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One-pion-exchange three-nucleon force and the A_y puzzle

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We consider a new three-nucleon force generated by the exchange of one pion in the presence of a $2N$ correlation. The underlying irreducible diagram has been recently suggested by the authors as a possible candidate to explain the puzzle of the vector analyzing powers A_y and iT_{11} for nucleon-deuteron scattering. Herein, we have calculated the elastic neutron-deuteron differential cross section, A_y, iT_{11}, T_{20}, T_{21}, and T_{22} below breakup threshold by accurately solving the Alt-Grassberger-Sandhas equations with realistic interactions. We have also studied how A_y evolves below 30 MeV. The results indicate that this new $3NF$ diagram provides one possible additional contribution, with the correct spin-isospin structure, for the explanation of the origin of this puzzle.

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The A_y puzzle (or, more appropriately, the puzzle of the vector analyzing powers) is probably the most famous of the open problems in $3N$ scattering at low energy. The problem with this observable has been observed quite early at the Tokio & Sendai Conference (Few-Body XI, 1986) [1], since at that time the first reliable Faddeev calculations with realistic $2N$ potentials were becoming available, thanks in particular to the employment of separable expansion methods that transform the $3N$ scattering equations of Alt, Grassberger, and Sandhas (AGS) [2] into an effective, multichannel Lippmann-Schwinger equation. This method of calculation has been pushed forward to obtain accurate results particularly by Koike *et al.* [3]. Since then, various alternative methods of solution of the $3N$ scattering equation have been developed and tested [4], and outstanding progresses have been made in the computational techniques in order to (1) include three-nucleon forces ($3NF$) in the $3N$ scattering equations [4–6]; (2) treat explicitly the Δ dynamics in the $3N$ system [7]; (3) provide a combined description of the $3N$ dynamics with Coulomb, realistic $2N$, and phenomenological $3N$ forces [8].

The puzzle was confirmed by these new approaches, and it turned out that the existing 2π-$3NF$ [9–11] provided a too small effect for A_y [4,5,8], and not always in the right direction. In absence of new $3NF$'s that could explain the puzzle, and since the $3N$ A_y is rather sensitive to the ^3P_j NN phase shifts, it was concluded in Ref. [12] (see also references therein) that such phases and the associated NN potentials derived from modern phase-shift analysis must be modified at low energy. These modifications can be achieved without affecting appreciably the $2N$ data because the low-energy $2N$ observables cannot resolve the ^3P_j phases uniquely due to the Fermi-Yang ambiguities. However, as has been argued in Ref. [13], it is not possible to increase the $3N$ A_y with reasonable changes in the NN potential; hence additional $3NF$'s of new structure have to be considered. Recently, an attempt has been made [14] using a purely phenomenological $3NF$ of spin-orbit type, constructed *ad hoc* to affect only the triplet-odd states of $2N$ subsystem. $3NF$ terms of pion-range/short-range form [15] have been reconsidered lately from the point of view of chiral perturbation theory [16] (χPT), which predicts for these terms a non-negligible role. Qualitatively, it was found that these terms somewhat affect A_y, but a quantitative conclusion could not be derived.

It is our intention to indicate here a possible solution of this puzzle in terms of a new $3NF$ proposed recently by the authors [17]. This force is generated by the one-pion-exchange diagram when one of the two nucleons involved in the exchange process rescatters with a third one while the pion is "in flight." The underlying diagram has been derived starting from a formalism [18] devoted to the explicit treatment of the pion dynamics in the $3N$ system. The resulting dynamical equation resembles a Faddeev-AGS equation, but entails in its inner structure the full complexity of the underlying four-body (πNNN) system. A gradual procedure to project out the pion degrees of freedom has been discussed in Ref. [19], where it has been shown that this formalism leads to irreducible $3NF$ diagrams, and includes in particular the $3NF$ we will evaluate here.

$3NF$ diagrams of the type derived in Ref. [17] and analyzed here are not new in the literature [20–22], but they have been discarded in modern few-nucleon calculations because of the presence of a cancellation effect that has been observed in Refs. [23,24] and discussed later from the point of view of effective chiral Lagrangians [25]. This cancellation involves meson retardation effects of the iterated Born term, and the irreducible diagrams generated by subsumming all time orderings involving the combined exchange of two mesons amongst the three nucleons. However, this cancellation is incomplete [17] and generates a three-nucleon force through a subtraction term of the type "t-v" with the $2N$ t matrix being pushed fully off the energy shell because of the presence of the pion. Obviously, some ambiguities and model dependencies should be expected, since the subtraction involves the t matrix in a region that is difficult to access and constrain, e.g., by $2N$ data. In addition, there are also possible model dependencies on how this "imperfect" cancellation should be specified, since the input $2N$ potential itself could possibly include—maybe in a somewhat hidden way—meson-retardation correction effects. Ideally, this new $3NF$ term should be constructed consistently with the specific aspects and details of the given $2N$ interaction.

The explicit expression of this force [17] is

$$V_3^{3N}(\mathbf{p},\mathbf{q},\mathbf{p}',\mathbf{q}';E)=\frac{f_{\pi NN}^2(Q)}{m_\pi^2}\frac{1}{(2\pi)^3}\left[\frac{(\boldsymbol{\sigma_1}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_3})+(\boldsymbol{\sigma_2}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_2}\cdot\boldsymbol{\tau_3})}{\omega_\pi^2}\right]\frac{\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-m_\pi\right)}{2m_\pi}$$
$$+\frac{f_{\pi NN}^2(Q)}{m_\pi^2}\frac{1}{(2\pi)^3}\frac{\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q'^2}{2\nu}-m_\pi\right)}{2m_\pi}\left[\frac{(\boldsymbol{\sigma_1}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_3})+(\boldsymbol{\sigma_2}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_2}\cdot\boldsymbol{\tau_3})}{\omega_\pi^2}\right]. \tag{1}$$

The full $3NF$ is given by summing over the cyclic permutations of the nucleons, $V^{3N}=V_1^{3N}+V_2^{3N}+V_3^{3N}$. The momenta $\mathbf{p},\mathbf{q}$ represent, respectively, the Jacobi coordinates of the pair "12" and spectator "3," while E is the $3N$ energy. We have set the pion-nucleon coupling constant to the traditional value $f_{\pi NN}^2/(4\pi)=0.078$, and have employed standard form-factors of monopole type to describe the effective, composite nature of the meson-baryon coupling:

$$f_{\pi NN}(Q)=f_{\pi NN}\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+Q^2}. \tag{2}$$

The chosen $2N$ model interaction implicitly determines the value of pion-nucleon cutoff in V^{3N}, e.g., $\Lambda_\pi=1.7$ GeV for the Bonn B potential. The transferred momentum $\mathbf{Q}=\mathbf{q}'-\mathbf{q}$ enters also in $\omega_\pi=\sqrt{m_\pi^2+Q^2}$. $\tilde{t}_{ij}$ denotes the subtracted t matrix between nucleons 1 and 2, defined according to the prescription

$$\tilde{t}_{12}(\mathbf{p},\mathbf{p}';Z)=c(Z)t_{12}(\mathbf{p},\mathbf{p}';Z)-v_{12}(\mathbf{p},\mathbf{p}'). \tag{3}$$

Other details can be found in Ref. [17].

In addition, we have introduced here the effective parameter $c(Z)$, which is the only adjustable quantity of this $3NF$. This parameter represents an overall correction factor for the far-off-the-energy-shell $2N$ t matrix entering this $3NF$ diagram. Ideally, $c(Z)$ should be one for a $2NF$ model able to provide a reliable extrapolation of the t matrix down to $Z\approx -160$ MeV. However, none of the existing $2N$ t matrices can guarantee such extrapolation since they are all constrained by experiments at the deuteron pole and at $Z\geqslant 0$. Furthermore, $c(Z)$ might also correct for possible model dependencies on how this imperfect cancellation manifest itself, as already observed at the beginning of this communication. On general grounds, one expects that with increasing energy in nd scattering, the factor $c(Z)$ should drift towards one, since the off-shell $2N$ t matrix in V^{3N} approaches gradually the energy region with experimental constrains.

To calculate the nd scattering observables below threshold, we have used the high-rank BBEST potential as two-body input [3,26]. With this separable representations of the Bonn B potential, it is possible to solve accurately the Faddeev-AGS scattering equations, and obtain results comparable (with errors less than 1%) to those obtained from a direct solution of the Faddeev equations using the original potentials as input. As an example, Fig. 1 shows the results we have obtained for A_y at 3 MeV with the BBEST potential and with the original Bonn interaction. There are three curves since the calculations have been performed using both the separable (one-dimensional) algorithm and the nonseparable (two-dimensional) method based on spline interpolation and Padé approximants, but the lines are practically indistinguishable. Similar tests have been made also before [27,28] on various occasions. Triangles represent the nd experimental data from Ref. [29].

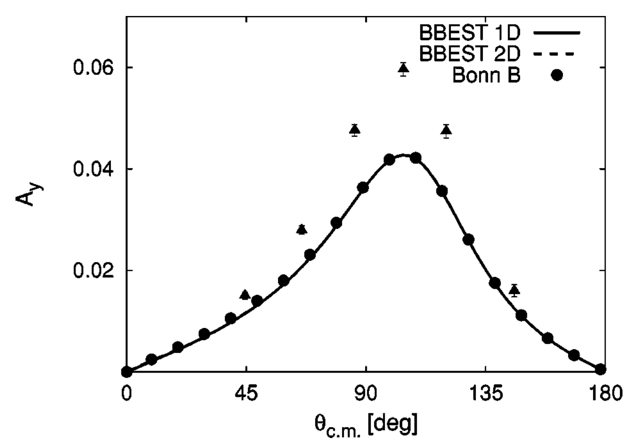

FIG. 1. The A_y puzzle, for nd scattering at 3 MeV (lab). Calculations with the Bonn B potential (dots), and with the high-rank BBEST potential (lines). With the BBEST potential there are two calculations, one obtained using the (nonseparable) two-dimensional approach, the other with the separable one-dimensional algorithm. The curves are not distinguishable. Data (triangles) are from Ref. [29].

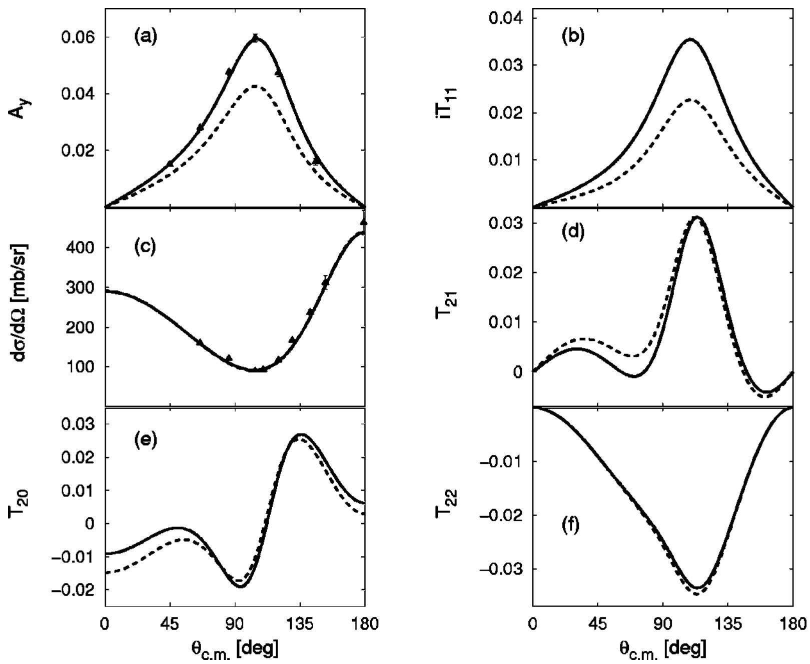

FIG. 2. 3*NF* effects on the differential cross section and analyzing powers, for *nd* scattering at 3 MeV. The dashed lines are results with no 3*NF*. Solid line exhibits 3*NF* effects as discussed in the text.

The three-nucleon forces can be incorporated in the scattering equation in a relatively simple way if the $2N$ input potential is of finite rank. We sketch the procedure for a rank-1 case: once the separable $2N$ t matrix is given, $t=|g_1\rangle\tau\langle g_1|$, and the (anti)symmetrized AGS equation have been rewritten in the Lovelace form,

$$X_{11}=Z_{11}+Z_{11}\tau X_{11}, \tag{4}$$

the above 3*NF* is incorporated into this one-dimensional integral equation with the driving term calculated as follows:

$$Z_{11}=\langle g_1|G_0P|g_1\rangle+\langle g_1|G_0V_1^{3N}G_0|g_1\rangle. \tag{5}$$

The first contribution is the standard $2N$ driving term, with G_0 and P being the free Green's function and the cyclic/anticyclic permutator, respectively, while the second takes into account the effects of the $3N$ force. This procedure is consistent with the formalism developed in Ref. [19] to include irreducible pionic effects in the $3N$ dynamics, and at the same time it represents an alternative to the standard procedure to include 3*NF* effects in the separable AGS equation [30].

The results obtained are shown in Fig. 2, for the differential cross section, and the analyzing powers A_y, iT_{11}, T_{20}, T_{21}, and T_{22}. The panel exhibits the results obtained with the BBEST potential, where a rank-4 expansion has been used for all states with $j\leqslant 2$, aside for the states ${}^1S_0$ (rank 5), ${}^3S_1$-${}^3D_1$ (rank 6), and ${}^3P_2$-${}^3F_2$ (rank 5).

The results without the 3*NF* are shown by the dashed lines, and are practically indistinguishable from the corresponding results for the Bonn B potential. The solid curve considers the additional contribution of the $3N$ force expressed in Eq. (1). We have used the subtraction method of Eq. (3), with t and v given by the Bonn B interaction while the parameter $c(Z)$ has been set to 0.73 for this energy. Results obtained with the PEST/Paris potential are similar.

In Fig. 3 we show how the puzzle evolves above the breakup threshold, up to 30 MeV. In the top panel we compare our theoretical calculations with the experimental data at 10 MeV [31], obtaining the correct reproduction for A_y with the 3*NF* when $c(Z)=0.735$. In the middle panel comparison is made with pd data at 18 MeV [32]. Comparison of nd calculations with pd data is somewhat questionable be-

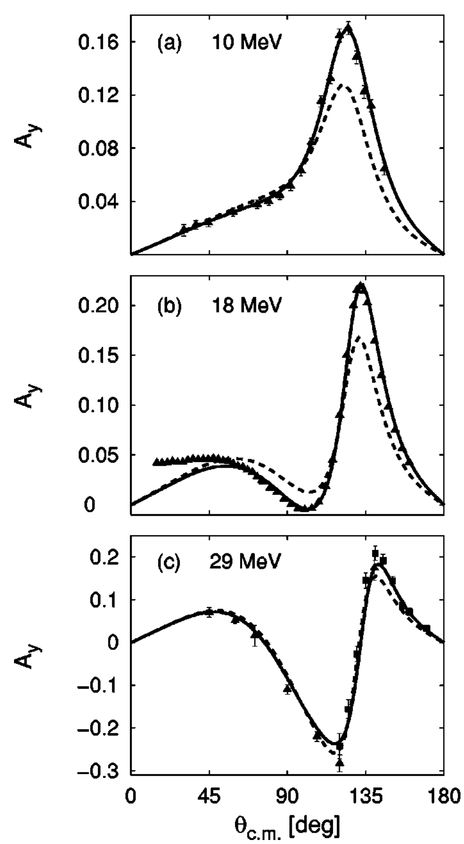

FIG. 3. Same as in Fig. 2 for A_y at 10, 18, and 29.6 MeV.

cause of the perturbations introduced by the Coulomb field. However, it is known that, aside for the angles in forward direction, one of the main effects of the Coulomb field in A_y can be approximately reproduced by comparing the charged data with calculations performed at energies lowered of about 0.5–0.7 MeV. For this reason the data are compared with calculations at 17.3 MeV. (However, care must be exercised in interpreting this fact as a Coulomb slow-down effect [33].) At this energy we obtain the reproduction of the observable with $c(Z)=0.773$. The botton panel shows the results obtained at 29.6 MeV, compared with the corresponding data at the same energy [34] (triangles), and with pd data at 30.2 MeV [35] (squares). The solid line has been obtained with $c(Z)=0.81$. It is evident that as the energy increases, the $c(Z)$ parameter shifts slowly towards one, as expected.

We checked also how the triton binding energy is affected, and found that here the effects are relatively small. With the highest possible rank, i.e., with a rank-5 representation in all states with $j\leqslant 2$, except the coupled states ${}^3S_1$-${}^3D_1$ (rank 6), and ${}^3P_2$-${}^3F_2$ (rank 7), the BBEST+3*NF* result is -8.137 while the corresponding 2*NF* result is -8.090 (MeV).

We observe that some unavoidable approximations entered in this study. For instance, in obtaining Eq. (1) we have ignored nucleonic recoil effects and have divided the subtracted t matrix by the pion mass, instead of ω_π, to get simpler expressions in partial waves. In addition, there might be a possible 3*NF* contribution of shorter range for the exchange of a ρ meson; this term would then counteract the one-pion-exchange 3*NF*, at least in the tensor part. Other uncertainties are related to the perturbative treatment of the pion dynamics in the AGS equation [19]. Finally, uncertainties about the fully off-the-energy-shell extrapolation of the *NN* t matrix entering in this 3*NF* forced us to introduce the effective parameter $c(Z)$.

As discussed in Refs. [17,19], the 3*NF* contribution analyzed in this study is just one class of irreducible diagrams generated by the pion dynamics, and in a more complete analysis also other classes of 3*NF* diagrams should be taken into account. Forces of the type TM [9], Brasil [10], Urbana [11], belong to another class and they complement the 3*NF* whose effects have been calculated here separately. Some of these 3*NF*'s give small corrections to A_y, but not always in the right direction [8,16]; their relevance, however, appears to be greater elsewhere (e.g., the binding energy of the triton). There are also additional 3*NF* terms of shorter range that might contribute, and in particular those obtained from χPT [16] appear very promising in providing an additional correction to A_y, possibly in the right direction.

To summarize and conclude: we have evaluated here for the first time a new ''pionic'' effect in the 3N system. The effect is a natural consequence of a recently developed theory [18,19] for the combined π-3N dynamics in the 3N system, and has been recast into a 3*NF* term of new structure by the authors [17]. The underlying 3*NF* diagram complements the extensively discussed 2π-3*NF* diagrams, and this complementarity shows up in the way this force affects the 3N observables: while the 2π-3*NF* terms have a large contribution on the 3N binding energy and little effects (in the considered energy range) on the vector analyzing powers, we have shown here that this new 3*NF* term greatly modifies in particular these two spin observables, and has the potential to provide in full the solution of the A_y puzzle. Conversely, we checked also that the same force produces smaller changes for the triton binding energy. Since both effects are clearly needed for describing the low-energy behavior of the 3N system, it will be important to investigate at this point what will be the effect of the *combined* treatment of these two forces.

Note added in proof. After this work was completed, we became aware of a recent paper by Schwamb and Arenhövel [36] that emphasizes the role of pionic retardations in calculating *NN* scattering processes above pion threshold. The relevant diagrams discussed in that paper (see Fig. 9), although applied to a simpler physical system and within a wider energy range, are clearly related to the dynamical approach we used in Ref. [17] to generate the OPE 3*NF* contribution that has been analyzed herein.

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