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^{5}He+α cluster model of ^{9}Be breakup

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We present here for the first time continuum-discretised-coupled-channels (CDCC) calculations for the elastic scattering and breakup of ^{9}Be+ ^{12}C. We employ a two-body cluster-folding (CF) model picture of ^{9}Be, which is considered as a ^{5}He core plus a ^{4}He cluster. The calculations agree well with data for elastic scattering and excitation of the $5/2^{-}$ resonance of ^{9}Be at 2.43 MeV. The cross section for excitation of the $1/2^{+}$ resonance at 1.68 MeV is not well described by our model, the calculations being considerably smaller than the data. This is as expected from structure calculations of ^{9}Be, which suggest that this state is almost exclusively of ^{8}Be+n cluster form.

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In recent years, the study of light weakly bound radioactive nuclei such as ^{6}He and ^{11}Li, made possible by the advent of radioactive beam facilities, has led to a resurgence of interest in the weakly bound stable nuclei, ^{6,7}Li and ^{9}Be. While the scattering of ^{6,7}Li has been widely studied by means of cluster-folding (CF) models within the continuum-discretised-coupled-channels (CDCC) framework, there has been little work in this direction on the scattering of ^{9}Be. This may be ascribed to the differing structure of the Li isotopes compared to ^{9}Be. While ^{6}Li and ^{7}Li are both well described by two-body cluster models ($\alpha+d$ and $\alpha+t$, respectively), ^{9}Be is considered to be a three-body, $\alpha+\alpha+n$, cluster. At present, three-body CDCC calculations are not possible, hence previous work on the reactions of ^{9}Be has been limited to conventional coupled-channels calculations where the resonant states of ^{9}Be are considered as rotational model states [1–8]. Such calculations are unable to include explicitly couplings to the continuum.

Recent measurements of the fusion of ^{9}Be with ^{64}Zn [9] and ^{208}Pb [10] targets have brought to light an interesting problem. While the ^{9}Be+ ^{208}Pb measurements show a large fusion suppression, ascribed to the influence of ^{9}Be breakup, the ^{9}Be+ ^{64}Zn fusion data show no evidence of such suppression. Conventional coupled-channels calculations that included couplings to the $5/2^{-}$ and $7/2^{-}$ resonances in ^{9}Be were unable to account for the fusion suppression observed with the ^{208}Pb target [10]. If we are to understand the effect of breakup on fusion for these two systems a realistic model of ^{9}Be that allows the inclusion of coupling to the continuum is required. In order to test such a model adequately, breakup data are required.

The motivation behind this work is to develop and test against existing elastic scattering and resonant breakup data a two-body cluster model of ^{9}Be that includes coupling to the two-body continuum. This model may then be used to investigate the differing behavior of the fusion cross sections for the ^{9}Be+ ^{208}Pb and ^{9}Be+ ^{64}Zn systems. We take the following approach to the construction of this model. While ^{9}Be is best described as an $\alpha+\alpha+n$ three-body cluster, one may also consider it to consist of a two-body ^{8}Be+n or ^{5}He+ α cluster. In fact, within the framework of the three-body system, structure calculations suggest that the ground state of ^{9}Be exhibits a large measure of ^{5}He+ α character [11]. Guided by this suggestion, we develop a two-body ^{5}He+ α CF model of ^{9}Be and use it to carry out CDCC calculations for the recent ^{9}Be+ ^{12}C data of Rudchik *et al.* [8]. These data include cross sections for the unbound $1/2^{+}$ and $5/2^{-}$ states in ^{9}Be at excitation energies of 1.68 and 2.43 MeV, respectively, and thus provide a severe test of our calculations. The use of a two-body cluster model allows us to carry out conventional CDCC calculations for ^{9}Be using CF model form factors, including couplings to the ^{5}He+ α continuum.

The CDCC calculations were carried out using the code FRESCO [12], version FRXP.14, in a manner analogous to that used previously for ^{6,7}Li [13,14]. The ^{5}He+ α CF model of ^{9}Be was constructed as follows. The ground state of ^{9}Be has spin and parity $3/2^{-}$. The ^{5}He core also has spin and parity $3/2^{-}$, which, coupled with the 0^{+} spin and parity of the α particle, naturally leads to a ground state configuration where the alpha particle has angular momentum $L=0$ with respect to the ^{5}He core. However, in order to account for reorientation of the highly deformed ^{9}Be nucleus an $L=2$ component was added to the ground state. The spectroscopic amplitudes of the $L=0$ and $L=2$ components were obtained from a shell model calculation [15] and were 0.81 and 0.5358, respectively. The ^{5}He+ α binding potential was of Woods-Saxon form, with radius parameter $r_b=1.115$ fm, where $R_b=r_b(5^{1/3}+4^{1/3})$, and diffuseness parameter $a_b=0.57$ fm. The well depth was adjusted to give the correct binding energy. The radius parameter was tuned so that the model ground state quadrupole moment matched the measured value of $Q=+5.3\pm0.3$ e fm^2 [16]. The model value of Q was calculated according to the following expression:

$$Q=-2\,\frac{2j-1}{2j+2}\beta\;\;AB\int_{0}^{\infty}r^{2}\phi_{0}(r)\phi_{2}(r)dr\;\;e\,\mathrm{fm}^{2},\qquad(1)$$

where A, B and $\phi_0(r)$, $\phi_2(r)$ denote the spectroscopic amplitudes and ^{5}He+ α relative motion wave functions for the $L=0$ and $L=2$ components of the ground state, respectively, and β is as given by Buck and Pilt [17]:

$$\beta = \frac{A_1^2 Z_2 + A_2^2 Z_1}{(A_1 + A_2)^2}. \tag{2}$$

Our calculations also included couplings to the $5/2^-$ and $7/2^-$ resonances at 2.43 and 6.76 MeV, respectively, and the $1/2^+$ resonance at 1.68 MeV. This last state was included, despite not being considered to show a large $^5\mathrm{He}+\alpha$ cluster structure [18,19], as data for excitation of this state were available and provided a good opportunity of investigating how much of the cross section could be attributed to this clustering mode. Detailed structure calculations [18,19] in fact suggest that the $1/2^+$ state is almost exclusively of $^8\mathrm{Be}+n$ cluster character, and as such is not expected to be well described by our model. The $5/2^-$ and $7/2^-$ resonances, as members of the $K=3/2$ ground state rotational band [20], might be reasonably expected to exhibit the same $^5\mathrm{He}+\alpha$ cluster structure as the ground state. In support of this expectation, α decay has been observed from both states [16,21]. That for the $7/2^-$ state was unambiguously identified as coming from decay into $^5\mathrm{He}+\alpha$ [21]. For the $5/2^-$ state the situation is less clear-cut. Measurements tabulated in Mikolas *et al.* [22] indicate that the fraction of decays from this state to $^8\mathrm{Be}_{\mathrm{g.s.}}+n$ is of the order of 10% or less. However, the remaining strength may be attributed to any one or combination of the decay channels: $^5\mathrm{He}+\alpha$, $^8\mathrm{Be}^*_{2^+}+n$, or directly to $\alpha+\alpha+n$.

The $5/2^-$ and $7/2^-$ resonances were considered as pure $L=2$ states and the $1/2^+$ as a pure $L=1$ state. In reality, all these states are unbound, the $^9\mathrm{Be}\rightarrow 2\alpha+n$ breakup threshold being at 1.57 MeV. However, as the $^9\mathrm{Be}\rightarrow {}^5\mathrm{He}+\alpha$ threshold is at 2.46 MeV, the $5/2^-$ and $1/2^+$ states were treated as bound within our model, the $7/2^-$ state being modeled as a resonant bin of width $\Delta E=3.0$ MeV centred at the resonance energy of 6.76 MeV. The $^5\mathrm{He}+\alpha$ binding potential for all these states was of the same Woods-Saxon form as the ground state (with the same geometry), the well depth being adjusted to give the correct binding energy for the $5/2^-$ and $1/2^+$ states and the correct resonance energy for the $7/2^-$ state.

In placing the $7/2^-$ resonance at 6.76 MeV we have followed a standard compilation of the properties of $^9\mathrm{Be}$ [16]. However, more recent work [20,23] suggests that the 6.76 MeV peak is in fact a doublet, consisting of the $7/2^-$ state at 6.38 MeV and a $9/2^+$ state at 6.76 MeV. Test calculations found that while the angular distribution for excitation of the $7/2^-$ state is affected by changing its excitation energy to 6.38 MeV, the other results are unaltered. As there are at present no data for the inelastic excitation of the $7/2^-$ state, we have kept a value of 6.76 MeV for the energy of this state.

A nonresonant $^5\mathrm{He}+\alpha$ continuum was also included in the calculations. The continuum was discretised into a series of momentum bins with respect to the momentum $\hbar k$ of the $^5\mathrm{He}+\alpha$ relative motion. The wave functions for these continuum bins were normalized to unity and the radius limiting their range was set at 45 fm. The model space was limited to $0.25\leqslant k\leqslant 0.75$ fm^{-1}, with $\Delta k=0.25$ fm^{-1}. The $0.0\leqslant k\leqslant 0.25$ fm^{-1} bins were omitted as tests showed that they had little effect on the result of the calculation. The calculations included the $L=0,1,2$ continuum, as tests again found that the inclusion of the $L=3$ continuum had little effect. For the $L=2$ continuum the binning scheme was slightly modified by the omission of the $0.25\leqslant k\leqslant 0.75$ fm^{-1} bin for $I^\pi=7/2^-$ to avoid double counting due to the presence of the $7/2^-$ resonance. The $^5\mathrm{He}+\alpha$ binding potential for the continuum bins employed the same Woods-Saxon geometry as the ground state.

As there are no optical model potentials available for $^5\mathrm{He}$, the $^5\mathrm{He}+{}^{12}\mathrm{C}$ and $\alpha+{}^{12}\mathrm{C}$ optical model potentials required for the CF model form factors were both obtained from the global α potential of Avrigeanu *et al.* [24]. The real and imaginary potential strengths of these potentials were then regarded as adjustable parameters which were varied to obtain the best fit to the elastic scattering data. Renormalization factors of 1.0 and 2.0 for the real and imaginary strengths, respectively, gave the best description of the elastic scattering angular distribution. Both Coulomb and nuclear couplings were included up to and including multipolarity $\lambda=2$ and the number of partial waves was limited to $\ell\leqslant 75$. All allowed reorientation couplings were included unless otherwise stated.

Coupling to the 2^+ state of $^{12}\mathrm{C}$ at 4.43 MeV was not included in our calculations. The effect of this coupling is implicitly included in the CF model optical potential, and recent calculations for $^7\mathrm{Li}+{}^{12}\mathrm{C}$ elastic scattering [25] found that explicitly including coupling to this state had little effect on the results. Thus, we believe that we are justified in omitting coupling to the $^{12}\mathrm{C}$ 2^+ state.

The data of Rudchik *et al.* [8] were measured using inverse kinematics with a 65 MeV $^{12}\mathrm{C}$ beam scattering from a $^9\mathrm{Be}$ target. This corresponds to $^9\mathrm{Be}$ scattering from a $^{12}\mathrm{C}$ target with a beam energy of 48.75 MeV, and our calculations were performed in this sense. The inverse kinematics technique was exploited extensively to measure the inelastic scattering to unbound states of $^6\mathrm{Li}$ and $^7\mathrm{Li}$ [26–28]. The results of our calculation including couplings to the $1/2^+$, $5/2^-$, and $7/2^-$ resonances and the $L=0,1,2$ $^5\mathrm{He}+\alpha$ continuum are compared with the data in Fig. 1, where they are denoted by the dotted lines. The dashed lines in Fig. 1 denote the results of a four channel calculation that includes couplings to the $1/2^+$, $5/2^-$, and $7/2^-$ resonances only. It can be seen that while the agreement with the elastic scattering and $5/2^-$ data of the calculation including couplings to the $^5\mathrm{He}+\alpha$ continuum is good, the $1/2^+$ data are not very well described, the calculations being approximately a factor of 10 too small. This suggests that the contribution of the $^5\mathrm{He}+\alpha$ clustering mode to this state is small, as expected from the structure calculations of Zahn [18] and Tanaka *et al.* [19] which suggest that this state is almost exclusively of $^8\mathrm{Be}+n$ cluster character.

The effect of coupling to the $^5\mathrm{He}+\alpha$ continuum can also be seen in Fig. 1, where its importance is clearly demonstrated. A comparison of the dashed and dotted curves shows that this coupling has an important effect on the $1/2^+$ and $5/2^-$ cross sections as well as the elastic scattering. Coupling to the continuum produces a redistribution of the $^5\mathrm{He}+\alpha$

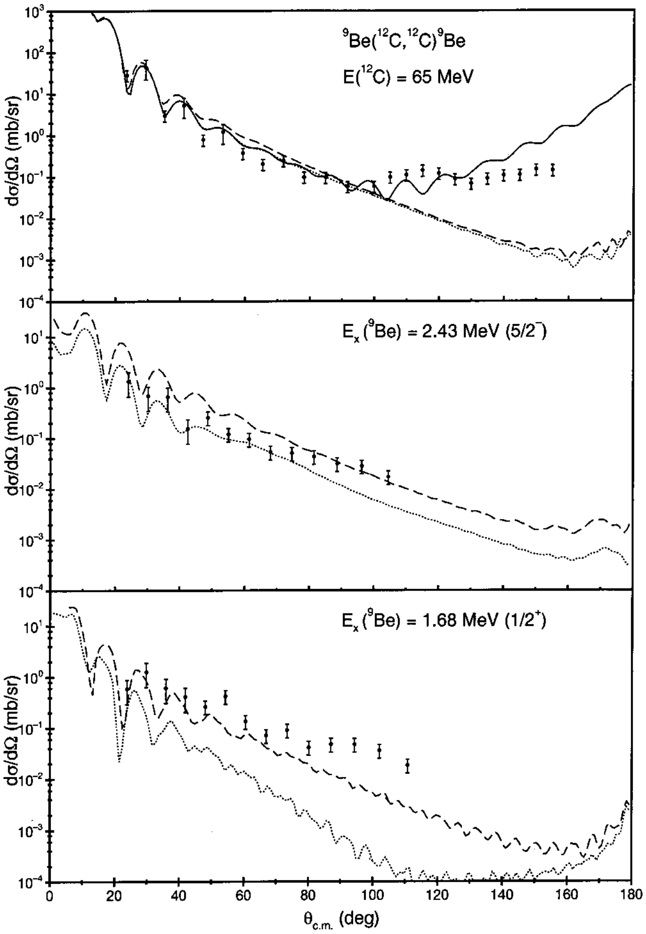

FIG. 1. Comparison of the CDCC calculations with the data for ^{9}Be+ ^{12}C scattering. The full curve denotes the result of the full calculation (1/2$^+$, 5/2$^-$, and 7/2$^-$ resonances and $L = 0,1,2$ continuum) including the ^{3}He cluster elastic transfer, while the dotted curves denote the results of the full calculation without the elastic transfer. The dashed curves denote the results of a four channel calculation that includes couplings to the 1/2$^+$, 5/2$^-$, and 7/2$^-$ resonances only.

breakup strength that leads to a reduction of the cross sections for scattering to the 1/2$^+$ and 5/2$^-$ resonances.

As ground state reorientation has been shown to be important in coupled-channels calculations using the rotational model of ^{9}Be [1], we investigated the effect of this coupling in our CF model description. Figure 2 compares the results of one channel calculations that include (dashed curve) and do not include (dotted curve) the ground state reorientation coupling. It can be seen that the effect of the reorientation coupling is to damp out the oscillations present in the calculated elastic scattering angular distribution, as found previously. The overall effect of this coupling is, however, considerably smaller than that produced by coupling to the ^{5}He+ α continuum.

It will be noted from Fig. 1 that for angles greater than about 100° our calculated elastic scattering angular distribution continues to reduce in magnitude, whereas the measured one shows a slight rise towards larger angles. Rudchik *et al.* [8] found that elastic transfer of a ^{3}He cluster made a significant contribution to the elastic scattering at backward angles. Therefore, we also performed a calculation including this effect. The optical potential within the ''transferred'' partition was just the bare CF model potential and no couplings were included within this partition. The ^{3}He cluster was bound in a standard Woods-Saxon potential well with a radius parameter $r_b = 1.25$ fm, where $R_b = r_b(9^{1/3} + 3^{1/3})$, and diffuseness parameter $a_b = 0.65$ fm, following Rudchik *et al.* [8]. The potential well depth was adjusted to give the correct binding energy. The ^{3}He cluster was considered to be in a ^{2}P$_{3/2}$ state with a spectroscopic amplitude of -1.224 [29].

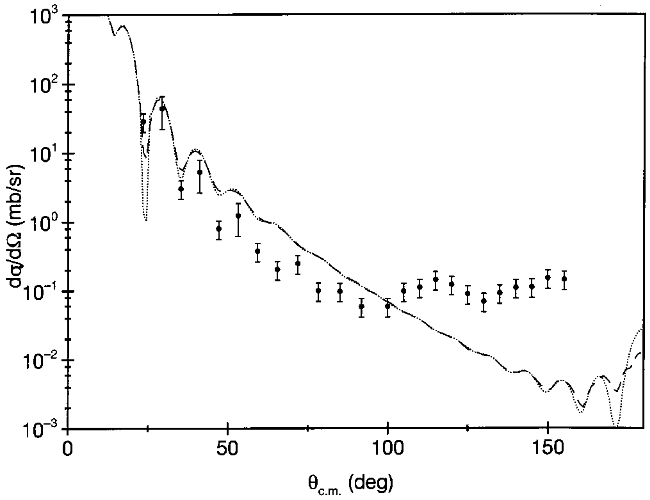

FIG. 2. Comparison of the results of one channel calculations that include (dashed curve) and do not include (dotted curve) the ground state reorientation coupling.

The result of our calculation including the ^{3}He cluster elastic transfer is denoted by the solid curve in Fig. 1. The addition of this coupling has a negligible effect on the calculated angular distributions for the 1/2$^+$ and 5/2$^-$ states and these are thus not shown in the figure. As Fig. 1 shows, the inclusion of the ^{3}He cluster elastic transfer leads to a significant improvement in the description of the elastic scattering data at angles greater than 100°, as found by Rudchik *et al.* [8].

One point of difference between the results of the calculations presented here and those of Rudchik *et al.* [8] using conventional rotational model form factors for the ^{9}Be resonant states is that our calculations produce angular distributions that are considerably less oscillatory. This is particularly noticeable for the 5/2$^-$ state. It would be of interest to obtain more detailed angular distributions in order to determine which is the more realistic calculation, as the current data are not sufficiently detailed to decide on this point.

To summarize, we have performed for the first time CDCC calculations for ^{9}Be scattering, using a two-body ^{5}He+ α CF model of ^{9}Be. Our calculations are able to provide a good description of the elastic scattering and 5/2$^-$ resonant breakup data. As expected from three-body structure calculations [18,19], which suggest that this state is almost exclusively of ^{8}Be+ n cluster form, our ^{5}He+ α model does not provide a good description of the 1/2$^+$ resonant breakup data. Our results suggest that any contribution to the $1/2^+$ state from the ^{5}He$+\alpha$ clustering mode is small, as the full calculation produces an angular distribution approximately an order of magnitude smaller than the measured one. At present there are no scattering data available for the $7/2^-$ resonant state. It would be of great interest to obtain such data in order to test the accuracy of our model description of this state.

Our calculations show that coupling to the ^{5}He$+\alpha$ continuum has an important effect on both the elastic scattering and resonant breakup cross sections. This is the first time that such couplings have been included in a calculation of ^{9}Be scattering. One channel calculations showed that while the effect of the ground state reorientation of ^{9}Be on the elastic scattering is significant, it is rather smaller than that of coupling to the ^{5}He$+\alpha$ continuum.

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[1] V. Hnizdo, K.W. Kemper, and J. Szymakowski, Phys. Rev. Lett. **46**, 590 (1981).
[2] B. Kamys, L. Jarczyk, Z. Rudy, A. Strzałkowski, H.H. Wolter, J. Lang, R. Müller, and J. Stromicki, Nucl. Phys. **A406**, 193 (1983).
[3] J. Cook and K.W. Kemper, Phys. Rev. C **31**, 1745 (1985).
[4] Y. Hirabayashi, S. Okabe, and Y. Sakuragi, Phys. Lett. B **221**, 227 (1989).
[5] J. Carter, Z.Z. Vilakazi, R.W. Fearick, V. Hnizdo, E. Muskat, K.W. Kemper, D.E. Trcka, and S.P. Van Verst, Nucl. Phys. **A591**, 349 (1995).
[6] E. Muskat, J. Carter, R.W. Fearick, and V. Hnizdo, Nucl. Phys. **A581**, 42 (1995).
[7] S. Roy, J.M. Chatterjee, H. Majumdar, S.K. Datta, S.R. Banerjee, and S.N. Chintalapudi, Phys. Rev. C **52**, 1524 (1995).
[8] A.T. Rudchik, O.A. Momotyuk, V.A. Ziman, A. Budzanowski, A. Szczurek, I. Skwirczyńska, S. Kliczewski, and R. Siudak, Nucl. Phys. **A662**, 44 (2000).
[9] S.B. Moraes, P.R.S. Gomes, J. Lubian, J.J.S. Alves, R.M. Anjos, M.M. Sant'Anna, I. Padrón, C. Muri, R. Liguori Neto, and N. Added, Phys. Rev. C **61**, 064608 (2000).
[10] M. Dasgupta, D.J. Hinde, R.D. Butt, R.M. Anjos, A.C. Berriman, N. Carlin, P.R.S. Gomes, C.R. Morton, J.O. Newton, A. Szanto de Toledo, and K. Hagino, Phys. Rev. Lett. **82**, 1395 (1999).
[11] K. Arai, Y. Ogawa, Y. Suzuki, and K. Varga, Phys. Rev. C **54**, 132 (1996).
[12] I.J. Thompson, Comput. Phys. Rep. **7**, 167 (1988).
[13] N. Keeley and K. Rusek, Phys. Lett. B **375**, 9 (1996).
[14] N. Keeley and K. Rusek, Phys. Lett. B **427**, 1 (1998).
[15] A.T. Rudchik and Yu.M. Tchuvil'skii, Ukr. Phys. J. **30**, 819 (1985) [in Russian].
[16] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).
[17] B. Buck and A.A. Pilt, Nucl. Phys. **A280**, 133 (1977).
[18] W. Zahn, Nucl. Phys. **A269**, 138 (1976).
[19] N. Tanaka, Y. Suzuki, K. Varga, and R.G. Lovas, Phys. Rev. C **59**, 1391 (1999).
[20] J.P. Glickman, W. Bertozzi, T.N. Buti, S. Dixit, F.W. Hersman, C.E. Hyde-Wright, M.V. Hynes, R.W. Lourie, B.E. Norum, J.J. Kelly, B.L. Berman, and D.J. Millener, Phys. Rev. C **43**, 1740 (1991).
[21] N. Soić, D. Cali, S. Cherubini, E. Costanzo, M. Lattuada, D. Miljanić, S. Romano, C. Spitaleri, and M. Zadro, Europhys. Lett. **41**, 489 (1998).
[22] D. Mikolas, B.A. Brown, W. Benenson, L.H. Harwood, E. Kashy, J.A. Nolen, Jr., B. Sherrill, J. Stevenson, J.S. Winfield, Z.Q. Xie, and R. Sherr, Phys. Rev. C **37**, 766 (1988).
[23] S. Dixit, W. Bertozzi, T.N. Buti, J.M. Finn, F.W. Hersman, C.E. Hyde-Wright, M.V. Hynes, M.A. Kovash, B.E. Norum, J.J. Kelly, A.D. Bacher, G.T. Emery, C.C. Foster, W.P. Jones, D.W. Miller, B.L. Berman, and D.J. Millener, Phys. Rev. C **43**, 1758 (1991).
[24] V. Avrigeanu, P.E. Hodgson, and M. Avrigeanu, Phys. Rev. C **49**, 2136 (1994).
[25] E.E. Bartosz, N. Keeley, P.D. Cathers, M.W. Cooper, K.W. Kemper, F. Maréchal, and K. Rusek, Phys. Rev. C **64**, 014606 (2001).
[26] M.F. Vineyard, K.W. Kemper, and J. Cook, Phys. Lett. **142B**, 249 (1984).
[27] M.F. Vineyard, J. Cook, and K.W. Kemper, Phys. Rev. C **31**, 879 (1985).
[28] J. Cook, M.N. Stephens, K.W. Kemper, and A.K. Abdallah, Phys. Rev. C **33**, 915 (1986).
[29] O.F. Nemets, V.G. Neudatchin, A.T. Rudchik, Yu.F. Smirnov, and Yu.M. Tchuvil'skii, *Nucleon Associations in Atomic Nuclei and Many-Nucleon Transfer Reactions* (Ukrainian Academy of Sciences, Institute for Nuclear Research, Kiev, 1988) [in Russian].