New distorted-wave impulse approximation calculation for inelastic scattering of deuterons at intermediate energies with the sudden approximation approach

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We perform a distorted-wave impulse approximation calculation of inelastic scattering of deuterons by nuclei using effective interactions in the framework of the sudden approximation at $E_d = 400$ MeV. Cross sections and spin observables are expressed in terms of amplitudes for the corresponding nucleon-nucleus scattering. The calculation is examined for the excitation of ^{12}C to the 2^+ (2.44 MeV), 3^- (9.64 MeV), and 1^+ (12.71 MeV) states and is found to give a reasonable description for most of the observables. Some discrepancies are found for the transition leading to the 1^+ state, suggesting the limitation of the applicability of the effective interaction. Contributions of the deuteron *D* state are studied. Relations between the deuteron-nucleus and proton-nucleus scattering observables are found and are studied against existing data.

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I. INTRODUCTION

Scattering of a composite projectile of weakly bound constituents by nuclei is well described by the adiabatic approximation at intermediate energies [1], since the period of the internal motion of the projectile is long enough compared to the projectile-nucleus collision time. This will allow us to describe the scattering in the framework of the impulse approximation which treats the scattering of the projectile as the individual scattering of the projectile constituents. For more details, we consider the scattering of a deuteron which is a well-known example of such weakly bound projectiles. From the above consideration, the deuteron scattering at intermediate energies will be described by the scattering of the constituent nucleons. For such descriptions, it is convenient to represent the distorted wave of the deuteron by the distorted wave of the proton and that of the neutron. Earlier, the sudden approximation was developed in this way by the use of the adiabatic approximation [2–4]. In this approximation, the deuteron distorted wave is represented by

$$\Psi^{sudden} = \int a(\boldsymbol{k})[\phi^{(+)}_{\boldsymbol{k}_p\nu_p}\phi^{(+)}_{\boldsymbol{k}_n\nu_n}]_{1\nu_d}\frac{d\boldsymbol{k}}{(2\pi)^3}\Psi_A, \qquad (1)$$

where Ψ_A denotes the target nucleus, $\phi^{(+)}_{\boldsymbol{k}_p\nu_p}$ ($\phi^{(+)}_{\boldsymbol{k}_n\nu_n}$) is the proton(neutron) wave function distorted by proton(neutron)-nucleus interactions, the symbol $[\cdots]$ describes the composition of the deuteron spin of the protons and the neutrons, and $a(\boldsymbol{k})$ is the Fourier component of the wave function of the deuteron internal motion, details of which will be given later. Here ν's are the *z* components of spins, and $\boldsymbol{k}_p$ and $\boldsymbol{k}_n$ are momenta related to the incident momentum $\boldsymbol{k}_d$ as

$$\boldsymbol{k}_p = \tfrac{1}{2}\boldsymbol{k}_d - \boldsymbol{k}, \quad \boldsymbol{k}_n = \tfrac{1}{2}\boldsymbol{k}_d + \boldsymbol{k}. \qquad (2)$$

Recently, elastic scattering of the deuteron has been investigated by this approximation at $E_d = 200$, 400, and 700 MeV for ^{40}Ca and ^{58}Ni targets [5], where the scattering amplitude of the deuteron is described in terms of those of the proton and the neutron. Calculated cross sections and vector analyzing powers have successfully reproduced measured ones to support the application of the approximation. This stimulates us to extend the theory for inelastic scattering of the deuteron, where experimental data are being accumulated including a number of spin observables [6–8].

In the conventional distorted-wave impulse approximation (DWIA) [9], the distortion of the deuteron wave is considered for the center-of-mass motion of the deuteron by assuming phenomenological optical potentials between the deuteron and the nucleus, while the excitation of the target nucleus is microscopically treated by using the *N*-*N* *t* matrix between the nucleon of the deuteron and the relevant nucleon of the target nucleus. On the contrary, in the sudden approximation, the scattering amplitude of the deuteron is composed of those of the proton and the neutron as in the elastic scattering, where the distortions are taken into account for the nucleon-nucleus scattering amplitude. Consequently, when the DWIA amplitudes for the inelastic scattering of the proton and the neutron are given, the sudden approximation provides the amplitude for the deuteron inelastic scattering, which is a new form of the DWIA amplitude for the deuteron scattering. The specific features of the new DWIA calculation are as follows. The inputs of the calculation of the deuteron scattering are given as those of the nucleon scattering: parameters of the nucleon optical potential, the initial and final wave functions of the target nucleus, and interactions

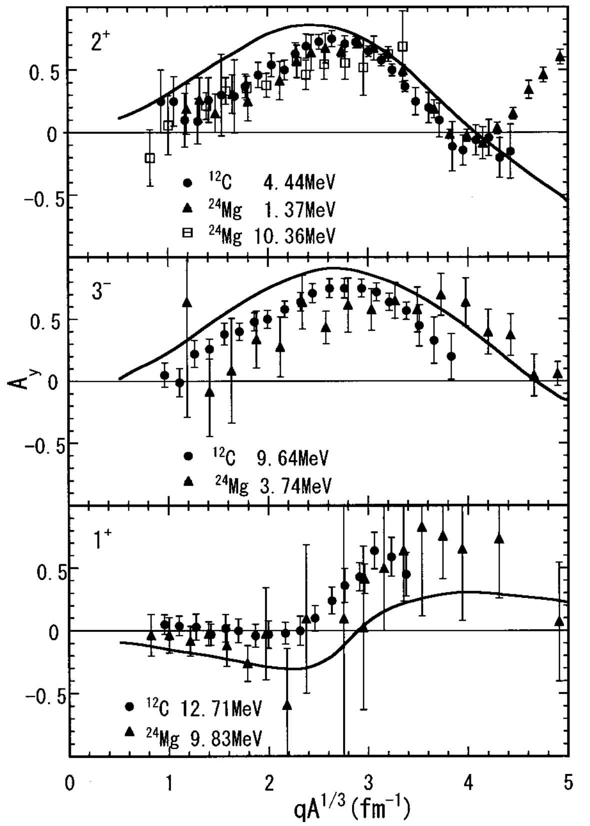

FIG. 1. Comparison of vector analyzing powers measured in (d,d') scattering at $E_d=400$ MeV with those in (p,p') scattering at $E_p=200$ MeV. The figures are taken from Ref. [10], where the solid lines represent the (p,p') mean experimental shapes for ^{12}C, ^{24}Mg, ^{28}Si, and ^{40}Ca targets.

for the excitation of the nucleus. Assume the potential parameters of the neutron to be the same as those of the proton except for the Coulomb interaction. Then, the inputs of the calculation are only those of the proton and can be examined by comparison of calculated quantities with experimental data for the proton inelastic scattering. Once these details of the proton scattering are provided, observables of the deuteron scattering are calculated uniquely in the case of the sudden approximation. Therefore, it will be interesting to compare the calculated observables with experimental data in the deuteron inelastic scattering for the criticism of the validity of the physical picture of the theory. On the other hand, such studies of the deuteron scattering will offer additional tests of the input of the proton scattering. The examination of tensor analyzing powers of (d,d') scattering is particularly valuable since it is not available in the (p,p') scattering.

It has been emphasized [10] that the vector analyzing powers measured in (d,d') scattering at $E_d=400$ MeV on light targets, for example ^{12}C and ^{24}Mg, are very similar to those in (p,p') scattering at $E_p=200$ MeV, for excitations of the nucleus to 2^+, 3^-, and 1^+ states except for at large momentum transfer, when their angular distributions are described as functions of $qA^{1/3}$ with the momentum transfer q and the target mass number A, as shown in Fig. 1. This suggests a strong correlation between the (d,d') scattering and the (p,p') one for the excitation of the same state of the nucleus. The sudden approximation will be favorable for understanding such a correlation between the analyzing powers, since the deuteron scattering amplitude is described in terms of the protons and the neutrons as was discussed above. In the present paper, we will examine as an example the ^{12}C$(d,d')^{12}$C* scattering at $E_d=400$ MeV by the sudden approximation, referring to the (p,p') one at $E_p=200$ MeV on the same target.

As in the elastic scattering, the scattering amplitude of the deuteron derived by Eq. (1) consists of two terms, namely, the single-collision term and the double-collision one. In the former, one of the constituent nucleons of the deuteron is scattered by the nucleus and the partner nucleon stays as a spectator, while in the latter the proton and the neutron are scattered successively by the nucleus. The former describes the typical impulse approximation scattering and the latter provides a correction to the approximation. In the elastic scattering, the measured cross section and analyzing powers have mostly been explained by the calculations which include only the single-collision term. Then, at present, we will also treat the single-collision term, and the correction due to double-collision effects is considered in a phenomenological way. Furthermore in the present calculation, contributions of the *D*-state admixture in the deuteron internal wave function, which have been neglected in the elastic-scattering calculation, are taken into account by the use of the recent information of the deuteron form factor obtained by analyses of electron scattering [11].

In the following sections, we will calculate the cross section σ, the vector and tensor analyzing powers, A_y and A_{yy}, the vector polarization of the emitted deuteron P^y, and the polarization transfer coefficient K_y^y, and discuss the isoscalar spin-flip signature $S_d^y(=\frac{4}{3}+\frac{2}{3}A_{yy}-2K_y^y)$ [6]. For final states of ^{12}C, we consider, as examples, two natural parity states 2^+ (4.44 MeV) and 3^- (9.63 MeV) and one unnatural parity state 1^+ (12.71 MeV). For the calculation of the inelastic-scattering amplitudes of the proton, we follow the DWIA calculation in Ref. [12], which provides satisfactory agreements with the measured cross section and vector analyzing power at $E_p=200$ MeV. For the proton optical potential, however, the parameter set taken from Ref. [13] is mostly employed in the present calculation, because it gives better agreements to the measured K_y^y providing similar agreements of σ and A_y to the previous calculation in the (p,p') scattering.

In the next section, we will derive the formulas of the scattering amplitudes and spin observables by the sudden approximation. In Sec. III, numerical results are presented and are compared with experimental data. They are also compared to the calculation by the conventional DWIA. The last section will be devoted to a summary. The details of the derivation of the formula including the deuteron *D* state are given in Appendix A, while the formulas for the spin observables are given in Appendix B. In Appendix C relations between deuteron-nucleus and nucleon-nucleus spin observables are derived within a simple approximation.

II. DERIVATIONS OF FORMULAS FOR SCATTERING AMPLITUDES AND SPIN OBSERVABLES

Following the basic idea given in the previous section, we will derive the transition matrix elements and the spin observables for the inelastic scattering of the deuteron by using the sudden approximation. The total Hamiltonian for the deuteron-nucleus system is given by

$$H = H_d + V_{dA} + H_A, \quad V_{dA} = V_{pA} + V_{nA}, \tag{3}$$

where H_d denotes the Hamiltonian of the deuteron which includes the deuteron-nucleus relative kinetic energy, H_A is the Hamiltonian of the target nucleus, and V_{pA} (V_{nA}) is the proton(neutron)-target interaction, $V_{pA} = \Sigma_{i=1}^{A} V_{pi}$ ($V_{nA} = \Sigma_{i=1}^{A} V_{ni}$). The scattering matrix for the inelastic scattering of the deuteron $A(d,d')A^*$ with the total energy E of the system is

$$T_{fi} = \langle \Psi_{A^*} \Phi_{d'} | V_{dA} | \Psi_{dA}^{(+)} \rangle, \quad \Psi_{dA}^{(+)} = (1 + D_{dA}) \Phi_{dA} \tag{4}$$

with

$$\Phi_{dA} = \Phi_d \Psi_A, \quad D_{dA} = \frac{1}{E - H + i\eta} V_{dA}, \tag{5}$$

where Ψ_A and Ψ_{A^*} denote the wave functions of the nucleus for the initial and final states. The deuteron wave function Φ_d is given by

$$\Phi_d(\boldsymbol{r}_p, \boldsymbol{r}_n) = \exp\{i \boldsymbol{k}_d \boldsymbol{r}_d\} \phi_{\nu_d}(\boldsymbol{r}), \tag{6}$$

where ϕ_{ν_d} is the internal wave function with the spin component ν_d (spin variables are omitted). The coordinates $\boldsymbol{r}_d = (\boldsymbol{r}_p + \boldsymbol{r}_n)/2$ and $\boldsymbol{r} = \boldsymbol{r}_n - \boldsymbol{r}_p$ are for the center-of-mass motion and the p-n relative one, respectively.

To avoid the complicated manipulation, we will develop the theory at present by neglecting the D-state component of the deuteron internal motion. In later numerical calculations, the D-state effect is fully included by using the formulas given in Appendix A. Introducing the Fourier expansion of Φ_d we write

$$\Phi_d(\boldsymbol{r}_p, \boldsymbol{r}_n) = \int \frac{d\boldsymbol{k}}{(2\pi)^3} a(\boldsymbol{k}) [\phi_{\boldsymbol{k}_p \nu_p}(\boldsymbol{r}_p) \phi_{\boldsymbol{k}_n \nu_n}(\boldsymbol{r}_n)]_{1\nu_d}, \tag{7}$$

where $\phi_{\boldsymbol{k}\nu}$ describes the nucleon plane wave with the momentum $\boldsymbol{k}$ and the spin component ν. In the sudden approximation, we take account of the nuclear distortions for each nucleon of the deuteron. We replace $\Psi_{dA}^{(+)}$ by $\Psi^{sudden} = \Phi_d^{sudden} \Psi_A$, where Φ_d^{sudden} is given by

$$\Phi_d^{sudden} = \int \frac{d\boldsymbol{k}}{(2\pi)^3} a(\boldsymbol{k}) [\phi_{\boldsymbol{k}_p \nu_p}^{(+)} \phi_{\boldsymbol{k}_n \nu_n}^{(+)}]_{1\nu_d}, \tag{8}$$

and $\phi_{\boldsymbol{k}_p \nu_p}^{(+)}$ ($\phi_{\boldsymbol{k}_n \nu_n}^{(+)}$) is the distorted wave due to the optical potential $U_{pA}(U_{nA})$. Equation (8) is consistent with the physical picture by the adiabatic approximation where the proton-neutron interaction is frozen while one of the nucleons interacts with the nucleus.

Now we redefine $\phi_{\boldsymbol{k}}^{(+)}$ so as to contain the spin variable as a Pauli spin matrix which operates on the deuteron spin wave function $\chi_{1\nu_d}$. Then Eq. (8) is rewritten as

$$\Phi_d^{sudden} = \int \frac{d\boldsymbol{k}}{(2\pi)^3} \phi_{\boldsymbol{k}_p}^{(+)} \phi_{\boldsymbol{k}_n}^{(+)} a(\boldsymbol{k}) \chi_{1\nu_d}, \tag{9}$$

where the order of $\phi_k^{(+)}$ and $a(k)$ is important when the deuteron D state is considered as in Appendix A. Then, the scattering matrix becomes

$$T_{fi}^{sudden} = \langle \chi_{1\nu_d'} | \hat{T}_d(\boldsymbol{k}_d', \boldsymbol{k}_d) | \chi_{1\nu_d} \rangle, \tag{10}$$

$$\hat{T}_d(\boldsymbol{k}_d', \boldsymbol{k}_d) = \int\int \frac{d\boldsymbol{k} d\boldsymbol{k}'}{(2\pi)^6} a(\boldsymbol{k}') [\hat{T}_p \hat{J}_n + \hat{T}_n \hat{J}_p] a(\boldsymbol{k}) \tag{11}$$

with

$$\hat{T}_p = \langle \Psi_{A^*} \phi_{\boldsymbol{k}_p'} | V_{pA} | \phi_{\boldsymbol{k}_p}^{(+)} \Psi_A \rangle, \quad \hat{J}_n = \langle \phi_{\boldsymbol{k}_n'} | \phi_{\boldsymbol{k}_n}^{(+)} \rangle, \tag{12}$$

and $\hat{T}_n$ and $\hat{J}_p$ are similarly defined. Here $\hat{T}$ and $\hat{J}$ are the operators in the nucleon spin space. The first one of Eq. (12) can be transformed by taking account of U_{pA}, the optical-potential part of V_{pA}, for the final-state proton wave function [14],

$$\hat{T}_p = \langle \Psi_{A^*} \phi_{\boldsymbol{k}_p'}^{(-)} | V_p^{eff} | \phi_{\boldsymbol{k}_p}^{(+)} \Psi_A \rangle, \quad V_p^{eff} = V_{pA} - U_{pA}, \tag{13}$$

which gives the distorted-wave Born approximation (DWBA) transition amplitude for the $A(p,p')A^*$ scattering when calculated on the energy shell.

As the first step of the calculation, we will extract the impulse-approximation term by assuming

$$\hat{J}_n = (2\pi)^3 \delta(\boldsymbol{k}_{n'} - \boldsymbol{k}_n). \tag{14}$$

This restricts the transition to the single collision; that is, the neutron(proton) remains as a spectator while the proton(neutron) interacts with the nucleus. More precise treatments of $\hat{J}_n$ or $\hat{J}_p$ produce the double-collision term [5]. Later, the correction to the approximation by Eq. (14) will be considered by taking account of the effect of recoil of the neutron(proton). Using Eq. (14), one can eliminate the integral over $\boldsymbol{k}'$ in Eq. (11).

For further developments, we will follow the prescription given in Ref. [5]. We take out the matrix element $\hat{T}_p$ or $\hat{T}_n$ from the integral over $\boldsymbol{k}$, replacing the matrix element by the on-energy-shell one at $\boldsymbol{k}=0$, say, $\overline{T_p}$ or $\overline{T_n}$, which gives the dominant contribution to the integral. Due to this simplification, the deuteron scattering amplitude is described by the nucleon scattering one, where the incident nucleon energy is a half of the deuteron incident one but the momentum transfer of the nucleon is the same as that of the deuteron. The

TABLE I. Correspondence of the operators in the deuteron spin space and those in the nucleon spin space.

$\mathcal{P}$	$\overline{\mathcal{P}}=P_1\mathcal{P}P_1$
1	$P_1=(3+\boldsymbol{\sigma}_n\boldsymbol{\sigma}_p)/4$
$\mathcal{P}_i=S_i$	$(\sigma_{ni}+\sigma_{pi})/2(=S_i)$
$\mathcal{P}_{ij}=\frac{3}{2}(S_iS_j+S_jS_i)-\frac{1}{2}\delta_{ij}$	$\frac{3}{4}(\sigma_{ni}\sigma_{pj}+\sigma_{nj}\sigma_{pi})-\frac{1}{2}\delta_{ij}\boldsymbol{\sigma}_n\boldsymbol{\sigma}_p$

validity of this simplification will be numerically examined in the next section. Then we get

$$\hat{T}_d=F(q)[\overline{T}_p+\overline{T}_n],\quad F(q)=\int d\boldsymbol{r}|\phi_d(\boldsymbol{r})|^2\exp\{-i\boldsymbol{q}\boldsymbol{r}/2\},\tag{15}$$

where $\boldsymbol{q}=\boldsymbol{k}_d-\boldsymbol{k}_d'$ is the momentum transfer and $F(q)$ is the form factor of the deuteron.

We now consider the formulas of spin observables in the present theory. A general expression for the spin observable K for the polarized deuteron beam with the unpolarized target has the form

$$K=\mathrm{Tr}[\hat{T}\mathcal{P}\hat{T}^\dagger\mathcal{P}']/I_0,\tag{16}$$

where $I_0=\mathrm{Tr}[\hat{T}\hat{T}^\dagger]$, Tr denotes the trace in the deuteron spin space and summation over the target magnetic substates, and $\mathcal{P}$ describes a 3×3 matrix in the deuteron spin space [15]. Since the present calculation is performed in the nucleon spin space, we will transform Tr into the trace operation in the nucleon spin space, tr, by introducing the projection operator on the spin triplet state, $P_1=(3+\boldsymbol{\sigma}_p\boldsymbol{\sigma}_n)/4$ as

$$K=\mathrm{tr}[\hat{T}\overline{\mathcal{P}}\hat{T}^\dagger\overline{\mathcal{P}'}]/I_0,\quad \overline{\mathcal{P}}=P_1\mathcal{P}P_1.\tag{17}$$

This relation, derived in Appendix B, can be understood by noting that the unity in the deuteron spin space, $\Sigma|\chi_{1\mu}\rangle\langle\chi_{1\mu}|$, becomes a projection operator P_1 in the nucleon spin space. Table I shows the correspondence of relevant operators in the spin space. Thus, for instance, the vector analyzing power of the deuteron is given by

$$I_0A_y=|F(q)|^2\mathrm{tr}\left[(\overline{T}_p\cdot\boldsymbol{1}_n+\boldsymbol{1}_p\cdot\overline{T}_n)\frac{\sigma_{py}+\sigma_{ny}}{2}\times(\overline{T}_p^\dagger\cdot\boldsymbol{1}_n+\boldsymbol{1}_p\cdot\overline{T}_n^\dagger)\frac{3+\boldsymbol{\sigma}_p\boldsymbol{\sigma}_n}{4}\right],\tag{18}$$

where $\boldsymbol{1}_p(\boldsymbol{1}_n)$ is the unit matrix in the proton(neutron) spin space. The above expression consists of two kinds of terms, namely, the direct terms and the cross ones. The former terms contain $\overline{T}_a$ and $\overline{T}_a^\dagger$ $(a=p,n)$ and are expressed in terms of the spin observables of the nucleon scattering, while the latter terms describe the interference between the proton amplitude $\overline{T}_p$ and the neutron one $\overline{T}_n$ and are not expressed by the nucleon spin observables. The cross terms actually give contributions of a similar size as the direct ones in the present calculation of deuteron spin observables.

Under the assumption of a genuine isoscalar property of the transition, i.e., a similarity of the proton and neutron transition matrix elements, the cross terms give an equal contribution as do the direct ones as proven in Appendix C. The assumption is rather well satisfied in the present calculation except at forward angles where Coulomb effects are significant. Thus, in this case, the deuteron-nucleus spin observables can be expressed entirely in terms of the nucleon-nucleus observables, the details of which are given in Appendix C. In particular, one can derive a relation between the vector analyzing power of the deuteron-nucleus scattering and the analyzing power/polarization of the proton-nucleus scattering as

$$A_y(d)=f(q)\,\tfrac{1}{4}\{3A_y(p)+P^y(p)\},$$

$$f(q)\equiv\frac{8}{3}\left(\frac{k_f}{k_i}\right)_d\left(\frac{k_i}{k_f}\right)_p\left(\frac{\mu_d}{\mu_p}\right)^2\frac{\sigma(p)}{\sigma(d)}F(q)^2,\tag{19}$$

where μ is the reduced mass, and d and p discriminate the projectile. The above formula is numerically examined in the next section and is found to give the qualitative explanation of the similarity between the deuteron analyzing power and the proton one experimentally observed (Fig. 1). As is shown in Appendix C, one can also derive a relation among the spin observables and the cross sections for the assumption of the isoscalar scattering amplitudes,

$$R\equiv[\tfrac{4}{3}+\tfrac{2}{3}A_{yy}(d)-K_y^y(d)]/f(q)=1,\tag{20}$$

which may be used as another test of the validity of the sudden approximation. This relation, too, will be studied below in our calculation and also in the experimental data.

III. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTS

As was discussed in the preceding sections, the observables in the (d,d') scattering are described by the amplitudes of the corresponding (p,p') and (n,n') scattering at the energy half of the deuterons. The (n,n') amplitudes are approximated by the (p,p') ones except for the Coulomb contribution. For the (p,p') calculation, we follow the one in Ref. [12]. In practice, the numerical calculations of the (p,p') and (n,n') scattering amplitudes are performed by the use of the computer code DWBA91 [16]. The wave functions of ^{12}C are taken from Ref. [17] for the positive-parity states and from Ref. [18] for the negative-parity ones and the interactions for the excitation of the nucleus are assumed to be the effective interactions given in Ref. [19]. The interactions were presented as the local equivalent of the N-N t matrix and then the DWBA calculation with such interactions may effectively give an equivalent of the DWIA calculation. The interactions between the nucleon 1 of the deuteron and the nucleon 2 of the target nucleus are given as

$$V_{12}^{eff}=V_{12}^{cent}+V_{12}^{so}+V_{12}^{tens},\tag{21}$$

where

TABLE II. Parameter set 1 and set 2 of the optical potentials for protons used in the calculation.

	Set 1[a]	Set 2[b]
V	-13.4 MeV	-4.87 MeV
r_0	1.20 fm	1.41 fm
a	0.643 fm	0.34 fm
W	-12.8 MeV	-16.5 MeV
r'	1.20 fm	1.05 fm
a'	0.637 fm	0.68 fm
$V_{\rm so}$	-13.4 MeV	-10.68 MeV
$r_{\rm so}$	0.93 fm	0.91 fm
$a_{\rm so}$	0.47 fm	0.52 fm
$W_{\rm so}$	0	-11.8 MeV
$r_{\rm wso}$	0	0.91 fm
$a_{\rm wso}$	0	0.52 fm

[a]Reference [12].
[b]Reference [13].

$$V_{12}^{cent}=V_0+V_\sigma\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_\tau\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+V_{\sigma\tau}(\boldsymbol{\sigma}_1\cdot\sigma_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2), \tag{22}$$

$$V_{12}^{so}=(V_{LS}+V_{LS\tau}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\boldsymbol{L}\cdot\boldsymbol{S}, \tag{23}$$

$$V_{12}^{tens}=(V_T+V_{T\tau}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)S_{12}, \tag{24}$$

and S_{12} is a spin-tensor operator. The form factors of the interactions are functions of r_{12}, where V^{cent} and V^{so} consist of sums of Yukawa functions and V^{tens} sums of ${r_{12}}^2\times$Yukawa functions. More details are given in Ref. [19]. For the proton optical potential, Ref. [12] used the parameters, set 1, of Table II with usual form factors. However, we will use the other ones, set 2, in the table [13] and the results are compared with those by the use of set 1 in some cases. The comparison of the calculated cross section σ and vector analyzing power A_y with the measured ones for the (p,p') scattering at $E_p=200$ MeV is shown in Fig. 2, where the angular range is limited by the momentum transfer $q=0-2.5\ \text{fm}^{-1}$, which corresponds to the angular range treated in the present (d,d') scattering. There the parameter sets 1 and 2 give almost similar results with satisfactory agreements with the experimental data. In detail, however, several discrepancies are seen between the calculated and the measured. The calculations underestimate the cross section at larger q in the 2^+-state and 3^--state transitions and overestimate that at smaller q in the 1^+- state transition. The angular distribution of the calculated analyzing power is shifted toward larger q compared to the data as seen typically in the 2^+-state transition. The calculated $K_y^y(p)$ will be discussed later together with the calculated $K_y^y(d)$ and $S_d^y(d)$ in the (d,d') scattering.

Figures 3, 4, and 5 show the results of the present calculation for the (d,d') scattering to the 2^+, 3^-, and 1^+ states, respectively, where set 2 is employed for the optical-potential parameters. The calculation is performed in two ways—with and without the deuteron D state. Both calculations reproduce the essential features of the experimental data [9], except for those of A_{yy} in the 1^+ state transition. In more detail, the calculated cross sections agree with the measured ones for their global shape of the angular distribution, although the magnitude of the cross section for 1^+ state is larger than that of the measured one. The calculated A_y shows similar features to those in the (p,p') scattering which was discussed above. That is, the angular distributions of the calculated A_y for three states have a tendency to shift toward larger angles compared to the data. The shift of A_y is enhanced for the 1^+-state transition although the characteristic of the shape of the calculated angular distribution is still consistent with the general shape of A_y in Fig. 1 which includes the data for both of the ^{12}C and ^{24}Mg targets. Such a similarity of the characteristics between the (p,p') scattering and the (d,d') one suggests that some of the discrepancies between the calculated and the measured in the (d,d') scattering will be solved by improving the input of the (p,p') calculation so as to fit the data. The agreement with the data is quite poor in the A_{yy} calculation for the 1^+-state transition. The use of the effective interaction will be responsible for this discrepancy, as will be discussed later in detail.

In the figures, contributions of the deuteron D state are mostly small for σ, A_y, and P^y but appreciable for A_{yy}. The calculated A_{yy} is decreased in a wide angular range by including the D state, remarkably for the 2^+-state and 3^--state transitions. The quantities K_y^y and S_d^y at larger angles are affected by the D state. This will be due to the spin flip by tensor interactions associated with the D state. Unfortunately,

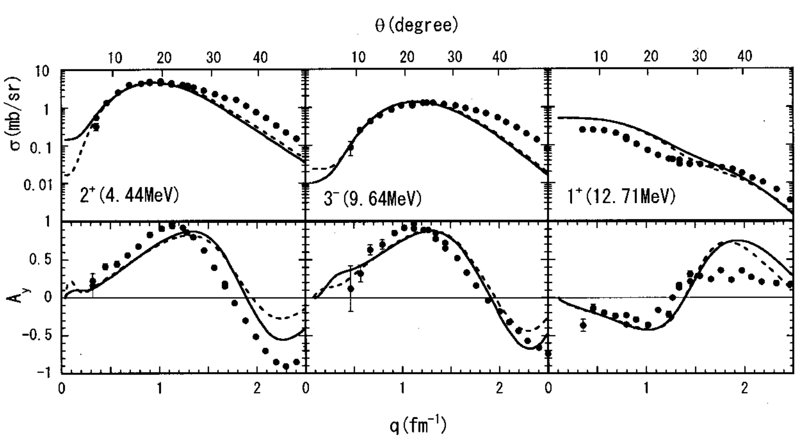

FIG. 2. Cross sections and vector analyzing powers in (p,p') scattering at $E_p=200$ MeV. The solid (dashed) lines are obtained by the potential parameters of set 1 (2) in Table II. The experimental data are taken from Ref. [12]. For the abscissa of the figure, the scale is represented by the scattering angle θ on the top and by the momentum transfer q in the bottom.

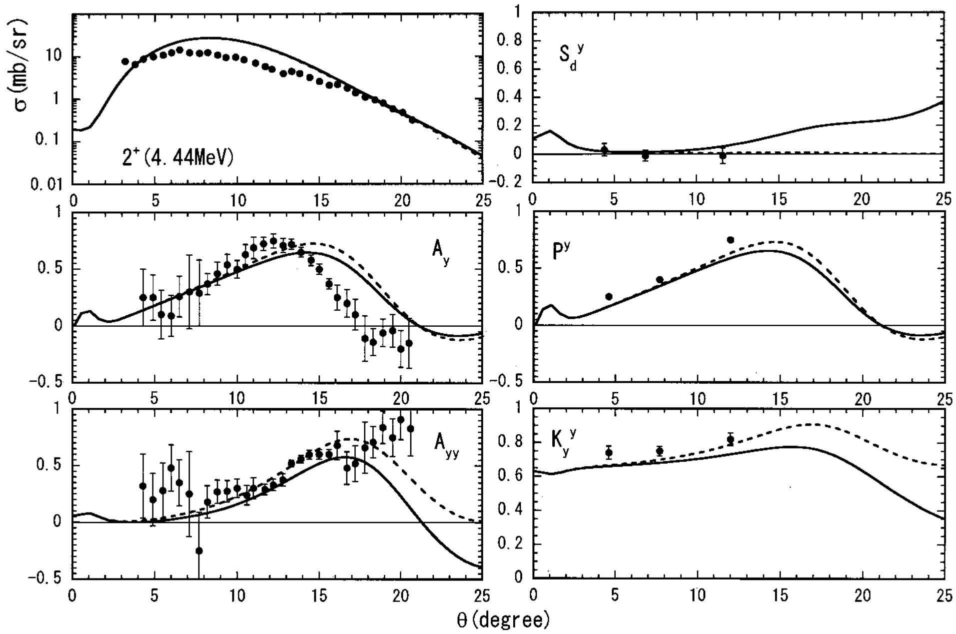

FIG. 3. Cross section and spin observables for $^{12}C(d,d')^{12}C(2^+$, 4.44 MeV) at $E_d=400$ MeV. The calculated quantities by the sudden approximation with the potential parameters of set 2 are compared with the experimental data (Ref. [9]). The solid lines include both the *S* and *D* states of the deuteron internal motion. The dashed lines include only the *S* state.

experimental data of these quantities are not available at large angles. Attempts at experimental observation of such effects will be interesting.

The calculations for the (d,d') scattering by the potential parameters of set 1 are not displayed in the figures since they are very similar to those by the parameters of set 2, except for K_y^y and then S_d^y in the 1^+-state transition. The comparisons of the calculated S_d^y and K_y^y between sets 1 and 2 in the 1^+-state transition are shown in Figs. 6(a) and 6(b), respectively, where the differences between the calculations are appreciable and the calculation is improved by using set 2, although by small amounts. In Fig. 6(c), we show K_y^y calculated by sets 1 and 2 for the (p,p') scattering to the 1^+ state, where the calculation by set 2 produces better agreement with the data than that by set 1. The difference between two calculations in K_y^y of the (d,d') scattering is apparently the reflection of the difference in the (p,p') scattering described above.

Figures 7, 8, and 9 show the correction due to the double-collision effect which is calculated according to the prescription described in Ref. [5]. One of the effects of the double-collision is to share the momentum transfer to the nucleus between the constituents of the deuteron. Although we neglect the double-collision term discussed in Sec. II, such sharing effects are taken into account by decreasing the momentum transfer of the proton or the neutron by small amounts. For the proton for example, the momentum transfer $\boldsymbol{q}_p$ is related to that of the deuteron $\boldsymbol{q}$ as

$$\boldsymbol{q}=(1-\alpha)\boldsymbol{q}_p, \tag{25}$$

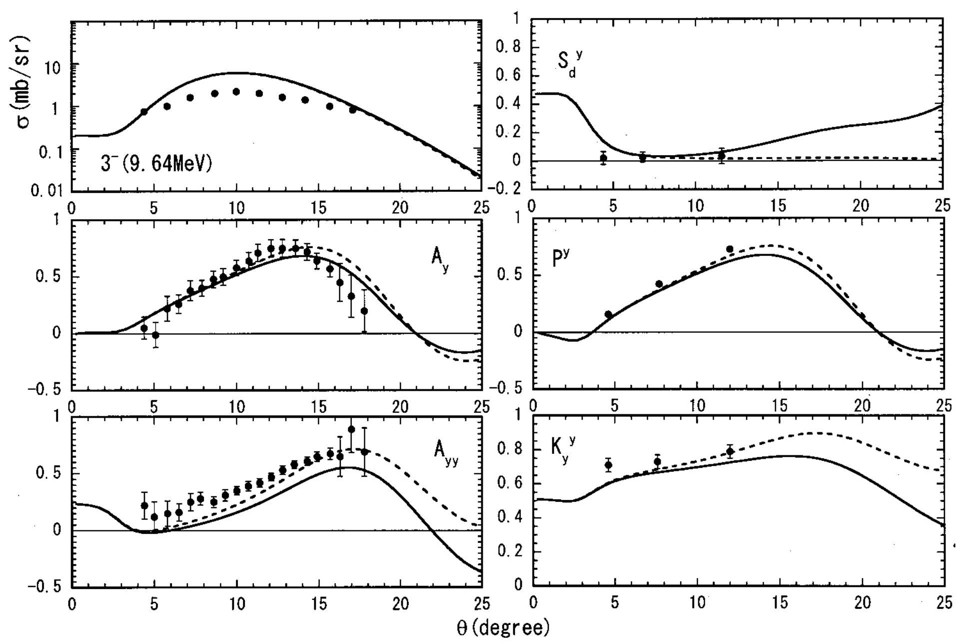

FIG. 4. Cross section and spin observables for $^{12}C(d,d')^{12}C(3^-$, 9.64 MeV) at $E_d=400$ MeV. See also the caption for Fig. 3.

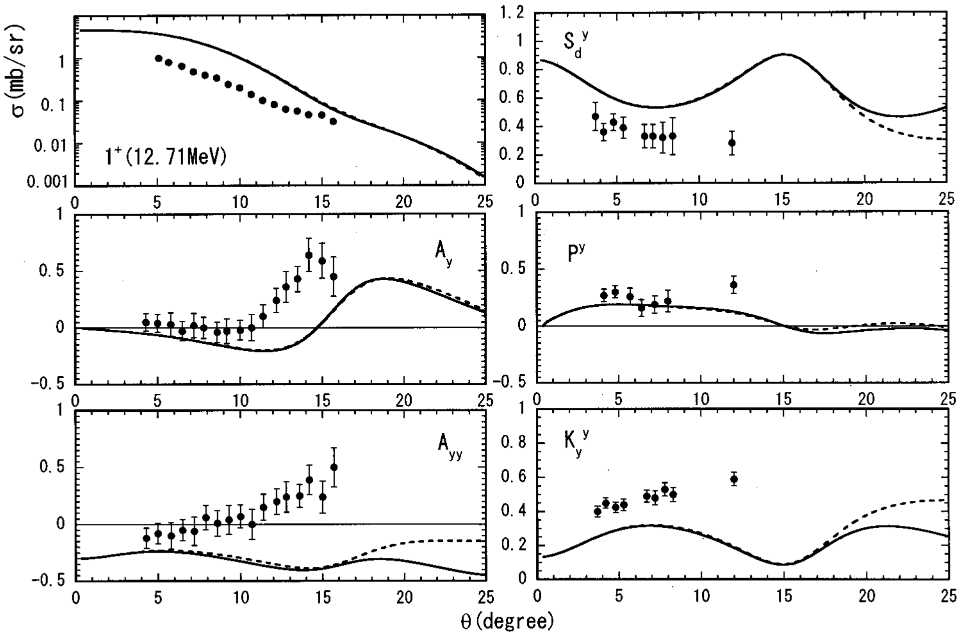

FIG. 5. Cross section and spin observables for ^{12}C(d,d')^{12}C$(1^+$, 12.71 MeV) at $E_d = 400$ MeV. See also the caption for Fig. 3.

where $-\alpha \boldsymbol{q}_p$ is the momentum transfer by the participating neutron [5]. In the elastic scattering, the choice of $\alpha = 0.07$ has produced good agreements with experimental data for the cross section and vector analyzing power. Then we adopt the same magnitude for α, to avoid ambiguities induced by arbitrary choices of α. As seen in the figures, the correction due to this effect produces appreciable improvements of the calculation for most observables of the (d,d') scattering.

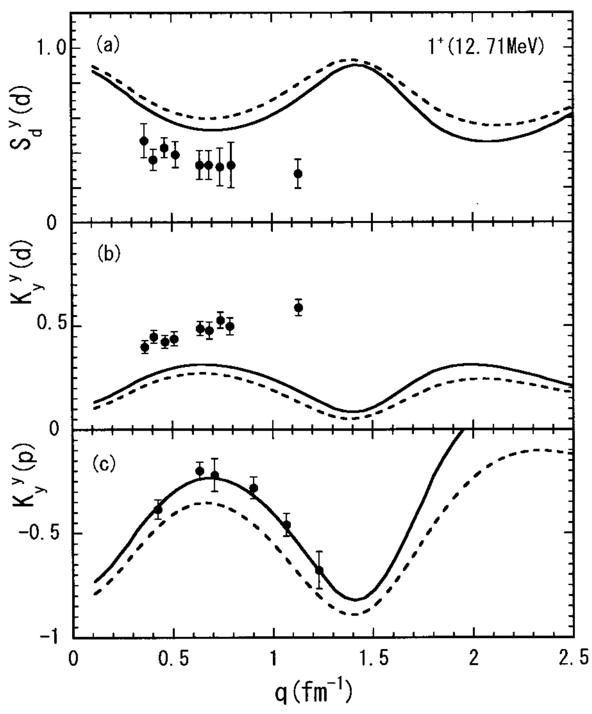

FIG. 6. $S_d^y(d)$ and $K_y^y(d)$ in (d,d') scattering at $E_d = 400$ MeV and $K_y^y(p)$ in (p,p') scattering at $E_p = 200$ MeV. The calculations by the parameters of set 1 (dashed lines) and set 2 (solid lines) are compared with the experimental data of S_d^y (a), K_y^y (b) in the (d,d') scattering, and those of K_y^y in the (p,p') one (c) for the 1^+ state.

Further, in Figs. 10, 11, and 12, we will compare our results with those by the conventional DWIA calculation [9], where the nuclear wave functions of ^{12}C are taken from the same source as ours. In the global viewpoint, both calculations have an almost similar quality in the agreement with the data for the 2^+-state and 3^--state transitions. However, the differences between the two calculations are quite remarkable in the 1^+-state transition. In comparison with the data, the calculated P^y by the present theory is better than the one by the conventional calculations, which has the opposite sign to that of the measured. On the other hand, for A_{yy} and K_y^y, the conventional calculation is successful in reproducing the data but the present calculation is not at most angles. To investigate the origin of this difference, we compare both calculations in the plane-wave (PW) limit, neglecting the distortions of the incident and outgoing waves. The comparison is shown, for example for σ, A_y, and A_{yy}, in Fig. 13. There, the calculated quantities are displayed for the 2^+-state and 1^+-state transitions for the convenience of the comparison between the natural parity transition and the unnatural parity one. In the 2^+-state transition, both calculations, conventional and present, provide almost similar results except for A_y and A_{yy} at larger angles. In the 1^+-state transition, however, two calculations produce remarkable differences in all of σ, A_y, and A_{yy}. In particular, A_{yy} by the present calculation has large negative values at all angles concerned, while the conventional one increases with the angle from small negative values at small angles to small positive ones up to $\theta \approx 14°$. This behavior of A_{yy} of the conventional calculation reproduces the global shape of the angular distribution of the measured A_{yy}. The features of A_{yy} of the distorted-wave calculations in Fig. 12 will be understood as the reflection of the above characteristics of A_{yy} in the PW calculation. Since in the PW limit the incident and final waves are described by the free deuteron in both calculations, the difference between two calculations is only in the treatment of the interactions for the excitations of the

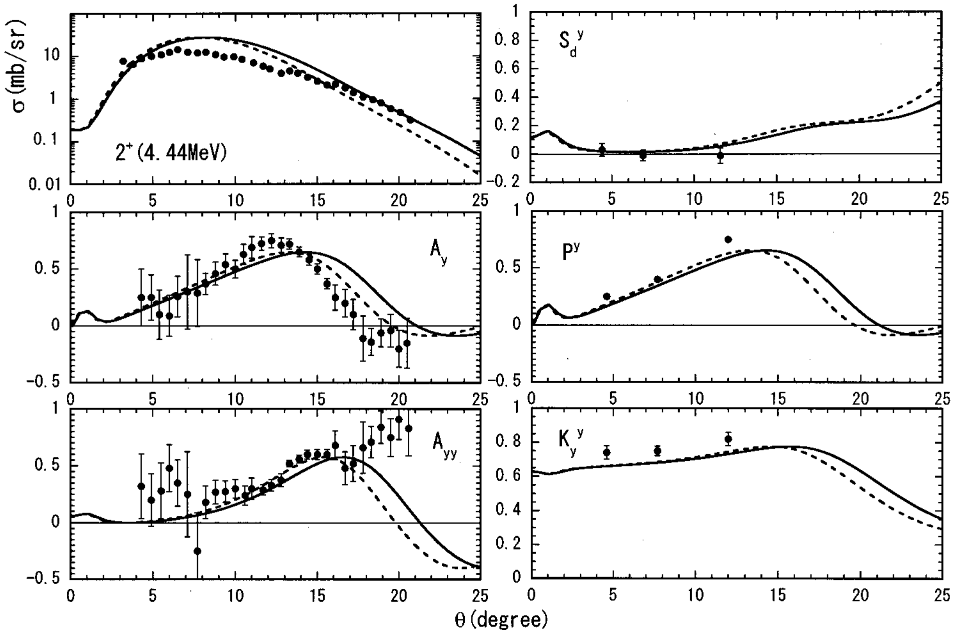

FIG. 7. Cross section and spin observables for $^{12}C(d,d')^{12}C(2^+$, 4.44 MeV) at $E_d=400$ MeV. The solid lines are calculated by the sudden approximation with the potential parameters in set 2 and the dashed ones include the double-collision effect. The experimental data are taken from Ref. [9].

nucleus. Therefore, the present analysis indicates that the *N-N t* matrix provides a better description of A_{yy} than the local equivalent interaction in the inelastic scattering to the 1^+ state. As was discussed in Ref. [20], the defect of the present effective interactions will be related to the procedure of the simulation of the empirical *N-N t* matrix by the local potentials, particularly for exchange scattering effects. Since the difference between the calculations by the *N-N t* matrix and the effective interaction is rather minor in the 2^+-state transition, the observables in the 1^+-state transition, particularly A_{yy}, will provide a critical examination of the validity of the interactions.

In Figs. 14 and 15, we study the validity of the approximate formula

$$A_y(d) \simeq f(q)\,\tfrac{1}{4}\{3A_y(p)+P^y(p)\} \simeq f(q)A_y(p), \qquad (26)$$

which was given in Sec. II, where the last similarity is obtained under the assumption

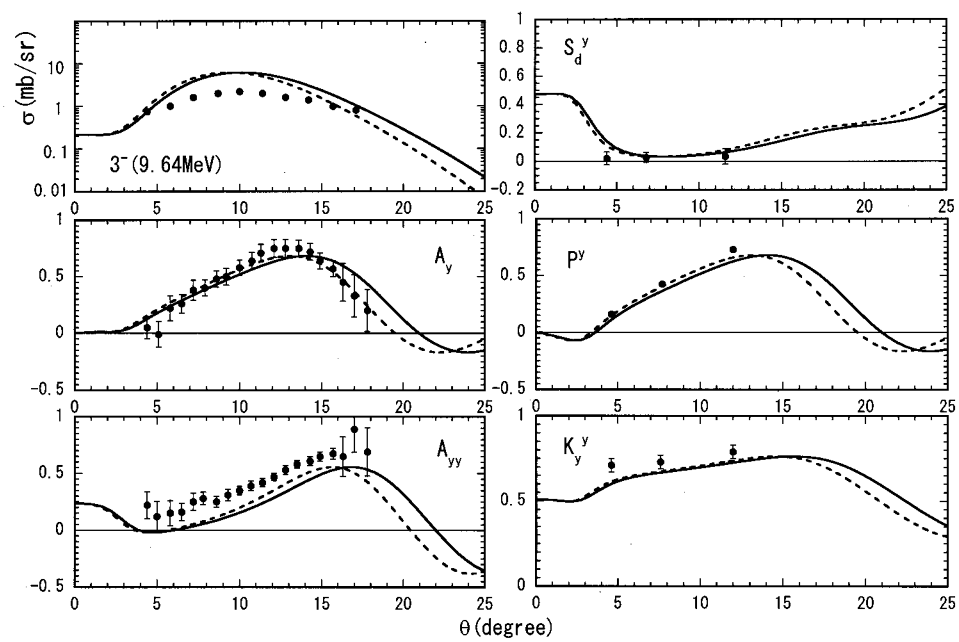

FIG. 8. Cross section and spin observables for $^{12}C(d,d')^{12}C(3^-$, 9.64 MeV) at $E_d=400$ MeV. See also the caption for Fig. 7.

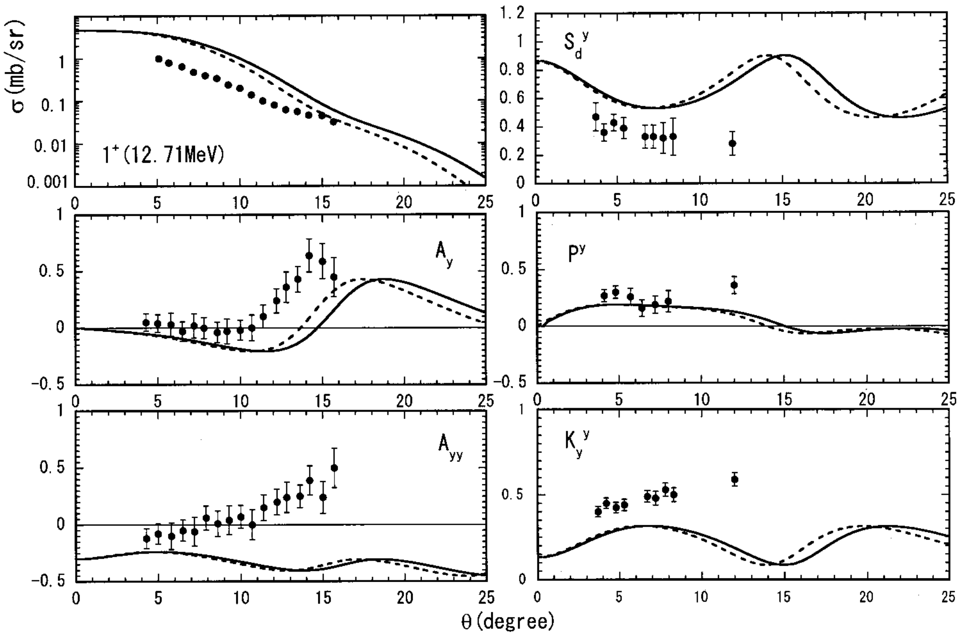

FIG. 9. Cross section and spin observables for ^{12}C$(d,d')^{12}$C$(1^+$, 12.71 MeV) at $E_d = 400$ MeV. See also the caption for Fig. 7.

$$P^y(p) = A_y(p). \tag{27}$$

The relation, Eq. (27), is valid for non-spin-flip transitions within the adiabatic approximation [21], the validity of which is numerically confirmed in the present case. In Figs. 14(a) and 14(b), $A_y(p)$ and $P^y(p)$ calculated without the approximation are shown for the 2^+-state transition and the 3^--state one, respectively, where both quantities are almost overlapped except at small q. For the spin-flip 1^+ state the relation (27) is largely violated as shown in Fig. 14(c). It is noted that the present (d,d') calculation reproduces this violation to some extent. In Fig. 15, $A_y(d)$ calculated by the first equality of Eq. (26) is compared with the realistic calculation in Figs. 15(a), 15(b), and 15(c) for the 2^+, 3^-, and 1^+ cases, respectively, showing the equality to be good approximations for all transitions. The second relation of Eq. (26) is examined in Figs. 15(d), 15(e), and 15(f) for the 2^+, 3^-, and 1^+ transitions, respectively. The right-hand side of the relation is a good description of the left-hand side for the former two transitions, while it is a rather poor one for the 1^+-state transition reflecting the violation of Eq. (27) for the 1^+ case. These will give the understanding as to why the measured A_y in the deuteron scattering resembles that in the proton scattering though in less of a grade for the 1^+ transition.

In Fig. 16 the calculated values of the ratio R given by Eq. (20) for the 2^+, 3^-, and 1^+ states are compared with the prediction by the assumption of the isoscalar scattering amplitude. The calculation gives for most angles

$$R \approx 1 \tag{28}$$

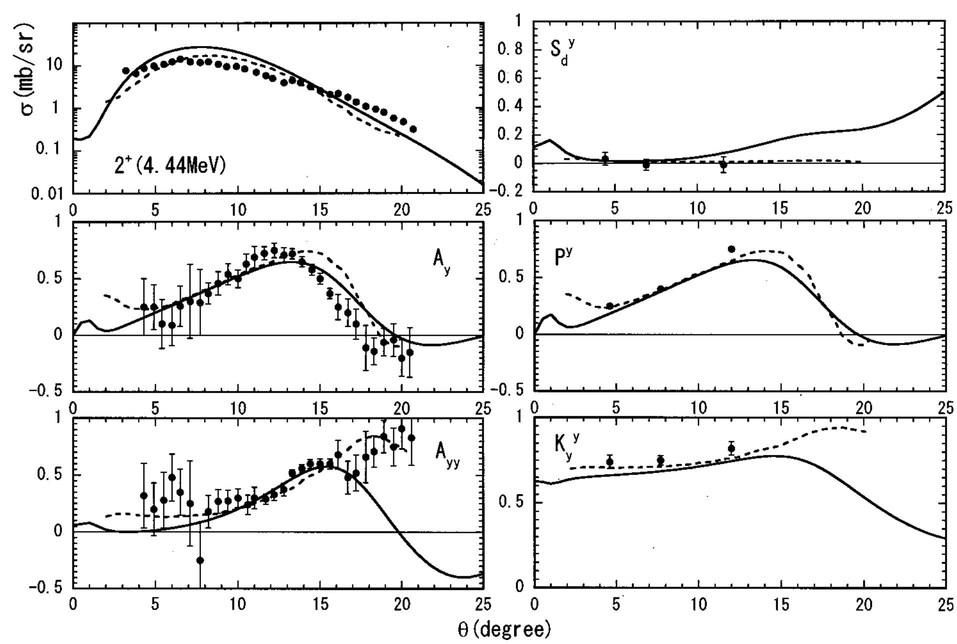

FIG. 10. Comparison between the calculations by the sudden approximation and those by the conventional DWIA for ^{12}C$(d,d')^{12}$C$(2^+$, 4.44 MeV) at $E_d = 400$ MeV. The solid lines are calculated by the sudden approximation with the parameters in set 2, including the D state contribution and the double-collision correction. The dashed ones are the conventional DWIA calculation taken from Ref. [9].

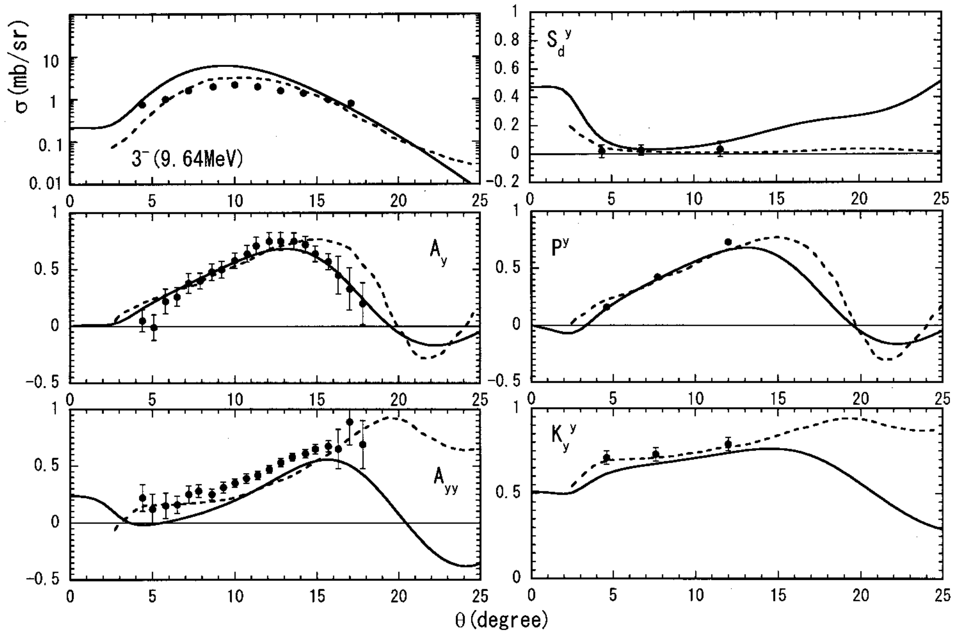

FIG. 11. Comparison between the calculations by the sudden approximation and those by the conventional DWIA for ^{12}C(d,d')^{12}C$(3^-$, 9.64 MeV) at $E_d=400$ MeV. See also the caption for Fig. 10.

and shows that the assumption leading to Eq. (20) is well satisfied except at forward angles where the Coulomb distortion effect is important. We also plot R obtained by using the empirical data for $A_{yy}(d)$ and $K_y^y(d)$, which is very close to 1 in agreement with the theoretical prediction for all of the transitions. This will support the sudden approximation approach. In the 1^+-state transition, each of the calculated A_{yy} and K_y^y shows considerable deviation from the measured one. Then the above result means that significant cancellations occur between the calculated A_{yy} and K_y^y in Eq. (20) for the transition. This indicates that the relation (20) provides a test for the rather inclusive nature of the theory.

Finally, we will examine the validity of the approximation used in Sec. II to remove the scattering matrix elements outside the integral on k. For the matrix element of the proton scattering for example, we vary artificially the final proton momentum by up to 20%. Such variations produce only very small changes in the magnitude of the matrix element except at small scattering angles. Then the off-shell matrix element will be replaced by the on-shell one in a good approximation. Further, the momentum dependence of the on-shell matrix element is studied. The increase of the incident momentum of the proton by 10% induces the increase of the magnitude of the matrix element by about 7% at the maximum. This is

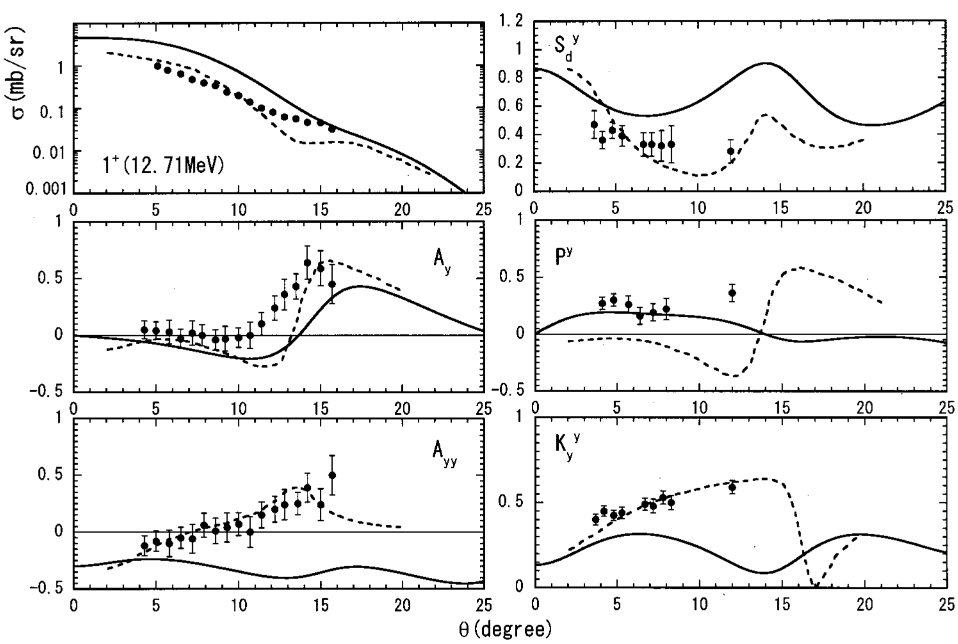

FIG. 12. Comparison between the calculations by the sudden approximation and those by the conventional DWIA for ^{12}C(d,d')^{12}C$(1^+$, 12.71 MeV) at $E_d=400$ MeV. See also the caption for Fig.10.

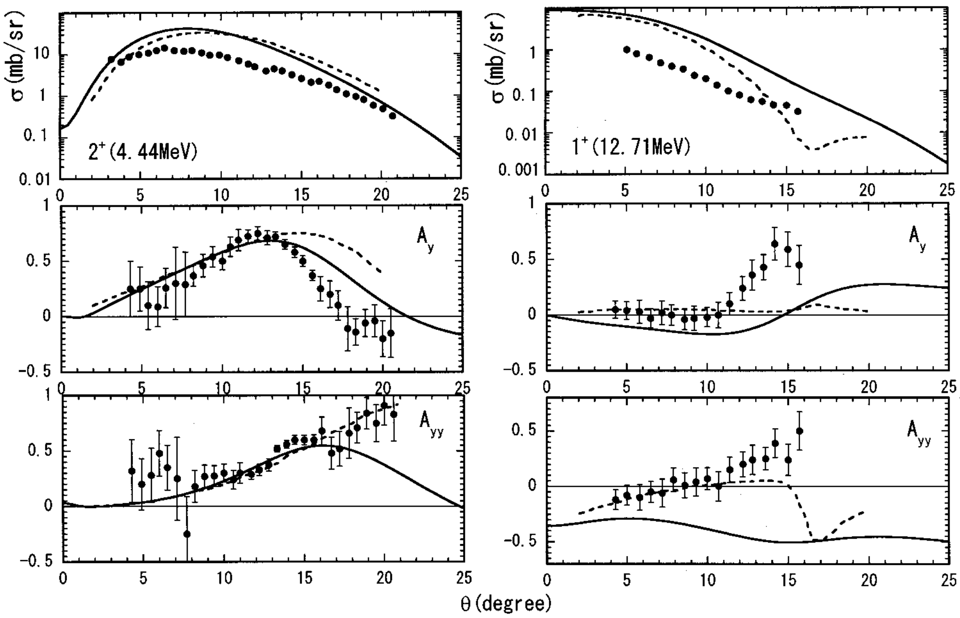

FIG. 13. Comparison between calculations by the effective interactions and those by the *N-N t* matrix for ^{12}C$(d,d')^{12}$C$(2^+$, 4.44 MeV and 1^+, 12.71 MeV) reactions at $E_d=400$ MeV in the plane-wave limit. The solid lines are for the calculations by the effective interactions and the dashed ones for those by the *N-N t* matrix. The latter calculations are taken from Ref. [9].

a slow variation compared to the momentum dependence of another factor inside the k integral, the form factor of the deuteron. These indicate that the removal of the scattering matrix element outside the k integral will not induce significant errors in the calculation.

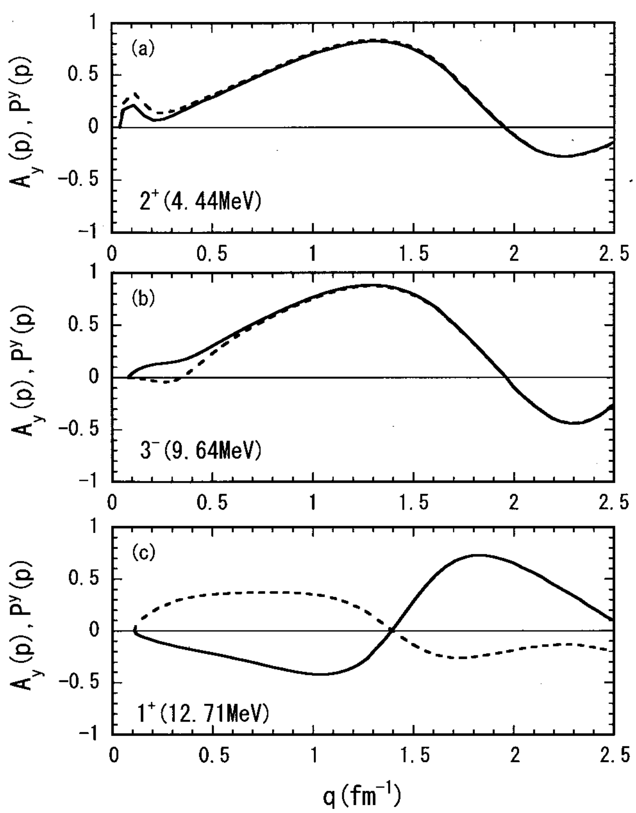

FIG. 14. Comparison between A_y and P^y for ^{12}C(p,p') at $E_p=200$ MeV. The solid $[A_y(p)]$ and dashed $[P^y(p)]$ lines are calculated by the DWIA and are for the (a) 2^+-, (b) 3^--, and (c) 1^+-state transitions.

IV. SUMMARY

A new DWIA calculation of the (d,d') scattering at intermediate energies is presented by the use of the sudden approximation, where the distortions of wave functions are considered for each proton and neutron of the deuteron. The transition amplitude of the (d,d') scattering is described in terms of the (p,p') and (n,n') amplitudes, which explain the correlation between the (d,d') vector analyzing powers and the (p,p') ones experimentally observed for light targets.

The numerical calculations are carried out at $E_d=400$ MeV for the ^{12}C target, for which the 2^+ (4.44 MeV), 3^- (9.64 MeV), and 1^+ (12.71 MeV) states are considered as the final states. We follow the calculation in Ref. [12] for the (p,p') amplitudes, for which the effective interactions [19] are employed for the excitation of the nucleus. The calculation describes well most of the measured quantities, σ, A_y, P^y, A_{yy}, K_y^y, and S_d^y, and it is found that the characteristics of the calculated A_y and K_y^y are the reflection of those of the corresponding quantities in the (p,p') scattering. The contribution of the D-state component of the deuteron internal wave function is found to be small for σ, A_y, and P^y but appreciable for A_{yy}, K_y^y, and then S_y^d.

The results of the calculation are compared with those by the conventional DWIA where the *N-N t* matrix is used for the excitation of the nucleus. The quality of the agreement with the data is almost similar for both calculations in the transitions to the 2^+ and 3^- states but is different in the transition to the 1^+ state. This indicates that some spin observables in the 1^+-state transition are sensitive to the details of the calculation due to the spin structure of the nuclear wave function.

The results in the PW limit by the present theory and the conventional one are compared to each other for the 2^+-state and 1^+-state transitions. The angular distribution of A_{yy} in the 1^+-state transition by the present calculation is quite dif-

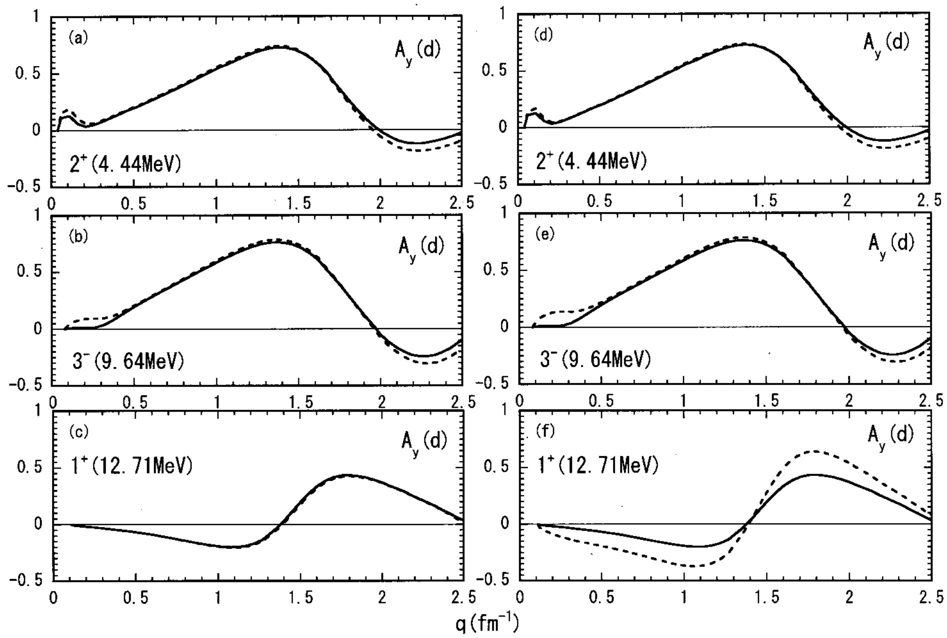

FIG. 15. Validity of the approximate formula for $A_y(d)$. (a), (b), and (c) plot the quantity $f(q)[3A_y(p)+P^y(p)]/4$ against $A_y(d)$ for the 2^+, 3^-, and 1^+ states, respectively, while (d), (e), and (f) plot $f(q)A_y(p)$ for the three states in the same manner.

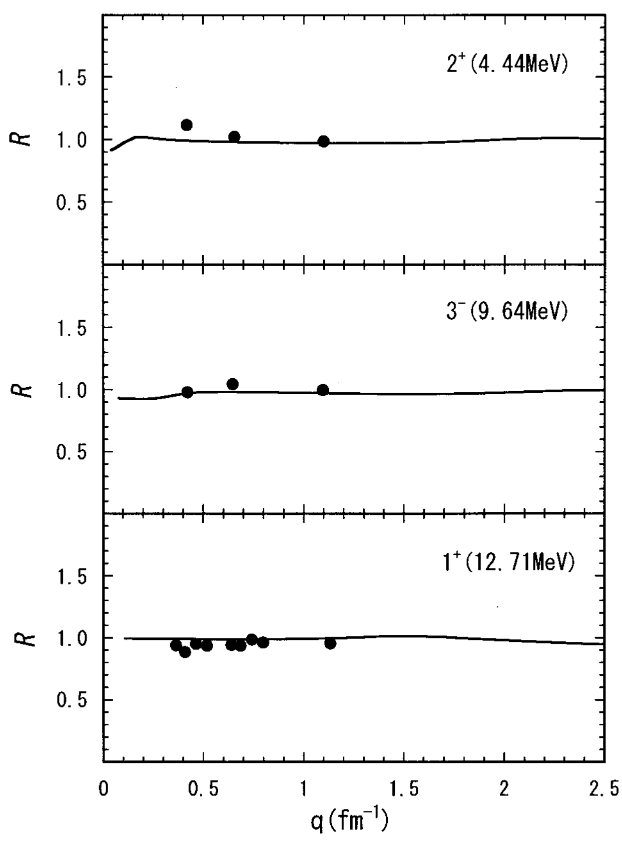

FIG. 16. Ratio R for the 2^+, 3^-, and 1^+ states of ^{12}C calculated by the sudden approximation and the one obtained by the use of the experimental data for A_{yy} and K_y^y.

ferent from that by the conventional calculation which is close to the measured one. This suggests the applicability of the present effective interaction to be substantially limited. Furthermore, in general, spin observables in the excitation of the 1^+ state by the deuteron scattering will provide a profitable examination of the nucleon-nucleon interaction, which is not available in the proton inelastic scattering. It will also be interesting to investigate if similar effects are observed in the excitation of other unnatural parity states.

Finally, it is concluded that the sudden approximation is a useful tool to describe the deuteron distorted wave and then examinations of the adiabatic approximation in other reactions will be valuable. Also experimental investigations at different energies are desired.

ACKNOWLEDGMENTS

The authors would like to express their thanks to Professor Y. Sakuragi for valuable discussions and to Dr. K. Tamura for providing us with his form factors of the deuteron.

APPENDIX A: SUDDEN APPROXIMATION INCLUDING THE *D* STATE OF THE DEUTERON

In this Appendix we give a detailed formula of the sudden approximation including the D state of the deuteron internal wave function. The full wave function of the deuteron is written as

$$\Phi_d(\boldsymbol{r}_n,\boldsymbol{r}_p)=\exp(i\boldsymbol{k}_d\boldsymbol{r}_d)\,\phi_\mu(\boldsymbol{r}), \tag{A1}$$

$$\phi_\mu(\boldsymbol{r})=\frac{1}{r}\{u(r)Y_{01\mu}(\hat{\boldsymbol{r}})+w(r)Y_{21\mu}(\hat{\boldsymbol{r}})\}=\xi(\boldsymbol{r})\chi_{1\mu}, \tag{A2}$$

where $u(r)$ and $w(r)$ are the standard S- and D-state radial wave functions of the deuteron, $\hat{\boldsymbol{r}}$ denotes the angular variables for the p-n relative coordinate, and

$$Y_{l\lambda\mu}(\hat{\boldsymbol{r}})=\sum_{m\nu}\langle lm1\nu|\lambda\mu\rangle Y_{lm}(\hat{\boldsymbol{r}})\chi_{1\nu}, \tag{A3}$$

$$\chi_{1\nu}=\sum_{\nu_n\nu_p}\left\langle\frac{1}{2}\nu_n\frac{1}{2}\nu_p\middle|1\nu\right\rangle\chi_{\nu_n}\chi_{\nu_p}, \tag{A4}$$

$$\xi(\boldsymbol{r})=\frac{1}{\sqrt{4\pi}}\frac{1}{r}\left\{u(r)+\frac{1}{\sqrt{8}}w(r)S_{12}(\hat{\boldsymbol{r}})\right\}, \tag{A5}$$

with $S_{12}(\hat{\boldsymbol{r}})=3(\boldsymbol{\sigma}_n\hat{\boldsymbol{r}})(\boldsymbol{\sigma}_p\hat{\boldsymbol{r}})-(\boldsymbol{\sigma}_n\boldsymbol{\sigma}_p)$ for the tensor operator. By Fourier transforming the internal wave function we obtain

$$\int d\boldsymbol{r}\exp(-i\boldsymbol{k}\boldsymbol{r})\phi_\mu(\boldsymbol{r}) =a(\boldsymbol{k})\chi_{1\mu}=a_0(k)Y_{01\mu}(\hat{\boldsymbol{k}})+a_2(k)Y_{21\mu}(\hat{\boldsymbol{k}}), \tag{A6}$$

where $\hat{\boldsymbol{k}}=\boldsymbol{k}/k$ and

$$a(\boldsymbol{k})=\frac{1}{\sqrt{4\pi}}\left\{a_0(k)+\frac{1}{\sqrt{8}}a_2(k)S_{12}(\hat{\boldsymbol{k}})\right\}, \tag{A7}$$

$$a_0(k)=4\pi\int r^2drj_0(kr)u(r)/r, \tag{A8}$$

$$a_2(k)=-4\pi\int r^2drj_2(kr)w(r)/r. \tag{A9}$$

The total wave function of the deuteron is now written in the form

$$\Phi_d(\boldsymbol{r}_n,\boldsymbol{r}_p)=\int\frac{d\boldsymbol{k}}{(2\pi)^3}\sum_\nu\sum_{\nu_n\nu_p}A_{\nu\mu}(\boldsymbol{k}) \times\left\langle\frac{1}{2}\nu_n\frac{1}{2}\nu_p\middle|1\nu\right\rangle\varphi_{\boldsymbol{k}_n\nu_n}(\boldsymbol{r}_n)\varphi_{\boldsymbol{k}_p\nu_p}(\boldsymbol{r}_p) \tag{A10}$$

with the amplitude

$$A_{\nu\mu}(\boldsymbol{k})\equiv\langle\chi_{1\nu}|a(\boldsymbol{k})|\chi_{1\mu}\rangle =\sum_{l=0,2}a_l(k)\sum_m\langle lm1\nu|1\mu\rangle Y_{lm}(\hat{\boldsymbol{k}}), \tag{A11}$$

which includes the coupling effects of the spin and the orbital motions due to the deuteron D state.

In the sudden approximation, the incident distorted wave of the deuteron in the present case is given by

$$\Phi_d^{sudden}\equiv\int\frac{d\boldsymbol{k}}{(2\pi)^3}\sum_\nu\sum_{\nu_n\nu_p}A_{\nu\mu}(\boldsymbol{k})\left\langle\frac{1}{2}\nu_n\frac{1}{2}\nu_p\middle|1\nu\right\rangle \times\varphi^{(+)}_{\boldsymbol{k}_n\nu_n}(\boldsymbol{r}_n)\varphi^{(+)}_{\boldsymbol{k}_p\nu_p}(\boldsymbol{r}_p), \tag{A12}$$

where $\varphi^{(+)}_{\boldsymbol{k}\nu}$ are the distorted nucleon wave functions with asymptotic momentum $\boldsymbol{k}$ and spin component ν. This gives Eq. (8) in Sec. II, where now the amplitude $a(\boldsymbol{k})$ involves $S_{12}(\hat{\boldsymbol{k}})$ coming from the D state. The scattering matrix for the deuteron inelastic scattering is given by

$$T_{fi}^{sudden}(\boldsymbol{k}'_d\mu',\boldsymbol{k}_d\mu)=\langle\chi_{1\mu'}|\hat{T}_d(\boldsymbol{k}'_d,\boldsymbol{k}_d)|\chi_{1\mu}\rangle, \tag{A13}$$

$$\hat{T}_d(\boldsymbol{k}'_d,\boldsymbol{k}_d)=\int\int\frac{d\boldsymbol{k}'}{(2\pi)^3}\frac{d\boldsymbol{k}}{(2\pi)^3}a(\boldsymbol{k}')\{\hat{T}_n(\boldsymbol{k}'_n,\boldsymbol{k}_n)\hat{J}_p(\boldsymbol{k}'_p,\boldsymbol{k}_p) +\hat{J}_n(\boldsymbol{k}'_n,\boldsymbol{k}_n)\hat{T}_p(\boldsymbol{k}'_p,\boldsymbol{k}_p)\}a(\boldsymbol{k}), \tag{A14}$$

where $\hat{T}_{n(p)}$ and $\hat{J}_{n(p)}$ are the same quantities as in Sec. II. If we apply the same approximation as in Sec. II, i.e., if we replace $\hat{J}$ by a momentum delta function and replace the variable $\boldsymbol{k}$ in $\hat{T}_{n(p)}$ by a representative, we finally obtain

$$\hat{T}_d=\int\frac{d\boldsymbol{k}}{(2\pi)^3}a\left(\boldsymbol{k}-\frac{1}{2}\boldsymbol{q}\right)(\overline{T}_n+\overline{T}_p)a(\boldsymbol{k}) =\int d\boldsymbol{r}e^{-i\boldsymbol{q}\boldsymbol{r}/2}\xi(\boldsymbol{r})(\overline{T}_n+\overline{T}_p)\xi(\boldsymbol{r}). \tag{A15}$$

Notice that the internal wave function ξ and the nucleon-nucleus scattering matrix $\overline{T}$ are not commutable because of the spin operators involved.

As we are interested in the effect of the deuteron D state in the spin observables, particularly in the tensor analyzing power, we extract from Eq. (A15) the parts which are proportional to the monopole and the quadrupole form factors of the deuteron:

$$\hat{T}_d=\hat{T}_0+\hat{T}_2+\Delta\hat{T},$$

$$\hat{T}_0=F(q)(\overline{T}_n+\overline{T}_p),$$

$$\hat{T}_2=G(q)\{S_{12}(\hat{\boldsymbol{q}})(\overline{T}_n+\overline{T}_p)+(\overline{T}_n+\overline{T}_p)S_{12}(\hat{\boldsymbol{q}})\}, \tag{A16}$$

where

$$F(q)\equiv\int drj_0(qr/2)\{[u(r))^2+[w(r)]^2\}, \tag{A17}$$

$$G(q)\equiv\frac{-1}{2\sqrt{2}}\int drj_2(qr/2)\left\{u(r)-\frac{1}{\sqrt{8}}w(r)\right\}w(r) \tag{A18}$$

are the deuteron form factors [22], while $\Delta\hat{T}$ denotes the part which is proportional to the difference between the monopole (quadrupole) form factor and the magnetic (spin transverse) form factor. It is of the order $O(w^2)$ and will be ignored in the calculation.

APPENDIX B: POLARIZATION OBSERVABLES FOR THE INELASTIC SCATTERING OF THE DEUTERON IN THE SUDDEN APPROXIMATION

In this appendix we collect formulas for some of the spin observables for the deuteron inelastic scattering including the D state. For the spin observables we adopt the definition in Ref. [15]. Let $\hat{T}$ be the deuteron-nucleus scattering matrix which is a matrix in the deuteron spin space. It depends on the magnetic quantum numbers of the target initial and final angular momentum states which are implicit. We first define the quantity

$$I_0 \equiv \mathrm{Tr}(\hat{T}\hat{T}^\dagger), \tag{B1}$$

which is related to the differential cross section by

$$\frac{d\sigma}{d\Omega} = \left(\frac{\mu}{2\pi}\right)^2 \frac{k_f}{k_i}\frac{1}{3}I_0, \tag{B2}$$

where μ is a reduced mass and $k_{i,f}$ the relative momenta in the initial and final states. In Eq. (B1), Tr denotes a trace over the deuteron spin states and a summation over the target initial/final magnetic substates. The spin observables are then defined as

$$I_0 A_i \equiv \mathrm{Tr}(\hat{T}\mathcal{P}_i\hat{T}^\dagger), \quad I_0 P^i \equiv \mathrm{Tr}(\mathcal{P}_i\hat{T}\hat{T}^\dagger),$$

$$I_0 A_{ij} \equiv \mathrm{Tr}(\hat{T}\mathcal{P}_{ij}\hat{T}^\dagger), \quad I_0 P^{ij} \equiv \mathrm{Tr}(\mathcal{P}_{ij}\hat{T}\hat{T}^\dagger),$$

$$I_0 K_i^j \equiv \mathrm{Tr}(\mathcal{P}_j\hat{T}\mathcal{P}_i\hat{T}^\dagger),$$

$$I_0 K_{ij}^k \equiv \mathrm{Tr}(\mathcal{P}_k\hat{T}\mathcal{P}_{ij}\hat{T}^\dagger), \quad I_0 K_k^{ij} \equiv \mathrm{Tr}(\mathcal{P}_{ij}\hat{T}\mathcal{P}_k\hat{T}^\dagger),$$

$$I_0 K_{ij}^{kl} \equiv \mathrm{Tr}(\mathcal{P}_{kl}\hat{T}\mathcal{P}_{ij}\hat{T}^\dagger), \tag{B3}$$

where i,j, etc., denote Cartesian coordinate components x,y, etc. In the following we fix the coordinate system so that the z axis is taken in the direction of the momentum transfer $\boldsymbol{q}$, while the y axis in the directions perpendicular to the scattering plane. Thus to compare with the experimental polarization data which normally refer to the incoming/outgoing directions in the coordinate system, one has to make a coordinate transformation for observables in the x and z directions. The operators $\mathcal{P}$ in the deuteron spin space are given in Table I.

In the sudden approximation the deuteron-nucleus scattering observables are expressed in terms of the nucleon-nucleus scattering matrix elements leading to the same transition in the target. To transform the trace in the deuteron spin space into the one in the nucleon spin space one may use the identity

$$\sum_\nu |\chi_{1\nu}\rangle\langle\chi_{1\nu}| = \sum_\nu (|\chi_n\rangle|\chi_p\rangle)_{1\nu}(\langle\chi_n|\langle\chi_p|)_{1\nu}$$
$$= \tfrac{1}{4}(3+\boldsymbol{\sigma}_n\boldsymbol{\sigma}_p) = P_1. \tag{B4}$$

By explicitly using the vector $|\chi_{1\nu}\rangle$ in Tr and using the above relation one obtains the relation (17) in the text. The operators $\overline{\mathcal{P}} \equiv P_1\mathcal{P}P_1$ are listed also in Table I.

We include in the deuteron-nucleus scattering matrix $\hat{T}$ the terms proportional to the monopole and the quadrupole form factors of the deuteron, i.e., $\hat{T}\to\mathcal{T}\equiv\hat{T}_0+\hat{T}_2$ defined in Eqs. (A16). The results for the spin observables K of Eq. (17) calculated in terms of the scattering matrix $\mathcal{T}$ have a general form

$$I_0 K = \mathrm{tr}[\overline{\mathcal{P}'}\mathcal{T}\overline{\mathcal{P}}\mathcal{T}^\dagger] = [F(q)]^2 D_{00} - F(q)G(q)D_{02}$$
$$+[G(q)]^2 D_{22}, \tag{B5}$$

where

$$D_{00} = \mathrm{tr}[\overline{\mathcal{P}'}\,\overline{T\mathcal{P}T^\dagger}],$$

$$D_{02} = \mathrm{tr}[\overline{\mathcal{P}'}\,\overline{T\mathcal{P}}(S_{12}\overline{T}^\dagger + \overline{T}^\dagger S_{12}) + \overline{\mathcal{P}'}(S_{12}\overline{T} + \overline{T}S_{12})\overline{\mathcal{P}T^\dagger}],$$

$$D_{22} = \mathrm{tr}[\overline{\mathcal{P}'}(S_{12}\overline{T} + \overline{T}S_{12})\overline{\mathcal{P}}(S_{12}\overline{T}^\dagger + \overline{T}^\dagger S_{12})] \tag{B6}$$

with $\overline{T}\equiv\overline{T}_n+\overline{T}_p$ and $S_{12}=S_{12}(\hat{\boldsymbol{q}})$. As noted in Sec. II, each observable is composed of the direct terms and the crossed ones. We denote them, respectively, by (d) and (c), i.e., $D = D^{(d)} + D^{(c)}$ for each of the contributions in Eq. (B6).

The direct terms are expressed entirely in terms of the nucleon-nucleus spin observables which are defined by

$$I_n \equiv \mathrm{tr}_n(\overline{T}_n\overline{T}_n^\dagger), \quad I_n A_{ni} \equiv \mathrm{tr}_n(\overline{T}_n\sigma_i\overline{T}_n^\dagger),$$

$$I_n P_n^i \equiv \mathrm{tr}_n(\sigma_i\overline{T}_n\overline{T}_n^\dagger), \quad I_n K_{ni}^j \equiv \mathrm{tr}_n(\sigma_j\overline{T}_n\sigma_i\overline{T}_n^\dagger) \tag{B7}$$

for the neutron matrix elements and, similarly, I_p, etc., for the proton matrix elements. The trace "tr_n" in the above expressions is taken only in the neutron spin space and should not be confused with "tr" which is taken in *both* the neutron and the proton spin spaces. The average/summation over the target magnetic states are also implied. For the crossed terms we define

$$J \equiv {\sum}' \ \mathrm{tr}_n'(\overline{T}_n)\mathrm{tr}_p'(\overline{T}_p^\dagger),$$

$$L_k \equiv {\sum}' \ \mathrm{tr}_n'(\overline{T}_n\sigma_k)\mathrm{tr}_p'(\overline{T}_p^\dagger\sigma_k) \quad (k=x,y,z),$$

$$A \equiv {\sum}' \ [\mathrm{tr}_n'(\overline{T}_n\sigma_y)\mathrm{tr}_p'(\overline{T}_p^\dagger) + \mathrm{tr}_n'(\overline{T}_n)\mathrm{tr}'(\overline{T}_p^\dagger\sigma_y)],$$

$$B \equiv i{\sum}' \ [\mathrm{tr}_n'(\overline{T}_n\sigma_x)\mathrm{tr}_p'(\overline{T}_p^\dagger\sigma_z) - \mathrm{tr}_n'(\overline{T}_n\sigma_z)\mathrm{tr}_p'(\overline{T}_p^\dagger\sigma_x)], \tag{B8}$$

where "tr′" denotes summation only over the nucleon *spin* states, while Σ' denotes a summation over the magnetic substates of the target initial/final states.

Below we list expressions for D's in Eq. (B6) for spin observables in the y direction (i.e., perpendicular to the scattering plane). Some of these observables have been recently measured [6–8]. As is clear from the expression for the observables, e.g., Eq. (18) in Sec. II, the results are symmetric under the interchange of the neutron and proton amplitudes; in particular, the direct terms are the sums of neutron and proton observables. Thus we do not explicitly show the whole expression but write only the first half of the terms and suggest the remaining terms by the notation $n\Rightarrow p$ or $n\Leftrightarrow p$. For instance, the quantities A_y, P^y, and K_i^j's appearing in the expression for $D^{(d)}$'s, refer to the spin observables for the *neutron*-nucleus scattering. We also use a simplified notation $K_i\,(i=x,y,z)$ for the quantity K_{ni}^i.

(1) $I_0=\mathrm{Tr}[\mathcal{T}\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot\frac{1}{8}\left(9+\sum_k K_k\right)+n\Rightarrow p, \tag{B9}$$

$$D_{00}^{(c)}=\frac{1}{4}\left(3J+2\sum_k L_k\right)+n\Leftrightarrow p, \tag{B10}$$

$$D_{02}^{(d)}=2I_n\cdot(-K_x-K_y+2K_z)+n\Rightarrow p, \tag{B11}$$

$$D_{02}^{(c)}=2(-L_x-L_y+2L_z)+n\Leftrightarrow p, \tag{B12}$$

$$D_{22}^{(d)}=I_n\cdot 2(9+4K_x+4K_y+7K_z)+n\Rightarrow p, \tag{B13}$$

$$D_{22}^{(c)}=2(12J+L_x+L_y+4L_z)+n\Leftrightarrow p. \tag{B14}$$

(2) $I_0A_{yy}=\mathrm{Tr}[\mathcal{T}\mathcal{P}_{yy}\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot\tfrac{1}{4}(-K_x+2K_y-K_z)+n\Rightarrow p, \tag{B15}$$

$$D_{00}^{(c)}=\tfrac{1}{4}(-L_x+2L_y-L_z)+n\Leftrightarrow p, \tag{B16}$$

$$D_{02}^{(d)}=-I_n\cdot\tfrac{1}{2}(9+K_x+7K_y+7K_z)+n\Rightarrow p, \tag{B17}$$

$$D_{02}^{(c)}=-(6J-L_x+2L_y+2L_z)+n\Leftrightarrow p, \tag{B18}$$

$$D_{22}^{(d)}=I_n\cdot\tfrac{1}{2}(9+13K_x+19K_y+7K_z)+n\Rightarrow p, \tag{B19}$$

$$D_{22}^{(c)}=(12J-L_x+2L_y-4L_z)+n\Leftrightarrow p. \tag{B20}$$

(3) $I_0P^{yy}=\mathrm{Tr}[\mathcal{P}_{yy}\mathcal{T}\mathcal{T}^\dagger]$.

This observable becomes identical to I_0A_{yy} in the sudden approximation.

(4) $I_0K_y^y=\mathrm{Tr}[S_y\mathcal{T}S_y\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot\tfrac{1}{2}(1+K_y)+n\Rightarrow p, \tag{B21}$$

$$D_{00}^{(c)}=\tfrac{1}{2}(J+L_y)+n\Leftrightarrow p, \tag{B22}$$

$$D_{02}^{(d)}=-2I_n\cdot(1+K_y)+n\Rightarrow p, \tag{B23}$$

$$D_{02}^{(c)}=-2(J+L_y)+n\Leftrightarrow p, \tag{B24}$$

$$D_{22}^{(d)}=-I_n\cdot(7+9K_x+7K_y+9K_z)+n\Rightarrow p, \tag{B25}$$

$$D_{22}^{(c)}=-2(8J-L_y)+n\Leftrightarrow p. \tag{B26}$$

(5) $I_0K_{yy}^{yy}=\mathrm{Tr}[\mathcal{P}_{yy}\mathcal{T}\mathcal{P}_{yy}\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot\tfrac{1}{2}(K_x+4K_y+K_z)+n\Rightarrow p, \tag{B27}$$

$$D_{00}^{(c)}=\tfrac{1}{2}(3J-2L_x+L_y-2L_z)+n\Leftrightarrow p, \tag{B28}$$

$$D_{02}^{(d)}=2I_n\cdot(5K_x+2K_y-K_z)+n\Rightarrow p, \tag{B29}$$

$$D_{02}^{(c)}=2(3J+2L_x-L_y-4L_z)+n\Leftrightarrow p, \tag{B30}$$

$$D_{22}^{(d)}=I_n\cdot(9+23K_x+29K_y+11K_z)+n\Rightarrow p, \tag{B31}$$

$$D_{22}^{(c)}=2(18J-2L_x+L_y-8L_z)+n\Leftrightarrow p. \tag{B32}$$

(6) $I_0A_y=\mathrm{Tr}[\mathcal{T}S_y\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot\tfrac{1}{4}(3A_y+P^y)+n\Rightarrow p, \tag{B33}$$

$$D_{00}^{(c)}=\tfrac{1}{4}(2A+B)+n\Leftrightarrow p, \tag{B34}$$

$$D_{02}^{(d)}=-I_n\cdot\tfrac{1}{2}(3A_y+5P^y)+n\Rightarrow p, \tag{B35}$$

$$D_{02}^{(c)}=-\tfrac{1}{2}(4A-B)+n\Leftrightarrow p, \tag{B36}$$

$$D_{22}^{(d)}=I_n\cdot 4P^y+n\Rightarrow p, \tag{B37}$$

$$D_{22}^{(c)}=2(A-B)+n\Leftrightarrow p. \tag{B38}$$

(7) $I_0P^y=\mathrm{Tr}[S_y\mathcal{T}\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot\tfrac{1}{4}(A_y+3P^y)+n\Rightarrow p, \tag{B39}$$

$$D_{00}^{(c)}=\tfrac{1}{4}(2A-B)+n\Leftrightarrow p, \tag{B40}$$

$$D_{02}^{(d)}=-I_n\cdot\tfrac{1}{2}(5A_y+3P^y)+n\Rightarrow p, \tag{B41}$$

$$D_{02}^{(c)}=-\tfrac{1}{2}(4A+B)+n\Leftrightarrow p, \tag{B42}$$

$$D_{22}^{(d)}=I_n\cdot 4A^y+n\Rightarrow p, \tag{B43}$$

$$D_{22}^{(c)}=2(A+B)+n\Leftrightarrow p. \tag{B44}$$

(8) $I_0K_{yy}^y=\mathrm{Tr}[S_y\mathcal{T}\mathcal{P}_{yy}\mathcal{T}^\dagger]$,

$$D_{00}^{(d)}=I_n\cdot A_y+n\Rightarrow p, \tag{B45}$$

$$D_{00}^{(c)}=\tfrac{1}{2}(A+B)+n\Leftrightarrow p, \tag{B46}$$

$$D_{02}^{(d)}=-I_n\cdot(A_y+3P^y)+n\Rightarrow p, \tag{B47}$$

$$D_{02}^{(c)}=-(2A-B)+n\Leftrightarrow p, \tag{B48}$$

$$D_{22}^{(d)}=I_n\cdot 2(-A_y+3P^y)+n\Rightarrow p, \tag{B49}$$

$$D_{22}^{(c)}=2(A-2B)+n\Leftrightarrow p. \tag{B50}$$

(9) $I_0K_y^{yy}=\mathrm{Tr}[\mathcal{P}_{yy}\mathcal{T}S_y\mathcal{T}^\dagger]$,

$$D_{00}^{(d)} = I_n \cdot P^y + n \Rightarrow p, \quad \text{(B51)}$$

$$D_{00}^{(c)} = \tfrac{1}{2}(A - B) + n \Leftrightarrow p, \quad \text{(B52)}$$

$$D_{02}^{(d)} = -I_n \cdot (3A_y + P^y) + n \Rightarrow p, \quad \text{(B53)}$$

$$D_{02}^{(c)} = -(2A + B) + n \Leftrightarrow p, \quad \text{(B54)}$$

$$D_{22}^{(d)} = I_n \cdot 2(3A_y - P^y) + n \Rightarrow p, \quad \text{(B55)}$$

$$D_{22}^{(c)} = 2(A + 2B) + n \Leftrightarrow p. \quad \text{(B56)}$$

APPENDIX C: RELATION BETWEEN DEUTERON-NUCLEUS AND NUCLEON-NUCLEUS SCATTERING OBSERVABLES

It has been noted [10] that the analyzing powers in deuteron-nucleus scattering are very similar to those of the corresponding nucleon-nucleus scattering when plotted against momentum transfer (Fig. 1). Our formulation is best suited for the study of these kinds of relationships. And then we will derive relations between deuteron-nucleus and nucleon-nucleus scattering observables within a reasonable assumption.

Let us consider the nucleon-nucleus scattering matrix which can be expanded in terms of the nucleon spin matrix as

$$\hat{T}_{NA} = \alpha_{N0} + \sum_{i=x,y,z} \alpha_{Ni}\sigma_{Ni} \quad (N = n,p), \quad \text{(C1)}$$

where α's are the complex parameters depending on the interaction and nuclear transition matrix elements. When we consider the isoscalar excitation in light $N=Z$ nuclei probed by the isoscalar particle, we may assume that these parameters are charge independent, i.e., $\alpha_n = \alpha_p$, etc. This approximation is indeed valid except at very forward angles where the effect of Coulomb interaction is important. By using this assumption and inserting the expression into Eq. (17), one can easily see that the crossed terms in the sudden approximation formula give an equal contribution as do the direct ones. As noted in Sec. II, therefore, the observables for the deuteron-nucleus scattering are entirely expressed in terms of those for the nucleon-nucleus scattering. If we further assume that the deuteron D-state contribution can be omitted we obtain the following relations:

$$I_0\Big|_d = \frac{1}{2} I_0\left(9 + \sum_i K_i^i\right)\Bigg|_p |F(q)|^2, \quad \text{(C2)}$$

$$I_0 A_y|_d = I_0(3A_y + P^y)|_p |F(q)|^2, \quad \text{(C3)}$$

$$I_0 P^y|_d = I_0(A_y + 3P^y)|_p |F(q)|^2, \quad \text{(C4)}$$

$$I_0 A_{yy}|_d = I_0(-K_x^x + 2K_y^y - K_z^z)|_p |F(q)|^2, \quad \text{(C5)}$$

$$I_0 K_y^y|_d = 2I_0(1 + K_y^y)|_p |F(q)|^2, \quad \text{(C6)}$$

$$I_0 K_{yy}^y|_d = 4I_0 A_y|_p |F(q)|^2, \quad \text{(C7)}$$

$$I_0 K_y^{yy}|_d = 4I_0 P^y|_p |F(q)|^2, \quad \text{(C8)}$$

$$I_0 K_{yy}^{yy}|_d = 2I_0(K_x^x + 4K_y^y + K_z^z)|_p |F(q)|^2. \quad \text{(C9)}$$

As the quantity I_0 is related to the cross section, the deuteron-nucleus scattering observables are given in terms of those for nucleon-nucleus scattering and the factor

$$f(q) \equiv 4\frac{I_0(p)}{I_0(d)}|F(q)|^2 = \frac{8}{3}\left(\frac{k_f}{k_i}\right)_d \left(\frac{k_i}{k_f}\right)_p \left(\frac{\mu_d}{\mu_p}\right)^2 \frac{\sigma(p)}{\sigma(d)}|F(q)|^2. \quad \text{(C10)}$$

For instance, for the analyzing power we obtain

$$A_y|_d = f \cdot \tfrac{1}{4}(3A_y + P^y)|_p \simeq f \cdot A_y|_p, \quad \text{(C11)}$$

where in the last relation we put $P^y \simeq A_y$ for nucleon-nucleus scattering. The latter relation is well satisfied for non-spin-flip transitions. In the case of spin-flip transitions such as the 1^+ excitation this relation is largely violated, which may be traced back to the spin-orbit distortion effect. By combining relations given above we can eliminate nucleon spin observables and express the deuteron observables in terms only of the factor (C10) as

$$\left(\tfrac{4}{3} + \tfrac{2}{3}A_{yy} - K_y^y\right)/f = (S_d^y + K_y^y)/f = 1, \quad \text{(C12)}$$

which may be used as an examination of the validity of the sudden approximation. Note that if one eliminates further the factor f using the expression (C9) for K_{yy}^{yy}, one obtains a relation $S_2 = 0$ for the double-spin-flip transition of Ref. [6], which is a natural consequence of the single-collision calculation.

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