Coulomb distortion effects for electron or positron induced (e,e') reactions in the quasielastic region

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In response to recent experimental studies we investigate Coulomb distortion effects on (e,e') reactions from medium and heavy nuclei for the case of electrons and positrons. We extend our previously reported full distorted-wave Born approximation treatment of Coulomb distortions to the case of positrons for the ^{208}Pb (e,e') reaction in the quasielastic region for a particular nuclear model. In addition, we use previously reported successful approaches to treating Coulomb corrections in an approximate way to calculate the Coulomb distortion effects for (e,e') reactions for both electrons and positrons for the case of a simple nuclear model for quasielastic knockout of nucleons. With these results in hand we develop a simple *ad hoc* approximation for use in analyzing experiments, and discuss methods of extracting the "longitudinal structure function" which enters into evaluation of the Coulomb sum rule. These techniques are generally valid for lepton induced reactions on nuclei with momentum transfers greater than approximately 300 MeV/c.

I. INTRODUCTION

A persistent problem in using electron scattering for investigating nuclear structure and nuclear properties, especially in the quasielastic region, is the large static Coulomb field of medium and heavy nuclei. The presence of the static Coulomb potential (of order 25 MeV at the surface of the ^{208}Pb nucleus) invalidates one of the primary attributes of electron scattering as usually presented. Namely, that in the electron plane-wave Born approximation, the cross section can be written as a sum of terms each with a characteristic dependence on electron kinematics and containing various bilinear products of the Fourier transform of charge and current matrix elements. That is, various structure functions for the process can be extracted from the measured data by so-called Rosenbluth separation methods. The trouble with this picture is that when Coulomb distortion of the electron (or positron) wave functions arising from the static Coulomb field of the target nucleus is included exactly by partial wave methods, the structure functions can no longer be extracted from the cross section, even in principle.

In the early 1990's, Coulomb distortion for the reactions (e,e') and $(e,e'p)$ in quasielastic kinematics was treated exactly by the Ohio University group [1–5] using partial wave expansions of the electron wave functions. Such partial wave treatments are referred to as the distorted-wave Born approximation (DWBA) since the static Coulomb distortion is included exactly by numerically solving the radial Dirac equation containing the Coulomb potential for a finite nuclear charge distribution to obtain the distorted electron wave functions. While this calculation permits the comparison of nuclear models to measured cross sections and provides an invaluable check on various approximate techniques of including Coulomb distortion effects, it is numerically

challenging and computation time increases rapidly with higher incident electron energy. Furthermore, the initial computer codes did not include the option of calculating positron induced reactions in an obvious manner although only the sign of the Coulomb distortion term in the Dirac equation needed to be changed. And, as noted above, it was not possible to separate the cross section into various terms containing the structure functions and develop insights into the role of various terms in the transition charge and current distributions.

In our DWBA investigations of (e,e') and $(e,e'p)$ reactions in the quasielastic region, we used a relativistic treatment based on the σ-ω model for the nucleons involved. In particular, for the $(e,e'p)$ reaction we use a relativistic Hartree single particle model for a bound state [6] and a relativistic optical model for an outgoing proton [7] combined with the free space relativistic current operator $J^{\mu}=\gamma^{\mu}+i(\kappa/2M)\sigma^{\mu\nu}\partial_{\nu}$. For the (e,e') case we solve for the continuum nucleon wave functions using the real bound state potential so as to maintain current conservation. Using these models, we compared our DWBA calculations with experimental data measured at various laboratories for (e,e') [1,2], and for $(e,e'p)$ [3–5] and have found excellent agreement with the data. We concluded that the relativistic nuclear models are in excellent agreement with the measured data and note that we do not need to invoke meson exchange effects and other two-body terms in the current that are necessary in a Schrödinger description that uses a nonrelativistic reduction of the free current operator [8]. However, other investigators use other nuclear models and our elaborate DWBA code cannot be easily modified to include different transition currents.

To avoid the numerical difficulties associated with DWBA analyses at higher electron energies and to look for a way to

still define structure functions, our group [9–11] developed an approximate treatment of the Coulomb distortion based on the work of Knoll [12] and the work of Lenz and Rosenfelder [13]. We were able to greatly improve some previous attempts along this line [14,15] where various additional approximations were made which turned out not to be valid. The essence of the approximation is to calculate the four potential A_μ arising from the lepton four current in the presence of the static Coulomb field of the nucleus. This is possible for momentum transfers greater than approximately 300 MeV/c in a limited spatial region which we take to be of order $3R$ where R is the nuclear charge radius. The Coulomb distortion is included in the four potential A_μ by the elastic scattering lepton phase shifts and by letting the magnitude of the lepton momentum include the effect of the static Coulomb potential. This last step leads to an r-dependent momentum. A key result of our approximation method is that the separation of the cross section into a "longitudinal" term and a "transverse" term is still possible.

We compared our approximate treatment of Coulomb distortion (which we will designate as approximate DW) to the exact DWBA results for the reaction $(e,e'p)$ and found good agreement (at about the 1–2 % level) near the peaks of cross sections even for heavy nuclei such as ^{208}Pb. With an improved parametrization of the elastic scattering electron phase shifts [11], we achieve quite good agreement away from the peaks in the cross sections. Using this approximate DW treatment of Coulomb distortions for the inclusive (e,e') reaction is much more difficult numerically since the direction of the outgoing nucleon has to be integrated over, and all the nucleons in the nucleus have to be knocked out. Therefore, we sought even more severe approximations in order to obtain a simple *ad hoc* method of calculating the structure functions for (e,e') reactions. In our earlier work, we found it necessary to use different *ad hoc* procedures for the longitudinal and transverse terms [10], although our investigation of the *ad hoc* procedure for the longitudinal terms was hindered by the fact that the longtitudinal contributions to the total cross section are usually considerably less than 50% and thus we did not have great sensitivity to the Coulomb corrections for the longitudinal structure function. In this paper we will use a simple nonrelativistic toy model to calculate the Coulomb corrections to the longitudinal structure function with our approximate DW methods that we applied to $(e,e'p)$ and then investigate the *ad hoc* treatment of the longitudinal structure function which is a key ingredient in investigating the Coulomb sum rule. After developing an improved *ad hoc* procedure using our toy model we compare it to the full DWBA calculation which we have now extended to include positron induced reactions.

II. APPROXIMATE TREATMENT OF COULOMB DISTORTION

Our approximate method of including the static Coulomb distortion in the electron wave functions is to write the wave functions in a plane-wave-like form [10]

$$\Psi^\pm(\mathbf{r}) = \frac{p'(r)}{p} e^{\pm i\delta(\mathbf{L}^2)} e^{i\Delta} e^{i\mathbf{p}'(r)\cdot\mathbf{r}} u_p,\qquad(1)$$

where the phase factor $\delta(\mathbf{L}^2)$ is a function of the square of the orbital angular momentum, u_p denotes the Dirac spinor, and the local effective momentum $\mathbf{p}'(\mathbf{r})$ is given in terms of the Coulomb potential of the target nucleus by

$$\mathbf{p}'(\mathbf{r}) = \left(p - \frac{1}{r}\int_0^r V(r)dr \right)\hat{\mathbf{p}}.\qquad(2)$$

The *ad hoc* term $\Delta = a[\hat{\mathbf{p}}'(r)\cdot\hat{r}]\mathbf{L}^2$ denotes a small higher order correction to the electron wave number which we have written in terms of the parameter $a = -\alpha Z(16 \text{ MeV}c/p)^2$. The value of 16 MeV/$c$ was determined by comparison with the exact radial wave functions in a partial wave expansion. We have examined the positron case ($Z \mapsto -Z$) and find that this parametrization works equally well when compared to the exact radial positron wave functions.

We calculate the elastic scattering phases and fit them to a function of the square of the Dirac quantum number κ used to label the phase shifts. We then replace the discrete values of κ^2 with the total angular momentum operator $\mathbf{J}^2$ which we subsequently replace by the orbital angular momentum operator $\mathbf{L}^2$ since the low κ terms where the difference between j and l is significant contribute very little to the cross section. Finally we replace the angular momentum operator squared by its classical value $(\mathbf{r}\times\mathbf{p})^2$. The removal of any spin dependence apart from what is in the Dirac spinor u_p is crucial for writing the cross section as the sum of a longitudinal and a transverse contribution.

Initially [9] we fitted the phases δ_κ to a quadratic function of κ^2 which worked reasonably well for lower electron energies, but with the prospect of new higher energy electron accelerators, we needed a fit to the phases that will work at higher energies. In addition, we wanted to avoid calculating all of the elastic phase shifts, particularly the very high κ values. We decided to make use of the fact that the higher κ phase shifts approach the point Coulomb phases which have a simple analytical form at high energy. The low κ phases, corresponding to orbitals which penetrate the nucleus, are linear in κ^2 which was the basis or our initial parametrization. The difficult phases to fit correspond to κ values of order pR which, from a classical point of view, correspond to scattering from the nuclear surface region and are known to make large contributions to the cross section. We were able to find a parametrization of the elastic scattering phases shifts in terms of κ^2 which has the correct large κ^2 behavior and becomes linear in κ^2 at low angular momentum, and since we have the correct large κ behavior, we need only calculate the exact scattering phase shifts for κ values up to order pr. After some investigation [11], we found that the following parametrization of elastic scattering phase shift describes the exact phase shifts very well:

$$\delta(\kappa) = \left[a_0 + a_2 \frac{\kappa^2}{(pR)^2} \right] e^{-1.4\kappa^2/(pR)^2}$$

$$- \frac{\alpha Z}{2}(1 - e^{-\kappa^2/(pR)^2}) \times \ln(1 + \kappa^2), \qquad (3)$$

where p is the electron momentum and we take the nuclear radius to be given by $R = 1.2A^{1/3} - 0.86/A^{1/3}$. We fit the two constants a_0 and a_2 to two of the elastic scattering phase shifts [$\kappa = 1$ and $\kappa = \text{Int}(pR) + 5$]. To a very good approximation, $a_0 = \delta(1)$ and $a_2 = 4\delta[\text{Int}(pR) + 5] + \alpha Z \ln(2pR)$, where $\text{Int}(pR)$ replaces pR by the integer just less than pR. Note that this parametrization only requires the value of the exact scattering phase shift for $\kappa = 1$ and $\kappa = \text{Int}(pR) + 5$. For this paper we have confirmed that this same parametrization works equally well for the positron phase shifts.

Using the new phase shift parametrization and the local effective momentum approximation, we construct plane-wave-like wave functions for the incoming and outgoing electrons. Since the only spinor dependence is in the Dirac spinor all of the Dirac algebra goes through as usual and we end up with a Møller-like potential given by

$$A_\mu^{\text{appro DW}}(\mathbf{r}) = \frac{4\pi e}{q^2 - \omega^2} e^{i(\delta_i\{[\mathbf{r} \times \mathbf{p}'_i(r)]^2\} + \delta_f\{[\mathbf{r} \times \mathbf{p}'_f(r)]^2\})}$$

$$\times e^{i(\Delta_i - \Delta_f)} e^{i\mathbf{q}'(r)\cdot\mathbf{r}} \bar{u}_f \gamma_\mu u_i, \qquad (4)$$

where the phase shift parametrization is given in Eq. (3) with κ^2 being replaced by $(\mathbf{r} \times \mathbf{p})^2$, the parameter Δ is given following Eq. (2), and the r-dependent momentum transfer is given by $\mathbf{q}'(r) = \mathbf{p}'_i(r) - \mathbf{p}'_f(r)$.

With this approximate DW four potential A_μ it is straightforward to calculate the $(e,e'p)$ cross sections and modified structure functions. We showed [11] that using this new phase shift (see Refs. [9,10] for details) we can reproduce the full DWBA cross sections for $(e,e'p)$ from medium and heavy nuclei very well.

III. APPLICATION TO THE INCLUSIVE PROCESS

In the plane wave Born approximation (PWBA), where electrons or positrons are described as Dirac plane waves, the cross section for inclusive quasielastic (e,e') processes can be written simply as

$$\frac{d^2\sigma}{d\Omega_e d\omega} = \sigma_M \left\{ \frac{q_\mu^4}{q^4} S_L(q,w) + \left[\tan^2\frac{\theta_e}{2} - \frac{q_\mu^2}{2q^2} \right] S_T(q,w) \right\}, \qquad (5)$$

where $q_\mu^2 = \omega^2 - \mathbf{q}^2$ is the four-momentum transfer, σ_M is the Mott cross section given by

$$\sigma_M = \left(\frac{\alpha}{2E} \right)^2 \frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}},$$

and S_L and S_T are the longitudinal and transverse structure functions which depend only on the momentum transfer q and the energy transfer ω. As is well known, by keeping the momentum and energy transfers fixed while varying the electron energy E and scattering angle θ_e, it is possible to extract the two structure functions with two measurements. As we will summarize below, our approximate treatment of Coulomb distortions still permit Rosenbluth-like separations but with Coulomb corrections which require the use of models.

For the inclusive cross section (e,e'), the longitudinal and transverse structure functions in Eq. (5) are bilinear products of the Fourier transform of the components of the nuclear transition current density integrated over outgoing nucleon angles. Explicitly, the structure functions for knocking out nucleons from a shell with angular momentum j_b are given by

$$S_L(q,\omega) = \sum_{\mu_b s_p} \frac{\rho_p}{2(2j_b+1)} \int |N_0|^2 d\Omega_p, \qquad (6)$$

$$S_T(q,\omega) = \sum_{\mu_b s_p} \frac{\rho_p}{2(2j_b+1)} \int (|N_x|^2 + |N_y|^2) d\Omega_p, \qquad (7)$$

where the nucleon density of states $\rho_p = pE_p/(2\pi)^2$, the z axis is taken to be along $\mathbf{q}$, and μ_b and s_p are the z components of the angular momentum of the bound and continuum state particles. The Fourier transfer of the nuclear current $J^\mu(\mathbf{r})$ is simply

$$N^\mu = \int J^\mu(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3r \qquad (8)$$

and the continuity equation has been used to eliminate the z component (N_z) via the equation $N_z = -(\omega/q)N_0$.

When we use our approximate Møller potential given in Eq. (4), we also can separate the cross section into longitudinal and transverse components since as noted previously, it is the Dirac spinor structure that leads to this result. However, when we use the approximate electron four potential along with current conservation to eliminate the z component of the current we run into a problem since the momentum transfer $\mathbf{q}'$ depends on r both in magnitude and direction. In addition, the phase factors depend on $\mathbf{r}$. To avoid generating additional terms we assume the direction of $\mathbf{q}'(r)$ is along the asymptotic momentum transfer $\mathbf{q}$ which defines the $\hat{z}$ axis, and neglect the dependence on $\mathbf{r}$ in the phases and in $\mathbf{q}'(r)$, when taking the divergence of $\mathbf{N}$. With this further approximation, current conservation implies $\omega N_0 + \mathbf{q}'(r)\cdot\mathbf{N} = 0$. Using these results, the approximate cross section for the inclusive reaction (e,e') can be written as

$$\frac{d^2\sigma}{d\Omega_e d\omega} = \sigma_M \left\{ \frac{q_\mu^4}{q^4} S'_L(q,w) + \left[\tan^2\frac{\theta_e}{2} - \frac{q_\mu^2}{2q^2} \right] S'_T(q,w) \right\} \qquad (9)$$

and the transform of the transition nuclear current elements which appears in S_L and S_T are given by

$$N_0^{\text{appro DW}} = \int \left(\frac{q'_\mu(r)}{q_\mu}\right)^2 \left(\frac{q}{q'(r)}\right)^2 e^{i\delta_f\{[\mathbf{r}\times\mathbf{p}'_i(r)]^2\}}$$

$$\times e^{i\delta_f\{[\mathbf{r}\times\mathbf{p}'_f(r)]^2\}} e^{i(\Delta_i-\Delta_f)} e^{i\mathbf{q}'(r)\cdot\mathbf{r}} J_0(\mathbf{r}) d^3r, \tag{10}$$

$$\mathbf{N}_T^{\text{appro DW}} = \int e^{i\delta_i\{[\mathbf{r}\times\mathbf{p}'_i(r)]^2\}} e^{i\delta_f\{[\mathbf{r}\times\mathbf{p}'_f(r)]^2\}}$$

$$\times e^{i(\Delta_i-\Delta_f)} e^{i\mathbf{q}'(r)\cdot\mathbf{r}} \mathbf{J}_T(\mathbf{r}) d^3r. \tag{11}$$

Due to the angular dependence in the phase factors in Eqs. (10) and (11), a multipole expansion of the approximate potential is not practical. Thus, $N_0^{\text{appro DW}}$ and $\mathbf{N}_T^{\text{appro DW}}$ have to be evaluated by carrying out a three-dimensional numerical integration. As we have shown for the $(e,e'p)$ case [11], this numerical integration reproduces the exact DWBA results very well. However, since the inclusive reaction (e,e') requires a sum over all occupied neutron and proton shells and a further integration over the directions of the outgoing nucleon, numerical integration is very time consuming. In order to have a more practical procedure we examine additional approximations that will allow the integration over the angular coordinates in Eqs. (10) and (11) to be done analytically.

We created such an *ad hoc* procedure in a previous paper [9], but we were comparing our *ad hoc* procedures to the exact DWBA calculation which was largely dominated by the transverse terms. Hence, our *ad hoc* procedures for the longitudinal term were not very well determined. In addition, our full DWBA calculation was only set up for electrons, so we could not check the *ad hoc* approximation for positrons. In order to address this matter, we created a simple toy model which assumes harmonic oscillator bound state protons and takes the outgoing continuum proton wavefunction to be a plane wave. Using this simple model to calculate the transition charge distributions allows us to calculate the longitudinal contribution to the cross section using the approximate DW expression for N_0 of Eq. (10) and to compare this result to various *ad hoc* proscriptions. Based on this investigation, coupled with our previous investigation of the transverse contributions which dominate the cross section at large electron scattering angles, we propose the following *ad hoc* expressions for the longitudinal and transverse structure functions:

$$N_0^{\text{ad hoc}} = \int \left(\frac{q'_\mu(r)}{q_\mu}\right)^2 \left(\frac{q}{q'(r)}\right)^2 e^{i\langle\delta(\kappa_i^2)+\delta(\kappa_f^2)\rangle}$$

$$\times e^{i\mathbf{q}'(r)\cdot\mathbf{r}} J_0(\mathbf{r}) d^3r, \tag{12}$$

$$\mathbf{N}_T^{\text{ad hoc}} = \left(\frac{p'_i(0)}{p_i}\right) \int e^{i\mathbf{q}'(r)\cdot\mathbf{r}} \mathbf{J}_T(\mathbf{r}) d^3r, \tag{13}$$

where $\langle\delta(\kappa_{i,f}^2)\rangle$ denotes an average over the angles of the vector $\mathbf{r}$. That is, $\langle\kappa_{i,f}^2\rangle = \langle(\mathbf{r}\times\mathbf{p}_{i,f})^2\rangle = r^2 p_{i,f}^2(3-\cos^2\theta_{p_{i,f}})/4$. Note that under this averaging, the Δ term goes to zero. This
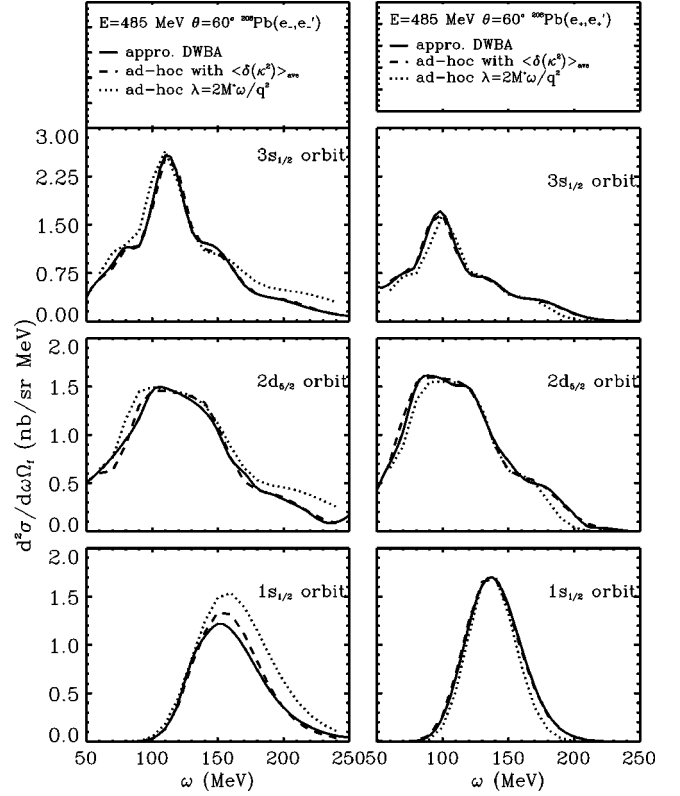

FIG. 1. Longitudinal contributions to the differential cross sections at a forward scattering angle for ^{208}Pb$(e_\pm,e'_\pm)$ for different bound state orbitals. The solid line is the approximate DW result, the dashed line is our *ad hoc* result and the dotted line is our previous LEMA$'$ approximation.

removes the angular dependence in the phase factors, and thus permits a multipole treatment of the matrix element as usual.

In the following figures for the longitudinal parts of the cross sections based on our simple model we will compare our new recommended longitudinal *ad hoc* result given in Eq. (12) to the result calculated by the full three-dimensional integration of Eq. (10) and to our previous *ad hoc* results called LEMA$'$ which we give below for convenience:

$$N_0^{\text{LEMA}'} = \left(\frac{p'_i(0)}{p_i} \int e^{i\mathbf{q}''(r)\cdot\mathbf{r}} J_0(\mathbf{r}) d^3r\right), \tag{14}$$

where $\mathbf{q}'' = \mathbf{p}''_i(r) - \mathbf{p}''_f(r)$, $p''(r) = p - (\lambda/r)\int_0^r V(r')dr'$, and the factor λ, which depends on the energy transfer ω, is given by $\lambda = (\omega/\omega_0)^2$ with $\omega_0 = q^2/1.4M$.

Clearly $N_0^{\text{ad hoc}}$ and $\vec{\mathbf{N}}_T^{\text{ad hoc}}$ represent a modified Fourier transform of the nuclear transition current. For comparison purposes, the approximation known as the EMA replaces $\mathbf{q}'(r)$ with $\mathbf{q}'(0)$ wherever it appears in Eqs. (10) and (11) for N_0 and N_T and the phases are neglected as usual. We find that for light nuclei the EMA is adequate, but it leads to large errors for nuclei as heavy as ^{208}Pb.

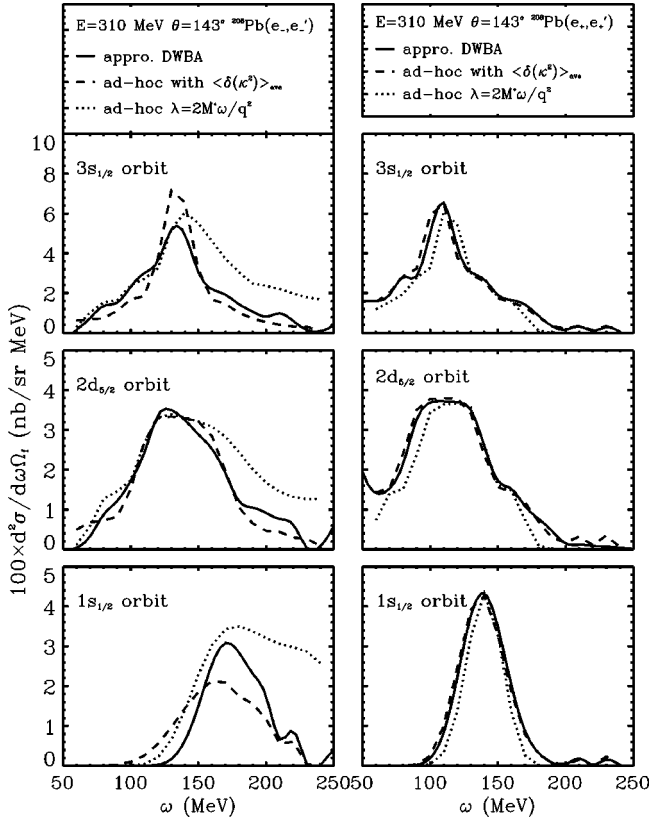In Fig. 1 we compare the two approximate calculations with the DW approximation for the longitudinal contribution

FIG. 2. Longitudinal contributions to the differential cross sections for ^{208}Pb($e_\pm, e'_\pm$) at a backward angle for different bound state orbitals. The solid line is the approximate DW result, the dashed line is our *ad hoc* DWBA result, and the dotted line is our previous LEMA$'$ approximation.

to the cross section for knocking protons out of various shells at a forward angle in ^{208}Pb by electrons or positrons. Note that while we use harmonic oscillator wave functions for all orbitals, we do use the binding energies of the orbitals that correspond to the values we find for our relativistic σ-ω model for ^{208}Pb. While the *ad hoc* result is not in perfect agreement with the full DW result, it clearly is in better agreement that the LEMA$'$ result and, for cases where the electron incident and final energy exceed 300 MeV is in reasonable agreement, particularly near the maxima. Note that the positron results are not very sensitive to which approximation is used.

In Fig. 2 we show similar results at a backward angle. We note that our *ad hoc* DWBA results for positrons tend to be in much better agreement with the DW result than the electron case. We again find that our new *ad hoc* approximation for the longitudinal contribution is considerably better than our previous LEMA$'$ result. We note that while the agreement between our *ad hoc* calculation and the DW calculation for knocking out protons from individual orbitals is not excellent, the discrepancies do not seem have a systematic tendency to be either low or high and we have reason to hope that when all the orbitals are added together as in the case of (e, e') reactions from nuclei that these discrepancies will tend to average out.
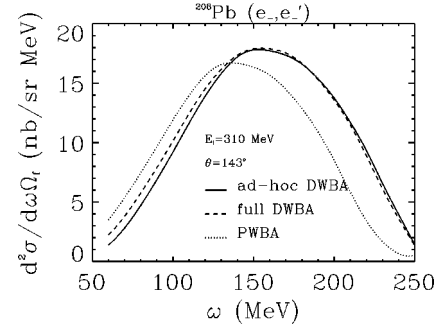

FIG. 3. The DWBA differential cross section for ^{208}Pb(e_-, e'_-) at 310 MeV and scattering angle $\theta = 143°$ compared to our *ad hoc* DWBA and to the plane wave result. The bound state and continuum neutron and proton orbitals are solutions to relativistic Hartree potential based on the σ-ω model.

IV. COMPARISON TO EXPERIMENT AND CONCLUSIONS

Based on our investigation of this simple toy model, we adopt our new *ad hoc* model for the longitudinal structure functions and return to our full nuclear model for investigating Coulomb corrections for ^{208}Pb (e, e') in the quasielastic region where the lepton can be electrons or positrons. Our first step is to reexamine our full DWBA calculation [3] and modify the code for the case of positrons. We were successful in doing this and can now compare the full DWBA calculation for electrons and positrons based on a realistic relativisitic nuclear model to our *ad hoc* treatment of Coulomb corrections which still permit a separation into longitudinal and transverse terms.

In Fig. 3 we compare the full DWBA calculation to the *ad hoc* result and the electron plane wave result (PWBA) for electrons with incident energy of $E_i = 310$ MeV and scattering angle of $\theta = 143°$. Note that this comparison is only a test of the our *ad hoc* transverse treatment which is unchanged from our previous work since the longitudinal contribution at such a large angle is at the few percent level. The agreement of the "plane-wave-like" *ad hoc* calculation with the full DWBA result is quite good, even though the outgoing electron energy is well below 300 MeV.

In Fig. 4, we perform a similar comparison for the case of positrons with incident energy of $E_i = 485$ MeV and scattering angle $\theta = 60°$. Again, the agreement is quite good, and unlike the backward angle scattering case, the longitudinal response contributes about 40% of the cross section.

We have examined a number of other cases, and the agreement shown in Figs. 3 and 4 is characteristic at these energies. As the lepton energies increase, the *ad hoc* approximation improves since the Coulomb distortion effects become smaller. We did notice in our investigations a general tendency that Coulomb distortion effects for positrons tend to be smaller than Coulomb distortion effects for electrons. This corresponds to an observation made many years ago when looking at inelastic lepton scattering from nuclei [16], where we noted that Coulomb distortion for positrons tends to saturate. As electrons pass near the nucleus, the attactive Coulomb interaction pulls them into regions of stronger po-
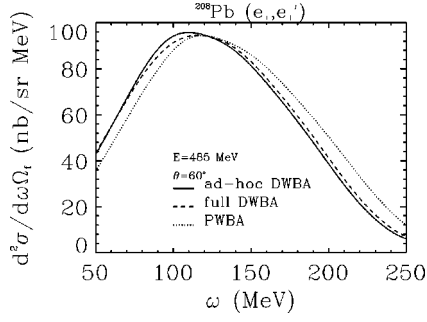
FIG. 4. The DWBA differential cross section for ^{208}Pb(e_+, e'_+) at 485 MeV and scattering angle $\theta = 60°$ compared to our *ad hoc* DWBA and to the plane wave result. The bound state and continuum neutron and proton orbitals are solutions to relativistic Hartree potential based on the σ-ω model.

tential which increases the Coulomb distortion effects, while positrons are pushed away from the region with a stronger potential.

With our capability of examining Coulomb distortion of both positrons and leptons with the full DWBA calculation and with our improved *ad hoc* procedure we can compare our model predictions to experiment. In Fig. 5, we compare our model calculations with Coulomb distortion included exactly and with our *ad hoc* method for quasielastic scattering of electrons of energy 383 MeV and positrons of energy 420 MeV both at a scattering angle of $\theta = 60°$ from ^{208}Pb to the experimental data from Saclay [17,18]. Note that in this and the following figure, we are plotting the total structure function $S_{total} = (d^2\sigma/d\omega d\Omega_f)/\sigma_M(E_i)$.

We first note that our *ad hoc* and exact DWBA results are in reasonable agreement although the lepton energy is somewhat low for our approximate result, and further that the positron and electron total structure functions have approxi-
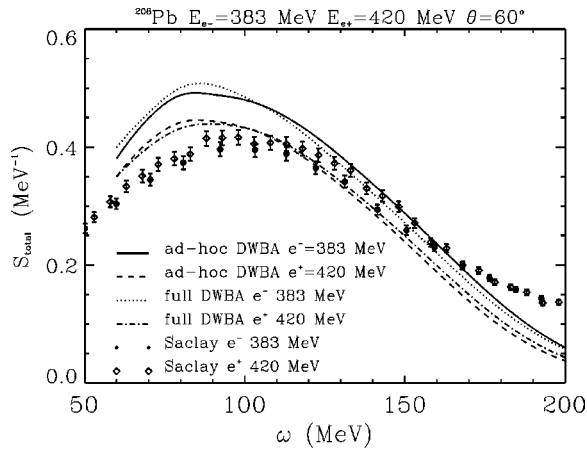

FIG. 5. The total structure function S_{total} generated by dividing the differential cross section by σ_M for ^{208}Pb$(e_\pm, e'_\pm)$ at a forward scattering angle of 60° with electrons of energy 383 MeV and positrons with energy 420 MeV. The theoretical curves correspond to the full DWBA calculation and to our *ad hoc* DWBA calculation. The data were taken at Saclay [17,18]. The bound state and continuum neutron and proton orbitals are solutions to relativistic Hartree potential based on the σ-ω model.
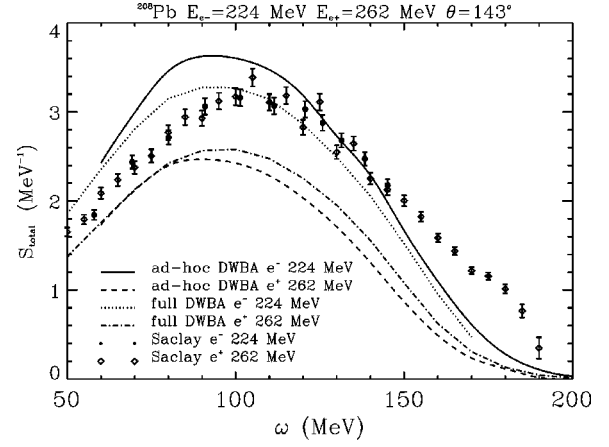

FIG. 6. The total structure function S_{total} generated by dividing the differential cross section by σ_M for ^{208}Pb$(e_\pm, e'_\pm)$ at a backward scattering angle of 143° with electrons of energy 224 MeV and positrons with energy 262 MeV. The theoretical curves correspond to the full DWBA calculation and to our *ad hoc* DWBA calculation. The data were taken at Saclay [17,18]. The bound state and continuum neutron and proton orbitals are solutions to relativistic Hartree potential based on the σ-ω model.

mately the same shape as a function of the energy transfer ω. However, they do not have the same magnitude as do the data from Saclay. The positron theory result is in reasonable agreement with the experimental data, but the electron result is approximately 15–20 % larger than the data.

In Fig. 6 we make a similar comparison except that now the scattering angle is $\theta = 143°$, and the electron incident energy is 224 MeV while the positron incident energy is 262 MeV. Again, when S_{total} is plotted the positron and electron shapes as a function of energy transfer ω are very similar, but again, unlike the experimental data, the magnitudes are quite different. At this backward scattering angle case, our electron result (DWBA) is in quite good agreement with the data. At these much lower energies, clearly our *ad hoc* approximation is beginning to fail, particularly for the electron case.

There is considerable interest in extracting the longitudinal contributions from (e, e') reactions from medium and heavy nuclei in order to investigate the Coulomb sum rule. Clearly, Coulomb distortion effects have to be handled properly. Our results indicate that we could use a Rosenbluth-like procedure in order to separate our "longitudinal" and "transverse" contributions to the cross section. However, these contributions depend on a modified (by Coulomb distortion) Fourier transform of the transition charge and current distributions. It is necessary to use a nuclear model to extract the longitudinal and/or tranverse structure functions from the data. It is not clear to us that a Rosenbluth-like procedure is the best way to proceed, since our *ad hoc* procedure is not accurate in the wings of the cross section distributions and in many cases, some of the Rosenbluth points fall on either the low ω or high ω side of the quasielastic peak. It seems that a better procedure might be to choose some semirealistic nuclear model for the process in question. Use Eqs. (12) and

(13) to calculate the structure functions and then fit the calculations to the available data using a least squares procedure to determine normalization factors N_L and N_T in front of the appropriate terms. The nuclear model should have the overall correct spatial and kinematic dependence, but the longitudinal or transverse strength will be determined by fitting these normalization factors. Once these factors are determined, one can use the same nuclear model weighted with these factors to calcualte the plane wave structure functions, thereby having ''measured'' the nuclear longitudinal and transverse response.

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