# **Semiclassical approach to sequential**  $\alpha$  **emission in** (96 MeV) <sup>16</sup>O+<sup>58</sup>Ni and  $(133 \text{ MeV})$   $^{16}O + ^{48}Ti$  deep inelastic collisions

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 $(96 \text{ MeV})$  <sup>16</sup>O+<sup>58</sup>Ni and  $(133 \text{ MeV})$  <sup>16</sup>O+<sup>48</sup>Ti reactions have been experimentally investigated by using coincident charged particle techniques. A closed-form theoretical approach, describing in a single picture the nonequilibrium component and the evaporation component of the angular correlation between particles and reaction residues emitted in a peripheral heavy-ion collision, is applied—in the hypothesis of sequential process—to the C- $\alpha$ , N- $\alpha$ , and O- $\alpha$  differential multiplicities for the <sup>16</sup>O+<sup>58</sup>Ni at 6 MeV/nucleon and  $16O+48Ti$  at 8.3 MeV/nucleon deep inelastic collisions. From this analysis some reaction mechanism information is deduced.

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### **I. INTRODUCTION**

In peripheral heavy-ion reactions at intermediate bombarding energies not exceeding 20 MeV/nucleon a dinuclear system can be formed with both the projectile and the target sticking together during a short time within a deep inelastic collision. The subsequent decay of this kind of dinuclear objects by light-particle sequential emission has been widely studied in the past  $[1]$ .

Many features of these emissions are explained by means of a simple theoretical approach in terms of breakup of the projectile and emission of particles from reaction residues  $[2-7]$ . The experimental observations spurred many theoretical models and approaches  $[5-7]$ .

In the case of peripheral collisions, where one observes the emission of two fragments close in *A* and *Z* to the ingoing partners, besides few light particles and clusters, energy and angular correlations between these particles and the fragments have been satisfactorily justified as due to a sequential emission from the detected projectilelike and the undetected targetlike fragments. A common feature is evident in these coincidence measurements, i.e., a *double forward-peaked* structure, showing a minimum close to the direction of the projectilelike fragment together with a marked asymmetry between emission probability at positive and negative angles  $[2,8]$ .

These observed features have been described in terms of a theoretical approach  $[9,10]$  recently revisited  $[11]$ , which accomodates in a simple way the nonequilibrium component together with the evaporative one of the sequential particle emission in peripheral heavy-ion collisions like *A*(*a*,*b*)*B*(*c*)*C*.

In this paper we outline a closed-form expression for the (*b*-*c*) multiplicity of a sequential process like  $A(a,b)B(c)C$ and show that even in the case of a sequential process, an important and remarkable nonequilibrium component in the particle emission is present. We also show how useful conclusions on the mechanism of a peripheral collision  $A(a,b)B$ can be drawn from the investigation of the (*b*-*c*) measured angular correlation around the forward angles.

In order to apply this *semiclassical* approach we have measured angular correlations of  $\alpha$  particles arising from the  $(96 \text{ MeV})$   $^{16}O+^{58}\text{Ni}$  and  $(133 \text{ MeV})$   $^{16}O+^{48}\text{Ti}$  deep inelastic collisions.

The paper is organized as follows. The semiclassical approach to particle-particle angular correlations is described in Sec. II, with its application given in Sec. III for both  $^{16}O$  $+$ <sup>58</sup>Ni and <sup>16</sup>O<sup>+48</sup>Ti reactions, and concluding remarks are finally proposed in Sec. IV.

### **II. SEMICLASSICAL APPROACH TO PARTICLE-PARTICLE ANGULAR CORRELATION**

To get the theoretical formulas of our approach  $[9-11]$ , let us start by considering a sequential process like  $A(a,b)B(c)C$  and assume that it proceeds through a given continuum state  $(\epsilon_B^*, J_B \pi_B)$  in the nucleus *B* to a narrow definite state ( $\epsilon_C^*$ ,  $J_C \pi_C$ ) in the final nucleus *C*.

In the following,  $\epsilon_X^*$  indicates the excitation energy of the state of definite spin  $J_X$  and parity  $\pi_X$  in the nucleus *X* and  $m_X$ , the *z* component of  $\tilde{J}_X$ . The pair (*xX*) has relative radial coordinate  $\vec{r}_x$ , momentum  $\vec{k}_x$ , velocity  $\vec{v}_x$ , and energy  $\epsilon_x$ . The spherical polar angles ( $\vartheta_b$ ,  $\varphi_b$ ) of  $\vec{k}_b$  are defined in the  $(A + a)$  center-of-mass (c.m.) system, while  $\tilde{k}_c$  has polar angles  $(\vartheta,\varphi)$  defined in the recoil center-of-mass (r.c.m.) system (rest frame of the nucleus *B*) and described in a  $xyz$ 

Deceased.

frame with the *x* axis and *z* axis parallel to the *x* axis and *z* axis of the c.m. frame.

In order that the  $A(a,b)B(c)C$  reaction be a sequential process, let us require that the excitation energy  $\epsilon_B^*$  of the intermediate system *B* formed in the first step of the threebody reaction be independent of the particle *c* angles and assume, moreover, that in the  $B \rightarrow c + C$  decay the nuclear interaction between *b* and *B* can be neglected; for simplicity, we suppose that the nuclei *A*,*a*,*b*, and *c* have spin zero and *b* and *c* are in the ground state.

To get the average value of the (*b*-*c*) angular correlation over the interval  $\Delta$  centered at  $\epsilon_B^*$ , let us split the S matrix into an equilibrium  $(E)$  and a nonequilibrium  $(NE)$  term as  $[12]$ 

$$
S = S^E + S^{NE} \tag{1a}
$$

with

$$
S^E = S - \langle S \rangle, \tag{1b}
$$

$$
S^{NE} = \langle S \rangle. \tag{1c}
$$

Moreover we suppose the phase of  $S^E$  and  $S^{NE}$  to be uncorrelated (so that their cross terms average out to zero) and we make the statistical assumption that in the energy interval  $\Delta$ around  $\epsilon_B^*$  there are many levels contributing to the *B* $\rightarrow$ *c*  $+C$  decay and that their widths and energies are randomly distributed so that interference terms generally vanish  $[13, 14]$ .

We also assume that the amplitude  $S^{NE}$  [see Eq. (1c)] is a very smoothly varying function of the excitation energy  $\epsilon_C^*$ within a region  $\Delta'(\sim \Delta)$ .

Finally, following restrictions and approximations of Ref. [11], the energy averaged  $(b-c)$  angular correlation can be expressed as

$$
\left\langle \frac{d^2 \sigma}{d \omega_b d \omega} \right\rangle = \left( \frac{d^2 \sigma}{d \omega_b d \omega} \right)^E + \left( \frac{d^2 \sigma}{d \omega_b d \omega} \right)^{NE} \tag{2}
$$

with

$$
\left(\frac{d^2\sigma}{d\omega_b d\omega}\right)^E = \sum_{m_C} \sum_{IJ_C} w_l(J_C) \left(\frac{T_l}{G}\right) \sum_{m_B} p_l(m_B, m_C; \omega_b, \omega) \Bigg|^2,
$$
\n(3)

$$
\left(\frac{d^2\sigma}{d\omega_b d\omega}\right)^{NE} = \sum_{m_C} \left| \sum_{U_C} \left\langle \mathcal{S}_l \right\rangle \sum_{m_B} p_l(m_B, m_C; \omega_b, \omega) \right|^2, \tag{4}
$$

where

$$
p_l(m_B, m_C; \omega_b, \omega) \equiv (-)^l F_{ba}(m_B, \omega_b) \cdot \langle l J_C, m_B
$$

$$
- m_C, m_C | J_B m_B \rangle Y_l^{m_B - m_C}(\omega). \quad (5)
$$

In Eq.  $(3)$  the quantity  $w_l$ , related to the relative density of the available states ( $\epsilon_C^*$ ,  $J_C\pi_C$ ) in the nucleus *C*, describes the probability of orbital angular momentum *l* transferred in the  $(B; J_B \epsilon_B^*) \rightarrow (cC; I J_C \epsilon_C^*)$  decay and we have assumed the

parametrization  $\langle |\mathcal{S}^E|^2 \rangle = T_l / G$ , where  $T_l$  is the opticalmodel transmission coefficient and *G* represents all decay modes energetically open for the  $B \rightarrow c + C$  decay [13,14].

Actually, by using the time-dependent scattering theory [15], it can be roughly assumed that the quantity  $(d^2\sigma)^{NE}$  is associated with a situation in which the dissociation of *B* into *c* and *C* is a fast process occurring in time scales by many orders of magnitude shorter than the typical time corresponding to the equilibrium decay process, described by  $(d^2\sigma)^E$ , whose long lifetime leads to the ''loss of memory'' of the formation of the decaying nucleus  $B$  [14]. For this reason the angular symmetry of the *c* emission from a statistical equilibrated system described by the  $(b-c)$  angular correlation  $(3)$ cannot be used as evidence for any particular model of dynamical effect.

On the contrary, one can deduce from the (*b*-*c*) angular correlation  $(4)$  that the memory of the first step of the sequential process  $A(a,b)B(c)C$  can be retained during the subsequent "fast"  $B \rightarrow c + C$  decay, so that the angular dependence of the particles *c* emerging from such a short-lived composite system can display a marked forward-backward asymmetry around the direction of the coincident projectile residue *b* or the beam axis.

Thus the study of the nonequilibrium sequential component of the particle emission can be seen as a powerful tool to probe the early stage of the peripheral collision besides a useful alternative technique to obtain reaction mechanism information complementary to the ones extracted by means of the angular distributions of the two-body reaction products.

When the interest in using the angular correlation method is mainly devoted to obtain information on the mechanism of the  $A(a,b)B$  reaction and on the polarization effects of the nucleus *B*, it is convenient to choose coordinate axes so that the *z* axis is along  $\vec{k}_b \times \vec{k}_a$  (perpendicular to the reaction plane) and the *x* axis along  $\tilde{k}_a$ .

Information on the polarization effects of the residual nucleus *B* induced by the first step of the sequential process  $A(a,b)B(c)C$  can also be obtained through the  $\varphi$  dependence of the differential multiplicity for the second step  $[11]$ .

A semiclassical expression for the (*b*-*c*) differential multiplicity has been treated and developed in Refs.  $[10,11,16]$ , which accounts for many of the observed features of the sequential emission of the high as well as low energy particles from the fragments excited in a peripheral heavy-ion reaction .

In this approach, we consider a semiclassical picture that assumes a coordinate rotation by means of the Euler angles to a more useful system chosen in describing the  $B \rightarrow c + C$ decay (in the restrictions and assumptions of Ref.  $[11]$ ), where the new quantization axis is oriented in the direction of  $J_B$  which is at a certain angle  $\Lambda$  with respect to the *z* axis and lies in a plane perpendicular to the reaction plane and to the direction of a unit vector  $\hat{k}_0$ , close to the recoil direction of the decaying nucleus  $B$  [17], corresponding to an angle  $\varphi_0 = (\pi/2 + \xi)$  with respect to the *x* axis.

Then the relative momentum  $\vec{k}_c$  of the pair (*cC*) has polar angles  $(\vartheta,\varphi)$  and  $(\Theta,\Phi)$  with respect to the space-fixed system and to the  $(\hat{k}_0 \times \hat{J}_B, \hat{k}_0, \hat{J}_B)$  axes, respectively. Since the polar angles of  $J_B$  axis with respect to the  $(x, y, z)$  axes are  $(\Lambda, \pi + \xi)$  (see Fig. 2 of Ref. [11]), we have

$$
\cos \Theta = \cos \Lambda \cos \vartheta - \sin \Lambda \sin \vartheta \cos (\varphi - \xi), \quad (6a)
$$

$$
\cot \Phi = \frac{\cos \Lambda \sin \vartheta \cos (\varphi - \xi) + \sin \Lambda \cos \vartheta}{\sin \vartheta \sin (\varphi - \xi)}.
$$
 (6b)

In the framework of the quantal treatment carried out in Ref.  $[18]$ , we assume the semiclassical replacement  $[18,19]$ 

$$
w_l(\mu) \sim \exp(-\alpha l^2) \exp(\beta \mu), \tag{7}
$$

where

$$
\alpha = (\mathcal{I} + MR^2)\hbar^2/2TT_CMR^2,
$$
  

$$
\beta = J_B\hbar^2/TT_C
$$

with  $M$ ,  $R$ , and  $T$  being the reduced mass, the radius, and the rigid-body moment of inertia of the pair (*cC*), respectively, and  $T_c$  is the nuclear temperature corresponding to the excitation energy  $\epsilon_C^*$  in the nucleus *C*.

In the *sharp cutoff* approximation for the coefficient  $T_l$ , converting the summation over  $l$  to an integral, we get [see Eq.  $(6a)$ ]

$$
[M(\vartheta, \varphi, \Lambda)]^{E} = C_{E} \exp(-\gamma \cos^{2} \Theta), \qquad (8)
$$

where  $C_E$  is independent of  $\vartheta$  and  $\varphi$  while the anisotropy coefficient  $\gamma$  is given by  $\gamma = \beta^2/4\alpha$ . We attribute the "direct" sequential  $B \rightarrow c + C$  decay described by  $\langle S \rangle$  [see Eqs. (1)] to a *prompt* emission of particles from peripheral regions of the nucleus *B* bearing in mind that in the classical limit the particles *c* while escaping from the rotating nucleus *B* get an additional velocity if emitted along the equatorial plane.

For an estimate of the NE (*b*-*c*) multiplicity we can therefore assume the emission of particles *c* in the equatorial plane with orbital angular momentum  $\vec{l}$  parallel to  $\vec{J}_B$  to dominate, and consequently we assume that the peripheral nature of the NE decay process is consistent with the hypothesis that only an "*l* window" centered at a certain  $l_0$  contributes. So for the energy-averaged element  $\langle S_l \rangle$  in the amplitude-phase representation

$$
\langle S_l \rangle = \eta(l) \exp[i \delta(l)],
$$

we can write near  $l=l_0$ 

$$
\langle S_l \rangle \sim \eta (l - l_0) \exp[i(l - l_0) \chi_0],\tag{9}
$$

where we have assumed the phase  $\delta(l)$  to be linear in *l* about  $l_0$  and

$$
\chi_0 \equiv \left[ \frac{\partial \delta(l)}{\partial l} \right]_{l_0} \tag{10}
$$

is the *quantal deflection function* somehow describing the ''classical trajectory'' of the particles *c* and the nucleus *C* in their mean field characterized by the phase shift  $\delta$  [20].

An estimate of the NE differential multiplicity can be written as follows:

$$
[M(\vartheta,\varphi,\Lambda)]^{NE} \sim |Q^{(+)}(\Phi)|^2 + h_0 |Q^{(-)}(\Phi)|^2, \quad (11)
$$

where we have defined the ''single source'' amplitude

$$
Q^{(\pm)}(\Phi) \equiv \sum_{l} \eta(l - l_0) \exp[i(l - l_0)(\chi_0 \pm \Phi)]. \quad (12)
$$

Recalling the peripheral nature of the direct NE decay process, if we express the amplitude  $\eta(l-l_0)$  as a Gaussian distribution  $[21]$ 

$$
\eta(l - l_0) \sim \exp[-(l - l_0)^2/4\lambda^2],
$$

following an analogous procedure as for  $M(\theta, \phi,\Lambda)$ , we finally obtain

$$
[M(\vartheta, \varphi, \Lambda)]^{NE} = C_{NE} \{ \exp[-\lambda^2 (\Phi + \chi_0)^2] + h_0 \exp[-\lambda^2 (\Phi - \chi_0)^2] \}, \quad (13)
$$

where  $C_{NE}$  englobes all the inessential constants independent of  $\vartheta$  and  $\varphi$ .

To obtain the final expression of the semiclassical (*b*-*c*) differential multiplicity, we shall assume that the spin orientation is governed by a distribution function  $L(\Lambda)$ , so that finally we have

$$
M(\vartheta, \varphi) = \{ M(\vartheta, \varphi) \}^{E} + \{ M(\vartheta, \varphi) \}^{NE} \tag{14}
$$

with

$$
M(\vartheta,\varphi)^{E} = \int d\Lambda L(\Lambda)[M(\vartheta,\varphi,\Lambda)]^{E}/\int d\Lambda L(\Lambda),
$$
\n(15)

$$
M(\vartheta,\varphi)^{NE} = \int d\Lambda L(\Lambda) [M(\vartheta,\varphi,\Lambda)]^{NE} / \int d\Lambda L(\Lambda),
$$
\n(16)

where  $M^E$  and  $M^{NE}$  are given by Eqs. (6), (8), and (13).

For simplicity we shall assume  $L(\Lambda)$  as a Gaussian distribution,

$$
L(\Lambda) = \exp[-(\Lambda - \Lambda_0)^2 / 2\Omega^2].
$$
 (17)

The in-plane differential multiplicity corresponds to  $\vartheta$  $= \pi/2$ . In this case Eqs. (6) become

$$
\cos \Theta = \sin \Lambda \cos(\varphi - \xi), \tag{18a}
$$

$$
\cot \Phi = \cos \Lambda \cot(\varphi - \xi). \tag{18b}
$$

As already shown in Ref.  $[11]$ , when the dealignment is sufficiently small  $(\Lambda \ll 1)$ , the NE in-plane  $(b-c)$  differential multiplicity is essentially given by a two component asymmetric (in general  $h_0 \neq 1$ ) pattern about the angle  $\xi = \varphi_0$  $-\pi/2$  (see Fig. 2 of Ref. [11]), peaked at the angles  $\varphi_1 = \xi$  $-\chi_0$  and  $\varphi_2 = \xi + \chi_0$ , respectively; moreover, if  $\chi_0 < \xi$  and  $h_0$ <1, the (*b-c*) coincidence events appear with maximum probability on the same side of the beam axis with respect to the direction of the ''detected'' projectile residue. The values of the in-plane coincidence cross section about  $\varphi_1$  and  $\varphi_2$ correspond to  $A(a,b)B$  reaction process with opposite polarization of *B*, which, in a qualitative picture, may somehow be explained by the assumption that only one type of ''semiclassical trajectory'' predominantly contributes to the inplane (*b*-*c*) angular correlation for either positive or negative angles with respect to the direction of the projectilelike nucleus  $b \mid 20,22 \mid$ .

In the cases when  $\Lambda \ll 1$  one can obtain an estimate of the angle  $\xi$  and the quantal deflection  $\chi_0$  by a simple inspection of the experimental in-plane angular correlation pattern around the "peak angles"  $\varphi_1$  and  $\varphi_2$ , using the expressions

$$
2\xi \approx \varphi_2 + \varphi_1, \qquad (19a)
$$

$$
2\chi_0 \approx \varphi_2 - \varphi_1. \tag{19b}
$$

Indeed here the deviation from left-right symmetry around a direction close to the one of the coincident projectile residue as well as the double forward-peaked shape in the angular correlation pattern does not necessarily imply that the light particles emerge from the contact zone between the two colliding nuclei (spatial-localization). Actually, in a simple optical picture, we can interpret the sums appearing in Eq.  $(12)$  [see also Eq.  $(13)$ ] as a beam of particles *c* emitted from an "*l*-window" centered about a mean value  $l_0$  and extended over a narrow width  $\Delta l \sim \lambda$  (*l* localization).

From the above rough picture we somehow idealize the time dependence of the NE  $B \rightarrow c + C$  decay; for example, the observed strongly forward-peaked in-plane angular correlation can be interpreted as an indication that the light particles *c* are emitted in decay times shorter than the rotational period of the nucleus *B*, corresponding to the time required for a hypothetical complete revolution of the  $(c+C)$  composite system. In a simple, classical picture we can use a wave packet description to estimate the average time interval occurring between *B* nucleus formation in the  $A(a,b)B$  peripheral collision and the  $B \rightarrow C + c$  *fast* emission. To this aim, let us consider the  $(C+c)$  composite system to rotate during the time  $\tau_0$  with angular momentum  $l_0$  and rotational frequency  $\omega_0 = \hbar l_0 / \mathcal{I}$ . If we assume that the "deflection" angle"  $\chi_0$  depends on  $\tau_0$  NE decay time, starting from a  $\tau_0=0$  when the  $k_c$  component in reaction plane is in the direction of  $\hat{k}_0$ , we get the following linear formula:

$$
-\chi_0 = \omega_0 \tau_0 = \frac{\hbar l_0}{\mathcal{I}} \tau_0.
$$
\n(20)

#### **III. EXPERIMENTAL RESULTS**

As an application of the above-mentioned theoretical approach, we analyze the C- $\alpha$ , N- $\alpha$ , and O- $\alpha$  differential multiplicities for the  $(96 \text{ MeV})$   $^{16}O + ^{58}\text{Ni}$  [23,8] and (133 MeV)  $16O+48$ Ti [24,25] deep inelastic collisions, respectively. We studied the *in-plane* and *out-of-plane* angular correlations  $(see, e.g. Ref. [11] and references therein) between project$ tilelike fragments  $(C, N, O)$  and  $\alpha$  particles coming from the  $(96 \text{ MeV})$   $^{16}O+^{58}\text{Ni}$  and  $(133 \text{ MeV})$   $^{16}O+^{48}\text{Ti}$ , respectively.

The  $\alpha$  particles associated to the  $(C, N, O)$  fragments are emitted by (Zn, Cu, Ni) intermediate nuclei during the sequential reaction

<sup>16</sup>O+<sup>58</sup>Ni
$$
\rightarrow
$$
(C,N,O)+(Zn<sup>\*</sup>,Cu<sup>\*</sup>,Ni<sup>\*</sup>)+ $Q_2\rightarrow$ (C,N,O)  
+(Ni,Co,Fe)+ $\alpha$ + $Q_3$ , (21)

while they are emitted by  $(Cr, V, Ti)$  in the sequential reaction

<sup>16</sup>O+<sup>48</sup>Ti
$$
\rightarrow
$$
(C,N,O)+(Cr\*,V\*,Ti\*)+ $Q_2\rightarrow$ (C,N,O)  
+ $(Ti, Sc, Ca) + \alpha + Q_3.$  (22)

# **A. The** <sup>16</sup>O<sup>+58</sup>Ni reaction at  $E_{lab}$ <sup>(16</sup>O)=96 MeV

The first experiment has been performed with a 96-MeV <sup>16</sup>O beam supplied by the MP Tandem facility in Strasbourg, to study the  $(C-\alpha)$ ,  $(N-\alpha)$ , and  $(O-\alpha)$  differential multiplicities for the  ${}^{58}\text{Ni}({}^{16}\text{O},\text{C})\text{Zn}(\alpha)$ Ni,  ${}^{58}\text{Ni}({}^{16}\text{O},\text{N})\text{Cu}(\alpha)$ Co, and  $^{58}\text{Ni}({}^{16}\text{O},\text{O})\text{Ni}(\alpha)$ Fe sequential processes [23,8].

The <sup>16</sup>O beam hit an isotopically enriched  $750-\mu$ g/cm<sup>2</sup>thick 58Ni target. The strongly energy damped projectile residues  $(C, N, O)$  ions were detected by a  $(\Delta E_{gas}, E_{silicon})$  telescope at  $\theta_{lab} = -35^\circ$ . Measurement of  $\alpha$  angular distributions have been performed by means of position-sensitive Si detectors (PSD), combined with an ionization chamber, together with a triple Si-telescope detector for small forward angles.

To extract the equilibrium and nonequilibrium sequential components, all other processes contributing to the  $\alpha$ emission, like, e.g., the  $\alpha$ 's coming from the C buildup contamination and the breakup events, were identified and removed  $[8]$ .

The *sequentiality* of the distribution so obtained is pointed out by the concentration of such events in *Q*-value windows that *do not depend* on the  $\alpha$  detection angles (see, e.g., Fig. 1) of Ref. [26]). The average values of  $(Q_2, Q_3)$  in MeV for  $(C-\alpha)$ ,  $(N-\alpha)$ , and  $(O-\alpha)$  coincidences are, respectively,  $(-38.4,-28.5), (-35.8,-25.8),$  and  $(-33.9,-24.8)$ . As a consequence, since the excitation energy of projectilelike particles is negligible and a major part of the kinetic energy is carried out by the  $\alpha$  particle, it follows that the  $(Zn, Cu,$ Ni) intermediate nuclei excitation energy does not appreciably depend on the  $\alpha$ -emission angle.

For the three coincidences the mean value of the excitation energy of the emitting target nucleus is about 35 MeV, a value lying in the continuum region of the excitation spectrum, and this allows us to apply to this reaction the semiclassical approach previously described. Moreover, approximating the impact parameter to the grazing one, and using the mean kinetic energy of the projectilelike fragments extracted from our data, we obtain a rough value of the angular momentum transferred in the first step of the reaction, which is about  $25\hbar$ . Such a value,  $58\text{Ni}$  spin being zero, gives us an estimation of the targetlike nucleus spin  $J_B$ . Figure 1 shows the in-plane differential multiplicity data for  $(C-\alpha)$ ,  $(N-\alpha)$ , and (O- $\alpha$ ) coincidences vs the  $\alpha$ -particle detection angle. As



FIG. 1. Best fit of the in-plane C- $\alpha$ , N- $\alpha$ , and O- $\alpha$  differential multiplicity data, for the sequential process  ${}^{16}O+{}^{58}Ni$  at 96 MeV laboratory energy [8]. The differential multiplicity, in units of  $10^{-2}$  sr<sup>-1</sup>, is plotted vs the in-plane  $\alpha$ -particle angle. The arrows indicate the directions of the projectilelike fragment  $(b)$  and targetlike fragment  $(B)$  with respect to the incident beam in the laboratory system,  $\phi_0$  being the direction of the average momentum transferred (see text).

these data have been referred to the rcm system, i.e., the c.m. system of  $(Zn, Cu, Ni)$  nuclei, they can be directly fitted by the theoretical formula  $(14)$ , represented by the solid lines; the dashed lines are the best fit of the equlibrated part of the differential multiplicity given by Eq.  $(15)$ .

The out-of-plane coincidence data shown in Fig. 2 are taken at backward angles; since in that angular region the nonequilibrium emission is negligible, these out-of-plane experimental data were employed to get the  $(C_E; \gamma; \Lambda_0; \Omega)$ parameters by means of the purely evaporative formula  $(15)$  $[11]$ .

In contrast to the case of the  $(C_E; \gamma; \Lambda_0; \Omega)$  parameters, the value of  $\phi_0$  obtained in the fitting prcedure cannot be determined to a sufficient accuracy, since in the present analysis the angular correlations given by Eq.  $(15)$  are not



FIG. 2. Best fit of the out-of-plane  $(C-\alpha)$ ,  $(N-\alpha)$ , and  $(O-\alpha)$ differential multiplicity data, for the  ${}^{58}\text{Ni}({}^{16}\text{O},\text{C})\text{Zn}(\alpha)$ Ni,  $58\text{Ni}({}^{16}\text{O,N})\text{Cu}(\alpha)$ Co, and  $58\text{Ni}({}^{16}\text{O,O})\text{Ni}(\alpha)$ Fe sequential processes at 96 MeV laboratory energy [23]. The differential multiplicity, in units of  $10^{-2}$  sr<sup>-1</sup>, is plotted vs the out-of-plane  $\alpha$ -particle angle.

sensitive to the choice of  $\phi_0$  within an angular interval of 30° around the values of the recoil directions of the  $\alpha$ -decaying nuclei, reported in Table I.

Best  $\chi^2$  values for  $C_E$ ,  $\gamma$ ,  $\Lambda_0$ , and  $\Omega$  are listed in Table I. Since  $\Lambda_0$  and  $\Omega$  are a measure of dealignment of the rotational axis of the  $\alpha$ -decaying nucleus along an axis normal to the reaction plane, one sees from an inspection of Table I that in a qualitative picture, the dealignment of Cr and Zn is small. The angular correlation data do not uniquely determine the quantities  $\gamma$  and  $\Lambda_0$  but rather define a range of possibilities, the values listed in Table I can therefore be considered as an estimate. In principle, the  $\xi$  parameter could be calculated in the same way, but the evaporative component is not as sensitive to the choice of  $\xi$  as the nonequilibrium one. As a matter of fact, the values obtained for  $\Lambda_0$  and  $\Omega$  mean that the targetlike nucleus rotational axis lies very close to the *z* axis, then both  $\xi$  and  $\chi_0$  can be evaluated using the approximate expressions (19), where  $\phi_1$  and  $\phi_2$  are the

Coincidences $(10^{-2} \text{ sr}^{-1})$	$C_E$ <sup>a</sup>	$\gamma$ <sup>a</sup>	$\Lambda_0$ <sup>a</sup>	$\Omega$ <sup>a</sup>	$\xi^{\text{b}}$	$\chi_0$ <sup>b</sup>
$C-\alpha$ $N-\alpha$ $O-\alpha$	$1.4 \pm 0.1$ $1.0 \pm 0.1$ $0.6 \pm 0.06$	$3.0 \pm 0.2$ $2.2 \pm 0.1$ $1.9 \pm 0.1$	$(19\pm13)$ ° $(6\pm4)$ ° $(19 \pm 13)^{\circ}$	$(13\pm2)$ ° $(24\pm2)$ ° $(13\pm2)$ °	$(-30\pm2)$ ° $(-41\overline{+}2)$ ° $(-43\pm2)$ °	$(-45\pm2)$ ° $(-45\pm2)$ ° $(-78\pm2)$ °
Coincidences $(10^{-2} \text{ sr}^{-1})$ $C-\alpha$ $N-\alpha$ $O-\alpha$		$C_{NE}$ $2.1 \pm 0.2$ $1.8 \pm 0.2$ $0.7 \pm 0.07$	$\lambda$ $2.3 \pm 0.2$ $2.5 \pm 0.3$ $2.2 \pm 0.2$	$h_0$ $0.30 \pm 0.04$ $0.49 \pm 0.06$ $0.43 \pm 0.05$	$\phi_R$ $(60\pm3)$ ° $(49\pm3)$ ° $(47 \pm 3)^{\circ}$	$\phi_0$ $(35\pm3)$ ° $(42\pm3)$ ° $(47 \pm 3)^{\circ}$

TABLE I. List of the parameters obtained in the analysis of the out-of-plane and in-plane angular correlations coming from the  $(96 \text{ MeV})$   $^{16}O+^{58}\text{Ni}$  reaction.

<sup>a</sup>The quantities obtained by fitting the experimental data by the evaporative formula (15).

<sup>b</sup>The quantities estimated from a simple inspection of the experimental angular correlation patterns by using the approximate expressions  $(19)$ .

 $\alpha$ -particle emission angle corresponding to the two peaks of the total differential multiplicity.

Finally,  $(C_{NE};\lambda;h_0)$  parameters were obtained by fitting the forward region experimental data by the formula  $(14)$ , where the above-determined values of  $(C_E; \gamma; \Lambda_0; \Omega; \xi; \chi_0)$ were inserted. From the analysis of the fit parameters reported in Table I, one easily infers that the spin direction is almost perpendicular to the reaction plane as we supposed in the theoretical approach. As a matter of fact, the average angle between the spin direction and the normal axis  $(\Lambda_0)$  is less than 20° for all three coincidences.

The nonequilibrium component consists of two bumps; the higher one is associated with the positive polarization, the lower to the negative polarization. The width of the peaks is related to the model parameter  $\lambda$ , which represents the width of the *l* window mainly contributing to the decay process; such a value does not exceed  $3\hbar$ , thus confirming that we are dealing with a peripheral process. Another interesting parameter is  $h_0$ , which is related to the probability  $p_0$  of positive polarization of the targetlike nucleus on a quantization axis perpendicular to the reaction plane (omitting the explicit indication of  $\omega_b$ ),

$$
p_0 = |f_{ba}(m_0)|^2 / [ |f_{ba}(m_0)|^2 + |f_{ba}(-m_0)|^2 ]
$$
  
=  $(1 + h_0)^{-1} = \begin{cases} 0.77 & (\text{C}-\alpha) \\ 0.67 & (\text{N}-\alpha) \\ 0.70 & (\text{O}-\alpha). \end{cases}$ 

According to Wilczynski's model of deep inelastic reactions  $[27]$ , which ascribes the energy dissipation to frictional forces arising in the projectile-target contact region, up and down polarization can be related to positive and negative deflection function, respectively. Then, the observed positive polarization can be explained by assuming  $[22]$  that only one kind of *semiclassical trajectory*, i.e., the *far-side* one, predominantly contributes to the nonequilibrium component of the sequential emission.

The half-angle between the two peaks  $\chi_0$  can be related to the lifetime of the emitting nucleus by Eq.  $(20)$ , where we approximated  $\mathcal I$  with the rigid body moment of inertia of the emitting nucleus  $[23]$ 

$$
\mathcal{I} \approx \mathcal{I}_{rigid} \approx 0.0137 A^{5/3} \hbar^2.
$$

The last parameter obtained by the fit is  $\xi$ , which is related to the direction  $\phi_0$  of the momentum transferred in the projectile-target interaction; if we had dealt with hard spheres, this direction would correspond to the recoil direction of the targetlike nucleus,  $\phi_R$ . As one can deduce from Table I, these angles are not equal but their difference decreases for decreasing projectile-target mass transfer.

# **B.** The <sup>16</sup>O<sup>+48</sup>Ti reaction at  $E_{lab}$ <sup>(16</sup>O) = 133 MeV

The second experiment we studied was the  ${}^{16}O+{}^{48}Ti$  reaction performed at the IRES MP tandem accelerator in Strasbourg, France. Since the mean excitation energy of the emitting targetlike nuclei is about 60 MeV, a value lying in the continuum region of the excitation spectrum, we could apply the same theoretical approach we used in the  $16$ O  $+$ <sup>58</sup>Ni reaction to this nuclear system. Following the same procedure adopted as in the  $16O+58$ Ni case, we give an estimation of about 27 $\hbar$  of the  $J_B$  targetlike nucleus spin.

The strongly energy damped projectile residues  $(C, N, O)$ were detected in a  $(\Delta E_{gas}, E_{silicon})$  telescope at  $\theta_{lab}$ =  $-30^{\circ}$  with respect to the beam direction, while the  $\alpha$ -particle angular distributions were measured by means of  $(\Delta E_{gas}, E_{silicon(PSD)})$  telescope and two  $(\Delta E_{silicon}, E_{Cs})$ telescopes for small forward angles. The  $(\Delta E_{gas}, E_{silicon})$ telescope, already used for the  $16O+58$ Ni measurement, is suitable for identifying charges of heavy ions, as shown in Fig. 3. Then we used the Vivitron accelerator and an early stage of the ICARE facility (whose complete configuration is made up of 48 telescopes), thus obtaining a good resolution in the emission angle, kinetic energy and *Z* of the detected particle, as well as the mass of the light charged particles by means of the time of flight technique. Eight telescopes are mainly devoted to the heavy-ion detection, the remaining 40 detect light charged particles, such as  $p$ 's and  $\alpha$ 's, 16 of



FIG. 3. Example of charge identification spectrum for the  $(133)$ MeV)  ${}^{16}O+{}^{48}Ti$  reaction.

which are devoted to the high energy particles emitted.

24 double  $\Delta E_{\text{Si}}-E_{\text{CsI(Tl)}}$  telescopes are devoted to the detection of fewer than 30-MeV charged particles, while 16 triple  $\Delta E_{\text{Si1}}\Delta E_{\text{Si2}}$ - $E_{\text{CsI(T)}}$  telescopes detect the light particles carrying higher energy. The eight  $\Delta E_{gas}$ - $E_{Si}$  telescopes are ionization chambers used to identify heavy fragments with *Z*<40.

Figure 4 shows the  $(C-\alpha)$ ,  $(N-\alpha)$ , and  $(O-\alpha)$  in-plane angular correlations in the recoil center of mass system, extracted after subtracting undesired contributions such as  $\alpha$ 's coming from projectile breakup and *C* buildup contamination. By means of a procedure similar to the one followed for the  $^{16}O+^{58}Ni$  system, the fit parameters for the  $^{16}O+^{48}Ti$ system were obtained and are shown in Table II, with the corresponding curves in the same figure.

By applying this procedure to the  ${}^{16}O+{}^{48}Ti$  system we get

$$
p_0 = \begin{cases} 0.83 & (C-\alpha) \\ 0.74 & (N-\alpha) \\ 0.74 & (O-\alpha), \end{cases}
$$

showing also in this case how only one kind of *semiclassical trajectory* plays a predominant role, namely the *far-side* one.

The  $\phi_R$  angle, given in the last three rows of the secondlast columns of Tables I and II denotes the angle of the recoil direction of the  $\alpha$ -decaying targetlike nucleus *B* with respect to the beam angle. From the tables we can note that the difference ( $\phi_0 - \phi_R$ )—which is the angular interval between



FIG. 4. Best fit of the in-plane (C- $\alpha$ ), (N- $\alpha$ ), and (O- $\alpha$ ) differential multiplicity data for the sequential process  ${}^{16}O+{}^{48}Ti$  at 133 MeV laboratory energy  $[18–20]$ . The differential multiplicity, in units of  $10^{-2}$  sr<sup>-1</sup>, is plotted vs the in-plane  $\alpha$ -particle angle. The arrows indicate the directions of the projectilelike fragment  $(b)$ and targetlike fragment  $(B)$  with respect to the incident beam in the laboratory system;  $\phi_0$  is the direction of the average momentum transferred (see text).

the direction of the average momentum transferred in the  $^{58}$ Ni( $^{16}$ O,*b*)*B* as well as  $^{48}$ Ti( $^{16}$ O,*b*)*B* and the recoil direction of the  $\alpha$ -decaying nucleus  $B$ —is larger for larger mass transfer in the reaction considered and increases with the relative energy between projectile and target at the  $V_C$  barrier

$$
V_C = Z_a z_A e^2 / R, \quad R = r_0 (A_a^{1/3} + A_A^{1/3}), \quad r_0 = 1.4 \text{ fm.}
$$
\n(23)

In addition, one can obtain a rough estimate of the in-plane

Coincidences $(10^{-2} \text{ sr}^{-1})$	$C_E$ <sup>a</sup>	$\gamma$ <sup>a</sup>	$\Lambda_0$ <sup>a</sup>	$\Omega$ <sup>a</sup>	$\xi^{\,b}$	$\chi_0^{\ b}$
$C-\alpha$ $N-\alpha$ $O-\alpha$	$0.4 \pm 0.04$ $0.3 \pm 0.2$ $0.23 \pm 0.02$	$3.8 \pm 0.2$ $3.3 \pm 0.2$ $3.0 \pm 0.2$	$(9\pm6)$ ° $(6\pm4)$ ° $(9\pm6)$ °	$(35 \pm 4)$ ° $(29\pm3)$ ° $(25\pm3)$ °	$(-35\pm2)$ ° $(-39\pm2)$ ° $(-35\pm2)$ °	$(-38\pm2)$ ° $(-39\pm2)$ ° $(-35\pm2)$ °
Coincidences $(10^{-2} \text{ sr}^{-1})$ $C-\alpha$ $N-\alpha$ $O-\alpha$		$C_{NE}$ $4.5 \pm 0.5$ $2.5 \pm 0.3$ $1.8 \pm 0.2$	$\lambda$ $3.5 \pm 0.4$ $3.3 \pm 0.4$ $2.6 \pm 0.3$	h <sub>0</sub> $0.20 \pm 0.02$ $0.35 \pm 0.04$ $0.36 \pm 0.04$	$\phi_R$ $(55 \pm 2)^{\circ}$ $(51 \pm 2)$ ° $(55 \pm 2)$ °	$\phi_0$ $(29.5 \pm 1)$ ° $(34.7 \pm 1)$ ° $(39 \pm 1)$ °

TABLE II. List of the parameters obtained in the analysis of the out-of-plane and in-plane angular correlations coming from the  $(133 \text{ MeV})$  <sup>16</sup>O+<sup>48</sup>Ti reaction.

<sup>a</sup>The quantities obtained by fitting the experimental data by the evaporative formula (15).

<sup>b</sup>The quantities estimated from a simple inspection of the experimental angular correlation patterns by using the approximate expressions  $(19)$ .

integrated sequential E and NE  $\alpha$  emissions for the processes considered here; in fact, from Eqs.  $(8)$  and  $(13)–(18)$ , we can get ( $\vartheta = \pi/2$ )

$$
\int_{-\pi}^{\pi} d\phi M(\phi) = M^{E} + M^{NE}
$$
\n(24)

with

$$
M^{E} \sim \pi C_{E} [1 - \exp(-\gamma)], \qquad (25)
$$

$$
M^{NE} \sim C_{NE}(1 + h_0) / \lambda. \tag{26}
$$

The values per out-of-plane unit angle of  $M^E + M^{NE}$  estimated within 30% are listed in Table III. Although NE processes contribute at the percentage level at low bombarding energy, they cannot be neglected at increasing bombarding energies.

#### **IV. SUMMARY AND CONCLUSIONS**

Differential multiplicities for the  ${}^{16}O+{}^{58}Ni$  reaction at 6 MeV/nucleon and for the  $16O+48Ti$  at 8.25 MeV/nucleon have been measured for deep inelastic events. A theoretical semiclassical approach, assuming the hypothesis of a twostep sequential process, is proposed to further analyze the

TABLE III. Values of rough approximations of  $M<sup>E</sup>$  and  $M<sup>NE</sup>$  for the  $(96 \text{ MeV})$  <sup>16</sup>O+<sup>58</sup>Ni and  $(132 \text{ MeV})$  <sup>16</sup>O+<sup>48</sup>Ti reactions.

		$^{16}O+^{58}Ni$	$^{16}$ O + $^{48}$ Ti	
Coincidences	$M^E$	$M^{NE}$	$M^E$	$M^{NE}$
$C-\alpha$	4.1	1.2	1.2	1.5
$N-\alpha$	2.8	1.1	0.9	1.0
$O-\alpha$	1.5	0.4	0.7	0.9

measured angular correlations between  $\alpha$  particles detected in coincidence with the deep inelastic projectilelike fragments C, N, and O.

From this analysis, we can see that the angular interval between the average transferred momentum in  $^{58}$ Ni( $^{16}$ O,*b*)*B* and  $^{48}$ Ti( $^{16}$ O,*b*)*B* reactions, respectively, and the recoil nucleus *B* direction increases with the transferred mass by  $16$ O nucleus to  $58$ Ni and  $48$ Ti nuclei. In the application to the  $16O+58$ Ni and  $16O+48$ Ti systems, the positive alignment parameters that have been deduced for the respective projectilelike fragments suggests that the *far-side* trajectory is dominant. The nonequilibrium component for the  ${}^{16}O+{}^{48}Ti$ reaction is quite large compared to the one extracted from the  $^{16}O+^{58}Ni$  reaction. Equation (20), applied to the two systems studied, gives the following values for  $\tau_0$  revolution times:

and

$$
\tau_0 = 3 \times 10^{-22} \text{ s},
$$

 $\tau_0 = 5 \times 10^{-22}$  s

where we used the  $l_0$  values calculated from our data, i.e., 2 $\hbar$  and 6 $\hbar$ . These estimates of  $\tau_0$  can be regarded as the lifetimes of the targetlike fragments, i.e., the ''decay times'' after the formation of Ni and Cr, for  ${}^{16}O+{}^{58}Ni$  and  ${}^{16}O$  $+$ <sup>48</sup>Ti systems, respectively.

The simple semiclassical approach used here seems to be able to reproduce many of the observed features of the sequential E and NE  $\alpha$  emission and to extract reaction mechanism information directly by applying formulas  $(15)$  and  $(16)$ to the analysis of the experimental angular correlation data. Of course, this model should be applied to other nuclear systems for further investigation of the reaction mechanism of deep inelastic collisions. To this aim, analysis of experimental data is still in progress.

- [1] W. U. Schröder and J. R. Huizenga, in *Treatise on Heavy-Ion Science*, edited by D. A. Bromley, Nuclear Sciences Vol. 2 (Plenum, New York, 1994); D. Guerreau, *Nuclear Matter and Heavy-Ion Collisions*, edited by M. Soyer, H. Flocard, B. Tamain, and M. Porneuf (Plenum, New York, 1989).
- [2] H. Ho, P. L. Gonthier, M. N. Namboodiri, J. B. Natowitz, L. Adler, S. Simon, K. Hagel, R. Terry, and A. Khodai, Phys. Lett. 96B, 51 (1980).
- [3] T. C. Awes, G. Poggi, C. K. Gelbke, B. B. Back, B. G. Glagola, H. Breuer, and V. E. Viola, Jr., Phys. Rev. C **24**, 89  $(1981).$
- [4] H. Ho, G. Y. Fan, P. L. Gonthier, W. Kühn, B. Lindl, A. Pfoh, L. Schad, R. Wolski, J. P. Wurm, J. C. Adloff, D. Disdier, V. Rauch, and F. Scheibling, Nucl. Phys. **A437**, 465 (1985).
- [5] B. Lindl, A. Brucker, M. Bantel, H. Ho, R. Muffler, L. Schad, M. G. Trauth, and J. P. Wurm, Z. Phys. A **328**, 85 (1987).
- [6] W. Terlau, M. Burgel, A. Budzanowski, H. Fuchs, H. Homeyer, G. Roschert, J. Uckert, and R. Vogel, Z. Phys. A **330**, 303  $(1988).$
- [7] R. Wada, M. Gonin, M. Gui, K. Hagel, Y. Lou, D. Utley, J. B. Natowitz, G. Nebbia, D. Fabris, R. Billerey, B. Cheynis, A. Demeyer, D. Drain, D. Guinet, C. Pastor, L. Vagneron, K. Zaid, J. Alarja, A. Giorni, D. Heuer, C. Morand, B. Viano, C. Mazur, C. Ngô, S. Leray, R. Lucas, M. Ribrag, and E. Tomasi, Nucl. Phys. **A328**, 85 (1987).
- [8] H. Ho, P. L. Gonthier, W. Kühn, A. Pfoh, L. Schad, R. Wolski, J. P. Wurm, J. C. Adloff, D. Disdier, A. Kamili, V. Rauch, G. Rudolf, F. Scheibling, and A. Strazzeri, Phys. Rev. C **27**, 584  $(1983).$
- @9# A. Strazzeri, Nuovo Cimento Soc. Ital. Fis., A **52A**, 323  $(1979).$
- [10] A. Strazzeri, Nuovo Cimento Soc. Ital. Fis., A 80A, 35 (1984).
- [11] A. Italiano, A. Trifiro, G. Pisent, and A. Strazzeri, Nuovo Ci-

mento Soc. Ital. Fis., A 110A, 781 (1997).

- [12] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. 96, 448 (1954).
- [13] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, MA, 1963).
- [14] W. Hauser and Feshbach, Phys. Rev. 87, 366 (1952).
- [15] F. L. Friedman and V. F. Weisskopf, *Niels Bohr and the Development of Physics* (McGraw-Hill, London, 1955).
- [16] P. A. Gottschalk and M. Weström, Nucl. Phys. A314, 232  $(1979).$
- [17] H. Eichner, H. Stehle, and P. Heiss, Nucl. Phys. A205, 249  $(1973).$
- [18] T. Ericson and V. Strutinsky, Nucl. Phys. **8**, 284 (1958).
- [19] M. Lefort, in *Proceedings of the International School of Physics ''Enrico Fermi*,'' Course LXII, Varenna, edited by H. Faraggi and R. A. Ricci (North-Holland, Amsterdam, 1976).
- [20] N. K. Glendenning, International Conference on Reactions between Complex Nuclei, Nashville, 1974, Vol. 2, p. 137.
- [21] V. M. Strutinsky, Sov. Phys. JETP **19**, 1401 (1964).
- [22] P. D. Bond, Phys. Rev. Lett. **40**, 401 (1978).
- [23] H. Ho, R. Albrecht, H. Damjantschitsch, F. J. Demond, W. Kühn, J. Slemmer, J. P. Wurm, D. Disdier, V. Rauch, F. Scheibling, and T. Døssing, Z. Phys. A 300, 205 (1981).
- [24] T. Bellot, Thesis, Strasbourg, IReS 97-35 (unpublished).
- [25] R. C. Barná, D. De Pasquale, A. Italiano, A. Trifiró, M. Trimarchi, C. Beck, T. Bellot, F. Haas, V. Rauch, M. Rousseau, O. Stezowski, and A. Strazzeri, Proceedings of the 9th International Conference on Nuclear Reaction Mechanisms, Varenna, 2000.
- [26] H. Ho, R. Albrecht, H. Dünnweber, G. Graw, S. G. Steadman, J. P. Wurm, D. Disdier, V. Rauch, and F. Scheibling, Z. Phys. A **283**, 235 (1977).
- [27] J. Wilczynski, Phys. Lett. **47B**, 484 (1973).