Neutron-proton quadrupole interaction in the nucleon-pair shell model

Yan-an Luo,[1,2,*] Jin-quan Chen,[3] Tai-fu Feng,[1] and Ping-Zhi Ning[1,2]
[1]*Department of Physics, Nankai University, Tianjin 300071, People's Republic of China*
[2]*Institute of Theoretical Physics, Chinese Academia Sinica, Beijing 100080, People's Republic of China*
[3]*Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China*
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The nucleon-pair shell model truncated to the *S-D* subspace is applied to nucleus ^{134}Ba for examining the effects on collectivity due to long-range *n-p* quadrupole forces. It is found that symmetric and mixed-symmetric states, consequently $M1$ transitions, are primarily controlled by the *n-p* quadrupole interaction. In comparison with surface-δ interaction, when the quadrupole *n-p* coupling is weak, the model can produce approximately the vibrational limit, and when the quadrupole *n-p* coupling is strong, the O(6) limit can be realized.

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It has been recognized that in the shell model description, quadrupole-quadrupole interactions are the most important long-range correlations. DeShalit and Goldhaber showed that the quadrupole *n-p* interaction is responsible for the strong mixing of the shell model configuration, and thus plays a major role in describing rotational spectra [1,2]. The importance of the *n-p* interaction can also be found in the algebra models. In the IBM-2 [3], the validity of the boson picture is attributed to the attractive *n-p* interaction, and the *n-p* interaction is also responsible for the transition from the vibrational to rotational spectra [4]. In the FDSM [5], if the Hamiltonian was chosen as $-P(\pi)\cdot P(\nu)$, the rotational spectra in SO(8)$\times$SP(6) can be realized, and even if one wants to reproduce the vibrational spectra in the *n-p* system, the *n-p* interaction has to be switched on to push the extra states out of the way, although the proton and neutron systems can produce two independent vibrational system by themselves [6]. Furthermore, in the FDSM, the exotic (protons and neutrons move out of phase) and normal (protons and neutrons move with phase coherent) states, termed as mixed-symmetric and symmetric states in the IBM-2 [7], consequently the $M1$ transitions, were also unified through the quadrupole *n-p* interaction [8].

The nucleon-pair shell model (NPSM) has attracted much interest recently [9–13]. Since the NPSM is also a fermion model and built from pairs, it should have the same physics as those of the FDSM. It is the aim of this paper to study the effect of the *n-p* quadrupole-quarupole interaction in the framework of the NPSM. As an example, the O(6) limit nucleus ^{134}Ba is used. We choose a rather simple Hamiltonian consisting of single particle term, surface-δ interaction (SDI) between like nucleons, and *Q-Q* interaction between protons (π) and neutrons (ν),

$$H=H_0-V(\nu)-V(\pi)-\kappa Q^2(\pi)\cdot Q^2(\nu), \tag{1}$$

$$H_0=\sum_{a\sigma}\epsilon_{a\sigma}n_{a\sigma}, \quad \sigma=\pi,\nu. \tag{2}$$

The single particle (SP) energies $H_0=H_0^{exp}(\pi)+H_0^{exp}(\nu)$, where $H_0^{exp}(\pi)$ and $H_0^{exp}(\nu)$ are fixed as the SP energies of the nuclei $^{133}_{51}\mathrm{Sb}_{82}$ and $^{131}_{50}\mathrm{Sn}_{81}$, respectively, are taken from Ref. [14] and listed in Table I.

The $E2$ transitional operator is

$$E2=e_\pi Q_\pi^2+e_\nu Q_\nu^2, \tag{3}$$

where e_π and e_ν are effective charges of the protons and neutrons. e_π is determined by the $B(E2;2_1^+\rightarrow 0_1^+)$ for ^{138}Ba, and is $2.1e$. From Ref. [15] we know that $e_\pi/e_\nu=1$ for 50–82 shells, thus $e_\nu=e_\pi=2.1e$ is used in this Brief Report.

The $M1$ transitional operator is

$$T(M1)=T(M1)_\pi+T(M1)_\nu,$$
$$T(M1)_\sigma=\sqrt{\frac{3}{4\pi}}(g_{l\sigma}\mathbf{L}_\sigma+g_{s\sigma}\mathbf{S}_\sigma), \tag{4}$$

where $g_{l\sigma}$ and $g_{s\sigma}$ are the orbital and spin g factors, which are fixed as those in Ref. [16], i.e., $g_{l\pi}=1.1$, $g_{l\nu}=-0.1$, $g_{s\pi}=3.910$, and $g_{s\nu}=-2.678$ (all in units of μ_N^2).

To show the effect of the quadrupole *n-p* interaction, we perform the following calculations. First, we fix the SDI strengths for protons (G_π) and neutron holes (G_ν) as 0.15 and 0.11 MeV, respectively. Then we let the quadrupole *n-p* interaction strength κ vary from 0.02 to 0.5 MeV. The reason for G_π being larger than G_ν is due to the fact that the SP level splitting for proton are larger than that for neutron holes [10].

Part of the important excitation energies are plotted against the *n-p* quadrupole interaction strength κ in Fig. 1, from which one can clearly see how the *n-p* quadrupole interaction affects the spectra. As those of the FDSM [8], we also predict a number of the 2^+ states in our present calculation.

From Ref. [10] we know that the explicit structure of the wave function expanded in terms of the nonorthogonal (but

TABLE I. The single particle (hole) energies for proton (neutron) for $^{133}_{51}\mathrm{Sb}_{82}$($^{131}_{50}\mathrm{Sn}_{81}$).

ϵ_π(MeV)	$g_{7/2}$	$d_{5/2}$	$d_{3/2}$	$h_{11/2}$	$s_{1/2}$
	0	0.963	2.69	2.76	2.99
ϵ_ν(MeV)	$d_{3/2}$	$h_{11/2}$	$s_{1/2}$	$d_{5/2}$	$g_{7/2}$
	0	0.242	0.332	1.655	2.434

*Email address: luoya@nankai.edu.cn

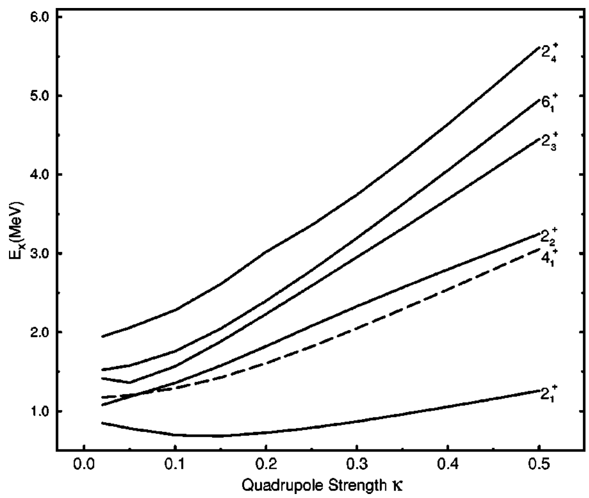

FIG. 1. The variations of the spectrum with the quadrupole n-p interaction.

normalized) multipair basis vectors is helpful and has very close correlation with the electromagnetic transitions, i.e., the strong $E2$ transition occurs between symmetric states, while the strong $M1$ transition occurs between mixed-symmetric and symmetric states. Hence, to gain a deeper understanding of the n-p quadrupole interaction in the NPSM, it will be certainly useful to investigate the detailed structure of the wave functions.

Before going further, we should also mention the meaning of symmetric and mixed-symmetric states in the shell model sense. We say that a state whose main component is $\psi = \alpha|D_\pi\rangle + \beta|D_\nu\rangle + \cdots$ is a symmetric one-D state if $\alpha \sim \beta$, and a mixed-symmetric one-D state if $\alpha \sim -\beta$. These two states are called partner states in FDSM [8]. Similar terminology applies to the two-D state, etc.

The main components of the wave functions for part of the states are given in Table II. To save space only the amplitudes larger than 0.2 are shown and the following shorthand notation is used for the multipair basis:

$$|(D_\pi^\dagger)^{n_\pi}(S_\pi^\dagger)^{N_\pi - n_\pi}(D_\nu^\dagger)^{n_\nu}(S_\nu^\dagger)^{N_\nu - n_\nu};JM\rangle \rightarrow |(D_\nu)^{n_\nu}(D_\pi)^{n_\pi};JM\rangle,$$
$$|(S_\pi^\dagger)^{N_\pi}(S_\nu^\dagger)^{N_\nu};JM\rangle \rightarrow |S;JM\rangle. \quad (5)$$

In the table the coefficients inside brackets are for the multipair basis states which occur twice and are distinguished by the intermediate angular momentum.

It is seen that a small n-p quadrupole interaction will cause a strong mixing between the proton and neutron states, and give rise to two new orthogonal partner states. For example, when $\kappa = 0.05$ MeV, the 2_1^+ state is $0.5481|D_\nu\rangle + 0.3329|D_\pi\rangle + \ldots$, and the 2_2^+ state is $-0.2356|D_\nu\rangle + 0.3403|D_\pi\rangle + \ldots$. The former is identified as the symmetric state and the latter the mixed-symmetric state. One can also see that the symmetry of the 2_2^+ and 2_3^+ states is opposite for κ. For $\kappa < 0.1$ MeV, the 2_2^+ state is s mixed symmetric one-D pair state with some admixture of two-D pair, and the 2_3^+ state is basically a symmetric two-D pair state. But from $\kappa = 0.1$ MeV on, the symmetry of the 2_2^+ and 2_3^+ states exchanges, i.e., the 2_2^+ state is basically a symmetric two-D pair state, while the 2_3^+ state is a mixed-symmetric one-D pair state with some admixture of the two-D pair components. Table II also shows that the number of the D pairs in the states $0_1^+, 2_1^+, \ldots$ increases with κ. This behav-

TABLE II. The main components of the eigenstates with the quadrupole n-p interaction strength.

κ	State	S	D_ν	D_π	$D_\pi D_\nu$	D_ν^2	D_π^2	$D_\pi^2 D_\nu$	$D_\nu^2 D_\pi$	$D_\nu^2 D_\pi^2$	D_π^3	$D_\pi^3 D_\nu$
0.05	0_1^+	0.6344			0.1837		−0.2743					
	2_1^+		0.5481	0.3329				−0.2065				
	2_2^+		−0.2356	0.3403		0.2016	0.3198					
	2_3^+		0.2395			0.5840				−0.2832		
	4_1^+						0.4069				0.5938	
	6_1^+										0.9220	
0.1	0_1^+	0.7264			0.3371		−0.1081					−0.1083
	2_1^+		0.5884	0.4158								
	2_2^+		−0.2153		0.2385	0.4026	0.3289					
	2_3^+		0.2786	−0.2593	0.2080	0.4804						
	4_1^+				0.2855		0.4127				0.4756	0.2297
	6_1^+							0.2503			0.8384	0.2686
0.2	0_1^+	0.8876			0.5451	0.1410	0.1481			0.1373 (0.1147)		
	2_1^+		0.6798	0.4980				0.2446	0.2294			
	2_2^+		−0.2923		0.4319	0.5044	0.4608					
	2_3^+		0.2445	−0.4077	0.2924	0.4334	−0.2042	−0.3174				
	4_1^+				0.4403	0.3135	0.4330				0.2843	0.2627
	6_1^+							0.3757			0.6986	0.3197

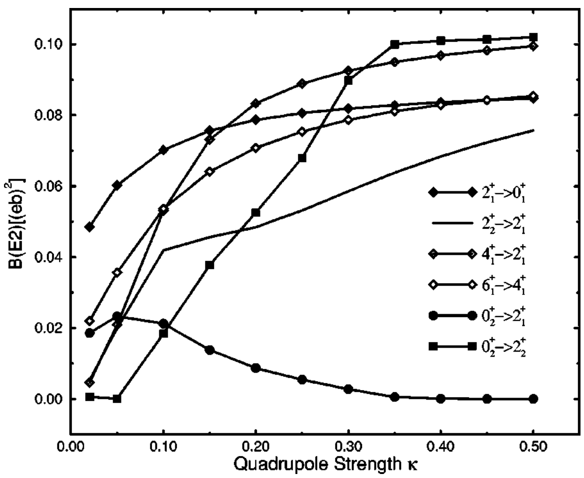

FIG. 2. The variations of the part of the $E2$ transition strength with n-p interaction κ.

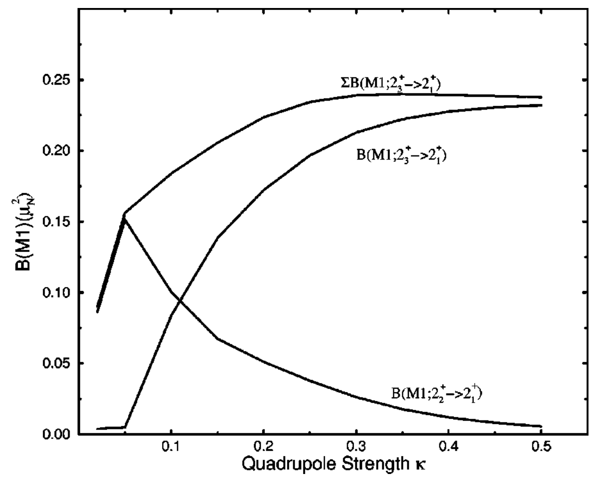

FIG. 3. The variations of the part of the $M1$ transition strength with the quadrupole n-p interaction.

ior is consistent with the well known fact that the n-p Q-Q force favors the formation of the D pairs. One can see that although the F spin [17] does not present in the fermion picture, the symmetric and mixed-symmetric states can also be unified through the n-p quadrupole interaction in the NPSM calculation.

Some important $E2$ transition rates with the n-p quadrupole interaction strength κ are displayed in Fig. 2. It is seen that, consistent with the analysis of the wave functions, the $E2$ transition between the yrast states as well as that between the 2_2^+ and 2_1^+ states are all increased rapidly with κ when κ is small, and then become gradually saturated. One can also see that $B(E2;0_2^+\to 2_2^+)$ increase against κ, while $B(E2;0_2^+\to 2_1^+)$ decrease with κ. For large κ value, $B(E2;0_2^+\to 2_2^+)$ is much larger than $B(E2;0_2^+\to 2_1^+)$, a typical feature of the O(6) limit.

From Ref. [8] we know that the $M1$ transition can only proceed via the partner states, and since the n-p quadrupole interaction is responsible for the splitting of the symmetric and mixed-symmetric states, it also control the $M1$ transitions. To show how the n-p quadrupole interaction affects the $M1$ transitions in the NPSM calculation, we present $B(M1;2_2^+\to 2_1^+)$, $B(M1;2_3^+\to 2_1^+)$ as well as the sum of these two $M1$ transitions in Fig. 3. One can see that in agreement with the detailed structure of the wave functions and the results of the FDSM (see Ref. [8]), the $M1$ transition can only proceed via the partner states. For $\kappa \leqslant 0.1$ MeV, the $M1$ transition mainly occurred between the 2_2^+ and 2_1^+ states. While after $\kappa = 0.1$ MeV, $B(M1;2_3^+\to 2_1^+)$ is dominant. That is to say that for the weak n-p coupling case, which is suitable for the vibrational limit, the $M1$ transitions are very different from those of the strong coupling case, which is suitable for the SO(6) limit. From Fig. 3 one can also see that the sum of the two $M1$ transition rates always increases with κ. When κ is large, it also becomes saturate as that of the $E2$ transitions between the yrast states.

From the above analysis, one can see that the n-p quadrupole interaction enhance the collectivity of low-lying states. As in the FDSM, it is also the n-p quadrupole interaction that is responsible for the splitting of the symmetric and mixed-symmetric states and produce the $M1$ transitions in the NPSM calculation.

In our present calculation, we also found that similar to that of the FDSM, even for the very large n-p quadrupole interaction, the NPSM truncated in the SD subspace still can not give rise to the rotational spectra in the 50–82 shell. For large κ value, only the SO(6) limit can be realized in the NPSM calculation. To show the effects that the n-p quadrupole interactions have in producing the vibartional and SO(6)

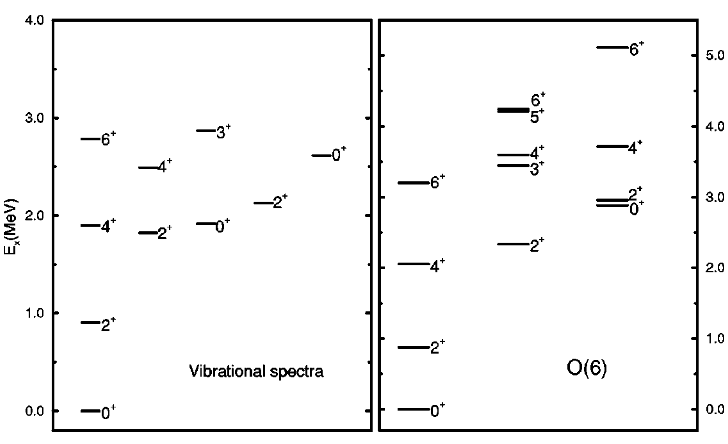

FIG. 4. The vibrational and O(6) limit spectra.

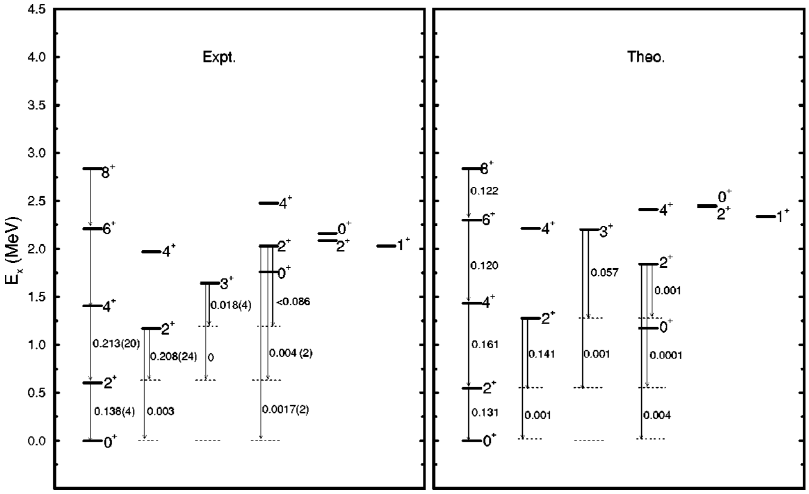

FIG. 5. The spectrum and transitions of ^{134}Ba. The parameters we used are $G_\pi = 0.169$ MeV, $G_\nu = 0.06$ MeV, and $\kappa = 0.212$ MeV. The effective charges for proton and neutron hole are $e_\pi = e_\nu = 2.1e$. The experimental $B(E2)$ values are taken from Ref. [18].

limit spectra, two examples are given in Fig. 4. The left one is for the vibrational spectra with the parameters fixed as $G_\pi = 0.2$ MeV, $G_\nu = 0.13$ MeV, and $\kappa = 0.12$ MeV, and the right one is for the SO(6) limit with $G_\pi = 0.15$ MeV, $G_\nu = 0.11$ MeV, and $\kappa = 0.3$ MeV. One can see that in comparison with the SDI strength, for the weak *n-p* quadrupole coupling, the vibrational spectra can be produced (except for the 2_3^+ state, which is too low in energy position), while for strong *n-p* quadrupole coupling, the SO(6) limit is realized. To check the vibrational mode and the SO(6) limit, the quadrupole moment for the vibrational states and the relative $B(E2)$ values for the SO(6) limit are investigated. For example, the quadrupole moment of the 2_2^+ state in the vibrational limit of the SO(8)$\times$SP(6) limit in the FDSM is $-4.038\,e$b, and it is $-4.197\,e$b in our present calculation. In the SO(6) limit, $B(E2;0_2^+ \to 2_1^+)/B(E2;0_2^+ \to 2_2^+)$ is 0.03, which implies that the $E2$ transition from the 0_2^+ to 2_1^+ state is forbidden, a typical feature of the SO(6) limit. This behavior can also be found in Fig. 2.

Finally, the detail fitting of ^{134}Ba is shown in Fig. 5. One can see that the spectra can be considered as in agreement with the experiments. For ^{134}Ba, two almost degenerate 2^+ states (2_3^+ and 2_4^+) share the $M1$ transition in experiments [18]; the sum of the two $B(M1)$ values is 0.20 μ_N^2. As those calculated in the IBM and FDSM [8,18], our predicted strong $M1$ transition occurs between the 2_3^+ and 2_1^+ state, which is $0.087\mu_N^2$, much smaller than the experiments. But if we use the effective orbital g factors as $g_{l\pi} = 1.2$, $g_{l\nu} = -0.2$ (all in units of μ_N^2) and keep $g_{s\pi}$ and $g_{s\nu}$ as before, a reasonable $B(M1)$ value between the 2_3^+ and 2_1^+ states is obtained, which is $0.171\mu_N^2$.

In summary, from the studying of the effects of the quadrupole *n-p* interaction in the framework of the NPSM truncated in the *SD* subspace, the general physics of the fermion model constructed from *S* and *D* pairs are obtained. It is found that the symmetric and mixed-symmetric states, consequently the $M1$ transitions are primarily controlled by the quadrupole *n-p* force. From the investigation of the competing feature of the surface-δ interaction and the quadrupole *n-p* interaction, we found that in comparison with the surface-δ interaction, when the quadrupole *n-p* coupling is weak, the results are similar to the vibrational limit, and when the quadrupole *n-p* coupling is strong, the O(6) limit can be realized.

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