Magnetic excitations in the nucleon-pair shell model

Yan-An Luo,[1,2] Xiao-Bing Zhang,[1] Feng Pan,[3,4] Ping-Zhi Ning,[1] and J. P. Draayer[4]
[1]*Institute of Physics, Nankai University, Tianjin 300071, People's Republic of China*
[2]*Institute of Theoretical Physics, Chinese Academia Sinica, Beijing 100080, People's Republic of China*
[3]*Department of Physics, Liaoning Normal University, Dalian 116029, People's Republic of China*
[4]*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803-4001*
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Magnetic transition strengths in ^{134}Ba are discussed within the framework of the nucleon-pair shell model (NPSM) truncated to the SD subspace. The S and D pairs are determined, respectively, by a variational method and a proton-neutron TDA approximation. The $M1$ and $M3$ transition strengths are found to be consistent with results from the proton-neutron interacting boson model calculations. The results confirm that the collective magnetic properties of ^{134}Ba are primarily due to the orbital motion of nucleons.

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It is known that magnetic excitations, which occur between mixed-symmetry states (MSSs) and symmetric ones, signal the existence of MSSs. This has been discussed extensively within the framework of the proton-neutron interacting boson model (IBM-2) [1] and the fermion dynamical symmetry model (FDSM) [2], as well as other theories [3]. The best known MSS is the 1^+ state that was first discovered in electron scattering experiments in well-deformed nuclei [4]. Recently, mixed-symmetry 1^+ states have also been found in O(6)-like nucleus ^{196}Pt [5,6] and ^{134}Ba [7], a transitional nucleus between O(6) and U(5) [8]. In addition to the mixed-symmetry 1^+ states, some higher-lying 2^+ states have also been identified as MSSs [7,9,10].

Since data on magnetic dipole moments and transitions are now readily available, every model should be tested as to its ability to make reasonable $M1$ predictions. This is especially so for models that champion collective modes since a proper description of the magnetic dipole properties of nuclei that display collective features has long been a challenging problem.

In our previous paper [11], the nucleon-pair shell model (NPSM) was proposed for a description of nuclear collective motion. This model uses ''realistic'' collective nucleon pairs with various angular momenta as the basic building blocks for wave functions. From Refs. [12–15] we know that although no dynamical symmetry is imposed in the NPSM, the model can reproduce the main results of two theories: IBM, which imposes a boson assumption, and FDSM, which is fermion-based.

In this report we study $M1$ as well as $M3$ excitations within the framework of the NPSM truncated to the SD subspace. ^{134}Ba is taken as an example since its $M1$ transitions are known experimentally. In order to account for as many experimental results as possible with as few parameters as possible, and at the same time avoid excessive computational requirements, we choose a rather simple Hamiltonian consisting of a surface delta interaction (SDI) [16] between like nucleons and a quadrupole-quadrupole interaction between the protons (π) and neutrons (ν),

$$H=H_0-V(\pi)-V(\nu)-\kappa Q_\pi^2 Q_\nu^2, \tag{1}$$

$$H_0=\sum_{a\sigma} \varepsilon_{a\sigma}\hat{n}_{a\sigma}, \tag{2}$$

$$V(\sigma)=V_{SDI}(\sigma)=4\pi G_\sigma \sum_{i>j=1}^{n} \delta(\Omega_{ij}),$$

$$T \quad \sigma=\pi,\nu \tag{3}$$

where ϵ_a and $\hat{n}_a$ are the nucleon single-particle energy and the number operator, respectively. The $E2$ transition operator is

$$T(E2)=e_\pi Q_\pi^2+e_\nu Q_\nu^2, \tag{4}$$

where e_ν and e_π are effective charges of proton and neutron hole, respectively. The $M1$ transition operator is

$$T(M1)=\sqrt{\frac{3}{4\pi}}\sum_{\rho=\pi,\nu}\left\{g_{l,\rho}^{\text{eff}}\sum_{i\in\rho} l_i+g_{s,\rho}^{\text{eff}}\sum_{i\in\rho} s_i\right\}, \tag{5}$$

and the $M3$ transition operator is

$$T(M3)=\frac{\sqrt{21}}{2}\sum_{\rho=\pi,\nu}\left\{g_{l,\rho}^{\text{eff}}\sum_{i\in\rho} r_i^2[Y^{(2)}(\hat{\mathbf{r}}_i)l_i]^{(3)} +2g_{s,\rho}^{\text{eff}}\sum_{i\in\rho} r_i^2[Y^{(2)}(\hat{\mathbf{r}}_i)l_i]^{(3)}\right\} \tag{6}$$

where $g_{l,\rho}^{\text{eff}}$ and $g_{s,\rho}^{\text{eff}}$ are the orbital and spin effective gyromagnetic ratios.

The building blocks of the NPSM in the SD subspace are ''realistic'' collective pairs

TABLE I. The single-particle (hole) energies for proton (neutron) for $^{133}_{51}$Sb$_{82}$ ($^{131}_{50}$Sn$_{81}$).

ϵ_π (MeV)	$g_{7/2}$	$d_{5/2}$	$d_{3/2}$	$h_{11/2}$	$s_{1/2}$
	0	0.963	2.69	2.76	2.99
ϵ_ν (MeV)	$d_{3/2}$	$h_{11/2}$	$s_{1/2}$	$d_{5/2}$	$g_{7/2}$
	0	0.242	0.332	1.655	2.343

TABLE II. The amplitude of the S pair (α_j) and D pairs ($\beta_{jj'}$) for neutron-hole.

$\alpha_{1/2}$	$\alpha_{3/2}$	$\alpha_{5/2}$	$\alpha_{7/2}$	$\alpha_{11/2}$
7.529×10^{-4}	5.116×10^{-3}	1.704×10^{-4}	-1.193×10^{-4}	-1.256×10^{-3}
$\beta_{1/2,3/2}=3.378\times10^{-1}$		$\beta_{1/2,5/2}=4.030\times10^{-2}$		
$\beta_{3/2,3/2}=-2.859\times10^{-1}$		$\beta_{3/2,5/2}=4.989\times10^{-2}$		$\beta_{3/2,7/2}=7.772\times10^{-2}$
$\beta_{5/2,5/2}=-2.172\times10^{-2}$		$\beta_{5/2,7/2}=4.229\times10^{-3}$		
$\beta_{7/2,7/2}=-1.120\times10^{-2}$				
$\beta_{11/2,11/2}=3.269\times10^{-1}$				

$$A_\nu^{r\dagger}=\sum_{cd} y(cdr)(C_c^\dagger\times C_d^\dagger)_\nu^r,\quad r=0,2 \tag{7}$$

where $y(cdr)$ are structure coefficients for the pair $A^{r\dagger}$. Many-body effects are included in the S pair via a variational technique, and in the D pair through a proton-neutron Tamm-Dancoff approximation. The single-particle energies (SP), $H_0=H_0^{\rm expt}(\pi)+H_0^{\rm expt}(\nu)$, where $H_0^{\rm expt}(\pi)$ and $H_0^{\rm expt}(\nu)$ were fixed by the single-particle levels of the neighboring odd-A ${}^{133}_{51}\mathrm{Sb}_{82}$ and ${}^{131}_{50}\mathrm{Sn}_{81}$ nuclei, respectively, reported in Ref. [17] and listed in Table I. The parameters used in the calculation were obtained by fitting to the ^{134}Ba experimental excitation energies: $G_\pi=0.139$ MeV, $G_\nu=0.056$ MeV, and $\kappa=0.144$ MeV.

To investigate the influence of the SP energy splitting on pair structure, we list the distribution coefficients of the S pair and D pair for neutron-hole in Table II, from which one can see that both the S pair and D pair favor the lowest SP levels, i.e., the lower the energy level, the larger the distribution coefficient.

To show the validity of the NPSM truncated to the SD subspace, calculated results of the spectra as well as $B(E2)$ ratios of ^{134}Ba with the effective charges fixed at $1.9e$ ($1.8e$) for proton (neutron-hole) are shown along with the corresponding experimental values in Fig. 1. The effective charges were determined by fitting to the $B(E2;2_1^+\to0_1^+)$ for ^{134}Ba. It can be seen from Fig.1 that this procedure gave results that are good agreement with experiment.

To study the magnetic transitions, we set the g factors to the same values as used in Ref. [18], i.e., $g_l^{\rm eff}=g_l^{\rm free}$ and $g_s^{\rm eff}=0.7g_s^{\rm free}$. The results are shown in Table III. For ^{134}Ba, there are two almost degenerate 2^+ states (2_3^+ and 2_4^+) that share the experimental $M1$ strength [7,10]. The sum of the two $B(M1)$ values is 0.20(2) μ_N^2. Table III shows that these results are in agreement with those of determined using IBM [10,19–22] and FDSM [2]. The strongest $M1$ transition between the 2^+ states is $B(M1;2_3^+\to2_1^+)=0.2749\mu_N^2$. To see the correlation between the M1 transition and the structure of the wave function, a few of the most important eigenstates are listed in Table IV, from which one can see that the $M1$ transition between two states is strong only if one is mixed symmetry while the other is symmetric. From Table V one can also see that the strongest $M1$ transition in our NPSM calculation is $B(M1;0_1^+\to1_1^+)=0.54\mu_N^2$, which is in agreement with the O(6) limit of the IBM-2 in which the scissors mode is described by the lowest 1^+ state. From Ref. [7], we know that experimentally the total $M1$ strength is 0.56(4) μ_N^2. Our calculated result is very close to this experimental value.

From Fig. 1 one can see that the 2_3^+ state is lower than the 1_1^+ state, which is in agreement with the experimental result that the lowest MSS for ^{134}Ba is the 2^+ state. From Ref. [7],

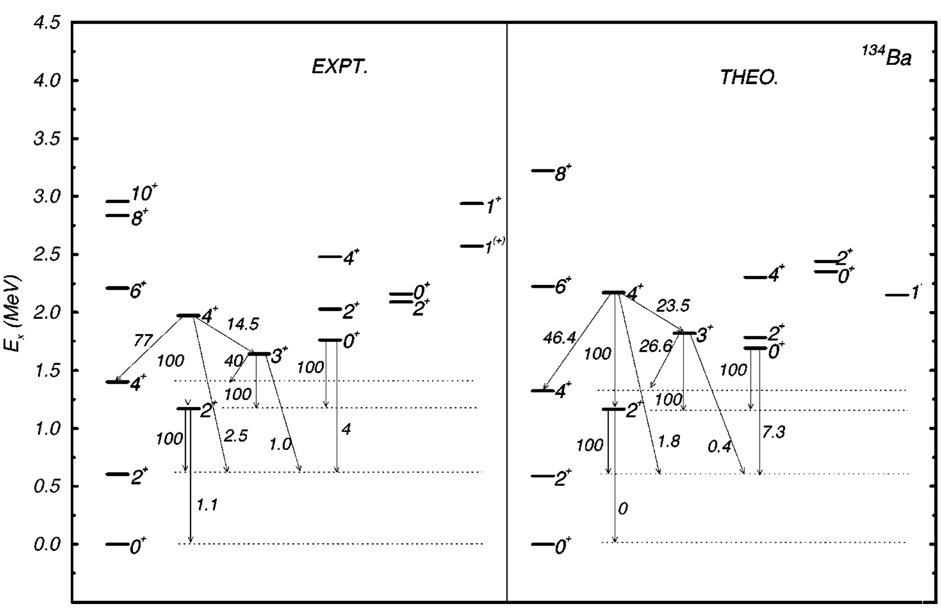

FIG. 1. The spectra and relative $B(E2)$ values for ^{134}Ba.

TABLE III. The $B(M1)$ values (in units of μ_N^2) for the 2^+ and 1^+ states.

theo.		expt.	
$B(M1;2_2^+\rightarrow 2_1^+)$	0.0557	$B(M1;2_2^+\rightarrow 2_1^+)$	0.0003(1)
$B(M1;2_3^+\rightarrow 2_1^+)$	0.2749	$B(M1;2_3^+\rightarrow 2_1^+)$	0.062(8)
$B(M1;2_4^+\rightarrow 2_1^+)$	0.1018	$B(M1;2_4^+\rightarrow 2_1^+)$	0.137(12)
$B(M1;2_5^+\rightarrow 2_1^+)$	0.0436	$B(M1;2_5^+\rightarrow 2_1^+)$	0.001(1)
$B(M1;0_1^+\rightarrow 1_1^+)$	0.5359	$B(M1;0_1^+\rightarrow 1^+)_{E_{1^+}=2571}$	0.081(12)
$B(M1;0_1^+\rightarrow 1_2^+)$	0.0450	$B(M1;0_1^+\rightarrow 1^+)_{E_{1^+}=2939}$	0.31(4)
$B(M1;0_1^+\rightarrow 1_3^+)$	0.0980	$B(M1;0_1^+\rightarrow 1^+)_{E_{1^+}=3027}$	0.039(8)
$B(M1;0_1^+\rightarrow 1_4^+)$	0.0133	$B(M1;0_1^+\rightarrow 1^+)_{E_{1^+}=3246}$	0.022(6)
		$B(M1;0_1^+\rightarrow 1^+)_{E_{1^+}=3327}$	0.075(15)
		$B(M1;0_1^+\rightarrow 1^+)_{E_{1^+}=3450}$	0.036(8)
$B(M1;1_1^+\rightarrow 2_2^+)$	0.2636	$B(M1;1^+\rightarrow 2_2^+)_{E_{1^+}=2571}$	0.096(18)
		$B(M1;1^+\rightarrow 2_2^+)_{E_{1^+}=2939}$	0.156(53)
$B(M1;1_1^+\rightarrow 2_1^+)$	0.0491	$\Sigma_i B(M1;1_i^+\rightarrow 2_1^+)$	0.101(32)

we know that the $M1$ transition between the 1^+ state and the quasi-γ band head exceeds that between the 1^+ and 0_1^+ for ^{134}Ba. The measured ratio, $R_{\mathrm{expt}}=B(M1;1^+\rightarrow 2_2^+)/B(M1;1^+\rightarrow 0_1^+)$, is 3.57(12) for the 1^+ state at 2571 keV and is 1.52(32) for the 1^+ state at 2939 keV. The 1^+ state at 2939 keV is the state with the largest experimental $B(M1)$ value. A similar decay pattern was found for the γ-soft ^{196}Pt nucleus [5,6]. From the NPSM calculation we see that the $B(M1;1_1^+\rightarrow 2_2^+)$ is indeed much larger than the $B(M1;1_1^+\rightarrow 0_1^+)$. The predicted ratio R_{theo} is 1.48.

In addition to the $1^+\rightarrow 2_2^+$ transition, the $M1$ transition from the 1^+ state to the 2_1^+ state, which is forbidden within the IBM framework, was observed in ^{134}Ba [7]. This transition was explained as a F-spin $E2$ transition in IBM-2 [5,6]. This interpretation is based on the $M1$ selection rules in the O(6) limit. However, there is no symmetry imposed in the NPSM and therefore the strict $M1$ selection rule is not valid in our model. It is thus of crucial importance to study whether a breaking of the O(6) dynamical symmetry may lead to a $M1$ transition between the 1^+ and the 2_1^+ states that is strong enough to account for the experimentally observed strengths. Using the parameters obtained by Puddu, Scholten, and Otsuka [23] in an IBM-2 calculation, the calculated $B(M1;1_1^+\rightarrow 2_1^+)$ and is about 0.0035 μ_N^2, while the experimentally observed strength is about $0.1\mu_N^2$. This calculated value is too small to explain the observed $1^+\rightarrow 2_1^+$ decay strength as a pure $M1$ transition [7]. Our NPSM calculation, as shown in Table III, yields $B(M1;1_1^+\rightarrow 2_1^+)$ is $0.0491\mu_N^2$. Though still small, it is comparable with the experimental value.

TABLE V. The contribution of spin and orbital part to $M1$ and $M3$ excitation.

		Proton		Neutron	
	$J_i^+\rightarrow J_f^+$	Spin	Orbit	Spin	Orbit
$M1$	$0_1^+\rightarrow 1_1^+$	0.0512	−0.9220	0.0534	0.8175
	$2_2^+\rightarrow 2_1^+$	0.0419	−0.5468	0.0374	0.4676
	$2_3^+\rightarrow 2_1^+$	−0.0671	1.1621	−0.0648	−1.0302
	$2_4^+\rightarrow 2_1^+$	−0.0436	0.7042	−0.0445	−0.6162
M3	$3_1^+\rightarrow 0_1^+$	−0.0729	−1.5642	0.1871	1.0092
	$3_2^+\rightarrow 0_1^+$	−0.1653	−2.4411	0.1126	0.8314
	$3_3^+\rightarrow 0_1^+$	−0.0532	−0.9654	0.0184	−0.0683
	$3_4^+\rightarrow 0_1^+$	−0.0769	−0.3033	−0.0810	−0.4779

The $M3$ transitions between 3^+ and 0_1^+ states were also studied. The results are consistent with the prediction of the IBM-2 calculation of Ref. [18], namely, the $M3$ transition strength is split between the first two 3^+ states with the $B(M3;3_1^+\rightarrow 0_1^+)$ equal to $6.1795\mu_N^2\,\mathrm{fm}^4$ and the $B(M3;3_2^+\rightarrow 0_1^+)$ equal to 11.5374.

The theoretical results on the spin contribution to the magnetic transition yield conflicting results. In Ref. [24] it is claimed that within an IBM-2 framework spin contributions to matrix elements of collective states approximately cancel, so the collective magnetic properties are due to the orbital motion of nucleons only. However, the spin contribution was

TABLE IV. Main components of part of eigenstates for ^{134}Ba. The coefficients inside the parentheses are for those multipair basis states that occur more than once, which are distinguished by the intermediate angular momentum.

State	S	D_ν	D_π	$D_\pi D_\nu$	D_ν^2	D_π^2	$D_\pi^2 D_\nu$	$D_\nu^2 D_\pi$	$D_\nu^2 D_\pi^2$	D_π^3	$D_\nu^3 D_\pi$	$D_\pi^3 D_\nu$
0_1^+	−1.0744			−0.6321	−0.5360				−0.2139			
0_2^+					−0.6112	0.2286	−0.2195	−0.4766	0.2176			
2_1^+		−0.9171	−0.5813				−0.2497	−0.2977				
							(−0.2617)	(−0.2634)				
2_2^+				−0.5368	−0.6982	−0.2881						
2_3^+		−0.4372	0.4262		−0.6043	0.2202	0.2902	−0.2537				
2_4^+							0.3746	0.3579	0.2286			0.2003
							(0.2033)	(0.4715)	(0.2522)			
									(0.2241)			
4_1^+				−0.6431	−0.6303	−0.4099						
6_1^+							0.5864	0.5100	0.2014	0.3654		0.2429

obviously included in the microscopic calculation of the IBM-2 (for example, see Ref. [25]). Moreover, IBM-2 results reported in Ref. [18] suggest that while the above conclusion, i.e., the collective magnetic properties are due to the orbital motion of nucleons only, is approximately true for $M1$ transitions, the spin contributions can have a strong influence over $M3$ strengths. To help clarify this matter, our results for the spin and the orbital parts for both $M1$ and $M3$ excitations are listed in Table V. One can see that although the spin contributions are not completely negligible, they are much smaller than the orbital part for both $M1$ and $M3$ excitations.

In summary, magnetic strength distributions for ^{134}Ba have been investigated within the framework of the NPSM truncated to the SD subspace. The SD pairs were determined, respectively, by variational and proton-neutron TDA methods. The results show that the $M1$ strength distribution can be reproduced rather well. And in agreement with IBM-2 calculations, the $M3$ transitions were found to be split between the first two 3^+ states. Concerning the relative contribution of the spin and the orbital parts to the $M1$ and $M3$ strengths, our results confirm earlier work suggesting that the collective magnetic properties are due primarily to the orbital motion of nucleons.

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