\vec{p}_\phi$ in c.m.s.

For a successful identification of ϕ mesons in the invariant mass spectrum of observed K^+K^- pairs, it is essential that momenta of daughter kaons do not change while leaving the hadronic fireball. Here we investigate two mechanisms that may change momenta of daughter kaons: rescattering in surrounding hadronic environment and change of momentum due to in-medium K meson potential.

The probability that the secondary kaon and antikaon leave a fireball without rescattering is determined by their mean free paths $\lambda_{K,\bar{K}}$, the time they need to reach the fireball border $\tau^{\pm}_{K,\bar{K}}$, and their velocities $\vec{v}^{\,\pm}_{K,\bar{K}}=\vec{p}^{\,\pm}_K/\omega^*_{K,\bar{K}}$ (in c.m.s.) as follows:

$$P_\lambda(t)=\exp\left[-v_K^+\int_t^{\tau_K^++t}\frac{dt'}{\lambda_K(t')}-v_{\bar{K}}^-\int_t^{\tau_{\bar{K}}^-+t}\frac{dt'}{\lambda_{\bar{K}}(t')}\right]. \tag{3.4}$$

Thus, the probability to register a ϕ meson in the kaon channel when it decays in medium can be expressed as

$$P_{\rm I}[1-P_\lambda(t)]+P_{\rm II}P_\lambda(t), \tag{3.5}$$

where $P_{\rm I}$ and $P_{\rm II}$ are probabilities of identifying a ϕ meson from rescattered, $P_{\rm I}$, and nonrescattered, $P_{\rm II}$, kaons.

Single rescattering of one of the kaons changes the momentum of the kaon pair by the vector $\Delta\vec{P}_{\rm scat}$, with $|\Delta\vec{P}_{\rm scat}|\sim p_T$ being an average thermal momentum of pions.

Correspondingly the invariant mass changes by $\Delta M^2_{\rm rescat} \sim p_T^2$. At high temperatures $(T\sim m_\pi)$, the average thermal momentum of pions is $p_T \sim 400$ MeV and hence a single rescattering may shift $M_{K^+K^-}$ far from the ϕ meson mass. In this case we put $P_{\rm I}=0$.

Neglecting in-medium effects, one takes $P_{\rm II}=1$, assuming that without a hard rescattering, all kaon pairs from ϕ decays can be identified. However the modification of particle properties can provide another mechanism preventing ϕ identification. In medium, kaons feel an effective mean-field potential inducing a change of their spectra, $\omega^*_{K,\bar K}(p) - \sqrt{m_K^2+p^2}$. Leaving the fireball, kaons have to come back to their vacuum mass shell. This changes the invariant mass and momentum of the pair. Going out of medium, the kaon stays on the same energy level and its momentum outside the fireball becomes

$$\vec q^{\,\pm}_\alpha = \vec p^{\,\pm}_K \sqrt{\omega^*_\alpha(p_K^{\pm},t)^2 - m_K^2}/p_K^{\pm 2} \quad (\alpha = K, \bar K),$$

where the kaon energy ω^*_α is evaluated at the moment of ϕ decay. As a result, the momentum of the kaon pair is changed by

$$\Delta \vec P_{\rm pot} \approx \Delta\omega_K^{*2} \vec p^{\,+}_K / p_K^{+2} + \Delta \omega_{\bar K}^{*2} \vec p^{\,-}_K/p_K^{-2},$$

where we put $\Delta\omega^{*2}_\alpha \ll p_K^{\pm 2}$ and $\Delta\omega^{*2}_\alpha = \omega^{*2}_\alpha(0) - m_K^2$. The invariant mass shift is then given by $\Delta M^2_{\rm pot} = (\Delta \vec P_{\rm pot})^2$.

In the nucleon-free, isospin symmetrical meson gas, the in-medium spectra of kaons and antikaons are identical, $\omega_K(p) = \omega_{\bar K}(p)$ and the invariant mass shift reduces to

$$\Delta M^2_{\rm pot} = -\frac{\Delta\omega_K^{*2}}{2p_K^{+2}p_K^{-2}}\left[\frac{1}{2}p_\phi^4 + 2\,\delta p^2 p_\phi^2 - 2(\delta p p_\phi)^2\right].$$

This expression can be interpolated between the limit cases $p_\phi \gg p_{KK}$ and $p_\phi \ll p_{KK}$ as $\Delta M^2_{\rm pot} \approx -\Delta\omega_K^{*2} 4p_\phi^2/(p_\phi^2 + 6p_{KK}^2)$. Therefore in the symmetrical case, $\Delta\vec P_{\rm pot}$ and $\Delta M^2_{\rm pot}$ vanish for small p_ϕ. Moreover, in the isospin symmetrical case kaons suffer only a small mass modification ($\lesssim 30$ MeV for $T\sim m_\pi$). Hence, the release of kaons from a small potential well formed inside a fireball does not affect the momentum and invariant mass of a pair strongly enough to prevent a ϕ meson reconstruction. For this case we take $P_{\rm II} \approx 1$.

In the case of rather strong isospin asymmetry and/or significant baryonic admixture, K^+ and K^- mesons can have quite different spectra in medium [28]. This may lead to a wide spread of the kaon pair invariant masses, making the reconstruction of ϕ mesons decaying inside the fireball impossible $(P_{\rm II}\to 0)$. Note that in this case, our results are insensitive to the details of kaon propagation in medium. They depend only on the total ϕ meson width via the function $D_\phi(t)$.

Finally, the probability that a ϕ meson created at position x_ϕ will be detected via the kaon channel is

$$\int_0^\infty dt \tilde\Gamma^*_{KK}(t) {\rm D}_\phi(t) P_\lambda(t) P_{\rm II}$$
$$= \frac{\tilde\Gamma^0_{KK}}{\tilde\Gamma^0_{\rm tot}} {\rm D}_\phi(\tau_\phi) + \int_0^{\tau_\phi} dt \tilde\Gamma^*_{KK}(t){\rm D}_\phi(t) P_\lambda(t) P_{\rm II}, \tag{3.6}$$

where τ_ϕ is the time of flight of a ϕ meson through the fireball, for $t>\tau_\phi$ we have $P_\lambda = 1 = P_{\rm II}$ and ${\rm D}_\phi(t) = {\rm D}_\phi(\tau_\phi)\exp[-\Gamma_{\rm tot}(t-\tau_\phi)]$. Momentum distribution of the kaon pairs from ϕ decays is then obtained by averaging expression (3.6) over all $\vec n_K$ directions in the ϕ meson rest frame, and integrating over the fireball volume with a primary ϕ distribution—see Eq. (3.1). Then, multiplying by the inverse branching ratio we express "reconstructed" ϕ distribution in the kaon channel as

$$\eta_K(p) = \left\langle {\rm D}_\phi(\tau_\phi) + \frac{\tilde\Gamma^0_{\rm tot}}{\tilde\Gamma^0_{KK}}\int_0^{\tau_\phi} dt {\rm D}_\phi(t)\tilde\Gamma^*_{KK}(t)\overline{P_\lambda(t)}P_{\rm II}\right\rangle,$$
$$\overline{P_\lambda(t)} = \int \frac{d\Omega_{\vec n_K}}{4\pi} P_\lambda(t). \tag{3.7}$$

Here, the brackets denote integration,

$$\langle \ldots \rangle = \int_\Sigma d^3\sigma^\mu p_\mu f_\phi(\vec x_\phi, \vec p)(\ldots).$$

Note that according to this definition $\eta_0(p) = \langle 1 \rangle$.

Consideration of the dimuon pair momentum distribution is more straightforward. With a change of total ϕ width in medium, the branching ratio of the $\phi\to\mu\mu$ decay $\Gamma_{\mu\mu}/\Gamma^*_{\rm tot}$ changes too. Therefore, the apparent ϕ distribution reconstructed in the muonic channel is

$$\eta_\mu(p) = \left\langle {\rm D}_\phi(\tau_\phi) + \tilde\Gamma^0_{\rm tot}\int_0^{\tau_\phi} dt {\rm D}_\phi(t)\right\rangle. \tag{3.8}$$

Note that for $\Gamma^*_{\rm tot} = \Gamma^0_{\rm tot}$ we have $\eta_\mu \equiv \eta_0$, for $\Gamma^*_{\rm tot} > \Gamma^0_{\rm tot}$ we have $\eta_\mu < \eta_0$, and if $\Gamma^*_{\rm tot} < \Gamma^0_{\rm tot}$ then $\eta_\mu > \eta_0$.

In our numerical estimates we will consider the ratio of the reconstructed ϕ meson distributions, Eqs. (3.7) and (3.8), which does not depend on the overall normalization,

$$\mathcal{R}(p) = \eta_K(p)/\eta_\mu(p). \tag{3.9}$$

For a direct comparison with experimental results on m_T distributions of identified ϕ mesons, one has to integrate over the rapidity interval accessible to the experiments, utilizing $p = \sqrt{m_T^2\cosh^2 y - m_\phi^2}$. The dependence of the ratio (3.9) on m_T reads

$$\mathcal{R}(m_T) = \langle \eta_K(p)\rangle_y / \langle \eta_\mu(p)\rangle_y. \tag{3.10}$$

We also define the ratio of the apparent and primary ϕ momentum distributions for K^+K^- and $\mu^+\mu^-$ decay channels as a function of m_T,

$$\mathcal{R}_{K,\mu}(m_T) = \langle \eta_{K,\mu}(p)\rangle_y / \langle \eta_0(p)\rangle_y.$$

IV. Φ DECAYS IN MEDIUM

Main hadronic decay channels of ϕ meson are $\phi\to K\bar{K}$ and $\phi\to\rho\pi$. Let us now consider the change of a ϕ decay width in a hot meson gas due to the modification of kaon, pion, and ρ-meson properties. In-medium properties of pions can be effectively incorporated by a small mass shift $m_\pi^*=m_\pi+\delta m_\pi$ with $\delta m_\pi \ll m_\pi$. Similar behavior is expected for kaons.

The spectral function of ρ meson in medium was extensively investigated in the context of dilepton production in heavy-ion collisions at SPS energies [29–32]. In baryonic matter, coupling of ρ mesons to resonance–nucleon-hole modes [29], together with the modification of pions [30], plays a dominant role. The ρ meson becomes very broad in medium and its spectral density strength is driven effectively to lower energies, in analogy to the Brown-Rho scaling picture [31]. In purely mesonic systems the ρ mass is found to be almost independent of the temperature due to the cancellation of $\pi-\pi$- and $\pi-a_1$-loop contributions [32]. The ρ width, on the other hand, is expected to increase in meson gas considerably, e.g., by 80 MeV at $T=150$ MeV and by 160 MeV at $T=180$ MeV [33].

The partial width of the $\phi\to K\bar{K}$ decay in medium depends on the kaon mass m_K^* as

$$\Gamma_{KK}^*=\Gamma_{KK}^0 p_{\rm c.m.}^3(m_\phi^2,m_K^*,m_K^*)/p_{\rm c.m.}^3(m_\phi^2,m_K,m_K), \tag{4.1}$$

where $p_{\rm c.m.}(s,m_1,m_2)$ is a kaon momentum in the rest frame of ϕ meson decay, obeying the equation $\sqrt{s}=\sqrt{m_1^2+p_{\rm c.m.}^2}+\sqrt{m_2^2+p_{\rm c.m.}^2}$. Assuming isospin symmetry we have $m_K=(m_{K^+}+m_{K^0})/2=495.6$ MeV. Then vacuum width is equal to $\Gamma_{KK}^0=\{\Gamma_{K^+K^-}+\Gamma_{K^0\bar{K}^0}\}/2=1.84$ MeV. Note that for $\delta m_K>0$ the in-medium width Γ_{KK}^* decreases fast and vanishes for $\delta m_K\approx 14$ MeV. The second hadronic decay channel $\phi\to\rho\pi$ is effected by the decrease of the ρ meson mass, increase of the ρ meson width, and the Bose-Einstein enhancement factor for pions. For the ϕ meson at rest we write

$$\Gamma_{\rho\pi}^*=\Gamma_{\rho\pi}^0\kappa_{\rho\pi}(m_\rho^*,\Gamma_\rho^*,T)/\kappa_{\rho\pi}(m_\rho^0,\Gamma_\rho^0,0), \tag{4.2}$$

and

$$\kappa_{\rho\pi}(m_\rho,\Gamma,T)=\int_{2m_\pi}^{m_\phi-m_\pi}\frac{d\omega}{\pi}[(\omega-m_\phi)^2-m_\pi^2]^{3/2}\times\frac{[1+n_\pi(\omega)]\omega\Gamma/(2m_\phi)}{[\omega-E_\rho(m_\rho)]^2+\omega^2\Gamma^2/(4m_\phi^2)}, \tag{4.3}$$

where m_ρ^* and Γ_ρ^* are in-medium ρ meson mass and width, respectively. Vacuum values are $m_\rho^0=770$ MeV, $\Gamma_\rho^0=150$ MeV, and $\Gamma_{\rho\pi}^0=0.75$ MeV. We use the constant width approximation for the ρ meson spectral density and denote $E_\rho(m_\rho)=(m_\phi^2+m_\rho^2-m_\pi^2)/2m_\phi$. The Bose-Einstein distribution of pions with temperature T is $n_\pi(\omega)$. In the zero-width limit we obviously have $\kappa_{\rho\pi}(\Gamma\to 0)=p_{\rm c.m.}^3(m_\phi^2,m_\rho,m_\pi)[1+n_\pi(m_\phi-E_\rho)]$. Numerical evaluation of Eq. (4.3) leads to the approximated relation

$$\Gamma_{\rho\pi}^*\approx\Gamma_{\rho\pi}^0\left(1-0.91\frac{\delta m_\rho}{100\ \text{MeV}}+0.25\frac{\delta\Gamma_\rho}{100\ \text{MeV}}+0.07\left[\frac{T}{100\ \text{MeV}}-1\right]\right)$$

valid for $\delta m_\rho=m_\rho^*-m_\rho\lesssim 200$ MeV, $\delta\Gamma_\rho=\Gamma_\rho^*-\Gamma_\rho^0\lesssim 200$ MeV, and 100 Mev$<T\lesssim 200$ MeV. The shift of a pion mass produces a minor effect and it is, therefore, neglected here. Finally, the total width of ϕ is given by $\Gamma_{\rm tot}^*=2\Gamma_{KK}^*+\Gamma_{\rho\pi}^*$. Here and below we do not consider the ϕ meson mass shift, which is small and cancels as soon as we consider the ratio of the momentum spectra, cf. Eqs. (3.9) and (3.10).

For completeness, we remark that the dilepton decay channel $\phi\to l^+l^-$ also suffers a modification in medium, because the vector-meson–photon coupling is suppressed by meson fluctuations [34,35]. For the ϕ-γ coupling, the suppression factor is determined only by kaon fluctuations $\chi_{\phi\gamma}=(1-2\langle|K|^2\rangle_T/f_\pi^2)$. Here $\langle|K|^2\rangle_T=\int d^3k\exp[-\omega_K(k)/T]\times[(2\pi)^3 2\omega_K(k)]^{-1}$, $\omega_K(k)=\sqrt{m_K^2+k^2}$, and $f_\pi=93$ MeV is the pion decay constant. Because of the large kaon mass this correction is negligibly small: $\langle|K|^2\rangle/f_\pi^2\sim 1\%$.

Finally, we specify how the in-medium mass and width of kaons and ρ mesons relax to their vacuum values during the fireball expansion. Assuming that $\delta m_{K,\rho}$ and $\delta\Gamma_\rho$ are proportional to the density of the system, we have

$$\delta m_{K,\rho}(t)=\delta m_{K,\rho}^0\frac{R_0^3}{R^3(t)},\qquad \delta\Gamma_\rho(t)=\delta\Gamma_\rho^0\frac{R_0^3}{R^3(t)},$$

where $\delta m_{K,\rho}^0$ and $\delta\Gamma_\rho^0$ are input parameters.

Before finishing this section we would like to point out that the estimations made here are valid only for an almost baryon-free fireball. In the presence of baryons the calculation of ϕ self-energy becomes more elaborate, cf. Ref. [7]. However, in this case the spectra of kaons and antikaons differ dramatically, inhibiting thereby the ϕ reconstruction even more. It corresponds to the case $P_{\rm II}=0$ when the reconstructed ϕ distributions (3.7) and (3.8) are the functions of a total width only. To simulate effectively the in-medium modification of $\Gamma_{\rm tot}^*$, we will use Eq. (4.1) and vary the kaon mass.

V. SPACE-TIME EVOLUTION OF THE FIREBALL

After the considerations above, let us now specify the model of the fireball expansion, which we will use in our numerical calculations. Assume a simple homogeneous spherical fireball with the constant density and temperature profiles. The ϕ meson momentum distribution

$$f_\phi(\vec{x},\vec{p})=\exp\left[-\frac{E_\phi-\vec{p}\cdot\vec{u}(\vec{x})}{T_0\sqrt{1-u^2(\vec{x})}}\right] \tag{5.1}$$

is determined by the temperature T_0, flow velocity profile $\vec{u}(\vec{x})=v_f\vec{x}/R_0$, and radius R_0. Time of flight of a ϕ meson and its daughter kaons through the medium contained in Eqs. (3.4) and (3.7) can be expressed as $\tau_\phi=\tau_R(\vec{v}_\phi,\vec{x}_\phi)$ and $\tau_K^\pm=\tau_{R+v_ft}(\vec{v}_K^\pm,\vec{x}_\phi+\vec{v}_\phi t)$, where $\tau_R(\vec{v},\vec{x})$ stands for a time, during which a particle with velocity $\vec{v}$ passes a distance from position $\vec{x}$ to the border of a sphere with the radius R. Since the fireball is expanding with radial velocity v_f, this time satisfies the equation $(\vec{x}+\vec{v}\tau)^2=(R+v_f\tau)^2$. This implies

$$\tau_R(\vec{v},\vec{x})=[\sqrt{(\vec{v}\cdot\vec{x}-v_fR)^2+(R^2-\vec{x}^2)(\vec{v}^2-v_f^2)}-(\vec{v}\cdot\vec{x}-v_fR)](\vec{v}^2-v_f^2)^{-1}. \tag{5.2}$$

Solution (5.2) is valid for $|\vec{x}|<R$ and $|\vec{v}|>v_f$. In the case $|\vec{v}|<v_f$ we put $\tau=\infty$.

During the expansion $R(t)=R_0+v_ft$, the fireball density drops as $\rho(t)=\rho_0R_0^3/R^3(t)$ and the temperature decreases as $T(t)=T_0R_0/R(t)$ as expected for the relativistic pion gas, cf. Ref. [36]. The kaon mean free path is $\lambda_K\propto 1/\rho$ and therefore

$$\lambda_K(t)=\lambda_K^0R^3(t)/R_0^3.$$

To incorporate the freeze-out effect we will assume that as soon as $T(t)\leqslant T_{\rm therm}$, kaons become free and $\lambda_K\rightarrow\infty$ as well as $P_{\rm II}\rightarrow 1$. The freeze-out (FO) time is then given by

$$\tau_{\rm FO}=\frac{R_0}{v_f}\left(\frac{T_0}{T_{\rm therm}}-1\right). \tag{5.3}$$

Parameters R^0, T^0, v_f, and λ_K^0 serve as input for numerical evaluations below. The parameter $T_{\rm therm}$ is used for an effective parametrization of the freeze-out time $\tau_{\rm FO}$. It should not be considered as a true freeze-out temperature, since our estimation is based on the simplified hydrodynamical description of a fireball.

The model set up here is a rather crude approximation. However, the final results are found to be rather insensitive to the details of hydrodynamical evolution of a fireball, being determined mainly by the values of $\Gamma^*_{\rm tot}R_0$, $\Gamma^*_{\rm tot}\tau_{\rm FO}$, and v_f. We shall vary the input parameters within a broad range to illustrate different possibilities.

VI. NUMERICAL ESTIMATES

In this section we perform numerical evaluation of our expressions for ϕ meson yields reconstructed via K^+K^- and $\mu^+\mu^-$ channels in central Pb+Pb collisions at 158 GeV/nucleon SPS energy.

First we investigate to what extent the rescattering of secondary kaons, enhanced by the in-medium modification of a ϕ meson width, can suppress the experimentally observed yield of ϕ mesons identified via K^+K^- channel.

We recall that results of Ref. [27] give a maximal suppression factor of 40% for the ϕ meson observation in the kaon decay channel.

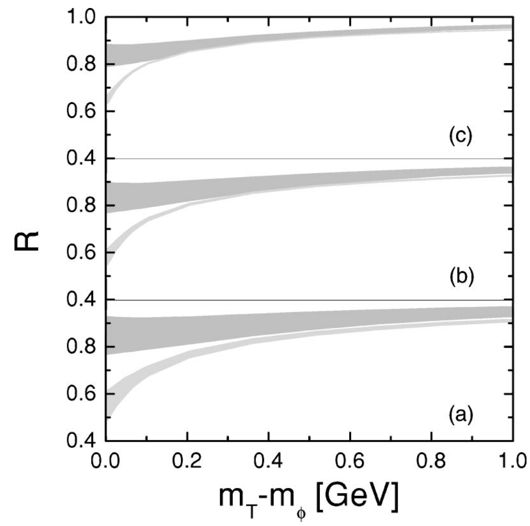

FIG. 1. Ratio (3.10) as a function of m_T calculated for three sets of parameters (T_0,v_f,R_0) without inclusion of in-medium modifications of kaons and ρ mesons. The upper gray area corresponds to variations of kaon mean free paths λ_K and freeze-out temperature $T_{\rm therm}$ as described in the text. The lower gray areas are obtained for the same set of parameters but with the common expansion velocity $v_f=0.1$.

In our evaluation we use several combinations of input parameters. Freeze-out temperatures T_0 of ϕ mesons distributed according to Eq. (5.1) vary between $T_{\rm chem}$ and $T_{\rm therm}$: (a) $T_0=150$ MeV, (b) $T_0=160$ MeV, (c) $T_0=170$ MeV. Size of the fireball R_0 at the stage of the ϕ freeze-out has to be comparable with ϕ meson mean free path λ_ϕ at the given temperature, $R_0\approx\alpha_R\lambda_\phi$ with $\alpha_R\sim 1$. For the different temperature parameters above we take, according to Ref. [4]; $\lambda_\phi^{(a)}=13$ fm, $\lambda_\phi^{(b)}=10$ fm, and $\lambda_\phi^{(c)}=7$ fm.

First we evaluate suppression factor (3.10) without any modifications of particle properties in medium, i.e., $\delta m_K^0=\delta\Gamma_\rho^0=0$. In this case we have $\mathcal{R}_\mu(m_T)\equiv 1$ and $\mathcal{R}(m_T)=\mathcal{R}_K(m_T)$. Flow velocities corresponding to selected temperatures T_0 are adjusted to reproduce the slope of the ϕ meson m_T distribution measured by the NA50 collaboration: $T_{\rm eff}=218$ MeV; $v_f^{(a)}=0.50$, $v_f^{(b)}=0.46$, $v_f^{(c)}=0.41$. We take $\alpha_R=1$ and vary the mean free path of kaons λ_K within the interval $0<\lambda_K^0<\lambda_K(T_0)$, where $\lambda_K(T_0)$ follows from estimations of Ref. [25]: $\lambda_K^{(a)}=2$ fm, $\lambda_K^{(b)}=1$ fm, and $\lambda_K^{(c)}=0.5$ fm. We vary also $T_{\rm therm}$ between 100 MeV and 80 MeV in agreement with analysis [20]. This translates into the interval of freeze-out time values 10 fm$<\tau_{\rm FO}<$20 fm. All these variations produce the upper gray areas shown in Fig. 1.

For all three sets of parameters (T_0,v_f,R_0) we observe that $\mathcal{R}(m_T)$ does not fall below 0.8 significantly. This is related to the large expansion velocity of the fireball. In this case $0.2<\Gamma_{\rm tot}^0\tau_{\rm FO}<0.4$ and ϕ mesons decay after the thermal freeze-out. To illustrate this effect we recalculate ratio $\mathcal{R}(m_T)$ for the same three cases fixing $v_f=0.1$, which corresponds to $\Gamma_{\rm tot}^0\tau_{\rm FO}\sim 1$. Results obtained ($\mathcal{R}\sim 0.6$) are shown as lower gray areas in Fig. 1.

To reproduce results of RQMD calculations described in

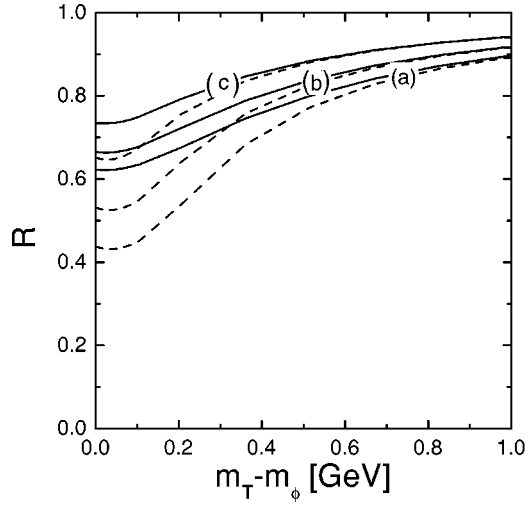

FIG. 2. Ratio (3.10) calculated for curves a–c with $\alpha_R=1.5$, $\lambda_\phi=13$ fm, and $\lambda_K^0=0.5$ fm. Solid lines are calculated with $T_{\rm therm}=80$ MeV and dash lines correspond to $T_{\rm therm}=40$ MeV. No medium effects are included.

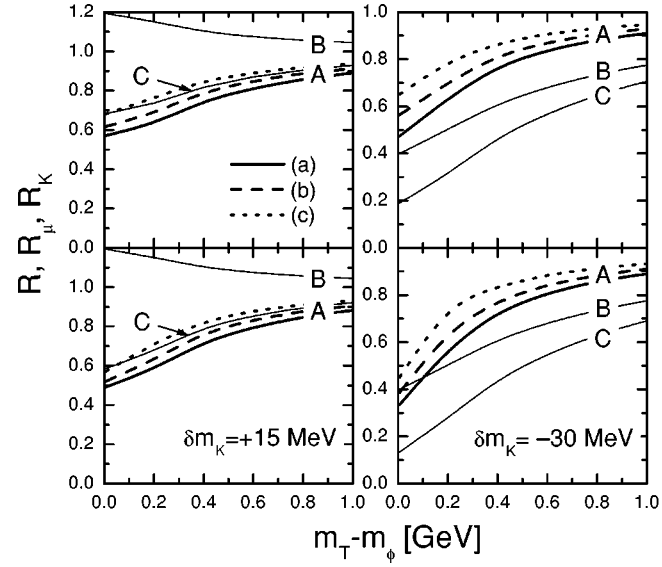

FIG. 3. Thick lines (label A) show the ratio $\mathcal{R}(m_T)$ calculated for in-medium modification of ϕ meson properties. Results for different parameters sets (a)–(c) are depicted by solid, dashed, and dotted lines, respectively. The left plane corresponds to the case when the kaon mass increases in medium, $\delta m_K^0=15$ MeV, whereas the right plane shows results for a decreasing kaon mass, $\delta m_K^0=-30$ MeV. Thin lines show suppression factors in the muon $\mathcal{R}_\mu$ (B) and kaon $\mathcal{R}_K$ (C) channels calculated for parameter set a. Upper plots are calculated for the case $P_{\rm II}=1$, lower plots correspond to $P_{\rm II}=0$. In all cases, modification of ρ meson properties in medium is taken into account.

Ref. [27] we take a somewhat larger size of a fireball with $\alpha_R=1.5$, freeze-out temperature $T_{\rm therm}=80$ MeV, and $\lambda_K^0=0.5$ fm. This corresponds to the lowest limit allowed by the analysis [20]. The results are shown in Fig. 2 by solid lines. The limiting scenario considered in [27], when the freeze-out volume is determined by the last kaon interactions, can be reproduced with $T_{\rm therm}=40$ MeV. This case is shown by dashed lines in Fig. 2. We take solid lines in Fig. 2 as a reference point for our further investigation of in-medium effects.

First we consider modifications of ρ meson properties. We choose ρ mass shift to be $\delta m_\rho^0=-200$ MeV for our three parameter sets. The ρ meson width depends on the temperature. Relying on Ref. [33], we take $\delta\Gamma_\rho^{(a)}=80$ MeV, $\delta\Gamma_\rho^{(b)}=180$ MeV, and $\delta\Gamma_\rho^{(c)}=200$ MeV. Comparison with Fig. 2 (solid lines) shows a slight decrease of ratio $\mathcal{R}(m_T)$, which corresponds to a small increase of the total ϕ meson width by 40% due to $\phi\to\rho\pi$ channel.

Figure 3 shows results obtained, taking into account the modification of K meson properties in medium.

First we investigate the case when $\phi\to K\bar{K}$ channel is closed initially ($\delta m_K^0=15$ MeV) and it opens only during the fireball expansion. Results are shown in the left part of Fig. 3, where the upper plot is calculated for $P_{\rm II}=1$ and the lower one corresponds to a strong suppression of the kaon channel in medium with $P_{\rm II}=0$. Comparing ratios $\mathcal{R}(m_T)$ in Fig. 2 and in Fig. 3 shown by a thick solid line A calculated for parameter set a, we observe that kinematical quenching of $\phi\to K^+K^-$ channel by increasing kaon mass, decreases ratio $\mathcal{R}(m_T)$ very slightly for both values of $P_{\rm II}$. This happens because the ϕ meson width becomes very small in this case and therefore the probability of ϕ meson decay inside the expanding fireball and consequently also the probability for rescattering of daughter kaons is small. Compare lines C, calculated for parameter set a, with a corresponding line in Fig. 2. Lines B in Fig. 3 (left side) show that $\mathcal{R}_\mu$ becomes larger than 1 for increasing kaon mass. Since increase of $\mathcal{R}_\mu$ by 10–20 % does not change considerably the effective m_T slope of $\langle\eta_\mu\rangle_y$ distribution, we do not need to readjust the flow velocity parameter. This increase of $\mathcal{R}_\mu$ leads in the end to a small decrease of $\mathcal{R}$.

Let us now consider the case when the ϕ meson width increases strongly in hadronic medium due to the increase of Γ^*_{KK}. We simulate this effect by the decrease of the kaon mass in medium, which can result, e.g., from rescattering of kaons on pions through K^* and heavier kaonic resonances [37]. Here we restrict ourselves to a rather conservative modification of kaon masses -30 MeV$<\delta m_K^0<0$, which corresponds to the ϕ width 4 MeV$\lesssim\Gamma^*_{\rm tot}\lesssim 20$ MeV. In this case $\mathcal{R}_\mu<1$ and at small m_T-m_ϕ region $\mathcal{R}_\mu$ can be suppressed by up to 40–60 %. Thus, for a given freeze-out temperature T_0 and total width $\Gamma^*_{\rm tot}$ we readjust flow velocity v_f^0 to reproduce the slope of the m_T distribution measured in the dimuon channel by NA50 [9]. For our three sets of parameters (T_0,R_0) we obtain new flow velocities: (a) $v_f^0=0.38$, (b) $v_f^0=0.35$, (c) $v_f^0=0.28$.

The right part of Fig. 3 shows our results obtained for $\delta m_K^0=-30$ MeV. The corresponding partial width is $\Gamma^*_{KK}\approx 10$ MeV and the total width is $\Gamma^*_{\rm tot}\approx 21$ MeV. This leads to $\Gamma^*_{\rm tot}R_0\sim\Gamma^*_{\rm tot}\tau_{\rm FO}\sim 2$, which provides a strong suppression of R_K as is shown for the parameter set a in Fig. 3 (right side, lines C). We find $\mathcal{R}_K(m_T\to m_\phi)\sim 0.2$ for $P_{\rm II}=1$ and ~ 0.15 for $P_{\rm II}=0$. However, since the ratio $\mathcal{R}_\mu(m_T)$ also shown in

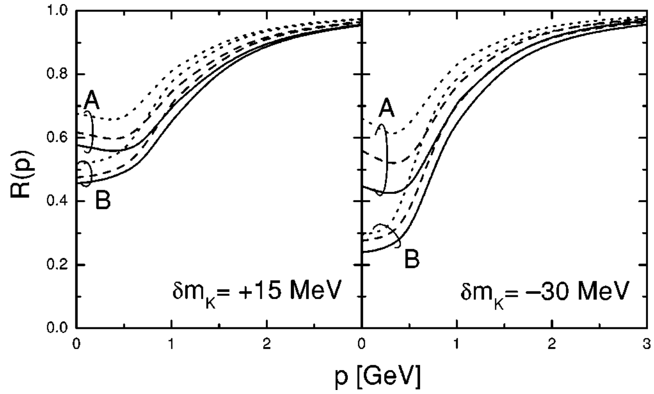

FIG. 4. The ratio (3.9) as a function of ϕ meson momentum. Curve styles correspond to those in Fig. 3 with readjusted flow velocities. Curves A are calculated for $P_{\rm I}=1$, whereas sets B are calculated with $P_{\rm I}=0$.

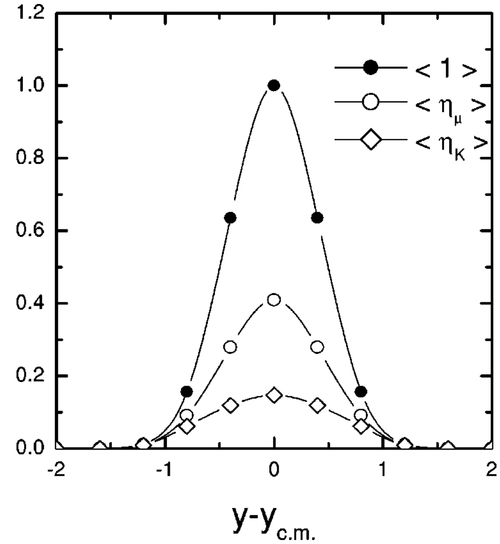

FIG. 5. Rapidity distributions $\langle 1\rangle(y)$ (full circles), $\eta_\mu(y)$ (open circles), and $\eta_K(y)$ (open diamonds) calculated for the parameter set a with $\delta m_K^0=-30$ MeV, flow velocity $v_f^0=0.38$, and $P_{\rm I}=0$.

Fig. 3 (lines B) is also suppressed, the resulting ratio $\mathcal{R}(m_T)$ remains on the level ~ 0.3 for small m_T-m_ϕ, provided we put $P_{\rm II}=0$ and ~ 0.5 for $P_{\rm II}=1$. Taking even larger values of the total decay width in the most preferable case a we obtain $\mathcal{R}(m_T\rightarrow m_\phi)\approx 0.28$ for $\Gamma^*_{\rm tot}=27$ MeV, $v_f^0=0.35$, and $\mathcal{R}(m_T\rightarrow m_\phi)\approx 0.23$ for $\Gamma^*_{\rm tot}=34$ MeV, $v_f^0=0.32$ (both for $P_{\rm II}=0$).

It is instructive to investigate ratio (3.9) as a function of the ϕ momentum. Averaging over rapidity mixes momenta within a broad interval, e.g., for $m_T=0$ momenta $100\lesssim p \lesssim 1000$ MeV, which partially washes out the final suppression effect. To illustrate this point we show ratio $\mathcal{R}(p)$ as a function of ϕ momentum in Fig. 4. For increasing kaon mass and correspondingly vanishing Γ_{KK} we obtain a decrease of $\mathcal{R}(p)$ at small momenta by 40–50 % (left plane). This is again a direct consequence of a rapid fireball expansion, which brings $\Gamma_{KK}(t)$ in the integral (3.7) quickly to its vacuum value. In the extreme case, for $p\rightarrow 0$ and $v_f\rightarrow 0$ we get $\mathcal{R}(p,v_f)\rightarrow 0$. For decreasing kaon mass the reduction is even stronger $\sim 70\%$. Momentum dependence of $\mathcal{R}$ at small momenta $p\lesssim p_f=m_\phi v_f\gamma_\phi$ is completely washed out by rapidity averaging.

In Fig. 5 we compare the original rapidity distribution of ϕ mesons $\eta_0(y)$ with the distributions that can be reconstructed via kaonic $\eta_\mu(y)$ and muonic $\eta_K(y)$ decay channels. Calculations are done for the parameter set a with $\delta m_K^0=-30$ MeV, flow velocity $v_f^0=0.38$, and $P_{\rm II}=0$. Distributions are normalized to give $\eta_0(y=0)=1$. We observe a considerable broadening of the rapidity distribution measured in the kaon channel.

VII. CONCLUSIONS

We have studied distributions of ϕ mesons in heavy-ion collisions at SPS energies reconstructed via hadronic K^+K^- and dilepton l^+l^- decay channels. The analysis of ϕ meson mean free path allows one to suppose that ϕ mesons decouple from the hadronic system at a somewhat earlier stage before the common breakup of the hadronic fireball. Therefore, kaon pairs originated from the ϕ decays inside a fireball can be rescattered or absorbed. Such kaon pairs will not contribute to a ϕ meson reconstruction, whereas the leptonic probes can leave a fireball freely. We derive the expressions (3.7) and (3.8) for the apparent momentum distribution of ϕ mesons in kaonic and muonic channels, respectively.

Within a simple model of a spherically expanding fireball, we investigate the dependence of a relative suppression factor of the hadronic channel with respect to the dileptonic one on parameters of the system and on the ϕ meson in-medium properties. For a vacuum ϕ meson width ~ 4 MeV the maximal suppression 0.6–0.8 is obtained for the fireball size and expansion time $R_0\sim\tau_{\rm FO}\sim 20$ fm. These values are in agreement with results of RQMD simulations [27]. The crucial parameter is the fast expansion of a fireball with $v_f \sim 0.4$–0.5 corresponding to the ϕ freeze-out temperature range $T_0\sim 150$–170 MeV.

Width of hadronic ϕ meson decay channels ($\phi\rightarrow K\bar{K}$ and $\phi\rightarrow\pi\rho$) can be modified in medium due to changes of the meson properties. We have found that increase of the $\pi\rho$ channel width due to the broadening of the ρ meson and decrease of the ρ meson mass leads alone to a tiny increase of the suppression.

The other possibility is the kinematical quenching of the kaon decay channel, which we simulate by simultaneous increase of K^+ and K^- masses. Since total width $\Gamma^*_{\rm tot}$ of the ϕ meson in medium becomes small (increase of the $\pi\rho$ channel width is not strong enough) ϕ mesons decay mainly outside the fireball, where vacuum properties of ϕ mesons are restored and rescattering of daughter kaons is negligible. Together with relative amplification of the muon decay channel by 20% the resulting suppression factor found for a quenched kaon decay channel was ~ 0.5.

The increase of the ϕ meson width in medium provides, on the other hand, a mechanism for strong suppression (~ 0.15) of the kaonic detection channel due to the enhancement of the ϕ decay probability inside a fireball, increasing thus the rescattering of daughter kaons. However, increase of the ϕ total width reduces simultaneously the branching ratio of the $\phi \rightarrow \mu^+\mu^-$ decay and suppresses the spectrum of ϕ mesons reconstructed via $\mu^+\mu^-$ decay channel. Obtained suppression of the muonic decay channel at the level $\sim$40–60 % requires a readjustment of the flow velocity to be compatible with the experimental slope of NA50. Adjusted flow velocity $v_f \sim 0.3$–0.4 for $\Gamma^*_{\rm tot} \sim 20$ MeV and $T_0 \sim 150$–170 MeV (compare to $v_f \sim 0.4$–0.5 obtained for a vacuum ϕ width) gives final net relative suppression factor of kaon channel to muon channel $\sim$0.3. This value is close to experimental observations at CERN SPS [8,9].

A strong increase of the ϕ meson in-medium width can take place if the kaon mass decreases in medium by 30 MeV. The mechanism for such kaon mass modification can be similar to that studied in Ref. [37].

We have found that reconstructed rapidity distributions of ϕ mesons become effectively wider, if in-medium properties of mesons and rescattering of kaons are taken into account.

Finally we suppose that to improve the understanding of experimental results on ϕ meson production in heavy-ion collisions at CERN SPS [8,9], further detailed investigations taking into account in-medium effects within transport or hydrodynamical models are necessary.

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