Measurements of cross sections for the fusion-evaporation reactions ^{204,206,207,208}Pb+⁴⁸Ca and ²⁰⁷Pb+³⁴S: Decay properties of the even-even nuclides ²³⁸Cf and ²⁵⁰No

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In preparation for recent experiments on the synthesis of superheavy nuclei with Z=114 in the reaction ²⁴⁴Pu+⁴⁸Ca, we modified the Dubna gas-filled recoil separator and its detection system and carried out bombardments of lead targets with ⁴⁸Ca projectiles. We studied excitation functions of the reactions ²⁰⁶Pb(⁴⁸Ca,1-4n) and ^{204,207,208}Pb(⁴⁸Ca,2n). Maximum cross sections for the evaporation of 1-4 neutrons in the complete fusion reaction ²⁰⁶Pb+⁴⁸Ca were measured to be $\sigma_{1n}=60$ nb, $\sigma_{2n}=500$ nb, $\sigma_{3n}=30$ nb, and $\sigma_{4n}=0.3$ nb. In the bombardment of an enriched ²⁰⁴Pb target, we simultaneously obtained excitation functions of the ^{204,206,207,208}Pb(⁴⁸Ca,2n) reactions induced on the isotopic admixtures present in the target material. The maximum cross sections for the evaporation of two neutrons from the compound nuclei ²⁵⁶No, ²⁵⁵No, and ²⁵²No were measured to be 2.1 μ b, 1.3 μ b, and 10 nb, respectively. The spontaneously fissioning even-even isotope ²⁵⁰No, with a half-life $T_{1/2}=36 \ \mu$ s, was identified for the first time in this experiment. In the reaction ²⁰⁷Pb+³⁴S, we measured the excitation function for the production of the 21-ms spontaneously fissioning isotope ²³⁸Cf, confirming our preliminary identification of this nuclide based on the results of cross bombard-ments.

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I. INTRODUCTION

Extensive studies of fusion-evaporation reactions of Pb target nuclei with ⁴⁸Ca projectiles have yielded valuable information on the formation and subsequent deexcitation of heavy compound nuclei. A characteristic of these reactions is that the proton and neutron numbers, Z and N, in both the target and projectile correspond to or lie near spherical shells. Moreover, the resulting compound nuclei themselves are close to the deformed N = 152 shell. In this respect, the $Pb + {}^{48}Ca$ system is of special interest when considering the production of superheavy nuclei in the vicinity of the predicted spherical shells $Z \ge 114$ and $N \ge 172$. To reach this superheavy region, complete fusion reactions of the actinide targets with the doubly magic ⁴⁸Ca projectile seem to be optimal, providing the closest approach to this nuclear domain due to neutron excess in both reaction partners and the large mass defect of ⁴⁸Ca [1].

The macroscopic component of the fission barrier of these heavy nuclei is close to zero, so their existence is governed largely by shell effects [2]. Thus the knowledge of trends in the radioactive properties of No isotopes with changing neutron number can provide information on the influence of the N=152 shell upon the structure and properties of heavy nuclei. In addition, fusion-evaporation reactions of Pb isotopes with ⁴⁸Ca provide more insight into the production of the heaviest nuclei. Thus, recently the ground-state bands of ²⁵⁴No and ²⁵²No have been identified up to spin 20 indicating that its fission barrier still exists at high angular momentum [3,4]. Further investigation of the fission barrier and its dependence on angular momentum is very important for understanding the mechanism of producing the heaviest shellstabilized nuclei since the fission barrier governs their survival probability.

The Pb +⁴⁸Ca reactions have been investigated by different groups for more than 20 years [5–13]. For the Pb +⁴⁸Ca system, the largest amount of experimental data is for the ²⁰⁸Pb(⁴⁸Ca, 2*n*) reaction [6–13], yet even in this case the discrepancy in measured maximum cross section values is an order of magnitude (see, e.g., [6–8]). The evaporation residues following the emission of one and three neutrons from the ²⁵⁶No compound nucleus were observed in [6,8,11–13]. In the ²⁰⁶Pb+⁴⁸Ca reaction, evaporation of two neutrons from the ²⁵⁴No compound nucleus was observed in [5,8,11], while the 3*n* evaporation product ²⁵¹No was observed only in [8]. As for the reaction ²⁰⁷Pb+⁴⁸Ca, the 3*n* channel was reported in [5]. No experimental data is available on the reaction of ⁴⁸Ca with ²⁰⁴Pb.

In this paper, we report the results from bombardments of various stable Pb nuclides with ⁴⁸Ca, experiments which were performed at the Dubna Gas-filled Recoil Separator in preparation for the synthesis of superheavy elements using the reactions ²⁴⁴Pu+⁴⁸Ca and ²⁴⁸Cm+⁴⁸Ca [1]. We studied the fusion-evaporation reactions ²⁰⁶Pb(⁴⁸Ca,1-4*n*) and ^{204,206,207,208}Pb(⁴⁸Ca,2*n*) with special attention to the radioactive properties of the even-even nuclides ²⁵²No and ²⁵⁰No. We also present the results of an experiment with the ²⁰⁷Pb + ³⁴S reaction, confirming the original assignment of the lightest spontaneously fissioning even-even nuclide ²³⁸Cf [14].

II. EXPERIMENTAL TECHNIQUE

Beams of ³⁴S and ⁴⁸Ca projectiles were delivered by the Dubna U400 cyclotron. The projectile energy was varied by

extracting the beam from the appropriate radius. The value of the beam energy was determined by measuring the energies of scattered ions and by a time-of-flight technique. Targets of isotopically enriched material were arranged on a rotating wheel to reduce heat and radiation damage by the beam. Experiments were performed with either pulsed or continuous cyclotron beams. For the pulsed-beam experiments, the beam was modulated with a 150 Hz frequency so that targets were exposed to the ~2.2 ms beam pulse during each ~6.7 ms beam cycle. Targets of ²⁰⁷Pb (93.2% ²⁰⁷Pb, 5.41% ²⁰⁸Pb, 1.39% ²⁰⁶Pb), ²⁰⁶Pb (95.9% ²⁰⁶Pb, 1.34% ²⁰⁸Pb, 2.76% ²⁰⁷Pb, and ²⁰⁴Pb (53.5% ²⁰⁴Pb, 15.9% ²⁰⁸Pb, 11.0% ²⁰⁷Pb, 19.6% ²⁰⁶Pb) with average thicknesses of 0.70, 0.38, and 0.20 mg/cm², respectively, were deposited by evaporation in vacuum on ~0.55 mg/cm² Cu substrates.

The Dubna Gas-filled Recoil Separator [15] was employed to separate in-flight evaporation residues (EVRs) recoiling out of the targets from beam particles, scattered ions, and transfer-reaction products. To set the magnetic field of the separator's dipole magnet for collecting specific EVRs on the focal-plane detectors, we used our measurements of the average charges of slow atoms moving in ~ 1 Torr of hydrogen [16]. The separated EVRs passed through a time-offlight (TOF) measurement system composed of two (start and stop) multiwire proportional chambers in a \sim 1.5-Torr pentane-filled module and were implanted in a positionsensitive detector array. The latter was composed of three 40×40 mm² silicon detectors, each with four 40-mm-high and 9.7-mm-wide strips. We obtained horizontal positions for the reaction products from the 12 strips and vertical positions from charge-splitting in the 40-mm-high resistive layer of the detectors. Compared with our previous design, the detection system was modified to improve both the detection effi-

ciency and background conditions [17]. The ²⁵²No EVRs produced in the ²⁰⁶Pb+⁴⁸Ca reaction at the excitation-function maximum had an initial energy of ~41 MeV, which was reduced to ~24 MeV [18] at implantation due to energy losses in the target, hydrogen, and pentane media. Since the corresponding implantation depth of EVRs in silicon (~1 mg/cm²) is considerably lower than the α -particle range, the detection efficiency for α particles by the focal-plane detectors is about 54% of 4π . To detect escaping α 's, a set of eight detectors of similar type, without position sensitivity, was mounted in a boxlike array around the perimeter of the focal plane detectors. Employing these side detectors increased α -particle detection efficiency to ~87%.

Some of the detector background is produced by lowionizing particles, e.g., protons, α 's, etc., which with some probability pass through the separator and reach the focal plane. These particles with ranges exceeding the focal-plane detector thickness of 300 μ m are detected by the TOF system with low efficiency. An array of three detectors of the same dimensions, without position sensitivity, was added 15 mm behind the focal-plane detectors. With these back detectors, we measured energy losses or residual energies of particles that passed through the focal-plane detectors. This allowed identification of the p, ²H, ³H, and α particles and measurement of the corresponding energy spectra. Employing the extra back detectors in "veto" mode suppressed the background in the focal-plane detectors when measuring the in-beam α -particle spectra.

 α -energy calibrations were performed using α emitters produced in the ^{nat}Yb+⁴⁸Ca reaction. Most of the strips of the focal-plane detectors had an α -energy peak resolution of 30–38 keV FWHM. When α particles escape the focal-plane detectors at varying angles they lose correspondingly a variable amount of energy traversing the detector's dead layer, the entrance window, and the pentane that fills the detection module. Thus, the strips of the side detectors generally had poorer α -energy resolution, with a peak FWHM of 90–160 keV. By using known event sequences from the products of the ^{nat}Yb+⁴⁸Ca and ²⁰⁶Pb+⁴⁸Ca reactions, we measured the FWHM position deviation Δ pos to be 1.0 mm for α - α sequences, 1.0–1.5 mm for No EVR- α and 0.7–1.3 mm for No and Cf EVR spontaneous fission (SF).

The collection efficiency of the separator was estimated using the code ANAMARI [19]. This computer code is based on the Monte Carlo technique. It is used to generate EVR events and calculate their trajectories in the separator and the corresponding spatial distribution at the focal plane. It accounts for reaction kinematics, energy losses, and multiple scattering in the target and the separator fill gas, equilibrium charge states of EVRs, and the magnetic-optical system of the separator. The calculations successfully reproduce the distributions of EVRs on the focal-plane detectors and their collection efficiencies for numerous reactions studied. We used collection efficiencies of 45% and 30% when calculating the cross sections of the reactions $Pb(^{48}Ca, xn)No$ and 207 Pb(34 S,3n) 238 Cf, respectively. We estimate that using the calculated efficiency values could systematically shift the measured cross sections within $\pm 30\%$ [19].

III. RESULTS

A. Cross sections of the fusion-evaporation reactions ^{204,206,207,208}Pb+⁴⁸Ca and the SF decay of even No isotopes

Cross sections for producing $^{250-254}$ No from 204 Pb and 206 Pb targets were measured at 48 Ca beam energy ranges of 213.5–219.4 MeV and 213.7–242.5 MeV, respectively. Energy losses of 48 Ca ions in the entrance window (1.34 mg/cm² or 0.71 mg/cm² of Ti), target backing, and target material were calculated using data in [20]. The systematic uncertainty in bombarding energy at the middle of the target was ~1%. Production cross sections for $^{251-254}$ No were calculated using the spectroscopy data of [21,22] and the data of the present work for 250 No (see below). The experimental fusion-evaporation cross sections for the reactions 206 Pb(48 Ca,1–4*n*) and 204,206,207,208 Pb(48 Ca,2*n*) are presented in Fig. 1 and Table I. The error bars represent statistical uncertainties; uncertainties in knowing the decay properties of the synthesized nuclei and detection efficiencies, in target thicknesses and isotopic compositions, as well as in determining beam doses.

An enriched ²⁰⁶Pb target was used to study the evaporation of 1-4 neutrons from the ²⁵⁴No compound nucleus. The 2n-evaporation product ²⁵²No was identified by detect-



FIG. 1. Experimental cross sections of the reactions $^{204,206-208}$ Pb(48 Ca,2*n*) (upper panel a) and 206 Pb(48 Ca,1-4*n*) (lower panel b). Excitation energies E_{min} corresponding to the Bass barriers [23] for 204,206,207,208 Pb+ 48 Ca reactions are shown by upward arrows. Results of the statistical model calculations are shown by solid lines. Calculated cross sections of the reaction 206 Pb(48 Ca,4*n*) 250 No with damping factors of 0.05, 0.061, and 0.08 MeV⁻¹ are shown by dashed, solid, and dotted lines, respectively.

ing α particles of known energies, correlations with the α decays of the daughter ²⁴⁸Fm, and correlations of α particle and spontaneous fission events with the known half-life relative to the EVR implantation time. The estimated contributions due to ²⁰⁷Pb(⁴⁸Ca,3*n*) and ²⁰⁸Pb(⁴⁸Ca,4*n*) reactions on target impurities did not exceed a few percent at the corresponding cross section maxima (see, e.g., [5,13]) and were ignored in calculating the 2*n* cross section. The maximum cross section of 515⁺⁸⁰₋₄₇ nb was measured at 217.1 MeV for the reaction ²⁰⁶Pb(⁴⁸Ca,2*n*)²⁵²No; the average cross section value of five measurements at 217–218 MeV was 489⁺³¹₋₂₁ nb.

In the present series of experiments, we detected a total of more than 10⁴ decays of ²⁵²No. From the sum distribution of the EVR- α and EVR-SF correlations, we obtained a half-life of 2.44±0.04 s for the decay of ²⁵²No. This agrees with the reported values: $T_{1/2}=2.30\pm0.22$ s [21,24], $2.25^{+0.18}_{-0.16}$ s [11], and 2.44 ± 0.12 s [25]. Assuming the electron capture (EC) branch $b_{\rm EC} \ge 50\%$ of ²⁵²Md, we set the upper limit for the EC branch of ²⁵²No at 10% (68% confidence level). The spontaneous-fission branch, $b_{\rm SF}=(32.2\pm0.5)\%$, which follows from the present data, is somewhat higher than $b_{\rm SF}=(26.9\pm1.9)\%$ or $(21.6\pm4.2)\%$ reported in [24,26], respectively. The corresponding partial SF half-life, $T_{\rm SF}$

=7.56±0.15 s, is included in the systematics presented in Fig. 2. For ²⁴⁸Fm, we obtained $T_{1/2}$ =32.9±1.8 s from α - α correlations with ²⁵²No, in agreement with [21].

We identified ²⁵¹No by the known energies of α particles correlated with α decays of the daughter ²⁴⁷Fm, taking into account its half-life. At 226.2 MeV bombarding energy, the cross section of the ²⁰⁶Pb(⁴⁸Ca,3*n*)²⁵¹No reaction reaches its maximum of 30⁺⁹₋₇ nb.

In determining cross sections for the reaction ²⁰⁶Pb(⁴⁸Ca, 1n)²⁵³No, contributions from the 2n products of the reactions with admixtures of ²⁰⁷Pb and ²⁰⁸Pb appeared to be considerable. The contribution from the reaction 207 Pb(48 Ca,2n) 253 No was determined from our experimental measurement of the excitation function and the certified isotopic composition of the target material. We also considered the fact that the α -particle spectrum of ²⁵³No overlaps the 8.1-MeV line of ²⁵⁴No [22], while the corresponding halflives differ by a factor of less than 2. To account for the contribution from ²⁵⁴No, we used our measured excitation function 208 Pb(48 Ca,2n) 254 No together with that from [12,13]. Impurities contributed as much as 50% of the α -particle yield of ²⁵³No at the 1*n* reaction maximum. Making the appropriate corrections, we obtained a maximum cross section of 58^{+16}_{-17} nb for the reaction 206 Pb(48 Ca,1*n*) 253 No at 217.4 MeV.

Analyzing the EVR-SF correlations detected in the bombardment of the ²⁰⁶Pb target with ⁴⁸Ca ions at higher energies, close to the expected cross section maximum of the 4n-evaporation channel, we isolated a new short-lived SF activity. We observed three correlated EVR-SF events at 237.7 MeV ⁴⁸Ca beam energy and seven events at 242.5 MeV. The resulting half-life measured for these ten events was $T_{1/2} = 26^{+12}_{-6}$ µs. In the same irradiations, we observed about 120 SF events from the decay of ²⁵²No. The number of decays this contributed to the $26 - \mu s$ activity was less than 0.006 for the fraction of the distribution of 2.44-s ²⁵²No SF events with short decay times. The short-lived activity was not observed at lower bombarding energies. Both the position of the excitation function and maximum cross section value for its production agree well with those expected for the 206 Pb(48 Ca,4n) 250 No reaction. Thus, the most probable origin of this new activity is the spontaneous fission of eveneven nuclide ²⁵⁰No. With the measured half-life value of 250 No, we set a cross section of $0.26^{+0.19}_{-0.13}$ nb for the reaction ²⁰⁶Pb(⁴⁸Ca,4*n*) at 242.5 MeV.

With the 84 μ s dead time of the electronics system, uncertainty in the radioactive-decay properties of ²⁵⁰No could significantly influence estimates of the ²⁰⁶Pb(⁴⁸Ca,4*n*) reaction cross section. Spontaneous fission as a predominant decay mode and $T_{1/2}=250\pm50~\mu$ s are reported for ²⁵⁰No in a single paper [29]. With the aim of determining more accurately the properties of ²⁵⁰No, we irradiated an enriched ²⁰⁴Pb target with ⁴⁸Ca ions at energies covering the range expected for the peak of the 2*n* reaction. One sequence with a correlation time of 143 μ s was detected at $E(^{48}Ca) = 213.5$ MeV, seven at 216.7 MeV with $T_{1/2}=41^{+24}_{-11}~\mu$ s, and three at 219.4 MeV ($T_{1/2}=39^{+47}_{-14}~\mu$ s). The sum distribution results in a half-life of $46^{+19}_{-11}~\mu$ s. For the 21 events,

Reaction	Laboratory energy	Excitation energy (MeV)	Cross section (nb)			
	(MeV)		1 <i>n</i>	2 <i>n</i>	3 <i>n</i>	4 <i>n</i>
⁴⁸ Ca+ ²⁰⁸ Pb	213.5	19.6		1870^{+430}_{-330}		
	216.7	22.3		2050^{+460}_{-340}		
	219.4	24.4		1190^{+290}_{-230}		
⁴⁸ Ca+ ²⁰⁷ Pb	213.5	19.8		670^{+460}_{-450}		
	216.7	22.4		1310^{+430}_{-410}		
	219.4	24.6		710 ± 370		
⁴⁸ Ca+ ²⁰⁶ Pb	212.7	19.8	29^{+25}_{-29}	100^{+20}_{-15}		
	213.5	20.4		308^{+88}_{-73}		
	216.7	23.0		500^{+110}_{-100}		
	217.0	23.2		488^{+76}_{-45}		
	217.1	23.3		515^{+80}_{-47}		
	217.4	23.6	58^{+16}_{-17}	489^{+31}_{-21}	≤0.9	
	218.1	24.1		475^{+74}_{-44}		
	219.4	25.2		327^{+91}_{-75}		
	219.9	25.7	54^{+38}_{-37}	325^{+61}_{-43}		
	223.9	28.9	≤15	166^{+37}_{-28}	$7.9^{+7.9}_{-4.3}$	
	226.2	30.7		56^{+14}_{-11}	30^{+9}_{-7}	
	233.5	36.6		$3.8^{+1.9}_{-0.9}$	18^{+9}_{-8}	
	237.7	40.0		$1.52^{+0.43}_{-0.34}$	1.70 ± 0.25	$0.11^{+0.12}_{-0.07}$
	239.1	41.2		$1.7^{+1.0}_{-0.7}$	$3.7^{+1.7}_{-1.2}$	≤0.24
	242.5	43.9		$1.37^{+0.20}_{-0.21}$	$1.53^{+0.30}_{-0.28}$	$0.26^{+0.19}_{-0.13}$
⁴⁸ Ca+ ²⁰⁴ Pb	213.5	20.6		$3.4^{+8.8}_{-3.2}$		
	216.7	23.2		$13.2^{+10.1}_{-6.7}$		
	219.4	25.4		$9.6^{+11.3}_{-6.4}$		
³⁴ S + ²⁰⁷ Pb	160.5	26.1			$0.12^{+0.13}_{-0.07}$	
	164.3	29.4			$0.82^{+0.27}_{-0.21}$	
	167.2	31.8			$1.36_{-0.33}^{+0.45}$	
	167.3	31.9			$1.57^{+0.32}_{-0.25}$	
	169.3	33.7			$3.1^{+1.2}_{-1.0}$	
	169.7	34.0			$2.62^{+0.73}_{-0.57}$	
	176.7	40.1			$2.10^{+0.62}_{-0.49}$	
	180.2	43.0			$0.35_{-0.14}^{+0.22}$	

TABLE I. Summary of measured fusion-evaporation cross sections.

including those observed in the ²⁰⁶Pb+⁴⁸Ca reaction, the half-life is 36^{+11}_{-6} µs. In the same irradiations, we observed about 270 SF events from the decay of ²⁵²No produced from the ²⁰⁶Pb isotope present in the target material. The possible background contribution to the observed 36-µs activity due to ²⁵²No SF events that could be detected with short decay times is less than 0.02. The corresponding excitation energies of the ²⁵²No compound nuclei produced at the three beam energies were 20.6, 23.2, and 25.4 MeV, respectively, calculated with mass values of [30]. These cover the energy range expected for the 204 Pb(48 Ca,2n) reaction. The observed energy dependence of the yield of the 36- μ s SF activity follows the excitation functions of the ${}^{206-208}$ Pb(48 Ca,2n) reactions measured in the same irradiation. The latter were obtained by detecting $^{252-254}$ No produced from the isotopic admixtures in the target material and agree well with the available experimental data. The corresponding EVRs are characterized by implantation energy, time of flight, and ionic charge, which are in accord with those measured simultaneously for the known EVRs with Z=102. Finally, the lower yield of this activity as compared with the other No isotopes produced via 2n evaporation reactions, its decay mode, and its half-life are consistent with those expected for ²⁵⁰No. In particular, spontaneous fission with a 14 μ s halflife has been predicted for ²⁵⁰No [31], which is in excellent agreement with this experimental result. On this basis, we assign the observed SF activity to the decay of the neutrondeficient even-even nucleus ²⁵⁰No produced in the reaction ²⁰⁴Pb(⁴⁸Ca,2n) with a maximum cross section of $9.5^{+7.6}_{-4.9}$ nb at 216.7-MeV (see Figs. 1 and 2). The partial α -decay halflife of ²⁵⁰No is calculated from systematics to be about 0.2 s [32], which corresponds to $b_{\alpha} \sim 2 \times 10^{-4}$.

In experiments with the 204 Pb target, we simultaneously obtained excitation functions for the 206,207,208 Pb(48 Ca,2*n*) reactions induced on the substantial isotopic admixtures in the target material. The maximum production cross section



FIG. 2. Partial SF half-lives of even-even nuclei with Z=98 to 106. Experimental data are taken from Refs. [14,21,22,27,28]. Solid symbols show the data of the present work for ²³⁸Cf and ²⁵⁰No.

of ²⁵²No was measured to be 500^{+110}_{-100} nb, which agrees both with the data of previous experiments and of our present measurements with ²⁰⁶Pb. Overlapping of the ²⁵³No and ²⁵⁴No α -particle spectra was taken into account when we determined the yields of these isotopes. The independent yield of ²⁵⁴No was calculated from the number of detected α particles of its daughter ²⁵⁰Fm taking into account α -branching values of (95±5)% [21] and 90% [6,10,21] for ²⁵⁰Fm and ²⁵⁴No, respectively. The corresponding contribution was subtracted from the sum α spectrum of ²⁵³No and ²⁵⁴No to obtain the ²⁵³No fraction. We obtained a maximum cross section of 2050⁺⁴⁶⁰₋₃₄₀ nb for the ²⁰⁸Pb(⁴⁸Ca,2n)²⁵⁴No reaction, in good agreement with data of [11–13], and 1310⁺⁴³⁰₋₄₁₀ nb for the reaction ²⁰⁷Pb(⁴⁸Ca,2n)²⁵³No.

B. Production of ²³⁸Cf

²³⁸Cf was originally produced in experiments employing a rotating wheel system and was identified through the cross bombardments ²⁰⁸⁻²⁰⁶Pb+³⁴S and ²⁰⁶Pb+³⁶S [14]. This identification was corroborated in the present experiments with the gas-filled recoil separator. We used the reaction ²⁰⁷Pb+³⁴S, which gave a maximum yield of the SF activity with $T_{1/2}=21\pm 2$ ms according to [14]. The ³⁴S-beam energy range of 160.5–180.2 MeV corresponded to excitation energies of the ²⁴¹Cf compound system ranging from 26.1 to 43.0 MeV. We detected about 260 SF events position correlated with EVR implants, with a half-life $T_{1/2}$ = $21.1^{+1.9}_{-1.7}$ ms, in agreement with the value given in [14]. The corresponding excitation function, which agrees well with that expected for the reaction ²⁰⁷Pb(³⁴S,3*n*)²³⁸Cf, is presented in Fig. 3 and Table I. The measured maximum



FIG. 3. Excitation function of the reaction 207 Pb(34 S,3*n*). The Bass barrier B_{fus} [23] is shown by an open arrow. The line is drawn to guide the eye.

cross section of $3.1^{+1.2}_{-1.0}$ nb exceeds the value reported in [14], yet both results are consistent within experimental uncertainties.

The partial α half-life for ²³⁸Cf can be estimated from the mass calculations of [30,33], which give similar Q_{α} values of 7.975 and 7.95 MeV, respectively. With these values, according to the formula of Viola and Seaborg with parameters from [32,33] defining the relation between T_{α} and Q_{α} , we obtain $T_{\alpha} \sim 10$ s and, therefore, $b_{\alpha} \sim 2 \times 10^{-3}$.

IV. DISCUSSION AND CONCLUSIONS

In the present work, we performed a systematic study of the production of No isotopes via ²⁰⁶Pb(⁴⁸Ca,1-4*n*) reactions in the excitation energy range $E^* = 19.8-43.9$ MeV and in ^{204,207,208}Pb(⁴⁸Ca,2*n*) reactions at $E^* \cong 20-25$ MeV. Note that excitation functions for the fusion-evaporation reactions ^{204,207}Pb(⁴⁸Ca,2*n*), ²⁰⁶Pb(⁴⁸Ca,1*n*), ²⁰⁶Pb(⁴⁸Ca,3*n*), and ²⁰⁶Pb(⁴⁸Ca,4*n*) were measured for the first time.

Cross sections for producing evaporation residues in complete-fusion reactions of stable lead isotopes with the various projectiles 40 Ar (2 and 3*n* evaporation channels), 48 Ca (2*n* channel), and 50 Ti (1 and 2*n* channels) versus the number of neutrons in the compound nuclei are shown in Fig. 4. For the Pb(48 Ca,2*n*) reactions, we note that changing the 208 Pb target to 206 Pb and then to 204 Pb results in a decrease of maximum cross section by a factor of ~4 and 40, respectively (see Table I and Figs. 1 and 4). Similar behavior of fusion-evaporation cross sections is observed in the



FIG. 4. Cross sections of fusion-evaporation reactions of stable Pb isotopes with various projectiles: 40 Ar (2n and 3n evaporation channels), 48 Ca (2n channel), and 50 Ti (1n and 2n channels) versus neutron number of compound nuclei. For the reactions with 40 Ar, the data of Refs. [34,35] are shown, where the excitation functions of the 206,208 Pb+ 40 Ar \rightarrow 2,3n reactions were measured with the same setup, and for the reaction 207 Pb(40 Ar,3n), an averaged value from [36] is shown. Cross section values of [37] were multiplied by a factor of 2 to bring these in correspondence with later results of the same authors for the reactions 208 Pb+ 50 Ti \rightarrow 1,2n [22].

⁵⁰Ti-induced reactions with Pb isotopes, which lead to Rf compound nuclei with the same neutron numbers. Reactions induced by the doubly magic ⁴⁸Ca show EVR production cross sections that are two orders of magnitude higher than the corresponding values for the reactions with ⁴⁰Ar and ⁵⁰Ti ions. A drastic drop in the cross sections for producing neutron-deficient EVRs in the reactions of Pb isotopes with ⁴⁰Ar, ⁴⁸Ca, and ⁵⁰Ti projectiles demonstrates an essential effect of neutron number in compound nuclei and the deformed shell N=152, which still manifests itself in the excited nuclei.

Experimental cross sections for the $^{204,206-208}$ Pb(48 Ca,*xn*) reactions were analyzed with a statistical model. To calculate the cross sections for the formation of No compound nuclei we used experimental data on the yields of fission fragments in the reaction 208 Pb+ 48 Ca, corresponding to the excitation energy range of 14–42 MeV [38].

In determining the survivability of the No EVRs with A = 250-254, we used experimental and calculated neutron binding energies B_n and calculated values of the fission barriers B_f . Neutron binding energies B_n in the No isotopes with N = 148 - 154 are in the range of 5.9-8.4 MeV [21,30]. Fission barriers were determined as

$$B_{\rm f}(E^*) = B_{\rm LD} - \delta W \cdot \exp(-\gamma_d \cdot E^*). \tag{1}$$

Here $B_{\rm LD}$ is the liquid-drop fission barrier, calculated in [39] to be about 1–1.2 MeV for the No isotopes under consideration, δW is the shell correction to the ground state poten-

tial energy as given in [40], and γ_d is the damping parameter used to describe the weakening of shell effects in the nuclear level density with an increase of the excitation energy E^* [41].

Results of the calculations for the reactions $^{204,206-208}$ Pb(48 Ca,2n) and 206 Pb(48 Ca,1-4n), are presented in Figs. 1(a) and 1(b), respectively, together with our measured EVR production cross sections. The values of the shell corrections for $^{250-256}$ No given in [40] and damping parameter $\gamma_d = 0.061 \text{ MeV}^{-1}$ that were used for calculating fission barriers agree well with the experimental data and allow us to describe our measured cross sections adequately without introducing extra parameters.

Shell effects and their damping with increased excitation of the compound nucleus largely determine production probabilities of the investigated evaporation residues $^{250-254}$ No. As Fig. 1(b) shows, the cross section of the 4n channel at $E^*>35$ MeV is very sensitive to the value of the damping factor γ_d . Varying γ_d by $\pm 20\%$ results in a variation of the cross section by a factor of 10. The same effect can be observed if the shell correction is varied by ± 1 MeV. Therefore, analysis of the present data defines the values of γ_d , δW and other model parameters quite stringently.

The production of No isotopes in the reactions Pb(⁴⁸Ca, *xn*) can be considered as a precursor to the study of the synthesis of heavier nuclides, in particular, of the superheavy elements. In complete fusion reactions of ⁴⁸Ca projectiles with actinide targets like ²⁴⁴Pu or ²⁴⁸Cm, the superheavy nuclides with Z=114,116 are produced in 3,4*n*-evaporation channels [1]. Here, the primary states of the compound nuclei with the excitation energies $E^* \approx 30-40$ MeV and their subsequent deexcitation have much in common with the studied No isotopes, whose survivability also is largely determined by the existence of nuclear shell effects. From this viewpoint, the results of the present studies can be helpful for refining the theoretical calculations of the production of superheavy nuclei in fusion-evaporation reactions of actinide targets with ⁴⁸Ca projectiles.

As for the decay properties of the investigated nuclides, we summarize as follows: In the reaction ${}^{207}\text{Pb} + {}^{34}\text{S}$, we studied the production of the 21-ms SF activity, which was previously assigned to the even-even isotope ²³⁸Cf [14]. The yield of the 21-ms activity and its dependence on the ³⁴S-beam energy agree well with what could be expected for the evaporation of three neutrons from the compound nucleus. Thus, our observations confirm our preliminary identification of ²³⁸Cf [14]. The new neutron-deficient eveneven nuclide ²⁵⁰No produced in the reactions ²⁰⁴Pb(48 Ca,2*n*) and 206 Pb(48 Ca,4n) undergoes spontaneous fission with the half-life of 36^{+11}_{-6} µs. The partial SF half-lives of even-even nuclei with Z=98 to 106 are shown in Fig. 2. The half-life of ²⁵⁰No (N=148) synthesized in our experiments is about 5 $\times 10^8$ times shorter than the partial SF half-life of ²⁵⁴No (N=152). In the No isotopes, the effect of the closed neutron shell N=152 on the spontaneous-fission half lives appears to be the strongest of all the known even-Z nuclei. This fact can apparently be explained by a rapid disappearance of the macroscopic part of the fission barriers with decreasing N and, accordingly, by an increased influence of the shell effects on the stability of No nuclei compared with lower-Z elements [2]. The considerable decrease of the SF half-lives of Fm and No isotopes while moving away from the N = 152 shell to lower or higher neutron numbers was explained in [42] by the lowering of the outer fission barrier below the ground state.

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