Prolate dominance of nuclear shape caused by a strong interference between the effects of spin-orbit and l^2 terms of the Nilsson potential

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(Received 17 March 2001; published 8 August 2001)

The origin of the dominance of prolate shapes over oblate ones of the ground states of atomic nuclei is investigated with the Nilsson-Strutinsky method. The number of prolate nuclei among all the deformed even-even nuclei is calculated as a function of the strengths of the spin-orbit and the l^2 terms of the Nilsson potential. The latter simulates a square-well-like radial profile of the mean potential. The proportion of prolate nuclei is 86% with the standard strengths corresponding to the actual atomic nuclei. By weakening the spin-orbit potential, the proportion oscillates strongly, having a local minimum value of 42% with about half of the standard strength and a local maximum value of 79% without the spin-orbit potential.

DOI: 10.1103/PhysRevC.64.037301 PACS number(s): 21.10.Ft, 21.10.Gv, 21.10.Ky, 21.10.Pc

A basic question in nuclear physics is why atomic nuclei have a strong tendency to deform into prolate shapes than into oblate ones.

Since the early days of the discovery of nuclear deformation [1], it has been usually believed that the nuclear deformation can be ascribed to the shell structure of nucleon's single-particle spectrum. One might suspect the existence of some unknown simple and direct correspondence between the prolate dominance and a feature of the Hamiltonian, e.g., a specific term of the elementary nucleon-nucleon interaction, in an analogous fashion as the tensor force causes a mixture of *d* wave in the wave function of a deuteron. However, it is probably sufficient at the present stage to confine the scope of the investigation to the mean single-particle potential, through which most of the possible causes affect the deformation.

There are two causes which favor prolate shapes *not* through the shell effect. One is the Coulomb repulsion between protons, which tends to deform the nucleus into an elongated shape rather than a flattened shape. This effect is, however, important only in heavy nuclei while the prolate dominance is present already in middle-weight nuclei. The other, argued by Zickendraht [2], is the difference of the volume element of the collective coordinates between prolate and oblate shapes, which can be identified with the difference of the available configuration space in spherical shell-model calculations. In mean-field approaches, this effect corresponds to that of an angular-momentum projection into zero-spin states. However, it does not seem to be essential because the prolate dominance can be reproduced without the projection in shell-correction [3] and mean-field [4] methods.

Let us mention three kinds of potentials which give rise to shell effects favoring prolate shapes.

The first kind is the anisotropic harmonic oscillator, which is the most simple approximation used for the nuclear mean-field potential. Concerning the sd shell nuclei, Bohr and Mottelson stated that prolate (oblate) shape is preferred in the beginning (end) of the major-shell filling due to the strong shape-driving effect of the particles (holes) in the $\Omega=\frac{1}{2}$ orbital [5]. This seems to suggest an equal number of prolate and oblate nuclei. According to Ref. [5], it is the spin-orbit potential which breaks this even situation by weakening the oblate-shape shell effect. A more general argument given by Castel *et al.* [6] is that the summation of the single-particle energies of an isotropic harmonic oscillator is decreased by extending one axis and shrinking the other two axes under volume-conservation condition neglecting detailed effects of the Pauli principle. (Their argument seems to apply only to the harmonic-oscillator potential contrary to the statements in their paper.) Therefore harmonic-oscillator potentials are expected to favor prolate shapes. A quantitative estimation of this effect is one of the aims of our study.

The second kind of potentials is those with square-well-like radial profile. The nuclear mean potential resembles the Woods-Saxon potential [7], whose radial profile is in between those of a square well and a harmonic oscillator. Frisk found [8] that such radial dependence is an origin of the prolate dominance from an analysis of classical periodic orbitals in an ellipsoidal cavity. By considering the volume conservation, he showed that the strength of the shell effect at the Fermi surface changes strongly in the prolate side while it stays almost constant in the oblate side as a function of the magnitude of deformation. Consequently, if the spherical shape is unstable, oblate shapes are equally unstable but there must be a more stable state in the prolate side.

The third kind is the spin-orbit potential, which is indispensable for the reproduction of the spherical magic numbers and is an important component of the nuclear mean potential. Its relation to the prolate dominance is suggested from an extensive Skyrme-Hartree-Fock calculation [4]: The energy difference between prolate and oblate minima exhibits a clear and abrupt change of behavior between $Z,N<40$ and $Z,N>50$ where Z and N are the numbers of protons and neutrons, respectively. In the former region prolate and oblate solutions appear evenly in the ground state, while in the latter region the oblate solutions have systematically higher energies than prolate ones. Between the two regions, the character of major shells changes from the harmonic oscillator type to the Mayer-Jensen type, the latter of which includes a high-j intruder in each major shell due to the spin-orbit potential. This parallelism suggests that the spin-orbit potential plays an essential role in giving rise to the prolate dominance.

In this paper we will examine how the situation of the prolate dominance changes when the radial profile of the potential and the strength of the spin-orbit potential are different from those of actual nuclei. For this purpose, we employ the Nilsson-Strutinsky method [9], which is a convenient and well-established method to reproduce nuclear shapes. The single-particle potential of the method is called the Nilsson or the modified oscillator potential and is expressed as

$$U(\mathbf{r})=\frac{1}{2}(\omega_\perp^2 x^2+\omega_\perp^2 y^2+\omega_\parallel^2 z^2)+2\hbar\omega_0 r_{\rm t}^2\sqrt{\frac{4\pi}{9}}\epsilon_4 Y_{40}(\hat{\mathbf{r}})$$
$$+2f_{ls}\kappa_N\hbar\mathring{\omega}_0\mathbf{l}_{\rm t}\cdot\mathbf{s}-f_{ll}\kappa_N\mu_N\hbar\mathring{\omega}_0(\mathbf{l}_{\rm t}^2-\langle\mathbf{l}_{\rm t}^2\rangle_N). \qquad (1)$$

The first term stands for an anisotropic harmonic oscillator potential, where the frequencies $\omega_\perp$ and $\omega_\parallel$ are expressed as functions of a quadrupole deformation parameter ϵ_2,

$$\omega_\perp=\omega_0\left(1+\frac{1}{3}\epsilon_2\right),\quad \omega_\parallel=\omega_0\left(1-\frac{2}{3}\epsilon_2\right),$$

while ω_0 is determined through a volume conservation condition $\omega_\perp^2\omega_\parallel=\mathring{\omega}^3$. The second term is a hexadecapole deformation potential. The third term is a spin-orbit potential, in which orbital and spin angular momenta are expressed as **l** and **s**, respectively. The subscript t means the usage of the stretched coordinates. The fourth term includes the square of the orbital angular momentum and is called the l^2 term or the l^2 potential hereafter. The Woods-Saxon-type radial dependence of the potential is approximated by the l^2 term. The standard values given in Table 1 of Ref. [10] are used for the parameters κ_N and μ_N, which are dependent on the total of the oscillator quanta N. The factors f_{ls} and f_{ll} are introduced in this paper to modify the standard potential, which is restored by putting $f_{ls}=f_{ll}=1$. A convenient feature of the Nilsson potential for our study is that the spin dependent and independent potentials can be changed independently unlike in the Woods-Saxon potential or the relativistic mean-field model [11].

We have utilized a program [12] which is based on the NICRA code [13] but is simplified for nonrotating axially symmetric states, which makes the calculation much faster. The pairing correlation is active for single-particle levels within $\pm1.2\hbar\omega_0$ from the Fermi level, while the strengths of the pairing force are determined such that the pairing gap for smoothed level density becomes $\overline{\Delta}=13A^{-1/2}$ MeV. The parameters of the macroscopic part [14] are $a_{\rm s}=17.9439$ MeV, $\kappa_{\rm s}=1.7826$, and $R_{\rm c}=1.2249A^{1/3}$ fm. See Ref. [13] for the details of the model.

Calculations with the above model have been done in the following way. We choose the values of reduction factors f_{ll} and f_{ls}. For each combination (f_{ll},f_{ls}), we calculate the total energy curve versus ϵ_2 ($-0.5\leqslant\epsilon_2\leqslant0.5$, with ϵ_4 optimized in $-0.16\leqslant\epsilon_4\leqslant0.16$ for each ϵ_2) for all the even-even nuclei with $8\leqslant Z\leqslant126$ and $8\leqslant N\leqslant184$ and between proton and neutron drip lines predicted by the Bethe-Weizsäcker mass formula [7]. The number of nuclei thus included is 1843. We neglect the possibility of triaxial deformations since nonaxial shapes are very rare for even-even nuclei [4]. The reduction factors are taken from a square area $-1\leqslant f_{ll}\leqslant1.5$ and $-1\leqslant f_{ls}\leqslant1.5$ with sampling spacings of $\Delta f_{ll}=\Delta f_{ls}=0.125$.

For each energy curve, we have to label the shape of the ground state as prolate or oblate. One has to be very careful in generating an algorithm for this purpose. Well-deformed nuclei usually have both prolate and oblate minima and thus each minima can be labeled without ambiguity. On the other hand, shape transitional nuclei often have several shallow minima in a large valley extending from oblate side to the prolate side. In such a situation, it is not meaningful to discuss which minima has the lowest energy. After examining a large number of energy curves, we have decided to consider only those nuclei which have both well-developed prolate and oblate minima. The practical procedures we finally adopted are as follows: (1) Draw a smeared energy curve obtained through a convolution with a weight function exp $[-(\Delta\epsilon_2/0.05)^2]$. (2) Separate the original (i.e., before the smearing) curve into valleys by regarding local maxima of the smeared curve as "watersheds." (3) For the minimum in each valley of the original curve, if $\epsilon_2<-0.05$ (>0.05) at the minimum and $\epsilon_2<0.1$ (>-0.1) at the right (left) end of the valley, regard the minimum as a clearly oblate (prolate) solution. (4) If a nucleus has both clearly oblate and clearly prolate solutions satisfying the above criteria and the deeper one is the oblate (prolate) one, count the nucleus as an oblate (prolate) nucleus. Denoting thus counted number of oblate (prolate) nuclei with $N_{\rm o}$ ($N_{\rm p}$), we define the proportion of prolate nuclei as $R_{\rm p}=N_{\rm p}/(N_{\rm p}+N_{\rm o})$. $R_{\rm p}$ may take values from 0 to 1. The denominator $N_{\rm p}+N_{\rm o}$ is about 900 on the average. Note that the smeared curve is used only to divide the curve into valleys and it does not affect the energy or the deformation of the minima.

Figure 1 shows the proportion of prolate nuclei $R_{\rm p}$ as a function of the reduction factors (f_{ll},f_{ls}) by means of contours for $R_{\rm p}$ and symbols for the locations of local maxima (triangles) and minima (squares). Fourth order polynomials in f_{ll} or f_{ls} are used for the interpolations to draw the contours and locate the extrema.

The standard nuclear potential corresponds to a point $f_{ll}=f_{ls}=1$, where $R_{\rm p}$ takes on 86%: Among about 900 even-even nuclei having both prolate and oblate minima, 86% are prolate in the ground state. One can say that the prolate dominance is reproduced with the standard Nilsson potential. It is also worth noting that this value of $R_{\rm p}$ is almost the largest value in the entire square area. The highest peaks are at $(f_{ll},f_{ls})=(0.4,1.4)$ and (0.9, 1.4) where $R_{\rm p}=89\%$. The point for the standard strengths is close to the third peak at (1.1, 1.0) where $R_{\rm p}=88\%$.

Our result is also in qualitative agreement with a calculation for metallic clusters [15], in which prolate ground states are found to be roughly twice as many as oblate ones in the

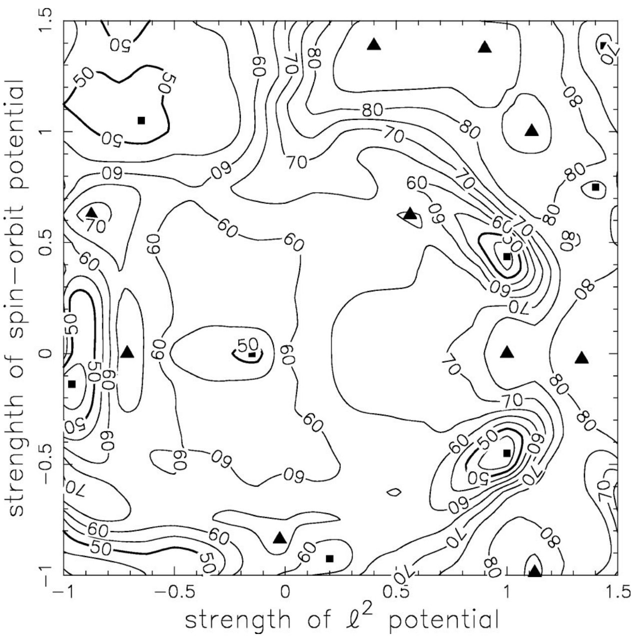

FIG. 1. The proportion of prolate nuclei R_{p}. The abscissa and the ordinate are the reduction factor of the strength of the l^2 potential (f_{ll}) and that of the spin-orbit potential (f_{ls}) relative to the standard values. Contours are for $R_{\mathrm{p}}=45,50,55,\ldots,80\,\%$ with labels aligned in the uphill direction. Thick curves are for $R_{\mathrm{p}}=50\%$. Solid triangles (squares) indicate the locations of some of the local maxima (minima).

framework of the jellium model with infinite square well potential. The corresponding region in Fig. 1 is $f_{ls}=0$ and $f_{ll}\sim 1$, where $R_{\mathrm{p}}\sim 70$–$80\,\%$.

The minimum value of R_{p} is obtained at $(f_{ll},f_{ls})=(-1,-0.125)$, where $R_{\mathrm{p}}=40\%$, i.e., 60% of the deformed nuclei are oblate. The increasing trend of R_{p} as a function of f_{ll} along $f_{ls}=0$ line implies that the attractive (repulsive) l^2 term favors prolate (oblate) shapes. This result supports the theory of Frisk.

On the other hand, the spin-orbit term cannot be regarded as favoring either prolate or oblate shapes, because R_{p} behaves roughly symmetrically between positive and negative values of f_{ls}. The most conspicuous fact concerning the spin-orbit term found in our study is a very strong interference with the l^2 term. In Fig. 1, by moving down from the point (f_{ll},f_{ls}) = (1,1) along a line $f_{ll}=1$, R_{p} takes on 86%, 42, 79, 41, and 81 % for $f_{ls}=1$, 0.44, 0, -0.46, and -1, respectively. One can see two oscillations in $-1\leqslant f_{ls}\leqslant 1$. Weakening the spin-orbit term by about 50% moves the proportion R_{p} from the highest peak at $f_{ls}=1$ to the bottom of a deep valley at $f_{ls}\simeq\frac{1}{2}$, where there are more oblate nuclei than prolate ones. A complete disappearance of the spin-orbit term moves the proportion to another high peak at $f_{ls}=0$ and recovers the prolate dominance. Combination of the two terms produces a situation which is beyond expectation from the independent effects of each term.

A prolate dominance as high as 80% is realized only for restricted combinations of the strengths of the two terms. It may not be a mere coincidence that the potential of actual nuclei matches one of such rare combinations. The same kind of subtle balance between the two terms has been discussed concerning the pseudospin symmetry [16–23], which holds when μ_N of the Nilsson potential is $\frac{1}{2}$ while the standard values of μ_N are between 0.5 and 0.6.

We think that the prolate dominance can be related to the pseudospin symmetry by rephrasing Frisk's idea, which was presented for spinless particles: An attractive l^2 term can cause prolate dominance if the spin is decoupled from the orbital motion. The prolate dominance occurs at $f_{ls}=\pm 1{,}0$ and $f_{ll}=1$. The real spin is decoupled at $f_{ls}=0$ while the pseudospin is decoupled at $f_{ls}=1$. The point at $(f_{ll},f_{ls})=(1,-1)$ might correspond to a similar situation in which another kind of spinlike quantity is decoupled. There are opinions that there seems to exist some simple explanation of the pseudospin symmetry in terms of the relativistic nature of the spin-orbit potential. If they would be approved finally, the prolate dominance of nuclear deformation might also be related with some relativistic aspect of the atomic nuclei.

Let us mention some other results from our calculations before concluding the paper. (1) The inclusion of optimized hexadecapole deformation has a tendency to favor prolate shapes. By setting $\epsilon_4=0$, R_{p} is reduced from 86% to 82% at $(f_{ll},f_{ls})=(1{,}1)$. An average of R_{p} over all the combinations of the reduction factors is decreased from 60% to 55%. (2) An almost pure harmonic oscillator potential (i.e., with $f_{ls}=f_{ll}=\epsilon_4=0$ and weakened pairing) produces R_{p} = 55%. This is an quantitative estimation of the tendency of prolate preference predicted by Castel *et al.* [6]

In summary, a strong interference is found between the effects of the spin-orbit and the l^2 terms of the Nilsson potential. The proportion of prolate nuclei among well-deformed even-even nuclei is more than 80% by using the standard strengths for the two terms. Multiplication of ± 1 or 0 to the strength of the spin-orbit term does not change the situation of prolate dominance. On the other hand, when the strength is multiplied by $\pm\frac{1}{2}$, the proportion is less than 50%, i.e., there are more number of oblate nuclei than prolate ones. The emergence of prolate dominance for restricted combinations of the strengths of the two terms is in a parallel situation with the decoupling of real or pseudospins from the orbital motion and can be understood by extending Frisk's idea to particles with spin.

We are planing to study possible changes due to (1) reductions of pairing force strengths and (2) a replacement of the Nilsson potential with the Woods-Saxon potential. It is also an interesting question to which region of Fig. 1 neutron-rich unstable nuclei, which are waiting for experimental studies, correspond. In such nuclei, both terms are expected to be more or less weakened compared with those for stable nuclei [24]. The potentials of drip-line nuclei may be in the oblate-favoring valley around $f_{ll}\sim 1$ and $f_{ls}\sim 0.5$ according to a result with the Skyrme Hartree-Fock-Bogoliubov calculations [25].

One of the authors (N.T.) thanks the Department of Energy's Institute for Nuclear Theory at the University of Washington for its hospitality during the completion of this work. He also thanks Professor J. P. Draayer and Professor J. Dobaczewski for valuable discussions. The authors are grateful to Professor Y. R. Shimizu for providing a computer program to perform the Nilsson-Strutinsky method calculation and kindly instructing them in its usage. Part of the numerical calculations have been performed using the computer systems of the Yukawa Institute for Theoretical Physics, Kyoto University.

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