Analysis of the $^{12}C+^{24}Mg$ reaction using a new coupling potential

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We introduce a new coupling potential to explain the experimental data for the $^{12}C+^{24}Mg$ system at numerous energies in a laboratory system from 16.0 MeV to 24.0 MeV. This new coupled-channels based approach involves replacing the usual first derivative coupling potential by a new, second-derivative coupling potential. This paper first shows and discusses the limitation of the standard coupled-channels theory in the case where one of the nuclei in the reaction is strongly deformed. Then, this new approach is shown to improve consistently the agreement with the experimental data and has made major improvement on all the previous coupled-channels calculations for this system.

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I. INTRODUCTION

The elastic and inelastic scattering of the light heavy-ion reactions, such as $^{16}O+^{28}Si$, $^{12}C+^{24}Mg$, and $^{12}C+^{12}C$ have been extensively investigated over the last 40 years and a large body of experimental data has been accumulated from the systematic studies of these reactions (see [1–3] and references therein). A variety of theoretical accounts, based on dynamical models or purely phenomenological treatments, have been proposed to explain the experimental data [1,4,5]. However, there appears no unique model that explains consistently the elastic and inelastic scattering data over wide energy ranges without applying any *ad-hoc* procedures.

Consequently, the following problems continue to exist for the light heavy-ion reactions [6,7]: (1) explanation of anomalous large angle scattering data; (2) reproduction of the oscillatory structure near the Coulomb barrier; (3) the out-of-phase problem between theoretical predictions and experimental data; (4) the deformation parameters (β values)—previous calculations require β values that are at variance with the empirical values and are physically unjustifiable.

The elastic and inelastic scattering data of the $^{12}C+^{24}Mg$ system have been studied extensively and some of the above-mentioned problems could not be accounted for [2,8–10]. The most extensive study for this system was carried out by Sciani *et al.* [2] who used *Q*-dependent potentials whose parameters had different values for the incoming and outgoing channels in the coupled-channels calculations. Without *Q*-dependent potentials, they observed that the theoretical calculations and the experimental data were completely out of phase and could not reproduce the experimental data. However, they overcame this problem by introducing these *Q*-dependent potentials. Nevertheless, not only were the parameters changing from energy to energy in an arbitrary way, but they also had to change the β value in order to optimize the fits.

It has been the practice to increase or decrease artificially the β value to obtain the magnitude of the 2^+ state data correctly in the standard coupled-channels calculations, without giving the physical justification other than stating that it is required to fit the data [2,8,9,12–14].

The out of phase between the theoretical predictions and the experimental data for the ground and 2^+ states has also been observed and without optimizing the β value, it has been impossible to obtain a simultaneous fit to the elastic and inelastic scattering data [2,11–14].

Therefore, building on a previous paper [6], which was outstandingly successful in explaining the experimental data for the $^{12}C+^{12}C$ system, we investigate the $^{12}C+^{24}Mg$ reaction that has been intensively investigated experimentally at energies near the Coulomb barrier [2,8–10]. The main feature of the experimental data is a strong oscillatory structure that cannot be explained in a wide energy range within the coupled-channels and distorted-wave Born approximation methods if any *ad hoc* procedures are not applied. In this paper, our aim is to explain the elastic and inelastic scattering data with empirical β value.

In the following section, we first introduce the standard coupled-channels model and show the results of these analyses in Sec. III for $E_{lab}=16.0$ MeV–24.0 MeV. Then, in Sec. IV, we introduce a new coupling potential to analyze the experimental data in the same energy range and show the results of these new coupled-channels calculations. Finally, Sec. VI is devoted to our summary and conclusion.

II. THE STANDARD COUPLED-CHANNELS CALCULATIONS

The interaction between ^{12}C and ^{24}Mg nuclei is described by a deformed optical potential. As shown in Fig. 1, the real potential is chosen as the square of a Woods-Saxon shape

$$V_N(r)=\frac{-V_0}{[1+\exp(r-R)/a]^2} \tag{1}$$

and the parameters are fixed as a function of energy to reproduce the experimental data over the whole energy range. The numerical values are shown in Table I.

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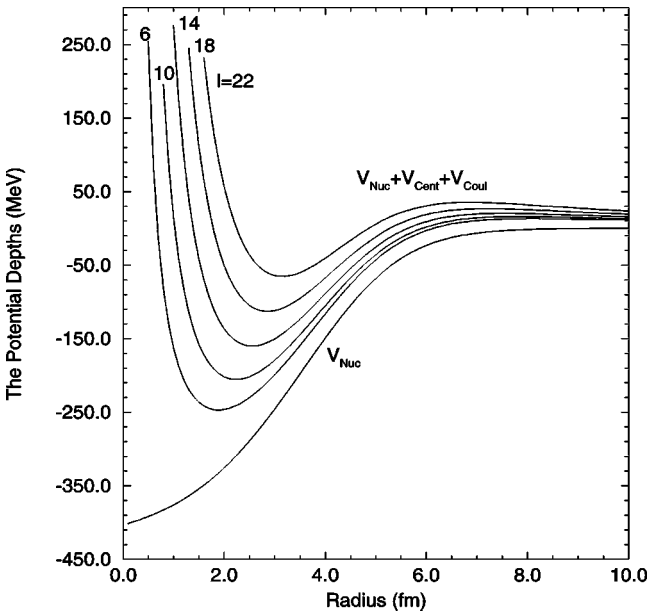

FIG. 1. Interaction potential between ^{12}C and ^{24}Mg is plotted for various values of the orbital angular momentum quantum number *l*.

The sum of the nuclear, Coulomb, and the centrifugal potentials is also shown in the same figure for various values of the orbital angular momentum quantum number *l*. The superposition of the attractive and repulsive potentials results in the formation of a potential pocket—the width and depth of the pocket depend on the orbital angular momentum. This pocket is very important for the interference of the barrier and internal waves, which produces the pronounced structure in the cross section. The effect of this pocket can be understood in terms of the interference between the internal and barrier waves that correspond to a decomposition of the scattering amplitude into two components, the inner and external waves [15,16].

The imaginary potential has the standard Woods-Saxon volume shape as in Eq. (2) and the depth increases linearly with energy as in Eq. (3),

$$W(r) = -\frac{W_0}{\{1+\exp[(r-R)/a]\}}, \tag{2}$$

$$W = 4.375E_{lab} - 67.0. \tag{3}$$

The other parameters of the real and imaginary potentials are fixed as a function of the energy and are not changed in the present calculations as shown in Table I.

TABLE I. The parameters of the real and imaginary potentials. *V* and *W* stand for the strengths of the real and imaginary parts respectively. R_c is the Coulomb radius.

V (MeV)	R_V (fm)	a_V (fm)	W (MeV)	R_W (fm)	a_W (fm)	R_c (fm)
427.0	4.486	1.187	Eq. (3)	1.386	0.286	5.174

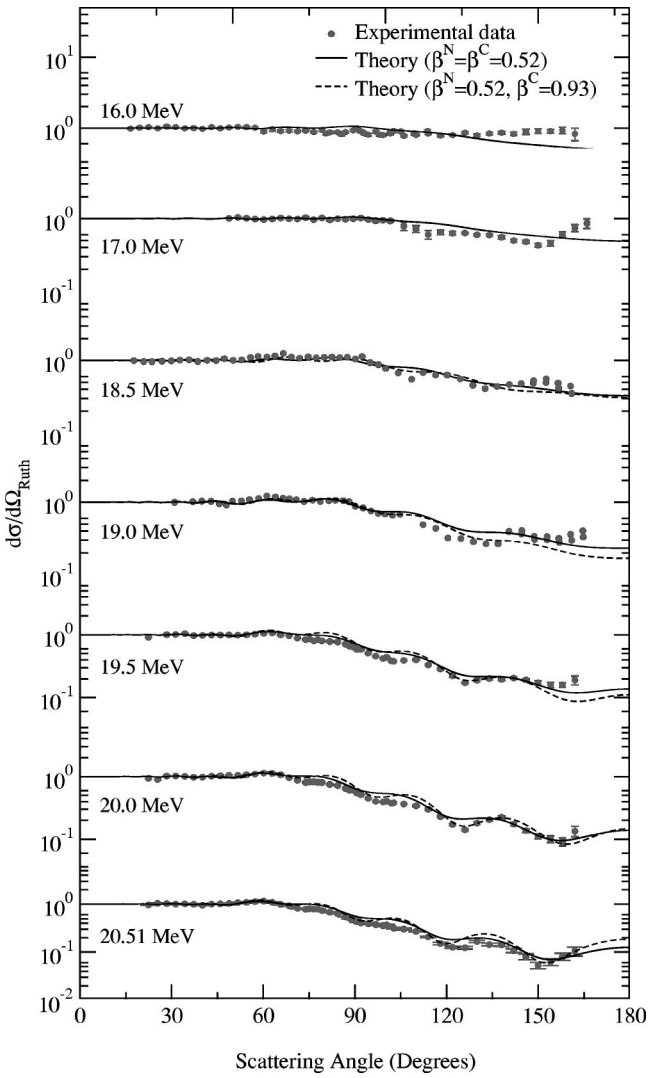

FIG. 2. Ground state results of the standard coupled-channels calculations with $\beta_2^N=\beta_2^C=0.52$ (solid lines) and with $\beta_2^N=0.52$, $\beta_2^C=0.93$ (dashed lines).

It is assumed that the target nucleus ^{24}Mg has a static quadrupole deformation and this assumption is taken into account by deforming the real potential in the following way:

$$R(\theta,\phi) = r_0 A_P^{1/3} + r_0 A_T^{1/3}[1+\beta_2 Y_{20}(\theta,\phi)], \tag{4}$$

where *P* and *T* refer to projectile and target nuclei, respectively, and β_2 is the deformation parameter of ^{24}Mg. In the present calculations, only target nucleus ^{24}Mg is deformed, although it is well known that the projectile ^{12}C is also a strongly deformed nucleus. However, when it is deformed, the number of channels increases and it makes the computational processing time insurmountable.

In our coupled-channels calculations, we shall use the exact value of β_2, derived from the deformation length δ. The invariant parameter in the coupled-channels formalism is in fact the deformation length, $\delta=\beta$R, or its value derived from the reduced electromagnetic transition probability $B(E2)$ rather than β itself.

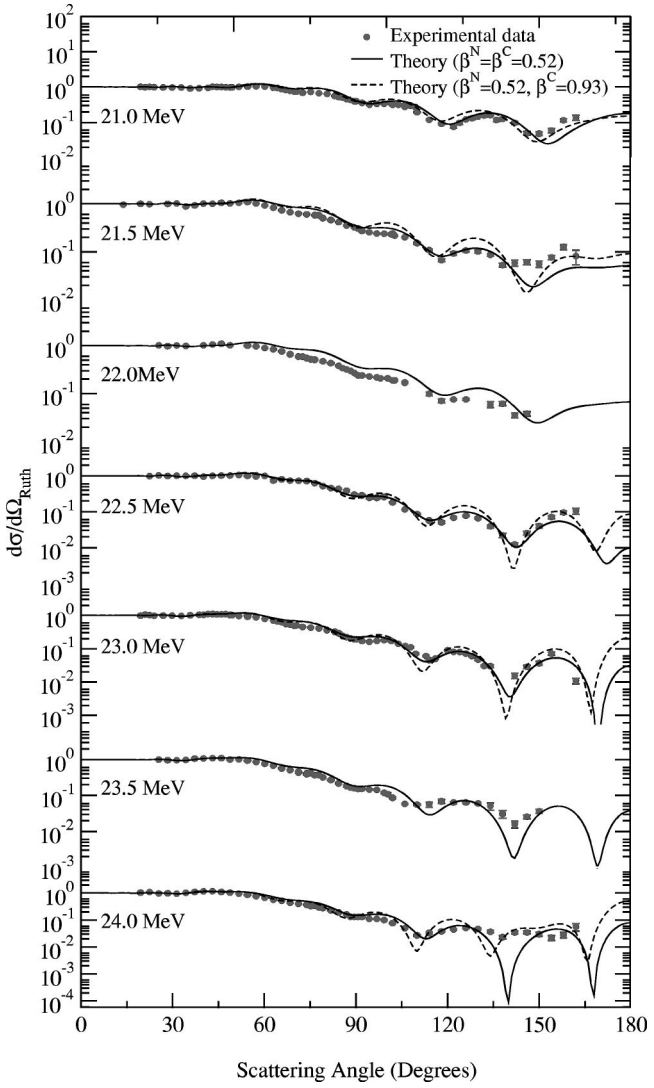

FIG. 3. Ground state results of the standard coupled-channels calculations with $\beta_2^N=\beta_2^C=0.52$ (solid lines) and with $\beta_2^N=0.52$, $\beta_2^C=0.93$ (dashed lines) (continued from Fig. 2).

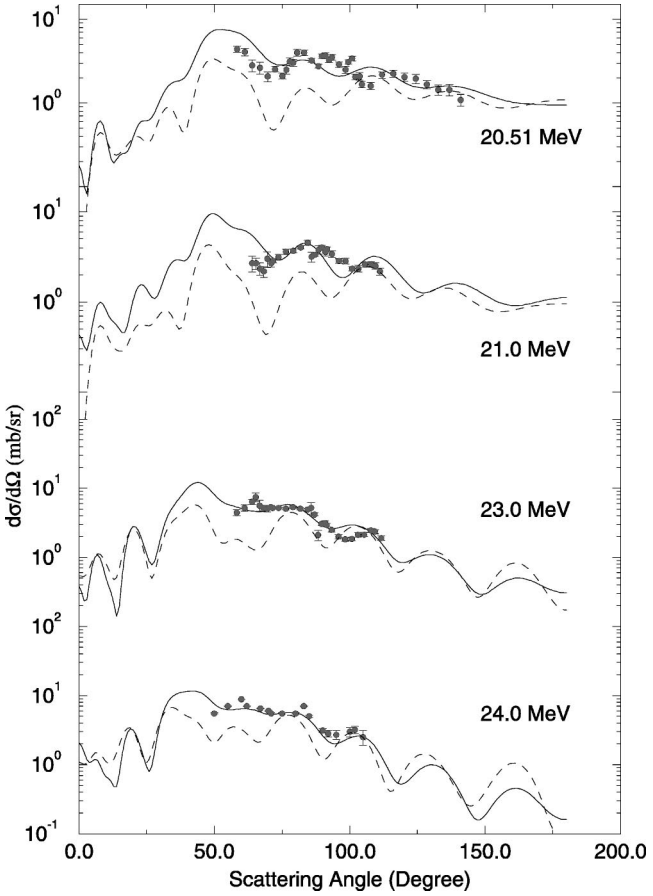

FIG. 4. 2^+ state results of the standard coupled-channels calculations with $\beta_2^N=\beta_2^C=0.52$ (dashed lines) and with $\beta_2^N=0.52$, $\beta_2^C=0.93$ (solid lines).

The actual value of $B(E2)$ is 430 e^2 fm^4 [17] and that of δ is between 1.48 fm [18] and 1.50 fm [8] for the target nucleus ^{24}Mg. We use δ=1.50 fm (β=0.52) in our coupled-channels calculations. For the Coulomb deformation, we assume $\beta_2^C=\beta_2^N$ [19].

In the present calculations, the first two excited states of the target nucleus ^{24}Mg, i.e., 2^+ (1.47 MeV) and 4^+ (4.12 MeV), are included and the 0^+-2^+-4^+ coupling scheme was employed. The reorientation effects for 2^+ and 4^+ excited states are also included. An extensively modified version of the code CHUCK [20] has been used in all the calculations.

III. RESULTS

Using the standard coupled-channels model, some of the results obtained using the empirical β values for the nuclear and Coulomb deformations ($\beta_2^N=\beta_2^C=0.52$) are shown in Figs. 2 and 3 for the ground state and in Fig. 4 for the first excited state with dashed lines, respectively. Although we obtained a good agreement with the experimental data for the ground state, it has not been possible to get the magnitude of the first excited state (2^+) correctly. The magnitudes of the 2^+ predictions are smaller than the measured experimental data and the minima and maxima observed in the experimental data are not reproduced correctly.

This has been a recurring problem in the earlier theoretical calculations, where numerous arbitrary values of the deformation parameter had to be used [2,8,9,12–14]. Varying the parameters and changing the shape of the real and imaginary potentials does not provide a global fit to the experimental data for both ground and 2^+ states. However, it is clear from these results that the problem is in the forward angle region, where two factors are very important [19,21]. The first one is the number of partial waves used in the calculations. Since the calculated cross section depends on the orbital angular momentum number l, the number of the partial waves used in the calculations should be checked whether they all contribute. This conception is examined in order to determine their effect on the results and it is observed that the number of the partial waves does not affect the results beyond a critical value.

The second factor, which is effective in this region, is the value of the Coulomb deformation parameter β_2^C. The sensitivity of the calculations to the β_2^C is checked and the β_2^C

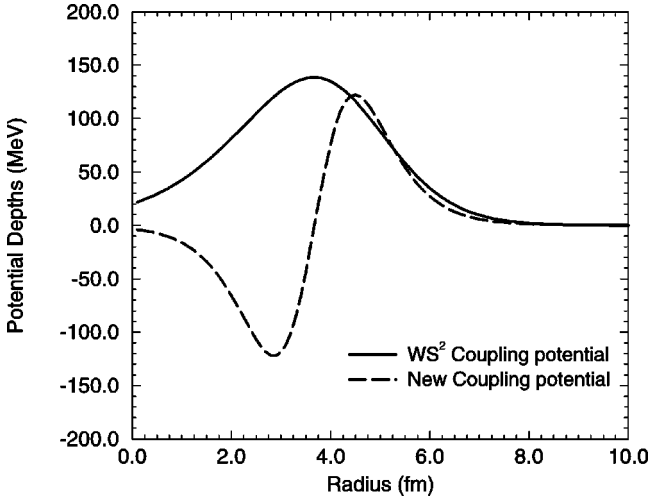

FIG. 5. Comparison of the *standard* coupling potential, which is the first derivative of the central potential, with our *new* coupling potential, parametrized as the second derivative of Woods-Saxon shape as in Eq. (5).

value required to fit to the data is found to be $\beta_2^C = 0.93$, which is larger than its actual value.

The results of these calculations using the exact β_2^N and increased β_2^C are shown in Figs. 2 and 3 for the ground state and in Fig. 4 for the first excited state with solid lines. The agreement is good for both the ground and the first excited states over the whole energy range although the magnitude and the phase oscillation problems persist at high energies for the 2^+ results.

IV. NEW COUPLING POTENTIAL

In the analyses of this reaction, our aim was to solve the out-of-phase problem and to reproduce the experimental data with empirical β value. We succeeded in achieving the former one, but failed to provide a solution to the latter.

Because of the limitations of the standard coupled-channels method in the analyses of this reaction, we use a new second-derivative coupling potential that has successfully explained the experimental data for the $^{12}C + ^{12}C$ reaction [6]. The standard and new coupling potentials are compared in Fig. 5 and the new coupling potential has the following shape:

$$V_C(r) = \frac{-V_{C_0}\, e^{(r-R)/a}(e^{(r-R)/a}-1)}{a^2[1+e^{(r-R)/a}]^3}, \qquad (5)$$

where $V_{C_0} = 185.0$ MeV, $R = 3.67$ fm, and $a = 0.62$ fm.

One possible interpretation of such a second-derivative coupling potential can be made if we express the total potential as a function of the radii for different orientations of the two colliding ^{12}C and ^{24}Mg nuclei. If $\theta_{P,T}$ are the angles between the symmetry axes and the axis joining the centers of the projectile and target, then the total potential, as an approximation, can be expressed in the following way:

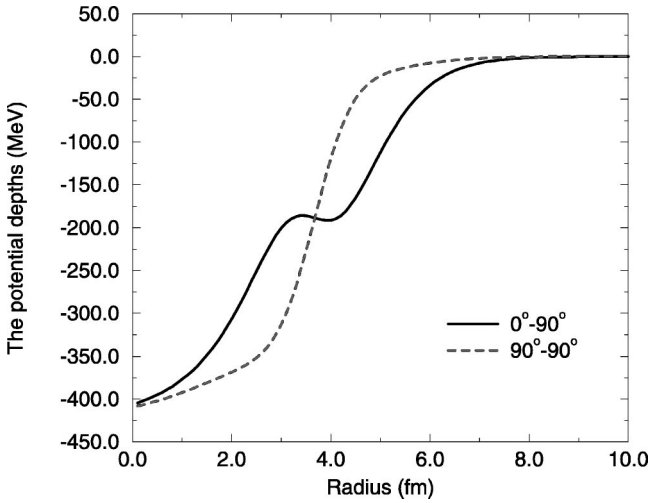

FIG. 6. Orientation potentials of the ^{12}C and ^{24}Mg nuclei at different angles.

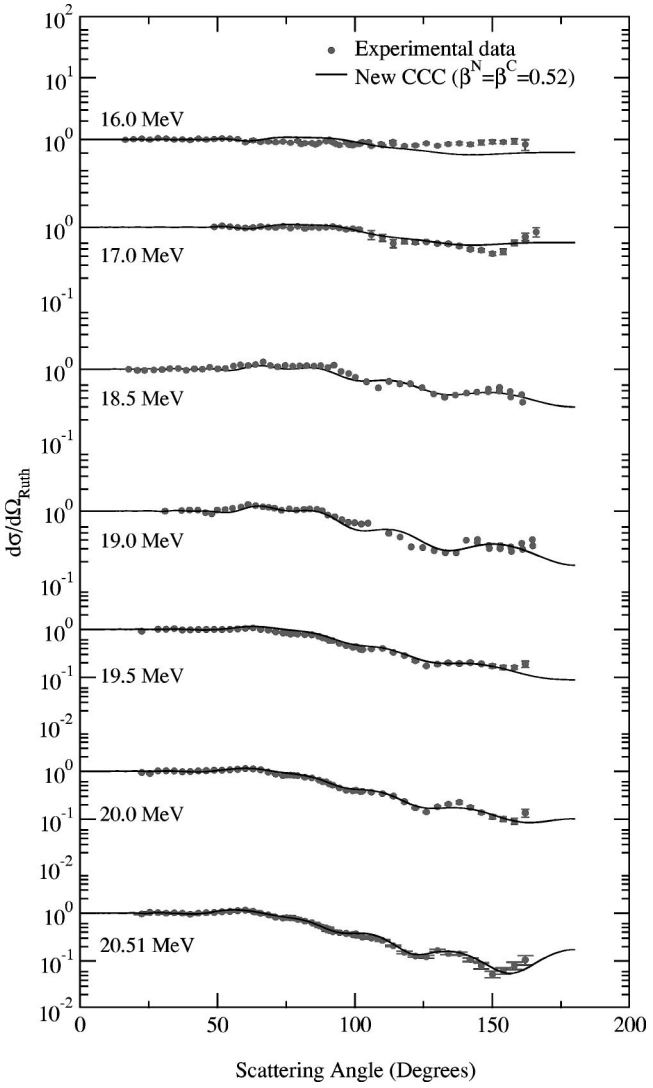

FIG. 7. Ground state results of the new coupled-channels calculations using the new coupling potential with the exact β value ($\beta_2^C = \beta_2^N = 0.52$).

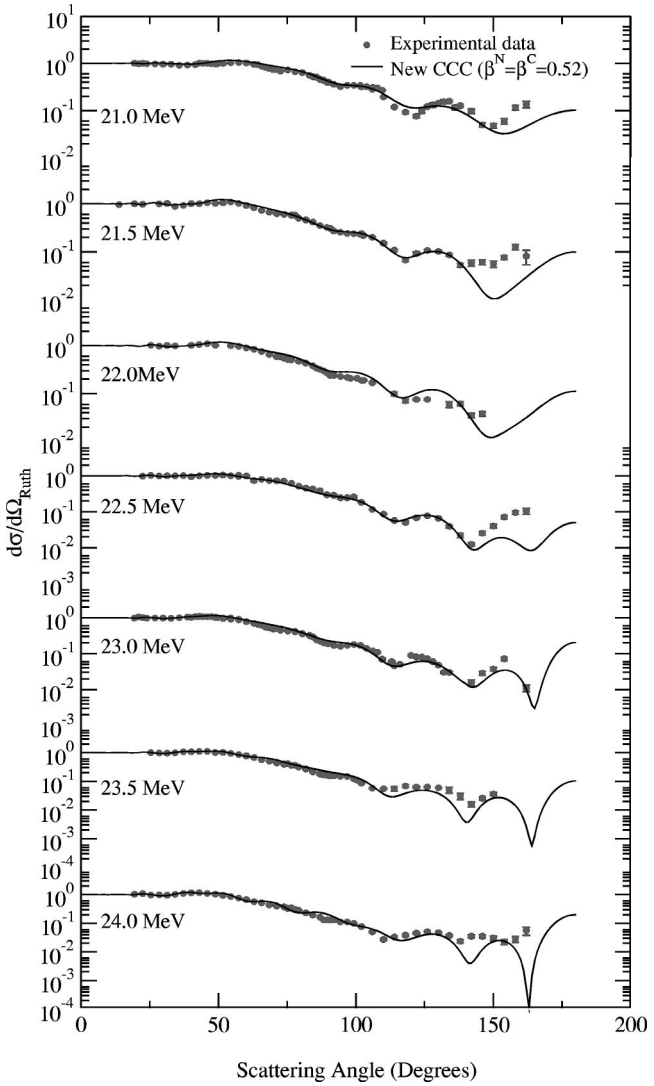

FIG. 8. Ground state results of the new coupled-channels calculations using the new coupling potential with the exact β value ($\beta_2^C=\beta_2^N=0.52$) (continued from figure 7).

$$V(r)=V_N(r)+\beta_2 R_P\frac{dV_C}{dR_P}Y_{20}(\theta_P,\phi_P)$$

$$+\beta_2 R_T\frac{dV_C}{dR_T}Y_{20}(\theta_T,\phi_T), \qquad (6)$$

where V_N is the nuclear potential and V_C is the new second-derivative coupling potential. The difference between Eq. (8) in Ref. [6] and Eq. (6) is due to the simultaneous mutual excitation of two nuclei. In Ref. [6], we took in to account the simultaneous mutual excitation of the projectile and target nuclei, therefore, there is an extra term to define it, whereas in Eq. (6) we do not have mutual excitation term since we just include the excitation of target nucleus.

The result for the $^{12}C+^{24}Mg$ system is shown in Fig. 6. A second local minimum is observed in the interaction potential for certain orientations. This feature has not been taken into account in the standard coupled-channels calculations. To investigate this minimum, we looked at the total inverted potential, i.e., the dynamical polarization potential plus the bare potential, obtained by the inversion of the S matrix [22]. Our analysis suggests that the new coupling potential points to the presence of the superdeformed configurations in the compound nucleus ^{36}Ar, as it has been speculated [23,24].

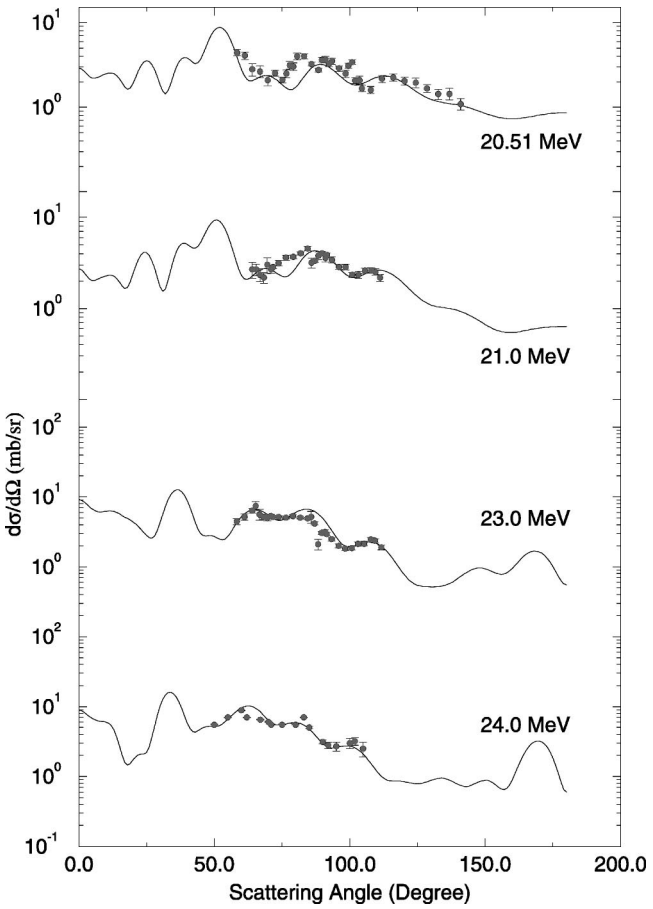

FIG. 9. 2^+ state results of the new coupled-channels calculations obtained using the new coupling potential with the exact β value ($\beta_2^C=\beta_2^N=0.52$).

V. RESULTS

The real and imaginary potentials in these new calculations have the same shapes and parameters as in the previous calculations [see Eqs. (1) and (2)] and the parameters of the new coupling potential are displayed in the caption of Fig. 5. We have analyzed the experimental data in the same energy range.

It is clearly seen from Figs. 7 and 8 for the ground state and Fig. 9 for the first excited state that the new second-derivative coupling potential with the exact β value ($\beta_2^N=\beta_2^C=0.52$) yields excellent agreement with the experimental data over the whole energy range studied. These figures show perfect fits with the experimental data; the phases of the oscillations and magnitudes in the 2^+ state data are well accounted for.

The comparison of the χ^2 values in Table II indicates that this new coupling potential has not only solved the out-of-

TABLE II. The numerical values of χ^2 for the standard and new coupled-channels calculations.

E_{lab}	Standard CC	New CC
16.0	1.24	1.4
17.0	1.37	0.59
18.5	0.67	0.34
19.0	1.44	0.8
19.5	1.29	0.7
20.0	1.63	0.4
20.51	2.48	0.5
21.0	2.45	1.6
21.5	3.46	3.1
22.0	8.2	2.3
22.5	1.7	3.0
23.0	3.96	1.4
23.5	5.69	3.5
24.0	6.59	4.8

phase problem and reproduced the experimental data with empirical β value, but it has also improved the quality of the fits.

VI. SUMMARY

We have shown a consistent description of the elastic and inelastic scattering of the $^{12}C+^{24}Mg$ system from 16.0 MeV to 24.0 MeV in the laboratory system by using the standard and new coupled-channels calculations. In the Introduction, we presented the problems that this reaction manifests. We attempted to find a consistent solution to these problems. However, within the standard coupled-channels method, we failed, as others did, to describe certain aspects of the data, in particular, the magnitude of the 2^+ excitation inelastic scattering data although the optical model and coupled-channels models explain perfectly some aspects of the elastic scattering data. We were compelled to increase the value of Coulomb deformation to reproduce the 2^+ data and such arbitrary uses of β have been practiced in the past without giving any physical justifications other than stating it is required to fit the experimental data.

We have obtained excellent agrement with the experimental data over the whole energy range by using a new coupling potential, which has been outstandingly successful in explaining the experimental data for the $^{12}C+^{12}C$ [6] and $^{16}O+^{28}Si$ [25] systems over a wide range of energies. The comparison of the results indicates that a global solution to the problems relating to the scattering observables of this reaction over a wide range of energies range has been provided by this new coupling potential. This work reveals that there is no reason for the coupling potential to have the same energy dependence as the central term. The work in order to derive the coupling potential explicitly from a microscopic viewpoint is still under progress and studies using this new coupling potential may lead to new insights into the formalism and also a new interpretation of such reactions.

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