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Role of dynamical particle-vibration coupling in reconciliation of the $d_{3/2}$ puzzle for spherical proton emitters

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It has been observed that the decay rate for proton emission from $d_{3/2}$ single particle state is systematically quenched compared with the prediction of a one-dimensional potential model although the same model successfully accounts for measured decay rates from $s_{1/2}$ and $h_{11/2}$ states. We reconcile this discrepancy by solving coupled-channels equations, taking into account couplings between the proton motion and vibrational excitations of a daughter nucleus. We apply the formalism to proton emitting nuclei ^{160,161}Re to show that there is a certain range of parameter set of the excitation energy and the dynamical deformation parameter for the quadrupole phonon excitation which reproduces simultaneously the experimental decay rates from the $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ states in these nuclei.

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Physics of nuclei close to the neutron and proton drip lines is one of the most active and exciting research areas of the current nuclear physics. Nuclei beyond the proton drip line are unstable against proton emission, but, since a proton has to penetrate the Coulomb barrier, their lifetime is sufficiently long to study their spectroscopic properties. Thanks to the recent experimental developments of production and detection methods, a number of ground-state as well as isomeric proton emitters have recently been discovered, which has stimulated many experimental and theoretical works [1–19].

For proton emitters in the $A\sim150$ region, proton emissions from the $1h_{11/2}$, $3s_{1/2}$, and $2d_{3/2}$ orbitals have been observed. It has been pointed out that a spherical calculation based upon a one-dimensional optical potential with spectroscopic factor estimated in the BCS approximation systematically underestimates the measured decay half-lives for proton emissions from the $2d_{3/2}$ state, while the same model works well for emissions from the $1h_{11/2}$ and $3s_{1/2}$ states in both odd-*Z* even-*N* nuclei and odd-*Z* odd-*N* nuclei [5,9]. For the ^{151}Lu nucleus, this discrepancy was attributed to the effects of oblate deformation of the core nucleus ^{150}Yb, which alter both the decay dynamics and the BCS occupation probability of the valence proton [9,15,20]. The coupled-channels calculations with $\beta_2\sim-0.15$ have successfully explained the measured decay half-lives for this nucleus [15]. However, for proton emitters such as ^{156}Ta, ^{160}Re, and ^{161}Re, the static deformation parameter of the core nuclei ^{155}Hf, ^{159}W, and ^{160}W is estimated to be $\beta_2=-0.053$, 0.080, and 0.089, respectively, based on the macroscopic-microscopic mass formula [21], and thus the deformation effects will be much smaller. Those nuclei are nearly spherical, and vibrational excitations should be considered instead of the deformation effects and the associated rotational excitations [22].

For vibrational nuclei, the excitation energy of collective excitations is in general significantly larger than that for rotational nuclei. This makes the channel coupling effects weaker in spherical nuclei. We notice, however, that the penetration probability is extremely sensitive to other degrees of freedom especially at deep barrier energies, regardless of the value of their excitation energies [23,24]. It is thus of interest and important to explore the role of collective core excitations during proton emission decays of spherical nuclei.

The aim of this Rapid Communication is to solve the coupled-channels equations for spherical proton emitters in order to investigate whether the effects of vibrational excitations of the daughter nucleus consistently account for the measured decay half-life from the $1h_{11/2}$, $3s_{1/2}$, and $2d_{3/2}$ states. We particularly study proton emissions from the $1h_{11/2}$ and $3s_{1/2}$ states of ^{161}Re [4] and from the $2d_{3/2}$ state of ^{160}Re [3], assuming that the vibrational properties are identical between the core nuclei ^{160}W and ^{159}W. We shall exclude in our study the proton emitter ^{156}Ta, since the experimental data are somewhat ambiguous due to the unknown β^+/EC branch. Since the properties of excited states of these proton-rich nuclei are not known, we shall study the dependence of the decay rate on the excitation energy as well as on the dynamical deformation parameter of the vibrational phonon excitation of the daughter nucleus. We will then extract a possible combination of these two from the experimental data. We will also discuss the dependence on the multipolarity of the phonon state.

We consider the following Hamiltonian for the spherical proton emitting systems:

$$H=-\frac{\hbar^2}{2\mu}\nabla^2+V_{coup}(\boldsymbol{r},\alpha)+H_{vib}, \tag{1}$$

where $\boldsymbol{r}$ is the coordinate for the relative motion between the valence proton and the daughter nucleus, and μ the reduced mass for this motion. α is the coordinate for the vibrational phonon of the daughter nucleus. It is related to the dynamical deformation parameter as $\alpha_{\lambda\mu}=\beta_\lambda/\sqrt{2\lambda+1}(a^\dagger_{\lambda\mu}+(-)^\mu a_{\lambda\mu})$, where λ is the multipolarity of the vibrational mode and $a^\dagger_{\lambda\mu}$ ($a_{\lambda\mu}$) is the creation (annihilation) operator of the phonon. $H_{vib}=\hbar\omega\Sigma_\mu a^\dagger_{\lambda\mu}a_{\lambda\mu}$ is the Hamiltonian for the vibrational phonon. In this paper, for simplicity, we do not consider the possibility of multiphonon excitations, but include only excitations to the single phonon state. The coupling Hamiltonian $V_{coup}(\boldsymbol{r},\alpha)$ consists of three terms, i.e., $V_{coup}(\boldsymbol{r},\alpha)=V^{(N)}_{coup}(\boldsymbol{r},\alpha)+V^{(ls)}_{coup}(\boldsymbol{r},\alpha)+V^{(C)}_{coup}(\boldsymbol{r},\alpha)$. The nuclear term reads

$$V^{(N)}_{coup}(\boldsymbol{r},\alpha)=-\frac{V_0}{1+\exp\left(\dfrac{r-R_0-R_0\,\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}})}{a}\right)}, \quad (2)$$

where the dot denotes a scalar product. We have assumed that the nuclear potential is given by the Woods-Saxon form. Notice that we do not expand the nuclear potential but include the couplings to all orders with respect to the phonon operator α [25,26]. The matrix elements of this coupling Hamiltonian are evaluated using a matrix algebra, as in Ref. [25]. As for the Coulomb $V^{(C)}_{coup}$ as well as the spin-orbit $V^{(ls)}_{coup}$ terms, the effects of higher order couplings are expected to be small [26], and we retain only the linear term. The Coulomb term thus reads

$$V^{(C)}_{coup}(\boldsymbol{r},\alpha)=\frac{Z_De^2}{r}+\frac{3Z_De^2}{r}\frac{1}{2\lambda+1}\left(\frac{R_c}{r}\right)^\lambda \times\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}) \quad (\text{for } r>R_c) \quad (3)$$

$$=\frac{Z_De^2}{2R_c}\left(3-\frac{r^2}{R_c^2}\right)+\frac{3Z_De^2}{R_c}\frac{1}{2\lambda+1}\left(\frac{r}{R_c}\right)^\lambda \times\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}) \quad (\text{for } r\leqslant R_c), \quad (4)$$

where Z_D is the atomic number of the daughter nucleus and R_c is the charge radius. For the spin-orbit interaction, we express it in the so-called Thomas form [19,27]

$$V^{(ls)}_{coup}(\boldsymbol{r},\alpha)=V_{so}\frac{1}{r}\frac{df}{dr}\boldsymbol{l}\cdot\boldsymbol{\sigma}+i\,V_{so}R_{so}\sum_\mu \alpha^*_{\lambda\mu} \times\left\{\left(\nabla\frac{df}{dr}Y_{\lambda\mu}(\hat{\boldsymbol{r}})\right)\cdot(\nabla\times\boldsymbol{\sigma})\right\}, \quad (5)$$

where $f(r)=1/[1+\exp((r-R_{so})/a_{so})]$. The last term in Eq. (5) can be decomposed to a sum of angular momentum tensors using a formula [28]

$$(\nabla g(r)Y_{\lambda\mu}(\hat{\boldsymbol{r}}))\cdot\boldsymbol{C} =-\sqrt{\frac{\lambda+1}{2\lambda+1}}\left(\frac{dg}{dr}-\frac{\lambda}{r}g(r)\right)[Y_{\lambda+1}\boldsymbol{C}]^{(\lambda\mu)} +\sqrt{\frac{\lambda}{2\lambda+1}}\left(\frac{dg}{dr}+\frac{\lambda+1}{r}g(r)\right)[Y_{\lambda-1}\boldsymbol{C}]^{(\lambda\mu)}. \quad (6)$$

In order to solve the coupled-channels equations, we expand the total wave function as

$$\Psi_{jm}(\boldsymbol{r},\alpha)=\sum_{l_pj_p}\sum_{nI}\frac{u^{(j)}_{l_pj_pnI}(r)}{r}|(l_pj_pnI)jm\rangle, \quad (7)$$

where

$$\langle\hat{\boldsymbol{r}},\alpha|(l_pj_pnI)jm\rangle=\sum_{m_pm_I}\langle j_pm_pIm_I|jm\rangle\mathcal{Y}_{j_pl_pm_p}(\hat{\boldsymbol{r}})\phi_{nIm_I}(\alpha), \quad (8)$$

ϕ being the vibrational wave function. We need to compute the coupling matrix elements of the operators $\alpha_\lambda\cdot T_\lambda$, where $T_{\lambda\mu}$ is either $Y_{\lambda\mu}$ or $[Y_{\lambda\pm1}(-i\nabla\times\boldsymbol{\sigma})]^{(\lambda\mu)}$. These are expressed in terms of the Wigner's 6-j symbol as [28]

$$\langle(l'_pj'_pn'I')jm|\alpha_\lambda\cdot T_\lambda|(l_pj_pnI)jm\rangle =(-)^{j_p+I'+j}\begin{Bmatrix} j & I' & j'_p\\ \lambda & j_p & I\end{Bmatrix} \times\langle\mathcal{Y}_{j'_pl'_p}\|T_\lambda\|\mathcal{Y}_{j_pl_p}\rangle\langle\phi_{n'I'}\|\alpha_\lambda\|\phi_{nI}\rangle. \quad (9)$$

For transitions between the ground and the one phonon states which we consider in this paper, the reduced matrix element $\langle\phi_{n'I'}\|\alpha_\lambda\|\phi_{nI}\rangle$ is given by β_λ. The reduced matrix elements for the operators T_λ are found in Ref. [29].

Our coupled-channels approach is based on the Gamow state wave function for resonances. This indicates that the channel wave functions $u^{(j)}_{l_pj_pnI}(r)$ have the asymptotic form of $N^{(j)}_{l_pj_pnI}(G_{l_p}(k_{nI}r)+i\,F_{l_p}(k_{nI}r))$ at $r\to\infty$ for all channels, where $k_{nI}=\sqrt{2\mu(E-n\hbar\omega)/\hbar^2}$ is the channel wave number, and F_{l_p} and G_{l_p} are regular and irregular Coulomb wave functions, respectively [14]. This method, however, requires us to solve the coupled-channels equations in the complex energy plane and out to large distances, which is quite time consuming and also may be difficult to obtain accurate solutions. A much simpler alternative approach has been proposed, which is based on the Green's function formalism [11,18,19,30–32]. This method was first developed for α decays by Kadmensky and his collaborators [30] and was recently applied to the coupled-channels problem for deformed proton emitters by Esbensen and Davids [19]. In this method, the coupled-channels equations are solved in the real energy plane and the solutions are matched to the irregular Coulomb wave functions G_{l_p} at a relatively small distance r_{match}, which is outside the range of nuclear couplings. From the solution of the coupled-channels equations $\Psi^{cc}_{jm}(\boldsymbol{r},\alpha)$ thus obtained, the outgoing wave function for the resonance Gamow state is generated using the Coulomb propagator as [18,19]

$$\Psi_{jm}(\boldsymbol{r},\alpha)=-\int d\boldsymbol{r}'d\alpha'\left\langle\boldsymbol{r}\alpha\left|\frac{1}{H_{coul}+H_{vib}-E-i\eta}\right|\boldsymbol{r}'\alpha'\right\rangle \times\left(V_{coup}(\boldsymbol{r}',\alpha')-\frac{Z_De^2}{r'}\right)\Psi^{cc}_{jm}(\boldsymbol{r}',\alpha'), \quad (10)$$

where η is an infinitesimal number and $H_{coul}=-\hbar^2\nabla^2/2\mu+Z_De^2/r$ is the Hamiltonian for the point Coulomb field. As is well known, the single particle Green's function is expressed in terms of the regular and the outgoing wave functions [18]. The asymptotic normalization factors $N^{(j)}_{l_p j_p nI}$ then read [19]

$$N^{(j)}_{l_p j_p nI} = -\frac{2\mu}{\hbar^2 k_{nI}} \int_0^\infty dr\, r F_{l_p}(k_{nI} r) \times \left\langle (l_p j_p nI) jm \left| V_{coup}(\boldsymbol{r},\alpha) - \frac{Z_D e^2}{r} \right| \Psi^{cc}_{jm} \right\rangle . \tag{11}$$

In this way, the effects of the long-range Coulomb couplings outside the matching radius r_{match} are treated perturbatively. Computing the asymptotic outgoing flux, the total decay width is found to be

$$\Gamma_j = \sum_{l_p j_p nI} \frac{\hbar^2 k_{nI}}{\mu} |N^{(j)}_{l_p j_p nI}|^2 \frac{1}{\langle \Psi^{cc}_{jm} | \Psi^{cc}_{jm} \rangle}, \tag{12}$$

where the normalization factor $\langle \Psi^{cc}_{jm} | \Psi^{cc}_{jm} \rangle$ is calculated inside the outer turning point. The decay half-life is then defined as

$$T_{1/2} = \frac{\hbar}{S_j \Gamma_j} \ln 2, \tag{13}$$

where S_j is the spectroscopic factor for the resonance state. If one assumes that the ground state of an odd-Z nucleus is a one-quasiparticle state, the spectroscopic factor S_j is identical to the unoccupation probability for this state and is given by $S_j = u_j^2$ in the BCS approximation [11].

Let us now numerically solve the coupled-channels equations for the resonant $1h_{11/2}$ and $3s_{1/2}$ states in ^{161}Re as well as the $2d_{3/2}$ state in ^{160}Re. We use the real part of the Becchetti-Greenless optical model potential for the proton-daughter nucleus potential [33]. The potential depth was adjusted so as to reproduce the experimental proton decay Q value for each value of the dynamical deformation parameter β and the excitation energy $\hbar\omega$ of the vibrational phonon excitations in the daughter nucleus. Following Ref. [11], we assume that the depth of the spin-orbit potential is related to that of the central potential by $V_{so} = -0.2 V_0$. The charge radius R_c is assumed to be the same as R_0 in the nuclear potential. The spectroscopic factor S_j in Eq. (13) is taken from Ref. [11]. This was evaluated in the BCS approximation to a monopole pairing Hamiltonian for single-particle levels obtained with a spherical Woods-Saxon potential.

We first discuss the effects of the quadrupole vibrational excitations. Figure 1 shows the dependence of the decay half-life for proton emissions from the three resonance states on the dynamical deformation parameter β_2 and on the excitation energy $\hbar\omega$ of the quadrupole mode. The experimental values for the decay half-life are taken from Refs. [3,4], and denoted by the dashed lines. The arrows are the half-lives in the absence of the vibrational mode. For the decays from the $3s_{1/2}$ and $1h_{11/2}$ states, if one takes into account uncertainty in the experimental Q value of the proton emission, these calculations for the decay half-life in the no coupling limit are within the experimental error bars [11]. In contrast, the calculation for the $2d_{3/2}$ state is still off from the experimental data by about 30% even when uncertainty of the decay Q value is taken into consideration [11]. The results of the coupled-channels calculations are shown by the solid lines in the figure. For each panel, with decreasing order, the four solid curves denote the decay half-life for $\hbar\omega = 0.6$, 0.7, 0.8, and 0.9 MeV, respectively. One notices that the channel coupling effects significantly enhance the decay half-life for the $2d_{3/2}$ state, while the effects are more marginal for the $3s_{1/2}$ and $1h_{11/2}$ states (notice the difference of the scale of the vertical axis in the figure).

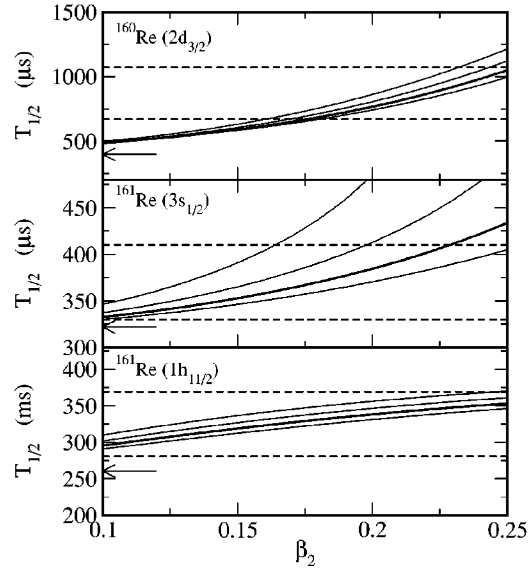

FIG. 1. The decay half-lives for proton emission from the $2d_{3/2}$ state of ^{160}Re and the $3s_{1/2}$ and $1h_{11/2}$ states of ^{161}Re as a function of the dynamical deformation parameter β_2 of the quadrupole vibrational excitation of the daughter nuclei. In the decreasing order, the curves are for $\hbar\omega_2 = 0.6$, 0.7, 0.8, and 0.9 MeV, respectively. The experimental data are taken from Refs. [3,4] and denoted by the dashed lines. The arrows indicate the results in the no coupling limit.

Provided that the vibrational properties are identical between the core nuclei ^{159}W and ^{160}W, one finds a range of

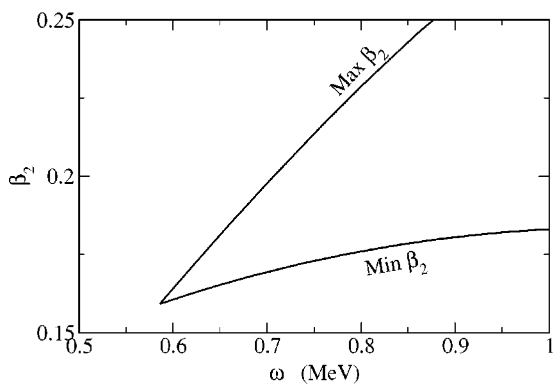

FIG. 2. The range of the dynamical deformation parameter β_2 and the excitation energy $\hbar\omega$ which simultaneously reproduces the experimental decay half-life for the $2d_{3/2}$ state of ^{160}Re and the $3s_{1/2}$ and $1h_{11/2}$ states of ^{161}Re.

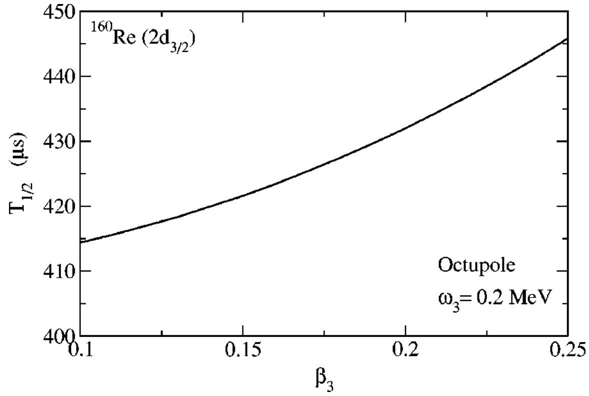

FIG. 3. The decay half-life for proton emission from the $2d_{3/2}$ state of ^{160}Re as a function of the dynamical deformation parameter of the octupole phonon excitation of the core nucleus. The excitation energy is set to be 0.2 MeV.

the dynamical deformation parameter which simultaneously reproduces the measured decay half-life for the $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ states. For instance, for $\hbar\omega=0.8$ MeV, the range reads $0.18\leqslant\beta_2\leqslant0.23$. The half-life for the $3s_{1/2}$ state is relatively sensitive to the phonon energy. For the phonon energy smaller than about 0.6 MeV, the curve raises too quickly as a function of the dynamical deformation parameter, and the consistent description among the decay rates for the three states is not possible. The range of dynamical deformation parameter which simultaneously reproduces the measured decay half-life for the three resonance states is plotted in Fig. 2 as a function of the excitation energy $\hbar\omega_2$. We see that the simultaneous description is possible only when $\hbar\omega_2$ is larger than 0.59 MeV. The value of β_2 larger than 0.25 would not be acceptable for spherical nuclei, but the minimum value of β_2 is always below this limit for $\hbar\omega\geqslant0.6$ MeV.

Lastly, we would like to discuss the dependence on the multipolarity of the phonon mode. Figure 3 shows the effects of octupole phonon excitations on the decay half-life from the $2d_{3/2}$ state of ^{160}Re. As an illustrative example, we take $\hbar\omega_3=0.2$ MeV, but results are qualitatively the same for different values of $\hbar\omega_3$. For the octupole vibration, the enhancement of the half-life is too small to account for the observed discrepancy between the experimental decay half-life and the prediction of the potential model with no coupling. As was noted before [2], the proton decay is very sensitive to the angular momentum of the proton state. The same seems to be true for the multipolarity of the collective excitation of the daughter nucleus.

In summary, we have solved the coupled-channels equations to take into account the effects of the vibrational excitations of the daughter nucleus during the proton emission. Applying the formalism to the the resonant $1h_{11/2}$ and $3s_{1/2}$ states in ^{161}Re and the $2d_{3/2}$ state in ^{160}Re, we have found that the experimental data for the decay half-lives for these three states can be reproduced if the quadrupole phonon excitation with $\hbar\omega_2\geqslant0.6$ MeV and $\beta_2\sim0.18$ is considered. This removes the discrepancy observed before between the experimental data and the prediction of the optical potential model calculation for the decaying $2d_{3/2}$ state in this mass region without destroying the agreement for the $1h_{11/2}$ and $3s_{1/2}$ states. A similar calculation with the octupole phonon was not satisfactory. In this paper, we estimated the spectroscopic factor in the BCS approximation for the monopole pairing Hamiltonian. It would be an interesting future work to consistently compute both the decay rate with the coupled-channels framework and the spectroscopic factor based on the phonon induced pairing mechanism proposed in Ref. [34].

Note added. After we submitted the paper, we noticed that Davids and Esbensen recently carried out independently similar works on the particle-vibration couplings in proton decays [35]. Although the Hamiltonian which they used was slightly different from ours, we both obtained very similar conclusions. Their systematic studies for several proton emitters are complementary to our study in this Rapid Communication. We also noticed that the first 2^+ state in ^{160}W was recently observed at 610 keV [36]. This value agrees with the one obtained in Fig. 2 in our paper, and confirms the usefulness of proton radioactivity to study the spectroscopic properties of proton rich nuclei. We thank Cary Davids for the correspondences.

[1] *Proceedings of the First International Symposium on Proton-Emitting Nuclei PROCON '99*, edited by J. C. Batchelder, AIP Conf. Proc. No. 518 (AIP, New York, 2000), and references therein.
[2] P.J. Woods and C.N. Davids, Annu. Rev. Nucl. Part. Sci. **47**, 541 (1997).
[3] R.D. Page *et al.*, Phys. Rev. Lett. **68**, 1287 (1992).
[4] R.J. Irvine *et al.*, Phys. Rev. C **55**, R1621 (1997).
[5] C.N. Davids *et al.*, Phys. Rev. C **55**, 2255 (1997).
[6] C.N. Davids *et al.*, Phys. Rev. Lett. **80**, 1849 (1998).
[7] J.C. Batchelder *et al.*, Phys. Rev. C **57**, R1042 (1998).
[8] K. Rykaczewski *et al.*, Phys. Rev. C **60**, 011301(R) (1999).
[9] C.R. Bingham *et al.*, Phys. Rev. C **59**, R2984 (1999).
[10] A.A. Sonzogni *et al.*, Phys. Rev. Lett. **83**, 1116 (1999).
[11] S. Åberg, P.B. Semmes, and W. Nazarewicz, Phys. Rev. C **56**, 1762 (1997); **58**, 3011 (1998).
[12] A.T. Kruppa, B. Barmore, W. Nazarewicz, and T. Vertse, Phys. Rev. Lett. **84**, 4549 (2000).
[13] B. Barmore, A.T. Kruppa, W. Nazarewicz, and T. Vertse, Phys. Rev. C **62**, 054315 (2000).
[14] E. Maglione, L.S. Ferreira, and R.J. Liotta, Phys. Rev. Lett. **81**, 538 (1998); Phys. Rev. C **59**, R589 (1999).
[15] L.S. Ferreira and E. Maglione, Phys. Rev. C **61**, 021304(R) (2000).
[16] E. Maglione and L.S. Ferreira, Phys. Rev. C **61**, 047307 (2000).
[17] L.S. Ferreira and E. Maglione, Phys. Rev. Lett. **86**, 1721 (2001).
[18] C.N. Davids and H. Esbensen, Phys. Rev. C **61**, 054302 (2000).
[19] H. Esbensen and C.N. Davids, Phys. Rev. C **63**, 014315 (2000).

[20] P.B. Semmes, Nucl. Phys. **A682**, 239c (2001).
[21] P. Möller, J.R. Nix, W.D. Myers, and W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[22] B. Barmore, A.T. Kruppa, W. Nazarewicz, and T. Vertse, Nucl. Phys. **A682**, 256c (2001).
[23] K. Hagino, N. Takigawa, M. Dasgupta, D.J. Hinde, and J.R. Leigh, Phys. Rev. Lett. **79**, 2014 (1997).
[24] C.H. Dasso, M. Lozano, and A. Vitturi, Phys. Rev. A **44**, 4743 (1991).
[25] K. Hagino, N. Rowley, and A.T. Kruppa, Comput. Phys. Commun. **123**, 143 (1999).
[26] K. Hagino, N. Takigawa, M. Dasgupta, D.J. Hinde, and J.R. Leigh, Phys. Rev. C **55**, 276 (1997).
[27] G. R. Satchler, *Direct Nuclear Reaction* (Oxford University Press, Oxford, 1983), p. 622.
[28] A. R. Edmonds, *Angular Momenta in Quantum Mechanics* (Princeton University Press, Princeton, NJ, 1957), Eqs. (5.9.17) and (7.1.6).
[29] P.-G. Reinhard and Y.K. Gambhir, Ann. Phys. (Leipzig) **1**, 598 (1992).
[30] S.G. Kadmensky, V.E. Kalechtis, and A.A. Martynov, Sov. J. Nucl. Phys. **14**, 193 (1972); S.G. Kadmensky and V.G. Khlebostroev, *ibid.* **18**, 505 (1974).
[31] V.P. Bugrov, S.G. Kadmensky, V.I. Furman, and V.G. Khlebostroev, Sov. J. Nucl. Phys. **41**, 717 (1985).
[32] V.P. Bugrov and S.G. Kadmensky, Sov. J. Nucl. Phys. **49**, 967 (1989); S.G. Kadmensky and V.P. Bugrov, Phys. At. Nucl. **59**, 399 (1996).
[33] F.D. Becchetti and G.W. Greenless, Phys. Rev. **182**, 1190 (1969).
[34] F. Barranco, R.A. Broglia, G. Gori, E. Vigezzi, P.F. Bortignon, and J. Terasaki, Phys. Rev. Lett. **83**, 2147 (1999).
[35] C. N. Davids and H. Esbensen, Phys. Rev. C **64**, 034317 (2001).
[36] A. Keenan *et al.*, Phys. Rev. C **63**, 064309 (2001).