Baryon number fluctuation and the quark-gluon plasma

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We show that ω_B or $\omega_{\bar{B}}$, the squared baryon or antibaryon number fluctuation per baryon or antibaryon, is a possible signature for the quark-gluon plasma that is expected to be created in relativistic heavy ion collisions, as it is a factor of 3 smaller than in an equilibrated hadronic matter due to the fractional baryon number of quarks. Using kinetic equations with exact baryon number conservation, we find that their values in an equilibrated matter are half of those expected from a Poisson distribution. Effects due to finite acceptance and nonzero net baryon number are also studied.

A new state of matter, the quark-gluon plasma, is expected to be formed in heavy ion collisions at ultrarelativistic energies, such as at the Relativistic Heavy Ion Collider (RHIC) that has just begun its operation at the Brookhaven National Laboratory. Many observables have been proposed as possible signatures for the quark-gluon plasma phase during the collisions [1], such as strangeness enhancement [2], J/ψ suppression [3], modification of high p_T particle spectrum [4], and M_T scaling [5] and double phi peaks [6] in the dilepton spectrum. Recently, event-by-event fluctuations of various particles have also attracted much attention [7]. Since the baryon and charge numbers of quarks are fractional, the net baryon number [8] and net charge [9] fluctuations per particle are smaller in the quark-gluon plasma than in the hadronic matter and have also been suggested as possible signatures for the quark-gluon plasma.

Because of conservation of total net baryon number, the net baryon fluctuation is finite only if one considers a part of the full phase space, e.g., in a finite rapidity range. Since the quark-gluon plasma is eventually converted to the hadronic matter, elastic scatterings of baryons and antibaryons would modify their phase space correlations and thus increase the net baryon fluctuation towards that expected from an equilibrated hadronic matter [10]. On the other hand, elastic scatterings of baryons and antibaryons do not change their numbers in the full phase space, so fluctuations of baryon and antibaryon numbers are alternative signatures for the quark-gluon plasma in relativistic heavy ion collisions.

In this study, we shall use a master equation with exact baryon number conservation to derive the baryon and antibaryon number fluctuations in an equilibrated matter, and to study the effects due to finite acceptance and nonzero net baryon number.

Due to the different fundamental units of baryon numbers in quark-gluon plasma and in hadronic matter, fluctuations of baryon and antibaryon numbers, like the fluctuation of the net baryon number, take very different values in the two phases of matter. To study dynamically the baryon number fluctuation in heavy ion collisions, we introduce a kinetic model that takes into account both production and annihilation of quark-antiquark or baryon-antibaryon pairs. We first consider the case of baryon-antibaryon production from and annihilation to two mesons, i.e., $m_1 m_2 \leftrightarrow B\bar{B}$, in a hadronic matter of net baryon number $s=0$ [11]. Following the for-

malism of Ref. [12] for the production of particles with conserving charges, we have the following master equation for the multiplicity distribution of $B\bar{B}$ pairs:

$$\frac{dP_n}{d\tau} = \frac{G}{V}\langle N_{m_1}\rangle\langle N_{m_2}\rangle(P_{n-1}-P_n)$$
$$-\frac{L}{V}[n^2 P_n - (n+1)^2 P_{n+1}]. \quad (1)$$

In the above, $P_n(\tau)$ denotes the probability of finding n pairs of $B\bar{B}$ at time τ; $G\equiv\langle\sigma_G v\rangle$ and $L\equiv\langle\sigma_L v\rangle$ are the momentum-averaged cross sections for baryon production and annihilation, respectively; N_k represents the total number of particle species k; and V is the proper volume of the system.

The equilibrium solution to Eq. (1) is

$$P_{n,\mathrm{eq}} = \frac{\epsilon^n}{I_0(2\sqrt{\epsilon})(n!)^2}, \quad (2)$$

where I_0 is the modified Bessel function, and

$$\epsilon \equiv \frac{G\langle N_{m_1}\rangle\langle N_{m_2}\rangle}{L}. \quad (3)$$

Using the generating function at equilibrium,

$$g_{\mathrm{eq}}(x) \equiv \sum_{n=0}^{\infty} P_{n,\mathrm{eq}} x^n = \frac{I_0(2\sqrt{\epsilon x})}{I_0(2\sqrt{\epsilon})}, \quad (4)$$

with $g(1)=\Sigma P_n=1$ due to normalization of the multiplicity probability distribution, it is straightforward to obtain all moments of the equilibrium multiplicity distribution [13]. For example, the mean baryon number per event is given by

$$\langle B\rangle_{\mathrm{eq}} = b_0\langle n\rangle = b_0\frac{\partial g(1)}{\partial x} = b_0\sqrt{\epsilon}\frac{I_1}{I_0} \simeq b_0\sqrt{\epsilon}. \quad (5)$$

In the above, b_0 is the fundamental unit of baryon number in the matter, and the argument of Bessel functions I_ν's is $2\sqrt{\epsilon}$. In obtaining the last expression in Eq. (5), we have kept only the leading term in $\sqrt{\epsilon}$, corresponding to the grand canonical limit, $\sqrt{\epsilon} \gg 1$, as baryons and antibaryons are abundantly produced in heavy ion collisions at RHIC [14,15].

The squared baryon number fluctuation is given by

$$D_B \equiv \langle B^2 \rangle - \langle B \rangle^2 = b_0^2 \left[\frac{\partial g(1)}{\partial x} + \frac{\partial^2 g(1)}{\partial x^2} - \left(\frac{\partial g(1)}{\partial x} \right)^2 \right]$$

$$= b_0^2 \left[\sqrt{\epsilon} \frac{I_1}{I_0} + \epsilon \left(\frac{I_2}{I_0} - \frac{I_1^2}{I_0^2} \right) \right] \simeq b_0^2 \frac{\sqrt{\epsilon}}{2}. \quad (6)$$

It is seen that the mean number of baryons is proportional to b_0 while their squared fluctuation is proportional to b_0^2. The squared baryon number fluctuation per baryon is thus given by

$$\omega_{B,\mathrm{eq}} = b_0 \left[1 - \sqrt{\epsilon} \left(\frac{I_1}{I_0} - \frac{I_2}{I_1} \right) \right] \simeq \frac{b_0}{2}. \quad (7)$$

The mean number of antibaryons $\langle \bar{B} \rangle$ and their fluctuation $\omega_{\bar{B}}$ are the same as those of baryons as a result of zero net baryon number. Since b_0 is 1/3 in quark-gluon plasma and 1 in hadronic matter, ω_B and $\omega_{\bar{B}}$ are smaller in the quark-gluon plasma than in an equilibrated hadronic matter by a factor of 3.

We note that the equilibrium multiplicity distribution in Eq. (2) is not Poisson, as pointed out earlier in Ref. [16]. The non-Poisson distribution results from the quadratic dependence on the multiplicity n in the loss term of the master equation of Eq. (1) due to baryon number conservation. A Poisson distribution is obtained if the dependence on the multiplicity n is linear, which corresponds to production of particles that do not carry conserved charges. The master equation of Eq. (1) also gives a Poisson distribution at early times ($\tau \rightarrow 0$) when the loss term can be neglected. This corresponds to either production of particles with conserved charges during the early stage of heavy ion collisions or particle production without chemical equilibration as in e^+e^- collisions. Since a Poisson multiplicity distribution gives

$$D_B^{\mathrm{Poisson}} = b_0 \langle B \rangle, \quad (8)$$

the squared baryon number fluctuation per baryon is

$$\omega_B^{\mathrm{Poisson}} = b_0, \quad (9)$$

which is a factor of 2 larger than that in an equilibrated quark-gluon plasma or hadronic matter. A similar result has been obtained previously by Gavin and Pruneau based on thermodynamic considerations [17].

Because of experimental limitations, only protons and antiprotons in a certain rapidity and momentum range are usually measured. Moreover, the net baryon number in heavy ion collisions is in general nonzero even at mid-rapidity due to the presence of projectile and target nucleons. To generalize the master equation to include these effects, we first con-

sider the case of two species of antibaryons, e.g., antiproton and antineutron production from meson-meson interactions, i.e., $m_1 m_2 \leftrightarrow B\bar{p}$ and $m_1 m_2 \leftrightarrow B\bar{n}$ in a hadronic matter of net baryon number $s \geq 0$ [11]. Defining $P^{k,l}$ as the probability of finding k number of antibaryon species 1 and l number of antibaryon species 2 in an event ($k,l = 0, \ldots, \infty$ for $s \geq 0$), we then have the following generalized master equation:

$$\frac{dP^{k,l}}{d\tau} = \frac{\epsilon_1 L_1}{V} (P^{k-1,l} - P^{k,l}) + \frac{\epsilon_2 L_2}{V} (P^{k,l-1} - P^{k,l})$$

$$- \frac{L_1}{V} [k(k+l+s)P^{k,l} - (k+1)(k+l+s+1)P^{k+1,l}]$$

$$- \frac{L_2}{V} [l(k+l+s)P^{k,l} - (l+1)(k+l+s+1)P^{k,l+1}]. \quad (10)$$

In the above, ϵ_j ($j = 1,2$) is defined as in Eq. (3) for the process involving antibaryon species j, and G_j and L_j denote the momentum-averaged cross sections for the gain and loss terms, respectively. We have neglected effects due to quantum statistics, as they are not expected to be significant [18].

The equilibrium solution to the above equation is given by the product of the equilibrium solution for the total $B\bar{B}$ pair number ($k+l$) and a binomial distribution, i.e.,

$$P_{\mathrm{eq}}^{k,l} = \left[\frac{\epsilon^{k+l+s/2}}{I_s(2\sqrt{\epsilon})(k+l)!(k+l+s)!} \right] \left[\frac{(k+l)! f_1^k f_2^l}{k! l!} \right], \quad (11)$$

with

$$\epsilon \equiv \epsilon_1 + \epsilon_2, \quad f_j = \frac{\epsilon_j}{\epsilon}. \quad (12)$$

The distribution in Eq. (11) can be understood intuitively as resulting from first obtaining the distribution for the total antibaryon number ($k+l$), and then selecting k number of antibaryon species 1 and l number of antibaryon species 2 from the total antibaryons with probabilities of f_1 and f_2, respectively.

At equilibrium, the generating function $g(y_1, y_2) \equiv \sum P^{k,l} y_1^k y_2^l$ is given by

$$g_{\mathrm{eq}}(y_1, y_2) = \frac{(f_1 y_1 + f_2 y_2)^{-s/2}}{I_s(2\sqrt{\epsilon})} I_s(2\sqrt{\epsilon}\sqrt{f_1 y_1 + f_2 y_2}). \quad (13)$$

The above formulas can be equally applied to study the effects due to finite phase space coverage in experiments. In this case, antibaryon species 1 and 2 denote antibaryons in and out of the observed phase space, respectively; and f_1 and f_2 are their respective fractions.

To generalize to N numbers of baryon and antibaryon species, we consider $m_1 m_2 \leftrightarrow B_i \bar{B}_j$ ($i,j = 1, \ldots, 2N$), where B_i

and $\bar{B}_j$ with $i, j = 1, \ldots, N$ denote baryon species i and anti-baryon species j in the observed phase space, respectively, and B_i or $\bar{B}_j$ with $i, j = N+1, \ldots, 2N$ denote baryon species $(i-N)$ and antibaryon species $(j-N)$ out of the observed phase space, respectively. The joint generating function for both baryons and antibaryons is defined by

$$g(x_1, \ldots, x_{2N}; y_1, \ldots, y_{2N}) \equiv \sum P^{\bar{B}_1, \ldots, \bar{B}_{2N}}_{B_1, \ldots, B_{2N}} \left(\prod_{i=1}^{2N} x_i^{B_i} \right)$$
$$\times \left(\prod_{j=1}^{2N} y_j^{\bar{B}_j} \right), \qquad (14)$$

with the summation over all baryon and antibaryon numbers, B_i's and $\bar{B}_j$'s $(i, j = 1, \ldots, 2N)$. Assuming that particle momentum distributions are thermal, the generating function at equilibrium is given by

$$g_{eq}(x_1, \ldots, x_{2N}; y_1, \ldots y_{2N})$$
$$= \frac{\left(\sum h_i x_i \right)^{s/2} \left(\sum f_j y_j \right)^{-s/2}}{I_s(2\sqrt{\epsilon})}$$
$$\times I_s \left(2\sqrt{\epsilon} \sqrt{\sum h_i x_i} \sqrt{\sum f_j y_j} \right), \qquad (15)$$

where

$$\epsilon \equiv \sum_{i,j} \epsilon_{ij}, \qquad (16)$$

with ϵ_{ij} defined for the process $m_1 m_2 \leftrightarrow B_i \bar{B}_j$ as in Eq. (3), and

$$h_i = \left(\sum_j \epsilon_{ij} \right) \bigg/ \epsilon, \quad f_j = \left(\sum_i \epsilon_{ij} \right) \bigg/ \epsilon. \qquad (17)$$

From Eq. (15), we obtain the mean numbers of baryons and antibaryons,

$$\langle B_i \rangle_{eq} = b_0 h_i \left(\sqrt{\epsilon} \frac{I_{s+1}}{I_s} + s \right) \simeq b_0 h_i \sqrt{\epsilon} \left(1 + \frac{2s-1}{4\sqrt{\epsilon}} \right), \qquad (18)$$

$$\langle \bar{B}_j \rangle_{eq} = b_0 f_j \sqrt{\epsilon} \frac{I_{s+1}}{I_s} \simeq b_0 f_j \sqrt{\epsilon} \left(1 - \frac{2s+1}{4\sqrt{\epsilon}} \right), \qquad (19)$$

and their fluctuations,

$$\omega_{B_i, eq} = b_0 \left[1 - h_i \left(\frac{\epsilon \left(\frac{I_{s+1}^2}{I_s^2} - \frac{I_{s+2}}{I_s} \right) + s}{\sqrt{\epsilon} \frac{I_{s+1}}{I_s} + s} \right) \right]$$
$$\simeq b_0 \left[1 - \frac{h_i}{2} \left(1 + \frac{2s-1}{4\sqrt{\epsilon}} \right) \right], \qquad (20)$$

$$\omega_{\bar{B}_j, eq} = b_0 \left[1 - f_j \sqrt{\epsilon} \left(\frac{I_{s+1}}{I_s} - \frac{I_{s+2}}{I_{s+1}} \right) \right]$$
$$\simeq b_0 \left[1 - \frac{f_j}{2} \left(1 - \frac{2s+1}{4\sqrt{\epsilon}} \right) \right]. \qquad (21)$$

In the above, we have kept only the first two leading terms in the expansions as $s/\sqrt{\epsilon} \ll 1$ for ultrarelativistic heavy ion collisions. We note that Eqs. (18), (19), (20), and (21) reduce to Eqs. (5) and (7) in the special case of full phase space coverage with only one baryon and antibaryon species ($h_1 = f_1 = 1$) and $s = 0$.

Keeping only the leading terms in Eqs. (20) and (21) leads to

$$\omega_{B_i, eq} \simeq b_0 \left(1 - \frac{h_i}{2} \right), \quad \omega_{\bar{B}_j, eq} \simeq b_0 \left(1 - \frac{f_j}{2} \right), \qquad (22)$$

where h_i and f_j ($i, j = 1, \ldots, N$) represent, respectively, the fraction of total baryons that are observed baryon species i and the fraction of total antibaryons that are observed antibaryon species j. Equation (22) thus shows that finite acceptance introduces a phase space correction factor $(1 - p/2)$, with p being the faction of the total baryon or antibaryon phase space that is observed, to the baryon or antibaryon number fluctuations in an equilibrated matter. We note that there is no such correction factor due to finite acceptance if the matter is far from equilibrium when the $B\bar{B}$ multiplicity distribution is Poisson.

From Eq. (15), we can also derive the following squared net baryon number fluctuation for baryon species i and antibaryon species j:

$$(\Delta N_b^{ij})^2 \equiv \langle (B_i - \bar{B}_j)^2 \rangle - \langle B_i - \bar{B}_j \rangle^2$$
$$= b_0 \langle B_i + \bar{B}_j \rangle \left(1 - \frac{h_i + f_j}{2} \right). \qquad (23)$$

The squared net baryon fluctuation per baryon and anti-baryon, $(\Delta N_b)^2 / \langle B + \bar{B} \rangle$, is thus a factor of 3 smaller in an equilibrated quark-gluon plasma than in an equilibrated hadronic matter, and is a useful observable as well [8,19]. If the phase space acceptance is the same for baryons and antibaryons, i.e., $h_i = f_j = p$, then the phase space correction factor is $(1-p)$ as given in Ref. [9].

We have shown that ω_B or $\omega_{\bar{B}}$, the squared fluctuation of baryon or antibaryon numbers per baryon or antibaryon, reflects the fundamental units of baryon number in the matter where they are produced. Since their expected values in the quark-gluon plasma are a factor of 3 smaller than those in an equilibrated hadronic matter, they can be used as signatures for the quark-gluon plasma formed in the initial stage of relativistic heavy ion collisions. Using a master equation with exact baryon number conservation, we have found that at equilibrium the baryon multiplicity distribution is not Poisson, and its squared fluctuation is half of that expected from a Poisson distribution. We have also calculated the correction factors due to finite acceptance in experiments. Because of the contamination from the projectile and target nucleons, $\omega_{\bar{B}}$ is preferred over ω_B.

Although the baryon and antibaryon fluctuations are smaller in the quark-gluon plasma formed in the initial stage of heavy ion collisions, their magnitudes will be affected during the hadronization and subsequent scatterings of baryons and antibaryons in the hadronic matter. In the simplest quark coalescence model for the hadronization of the quark-gluon plasma to the hadronic matter, where three quarks form a baryon and a quark-antiquark pair forms a meson, the baryon number fluctuation is increased as a result of the finite fraction of quarks that form baryons. The correction factor to the baryon fluctuation in this hadronization scenario can be similarly derived as for finite acceptance. On the other hand, if hadronization is through the formation of strings and their subsequent fragmentation [20], then quark-antiquark pairs can be produced and annihilated during the string fragmentation and such additional processes usually also increase the baryon and antibaryon fluctuations. Furthermore, anomalous fluctuations could arise if the phase transition is first order [21,22] or the quark matter is at the QCD tricritical point [7,23]. As to the effects of hadronic scatterings, elastic scatterings of baryons and antibaryons are not expected to affect baryon and antibaryon number fluctuations in the full phase space. However, baryon-antibaryon pair production and annihilation tend to increase the fluctuations towards the values expected from a hadronic matter [24]. The determination of baryon and antibaryon fluctuations in heavy ion collisions could be further complicated by the existence of gluonic baryon junctions [25–27], so that baryon numbers are not necessarily carried fractionally by quarks but instead by the baryon junctions. In order for the baryon number fluctuation to be a viable signature for the quark-gluon plasma, quantitative studies of above effects are needed.

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