Multiplicity distributions associated to subthreshold events in heavy-ion collisions

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Subthreshold events (pion production, for instance, at energies $E < m_\pi$) in heavy-ion collisions are treated as rare cluster-cluster collision events. On kinematical grounds such events are forbidden in free nucleon-nucleon and even nucleon-nucleus collisions. We show, by using effective mass clustering arguments, that the associated distribution $P_c(n)$, when the pion trigger is present, and the unconstrained distribution $P(n)$, when there is no pion trigger, are related by the universal relation $P_c(n) = (n^2/\langle n^2\rangle)P(n)$. This relation, and in particular an improved version taking into account the number of clustering nucleons, is in fair agreement with data. Moreover, it allows information to be extracted from the multiplicity data on the number of nucleons involved in subthreshold meson production processes.

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I. INTRODUCTION

Production of particles, pions or other hadrons, energetic photons, or fast protons in nucleus-nucleus collisions at energies per nucleon well below the free nucleon-nucleon threshold for such production [1] gives clear evidence for nucleonic correlations in nuclear matter. These correlations may simply reflect the fermionic nature of the nucleons or they may be seen as true collective effects, like clustering of nucleons (see [2] for a review).

By clustering processes we mean here pion production mechanisms involving subsystems with a number of nucleons between two and the mass number *A*, the simplest example being the interaction of a nucleon with a deuteron. In fact, schematically we can represent pion production in the $pd \rightarrow \pi^0 pd$ reaction via resonance formation as in the diagram of Fig. 1(a). The threshold for the mentioned reaction is lower than the free nucleon threshold [Fig. 1(b)]. In contrast to this diagram, the one of Fig. 1(a) is kinematically allowed at subthreshold production energies, because the two nucleons of the deuteron are correlated. Basic diagrams [3–6], like the one in Fig. 1(b), require a medium to be nonvanishing for energies below the production energy threshold.

Higher-mass resonances can be obtained if larger clusters supply the required energy. Our emphasis here is on clustering rather than on the specific mechanism of pion production (like higher-mass resonances [7,8]). The mechanisms or models that describe pion production are the same below and above threshold [the same exchange and isobar excitation are depicted in both diagrams (a) and (b) of Fig. 1]. However, by definition of threshold and due to energy conservation, some processes are forbidden for subthreshold energies, unless they are distorted by medium effects. At those energies, initial state interactions, such as the ones producing a bound deuteron, as in Fig. 1(a), or a generic *NN* scattering state, generate intermediate states with off-shell particles, thus enabling subthreshold particle production.

In order to see what the problem is in a simplified manner, let us write the laboratory energy E_L of nucleus *A* as

$$E_L = A\langle E\rangle_N, \tag{1}$$

where $\langle E\rangle_N$ is the average nucleon energy. If one assumes that a collision results from the superposition of nucleon-nucleon collisions and ignores Fermi momentum and binding energy, the threshold kinetic energy per nucleon to produce, say, a pion is

$$E_{th} \equiv \langle E\rangle_N - m_N|_{th} = 2m_N\left[\left(1+\frac{m_\pi}{2m_N}\right)^2 - 1\right] \simeq 2m_\pi \simeq 280 \text{ MeV}. \tag{2}$$

This is the free nucleon threshold. In nuclear matter one may have nucleons with energy above $\langle E\rangle_N$ and the threshold becomes lower than (2).

The lowering of the threshold (2) can be very easily visualized if one allows for clustering of nucleons. If $\alpha \leqslant A$ nucleons of nucleus *A*—with mass number *A*—collide with one nucleon of nucleus *B*—with mass number *B*—(or vice versa) the threshold energy per nucleon becomes

$$E_{th} \simeq \frac{1+\alpha}{\alpha} m_\pi \geqslant m_\pi \simeq 140 \text{ MeV}. \tag{3}$$

This is, of course, the threshold for free nucleon-nucleus collisions.

As experimental production of pions occurs even at energies below m_π, this requires clustering from both nuclei, $\alpha \leqslant A$ and $\beta \leqslant B$,

$$E_{th} \simeq \frac{\alpha+\beta}{\alpha\beta} m_\pi, \tag{4}$$

with the absolute threshold (minimum value of E_{th}), naturally occurring for $\alpha = A$ and $\beta = B$,

$$E_{th} \simeq \frac{A+B}{AB} m_\pi. \tag{5}$$

In the case of ^{12}C-^{12}C interactions, for instance, this threshold corresponds to $\simeq 23$ MeV. The binding energies will affect this value within less than a keV, provided they

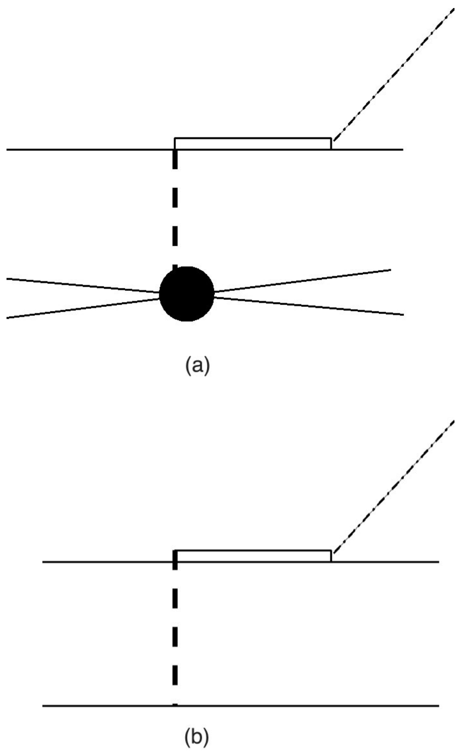

FIG. 1. Pion production mechanisms mediated by resonance formation in the medium (a) and in free nucleon scattering (b).

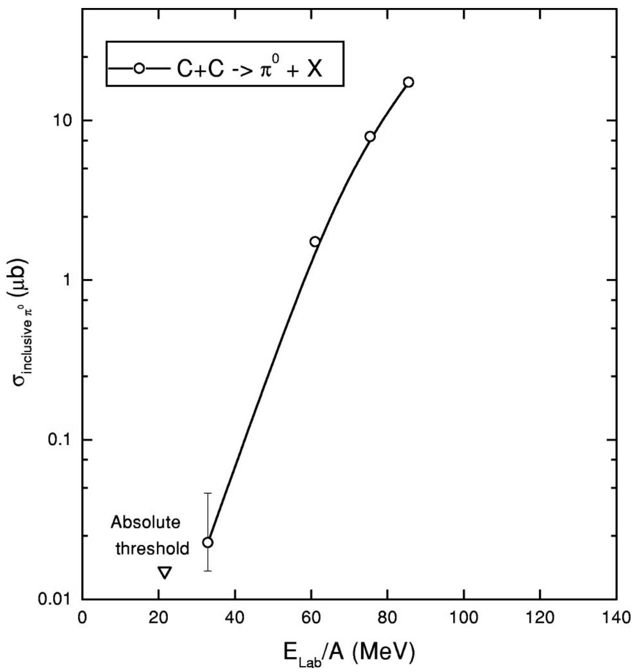

FIG. 2. Experimental cross section for ^{12}C-^{12}C from the fourth reference in [1]. The full line is only to guide the eye.

are considered simultaneously in the final and initial states. This lower bound cannot be obtained in a simple way from Fermi-motion-based arguments.

Independently of the underlying model for the nucleus-nucleus collisions, the point we would like to make is that these sub NN threshold events are *rare*, in the sense that their probability of occurrence is very small. While total inelastic cross sections are of the order of several millibarns, the cross sections we are talking about here are of the order of the microbarn or nanobarn. In other words, by imposing kinematic restrictions through lowering the energy available in the system, one moves to the tails of the fermionic distributions or requires simultaneous clustering, and the events become rare, the probability of occurrence being very small.

It is interesting to note that as one unconstrains the kinematics, i.e., increases the energy, the π production cross section increases very rapidly, reaching millibarn values for $E_{th}>2m_\pi$ (see Fig. 2). That is the region of (free) nucleon-nucleon interactions where the Glauber approach becomes valid.

In the following section we make a short discussion on rare events based on Ref. [9], where clustering in the nucleus is considered to account for proton-nucleus scattering. Here, however, the ideas of Ref. [9] are adapted to the different situation of nucleus-nucleus scattering, where we are interested in clustering processes in *both* colliding nuclei, instead of the clustering in only one. In Sec. III we show results and in Sec. IV we present a discussion and conclusions.

II. CLUSTERING EFFECTS AND RARE EVENTS

For a multicollision nucleon-nucleus scattering process, where $N(\nu)$ is the number of events proceeding through ν elementary collisions, denote by τ_c the probability of clustering of nucleons in the nucleus. Then $\binom{\nu}{0}(1-\tau_c)^\nu N(\nu)$ is the number of events proceeding without any clustering mechanism in the scattered nucleus, $\binom{\nu}{1}\tau_c(1-\tau_c)^{\nu-1}N(\nu)$ is the number of events where one clustering mechanism occurs inside the nucleus in a total of $N(\nu)$ events with ν elementary collisions, and so on.

Next, considering nucleus-nucleus scattering, since clustering may occur independently in both scattered nuclei, we obtain in the small τ_c approximation

$$N(\nu)=[(1-\tau_c\nu)^2+2(1-\tau_c\nu)(\tau_c\nu)+(\tau_c\nu)^2+\cdots]N(\nu). \tag{6}$$

One sees that the first term means the normal one-nucleon contribution to the production process (i.e., no clustering in either nucleus, corresponding to Glauber-type scattering, and implying $E_{th}\simeq 2m_\pi$), the second term means clustering in one of the nuclei (thus reducing E_{th} to $E_{th}\simeq m_\pi$), and the third term

$$N_c(\nu)=\tau_c^2\nu^2N(\nu) \tag{7}$$

represents what interests us, namely, the number of events with nucleon clustering in both nuclei (further reducing $E_{th}<m_\pi$). In other words, when ν collisions occur, the probability of having clustering in both nuclei at the same time is $(\tau_c\nu)^2$, since $\tau_c\nu$ is the probability of clustering in one of the nuclei alone, for small τ_c. The events described by Eq. (7) are doubly rare events.

The clustering probability τ_c is defined here in an average sense, not distinguishing between the two-, three-, and four-body (or higher) clustering cases. The nature of the clustering is of course determined by the underlying nuclear microscopic interaction (a test of which is beyond the scope of this paper) and will depend on the nucleus-nucleus reaction considered. Nevertheless, we will show in the following that this definition is operational, since it actually enables a simple procedure of extracting the knowledge on the nature of the clustering (two-, three-, four-body) involved in a particular reaction from its data analysis.

If one makes the simplest assumption that the number ν of collisions is a measure of the number n of produced particles, by normalizing Eq. (7) we obtain the τ_c independent, universal relation

$$P_c(n)=\frac{n^2}{\langle n^2\rangle}P(n), \tag{8}$$

where $P(n)$ is the unconstrained particle multiplicity distribution, $\langle\cdots\rangle$ denotes the average value with respect to the $P(n)$ distribution, and $P_c(n)$ is the multiplicity distribution associated with the rare event (π emission below the proton-nucleus threshold).

Note that

$$\Sigma_{n=0}P_c(n)=\Sigma_{n=0}P(n)=1, \tag{9}$$

and that [9]

$$\langle n\rangle_c=\frac{C_3}{C_2}\langle n\rangle, \tag{10}$$

where $C_q\equiv\langle n^q\rangle/\langle n\rangle^q$. Equation (10) implies that

$$\langle n\rangle_c>\langle n\rangle. \tag{11}$$

It is also clear that

$$\begin{aligned}P_c(n)<P(n)\Leftarrow n^2<\langle n^2\rangle,\\ P_c(n)>P(n)\Leftarrow n^2>\langle n^2\rangle,\end{aligned} \tag{12}$$

i.e., the distributions must cross at some point n, corresponding to $n^2=\langle n^2\rangle$.

III. COMPARISON WITH MULTIPLICITY DATA

In Fig. 3 we test Eq. (8) for the particular case of the data of Ref. [10] for collisions of ^{36}Ar on ^{27}Al. The data $P(n)$ [10] on the inclusive charged particle distribution (nucleons, etc.) are shown by the open circles. The data $P_c(n)$ [10] on the equivalent distribution when a subthreshold pion, at $E=95$ MeV/nucleon, is detected are shown by the open squares. For the description of $P(n)$ (full line) the generalized gamma function was used:

$$P(n)=\frac{1}{\langle n\rangle}\frac{\mu}{\Gamma(k)}\left[\frac{\Gamma(k+1/\mu)}{\Gamma(k)}\right]^{k\mu}\left(\frac{n}{\langle n\rangle}\right)^{k\mu-1}\times\exp\left[-\left(\frac{\Gamma(k+1/\mu)}{\Gamma(k)}\frac{n}{\langle n\rangle}\right)^{\mu}\right] \tag{13}$$

with parameters $k=0.77$, $\mu=2.0$, $\langle n\rangle=4.4$. To describe $P_c(n)$ Eq. (8) was used. Qualitatively, Eq. (8), implying Eqs. (11) and (12), satisfies the data: the average multiplicity, with a trigger on π, is larger, $\langle n\rangle_c\simeq 9$, while $\langle n\rangle\simeq 4.4$ [11], the two distributions cross at $n^2=\langle n^2\rangle\simeq 49>\langle n\rangle^2\simeq 19.4$ (the inequality $\langle n^2\rangle>\langle n\rangle^2$ has to be satisfied), and the distribution is independent of the rare event trigger [12].

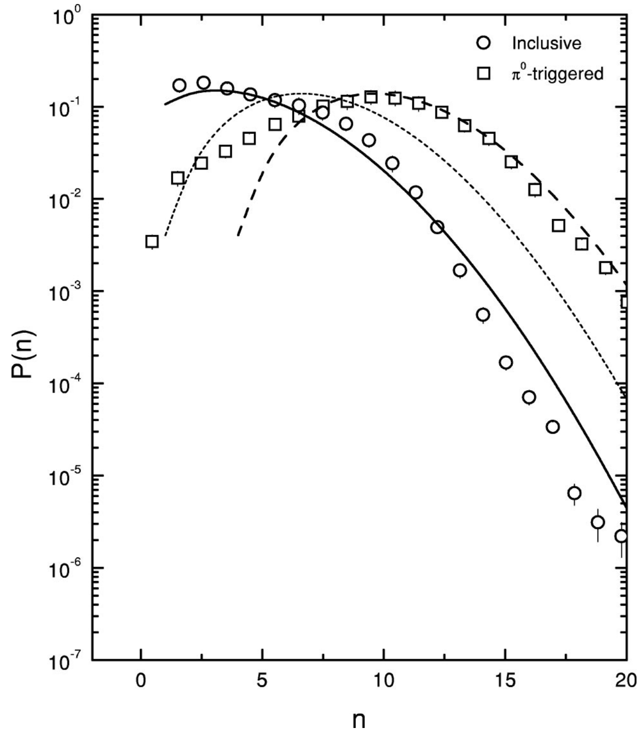

FIG. 3. Data for ^{36}Ar on ^{27}Al collisions taken from Ref. [10]. The full curve is the fit to $P(n)$, the short-dashed line is $P_c(n)$ given by Eq. (9), and the long-dashed line is $P_c(n)$ given by Eq. (15).

The results (short-dashed line) show, however, that the agreement of Eq. (8) with the rare-event-triggered distribution is not quantitative enough to allow a correct description of the crossing point with the inclusive distribution curve. One should keep in mind that pion absorption is present and it was not taken into account in Eq. (8). The multiplicity associated with an energetic photon, for instance, would be better to test Eq. (8) as the photon is less affected by absorption.

There is still another correction to Eq. (8) as discussed in Ref. [13]. When clustering occurs, since clusters contain more nucleons, naturally more particles are produced, implying an increase in multiplicity. To take this effect into account, Eq. (8) is modified to become

$$P_c(n+\delta)=\frac{n^2}{\langle n^2\rangle}P(n), \tag{14}$$

where δ traces the average number α (>1) of nucleons in the clusters, according to

$$\delta\simeq 2\alpha-2. \tag{15}$$

For $\alpha=1$, $\delta=0$ as it should (no clustering) and Eq. (14) reduces to Eq. (8).

Using Eq. (4), with $\alpha \simeq \beta$, and requiring

$$E_{th} = \frac{2}{\alpha} m_\pi > 95 \ \text{MeV/nucleon}, \tag{16}$$

one estimates

$$\alpha \leqslant 3 \tag{17}$$

and, from Eq. (15),

$$\delta \leqslant 4. \tag{18}$$

In Fig. 3 we also included the description of the pion-triggered data $P_c(n)$ by means of Eq. (14) with $\delta = 3$ (long-dashed line). In comparison with the $P_c(n)$ results given by Eq. (8) (short-dashed line), improvement is achieved with the curve corresponding to Eq. (14). Indeed, the shift by δ originating in cluster formation better describes the crossover point of the constrained and unconstrained multiplicities. Moreover, the value of the parameter δ ($\delta = 3$), which is found to be consistent with the distribution data, is also consistent with Eqs. (17) and (18), originating only from the observed subthreshold energy. Thus, Eq. (18) constitutes an important bound constraining the overall behavior of the data. In conclusion, clustering arguments directly relate the observed subthreshold production energies to the behavior of the distribution data.

IV. DISCUSSION AND CONCLUSIONS

The underlying physical picture in nucleus-nucleus collisions at subthreshold energies is that the corresponding wavelength scale of the process is large enough for clusters of nucleons in the nucleus to recoil as a whole. At these energies correlations due to nucleon-nucleon interactions may dominate Pauli blocking or Fermi-motion correlations. An absolute threshold may be calculated corresponding to a recoil of the complete nucleus as a whole [Eq. (5)]. The deviation of the physical threshold from the absolute one is due to clustering formation involving only some of the constituent nucleons. The clustering also affects the distribution data for particle production events triggered by rare events such as pion production.

Therefore, rare-event-triggered measurements give direct information on the number of nucleons involved in particle production at threshold. Accordingly, few-nucleon production reactions, near threshold, such as $pd \rightarrow \pi^\circ\, {}^3\text{He}$ and $pd \rightarrow \eta\, {}^3\text{He}$, for example, which may be exactly calculable at present, are worth investigating experimentally at existing strong focusing synchroton facilities (COSY, CELSIUS), since they may confirm the proposed relation between threshold energies and number of nucleons involved in the production mechanisms. We note here also that the failure already found in Ref. [14] to describe by means of the impulse approximation the ${}^3\text{He}(\pi,\eta){}^3\text{H}$ reaction below the free production threshold for forward scattering is already evidence for a cluster-enhanced rare event.

As we mentioned before, it is not our purpose to test models of interacting nuclei but to test the validity of Eq. (8) and its improved version Eq. (14) when the kinematic constraints are such that occurrence of event c (pion, fast proton, etc.) is rare. However, Eq. (14), which in any case is approximate, as α and β may fluctuate, makes sense only in a model with interaction via clusters. This is similar to what is proposed in [13] for cumulative effects at very high energies. Models with independent *NN* collisions ($\alpha = 1$) in a medium (based on Fermi-motion distribution arguments alone) do not give support to the arguments leading to Eq. (14). So, although relation (10) is universal, Eq. (14) depends upon assumptions concerning the type of interaction involved and thus can distinguish between models.

The fact that particles can be produced *below threshold* via a clusterization mechanism raises still another important question, namely, whether particle production normally forbidden by Zweig's rule can occur in an environment where (microscopic) clusterization takes place. In particular, this effect may have important consequences for ϕ meson production in heavy-ion collisions at Super Proton Synchrotron and Large Hadron Collider energies. It remains also to be clarified whether this effect has any consequences for J/ψ production at collider energies.

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