Isospin breaking in neutron β decay and SU(3) violation in semileptonic hyperon decays

N. Kaiser
Physik Department T39, Technische Universität München, D-85747 Garching, Germany
(Received 22 February 2001; published 2 July 2001)

Present precision measurements of the neutron lifetime lead to a CKM matrix element $|V_{ud}|$, which is 3 standard deviations off the value inferred from heavy quark decays, etc. We investigate the possibility whether isospin-breaking effects in the neutron-to-proton vector current transition matrix element $\langle p|V_0^+|n\rangle = 1+\delta g_V$ could eventually close this gap. For that we calculate in chiral perturbation theory the effect of pion and kaon loops on the matrix element $\langle p|V_0^+|n\rangle$ taking into account the mass differences of the charged and neutral mesons. We find a negligibly small isospin-breaking effect of $\delta g_V \simeq -4\times 10^{-5}$. The crucial quantity in the analysis of neutron β-decay precision measurements is thus the radiative correction term Δ_R. Furthermore, we calculate in heavy baryon chiral perturbation theory the SU(3) breaking effects on the vector transition charges of weak semileptonic hyperon decays. We find for these quantities channel-dependent relative deviations from the SU(3) limit which range from -10% to $+1\%$.

DOI: 10.1103/PhysRevC.64.028201 PACS number(s): 12.15.$-$y, 12.20.Ds, 13.40.Dk, 13.40.Ks

In the framework of the electroweak standard model the neutron β decay $n\to pe^-\bar{\nu}_e$ is described by only two parameters. These are the quark-mixing (CKM) matrix element V_{ud} and the ratio of the axial-vector and vector coupling constants g_A/g_V. The (unitary) 3×3 CKM matrix appears already at the fundamental level of the weak interaction of quarks and it expresses the fact that weak eigenstates and mass eigenstates of quarks are not identical. The ratio $g_A/g_V \neq 1$ on the other hand reflects nontrivial nucleon structure which has its origin in the strong interaction (i.e., QCD).

Experimentally, both parameters can be determined from the observables in polarized neutron β decay. The count rates of electrons with momentum parallel or antiparallel to the neutron spin define the (experimental) asymmetry of the electron spectrum $A=(N^\uparrow - N^\downarrow)/(N^\uparrow + N^\downarrow)$. In a recent precision measurement at ILL using the PERKEO II spectrometer the value $A=-0.1189\pm 0.0008$ [1] has been obtained for the electron asymmetry. Via the theoretical relation $A=2r(1-r)/(1+3r^2)$, $r=g_A/g_V$ the ratio of the axial-vector and vector coupling constants has been deduced as $g_A/g_V=1.2740\pm 0.0021$ [1]. The inverse neutron lifetime τ_n^{-1}, on the other hand, is proportional to $|V_{ud}|^2(g_V^2+3g_A^2)$ and therefore gives complementary information. The proportionality factor is the product of the squared Fermi-coupling constant G_F^2 (known from muon-decay $\mu^-\to e^-\bar{\nu}_e\nu_\mu$), a three-body phase space integral depending only on the neutron-proton mass difference $M_n - M_p$ and the electron mass m_e, and a radiative correction term $1+\Delta_R$ to be discussed later. From the world average of the neutron lifetime $\tau_n=(885.8\pm 0.9)$ sec (see Table 3 in Ref. [2]) the value of the CKM matrix element $|V_{ud}|=0.9713\pm 0.0014$ [2] has been obtained using the precise empirical ratio g_A/g_V and the conserved vector current (CVC) hypothesis which implies $g_V=1$. This value of $|V_{ud}|$ as extracted from neutron beta decay alone, is marginally consistent with the one coming from an analysis of superallowed $0^+\to 0^+$ Fermi transitions in nuclei which gives $|V_{ud}|=0.9740\pm 0.0010$ [3,4] (see also Table 4 in Ref. [2]).

According to the unitarity of the CKM quark-mixing matrix in the standard model $|V_{ud}|$ is equal to $\sqrt{1-|V_{us}|^2-|V_{ub}|^2}$. With $|V_{us}|=0.2196\pm 0.0023$ from the analysis of K_{e3} decays [4] and the information from B decays $|V_{ub}|/|V_{cb}|=0.08\pm 0.02$ and $|V_{cb}|=0.0395\pm 0.0017$ [2,4], one obtains this way $|V_{ud}|=\sqrt{1-|V_{us}|^2-|V_{ub}|^2} =0.9756\pm 0.0004$ [2]. The previously mentioned result for $|V_{ud}|$ derived from neutron beta decay differs from the one inferred from unitarity by about 3 standard deviations. Even though the relative deviation of both $|V_{ud}|$ values is small (about 4 permille) it is taken seriously and considered as a possible hint at physics beyond the standard model. In fact Ref. [2] has deduced from this deviation bounds on the masses, mass ratio, and mixing angle of new weak gauge bosons coupling to right-handed currents.

However, it should be kept in mind that isospin symmetry is not a perfect symmetry of the standard model and a small 4 permille deviation of the vector coupling constant from unity, $g_V=0.996$, would immediately resolve the above mentioned discrepancy. It is the purpose of the present paper to investigate this possibility. The result of our (exploratory) calculation is that isospin-breaking effects in g_V are typically a factor 100 smaller. Therefore with respect to the accuracy of present and upcoming neutron β-decay experiments one can safely use the CVC (or isospin symmetry) relation $g_V=1$.

Let us first define the V-A transition current matrix element relevant for neutron beta decay

$$\langle p|V_\mu^+ - A_\mu^+|n\rangle = \bar{u}_p\gamma_\mu(g_V - g_A\gamma_5)u_n = (g_V, g_A\vec{\sigma}). \quad (1)$$

Here, $V_\mu^+ = \bar{u}\gamma_\mu d$ and $A_\mu^+ = \bar{u}\gamma_\mu\gamma_5 d$ are the charge-raising vector and axial-vector currents expressed in terms of the up- and down-quark fields. $u_{p,n}$ denote Dirac-spinors for a proton and a neutron (at rest) and $\vec{\sigma}$ is the usual Pauli spin-vector. In the right-hand side of Eq. (1) one has already neglected the small four-momentum transfer between the neutron and the proton as well as the related nucleon form factor effects. The latter are at most of the size $(E_{\max} r_N)^2/3 \simeq 1\times 10^{-5}$ with $E_{\max}=M_n-M_p=1.293$ MeV the maximal energy transfer and $r_N \simeq 0.9$ fm a typical nucleon (electromagnetic) root mean square radius. Of the same size is the correction from the weak magnetism $(\mu_p-\mu_n)(E_{\max}/M_p)^2$ with $\mu_p=2.793$ and $\mu_n=-1.913$ [4] the proton and neutron magnetic moments.

The standard model has two sources of isospin symmetry violation: the mass difference of the up and down quarks and electromagnetic corrections. Prominent manifestations thereof in the hadron spectrum are the mass differences of the charged and neutral pions and kaons $m_{\pi^+}-m_{\pi^0}=4.6$ MeV and $m_{K^0}-m_{K^+}=4.0$ MeV [4]. The π^+-π^0 mass-splitting (a sizable 3.4% effect) is almost entirely due to the electromagnetic interaction. A calculation of the corresponding one-photon loop self-energy diagram employing a simple vector-meson-dominance expression $F_\pi(t)=m_\rho^2/(m_\rho^2-t)$ for the pion charge form factor $F_\pi(t)$ gives for the π^+-π^0 mass splitting

$$m_{\pi^+}-m_{\pi^0}=\frac{\alpha}{2\pi}m_\rho[x+2x^3\ln 2x-(2x^2+1)\sqrt{x^2-1}\times\ln(x+\sqrt{x^2-1})]=4.3\ \text{MeV}, \tag{2}$$

with $\alpha=1/137.036$ the fine structure constant, $m_\rho=769$ MeV the (neutral) ρ-meson mass, and $x=m_\rho/2m_{\pi^0}=2.85$. The K^0-K^+ mass splitting (a 0.8% effect) is of different composition. Electromagnetic effects contribute about -2.2 MeV and the remaining 6.2 MeV are attributed to the up- and down-quark mass difference. One of the main aspects of nucleon structure at low energies is the meson cloud surrounding the nucleon. In the weak $n\to p$ vector current transition neutral virtual mesons are converted into positively charged ones of slightly different mass and this induces some (small) deviation of g_V from unity.

The systematic method to quantify such an isospin-breaking effect is chiral perturbation theory (for a review see Ref. [5]). Observables are calculated with the help of an effective field theory formulated in terms of the Goldstone bosons (π,K,η) and the low-lying baryons. A systematic expansion in small external momenta and meson masses is possible. For the problem considered here this means that one has to compute the quantity δg_V defined by the matrix element $\langle p|V_0^+|n\rangle = g_V = 1+\delta g_V$ from the one-loop diagrams shown in Fig. 1. The relevant effective Lagrangians, Feynman rules, and loop functions can be found in Ref. [5]. It is instructive to present first results for δg_V which follow from the individual pion one-loop diagrams (a)–(e) shown in Fig. 1. One finds

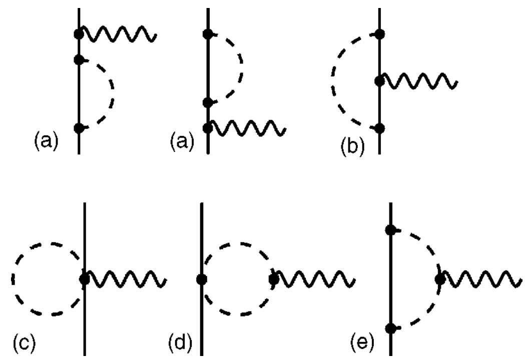

FIG. 1. One-loop diagrams for δg_V. Dashed lines represent pions, kaons, or etas. The wiggly line denotes the external charged vector field, i.e., the W^+ boson.

$$\delta g_V^{(a)}=-\frac{g_A^2 m_{\pi^+}^2}{(4\pi f_\pi)^2}\left(3\ln\frac{m_{\pi^+}}{\lambda}+1\right)-\frac{g_A^2 m_{\pi^0}^2}{2(4\pi f_\pi)^2}\times\left(3\ln\frac{m_{\pi^0}}{\lambda}+1\right), \tag{3}$$

$$\delta g_V^{(b)}=-\frac{g_A^2 m_{\pi^0}^2}{2(4\pi f_\pi)^2}\left(3\ln\frac{m_{\pi^0}}{\lambda}+1\right), \tag{4}$$

$$\delta g_V^{(c)}=-\frac{m_{\pi^+}^2}{(4\pi f_\pi)^2}\ln\frac{m_{\pi^+}}{\lambda}-\frac{m_{\pi^0}^2}{(4\pi f_\pi)^2}\ln\frac{m_{\pi^0}}{\lambda}, \tag{5}$$

$$\delta g_V^{(d)}=\frac{1}{(4\pi f_\pi)^2(m_{\pi^+}^2-m_{\pi^0}^2)}\left[m_{\pi^+}^4\left(\ln\frac{m_{\pi^+}}{\lambda}-\frac{1}{4}\right)-m_{\pi^0}^4\left(\ln\frac{m_{\pi^0}}{\lambda}-\frac{1}{4}\right)\right], \tag{6}$$

$$\delta g_V^{(e)}=\frac{g_A^2}{(4\pi f_\pi)^2(m_{\pi^+}^2-m_{\pi^0}^2)}\left[m_{\pi^+}^4\left(3\ln\frac{m_{\pi^+}}{\lambda}+\frac{1}{4}\right)-m_{\pi^0}^4\left(3\ln\frac{m_{\pi^0}}{\lambda}+\frac{1}{4}\right)\right]. \tag{7}$$

Here, we have used dimensional regularization and minimal subtraction (see Appendix B in Ref. [5]) to evaluate divergent loop integrals. The renormalization scale $\lambda\sim 1$ GeV does not play a role for the final result since the total sum of the five terms in Eqs. (3)–(7) is in fact λ independent. $f_\pi=92.4$ MeV is the weak pion decay constant. We use here $g_A=1.3$ which corresponds via the Goldberger-Treiman relation $g_{\pi N}=g_A M_p/f_\pi$ to a strong πNN coupling constant of $g_{\pi N}=13.2$ which is consistent with present empirical determinations [6]. Summing up the five terms given in Eqs. (3)–(7) and expanding in the pion mass-splitting one gets

$$\delta g_V^{(\pi\text{-loop})} = \frac{3g_A^2+1}{(4\pi f_\pi)^2}\left\{ \frac{m_{\pi^+}^2 m_{\pi^0}^2}{m_{\pi^+}^2 - m_{\pi^0}^2}\ln\frac{m_{\pi^+}}{m_{\pi^0}} - \frac{1}{4}(m_{\pi^+}^2 + m_{\pi^0}^2)\right\}$$
$$\simeq -\left(g_A^2+\frac{1}{3}\right)\left(\frac{m_{\pi^+}-m_{\pi^0}}{4\pi f_\pi}\right)^2$$
$$= -3.2\times 10^{-5}. \tag{8}$$

Omitted terms of the order $(m_{\pi^+}-m_{\pi^0})^4$ are numerically irrelevant. Note that Eq. (8) has been derived with isospin-symmetric interaction vertices and isospin-violating pion propagators. According to QCD sum rule calculations [7] the charged and neutral pion-nucleon coupling constants squared differ by at most 0.5%. This is also confirmed by the phenomenology of charge independence breaking in the singlet NN-scattering lengths [8]. The expression in Eq. (8) therefore represents indeed the dominant isospin-breaking effect of the nucleon's pion cloud. The same set of one-loop diagrams (see Fig. 1) with pions replaced by kaons gives a further contribution of the form

$$\delta g_V^{(K\text{-loop})} = \frac{6DF-D^2+3F^2+1}{(8\pi f_\pi)^2}\left\{ \frac{2m_{K^0}^2 m_{K^+}^2}{m_{K^0}^2 - m_{K^+}^2}\ln\frac{m_{K^0}}{m_{K^+}} - \frac{1}{2}(m_{K^0}^2 + m_{K^+}^2)\right\}$$
$$\simeq -\frac{1}{6}(6DF-D^2+3F^2+1)\left(\frac{m_{K^0}-m_{K^+}}{4\pi f_\pi}\right)^2$$
$$= -0.7\times 10^{-5}. \tag{9}$$

Here, $D\simeq 0.8$ and $F\simeq 0.5$ denote the SU(3) axial vector coupling constants with $g_A = D+F$. As expected the kaon cloud effect is considerably smaller than the pion cloud effect. Further pion-loop diagrams (a), (b), and (e) with intermediate spin-isospin-3/2 $\Delta(1232)$ excitation give rise to a (relatively small) contribution of the form

$$\delta g_V^{(\pi\Delta\text{-loop})} = \frac{g_A^2}{\zeta^2-1}\left[\frac{\zeta}{\sqrt{\zeta^2-1}}\ln(\zeta+\sqrt{\zeta^2-1})-1\right] \times\left(\frac{m_{\pi^+}-m_{\pi^0}}{4\pi f_\pi}\right)^2$$
$$= +0.4\times 10^{-5}, \tag{10}$$

where $\zeta = \Delta/m_{\pi^0} = 2.17$. Here, $\Delta = 293$ MeV denotes the delta-nucleon mass-splitting and we have used the empirically well-satisfied coupling constant relation $g_{\pi N\Delta} = 3g_{\pi N}/\sqrt{2}$. The fact that $\delta g_V^{(\pi,K,\pi\Delta\text{-loop})}$ scale with the square of the meson-mass splittings is consistent with the theorem of Behrends and Sirlin [9]. Their estimate $\delta g_V \simeq 10^{-6}$ appears, however, too small in comparison to the result of our explicit calculation.

In summary, we find a negligibly small isospin-breaking effect of $\delta g_V \simeq -4\times 10^{-5}$, which does not help to resolve the presently existing discrepancy between various determinations of $|V_{ud}|$. Stated differently, we can conclude that the CVC relation $g_V^2 = 1$ holds with an accuracy of 10^{-4} or better. The crucial quantity in the analysis of neutron β-decay precision experiments is therefore the radiative correction term Δ_R calculated by Sirlin [3,10],

$$\Delta_R = \frac{\alpha}{2\pi}\left(3\ln\frac{M_Z}{M_p} + \ln\frac{M_Z}{M_A} + 2C_{\text{Born}} + \cdots\right)$$
$$= (2.46\pm 0.09)\times 10^{-2}. \tag{11}$$

Here, $M_Z = 91.19$ GeV is the Z^0 mass and M_A an arbitrary mass parameter introduced in the nucleon axial form factor in order to cope with infrared divergences. In practical applications M_A is allowed to vary in the range 0.4 GeV$<M_A<$1.6 GeV [3,11]. We intend to take a fresh look at radiative corrections in neutron β decay using the systematic framework of chiral perturbation theory with the aim of improving on the quantity Δ_R.

While isospin breaking effects in neutron beta decay turn out to be negligibly small, one expects much larger effects from SU(3) violation (i.e., the mass difference between the strange and the up and down quarks) in strangeness-changing semileptonic weak hyperon decays. Considering the matrix elements of the charged strangeness-changing vector current $\bar{u}\gamma_\mu s$ at momentum transfer zero, the Ademollo-Gatto theorem [12] asserts, however, that for these quantities SU(3) breaking effects start first at quadratic order in the quark mass difference $m_s - m_{u,d}$. In order to quantify these SU(3) breaking effects we calculate here in heavy baryon chiral perturbation theory the leading order contributions arising from (π,K,η)-loop diagrams (see Fig. 1). In such a calculation SU(3) symmetry breaking originates entirely from the different masses of the eight pseudoscalar Goldstone bosons (π,K,η). The quantities of interest are the matrix elements of the strangeness-changing $(s\to u)$ weak vector current density evaluated in baryon states which differ by one unit of strangeness

$$\langle B'|\bar{u}\gamma_0 s|B\rangle = g_V(B\to B')[1+\delta_V(B\to B')], \tag{12}$$

with $B\in\{\Lambda,\Sigma^0,\Sigma^-,\Xi^0,\Xi^-\}$ and $B'\in\{p,n,\Lambda,\Sigma^+,\Sigma^0\}$. The numbers $g_V(B\to B')$ correspond to the results in the exact SU(3) limit and they read

$$g_V(\Lambda\to p) = -\frac{\sqrt{6}}{2},\quad g_V(\Sigma^0\to p) = -\frac{\sqrt{2}}{2},$$
$$g_V(\Sigma^-\to n) = -1,$$
$$g_V(\Xi^0\to\Sigma^+) = 1,\quad g_V(\Xi^-\to\Lambda) = \frac{\sqrt{6}}{2},$$
$$g_V(\Xi^-\to\Sigma^0) = \frac{\sqrt{2}}{2}. \tag{13}$$

The quantities $\delta_V(B\to B')$ measure for a specific transition the relative deviation from SU(3) symmetry. Evaluation of the one-loop diagrams shown in Fig. 1 leads to the following expression for the SU(3) breaking effect:

$$\delta_V(B\to B') = \frac{1}{(8\pi f_\pi)^2}\Bigg\{[3+\alpha(B\to B')]\left[\frac{m_K^2 m_\pi^2}{m_K^2-m_\pi^2}\ln\frac{m_K}{m_\pi} - \frac{1}{4}(m_K^2+m_\pi^2)\right] + [3+\beta(B\to B')] \times\left[\frac{m_\eta^2 m_K^2}{m_\eta^2-m_K^2}\ln\frac{m_\eta}{m_K} - \frac{1}{4}(m_\eta^2+m_K^2)\right]\Bigg\}, \quad (14)$$

with the channel-dependent coefficients $\alpha(B\to B')$ and $\beta(B\to B')$ given by

$$\alpha(\Lambda\to p) = 9D^2+6DF+9F^2, \quad \beta(\Lambda\to p) = (D+3F)^2, \quad (15)$$

$$\alpha(\Sigma^0\to p) = \alpha(\Sigma^-\to n) = D^2-18DF+9F^2,$$
$$\beta(\Sigma^0\to p) = \beta(\Sigma^-\to n) = 9(D-F)^2, \quad (16)$$

$$\alpha(\Xi^-\to\Lambda) = 9D^2-6DF+9F^2,$$
$$\beta(\Xi^-\to\Lambda) = (D-3F)^2, \quad (17)$$

$$\alpha(\Xi^0\to\Sigma^+) = \alpha(\Xi^-\to\Sigma^0) = D^2+18DF+9F^2,$$
$$\beta(\Xi^0\to\Sigma^+) = \beta(\Xi^-\to\Sigma^0) = 9(D+F)^2. \quad (18)$$

The channel-independent terms in Eq. (14) proportional to the coefficient 3 stem from the (π,K,η)-loop diagrams (c) and (d) in Fig. 1 with no internal heavy baryon propagator. For the numerical evaluation of $\delta_V(B\to B')$ we use for the axial-vector coupling constants $D=0.8$, $F=0.5$, and for the meson masses $m_\pi=139.57$ MeV, $m_K=493.68$ MeV, $m_\eta=\sqrt{(4m_K^2-m_\pi^2)/3}=564.33$ MeV (the value from the GMO relation which deviates only by 3.1% from the physical η mass). This input gives numerically

$$\delta_V(\Lambda\to p)\simeq -10\%, \quad \delta_V(\Sigma^0\to p) = \delta_V(\Sigma^-\to n)\simeq +1\%, \quad (19)$$

$$\delta_V(\Xi^-\to\Lambda)\simeq -6\%,$$
$$\delta_V(\Xi^0\to\Sigma^+) = \delta_V(\Xi^-\to\Sigma^0)\simeq -10\%. \quad (20)$$

One notices that the deviations from SU(3) symmetry are sizable and strongly channel dependent (for results of a relativistic calculation, see Ref. [13]). Clearly, these SU(3) breaking effects in the weak vector transition matrix elements should be included in the analysis of the strangeness-changing semileptonic hyperon decays. On the theoretical side one should attempt to further improve the predictions for $\delta_V(B\to B')$ by performing next-to-leading (or even higher) order calculations in baryon chiral perturbation theory.

Let us finally make a comparison to SU(3) breaking effects in the axial vector coupling constants D and F. Chiral logarithmic corrections of the form $(m_K/4\pi f_\pi)^2\ln(m_K/\lambda)$ to D and F have been calculated in Ref. [14]. With inclusion of only octet baryons in intermediate states the tree-level couplings D and F had to be reduced by about 30% and the further inclusion of decuplet intermediate states increased these couplings again somewhat. The chiral corrections to the vector couplings $\delta_V(B\to B')$ considered here behave differently. First, the one-loop results are finite (i.e., independent of the renormalization scale λ), free of possible counterterm contributions and secondly the relative deviations from the SU(3) limit do not exceed 10%. Of course only complete higher order calculations allow to judge the accuracy of the present one-loop results. In order test the SU(3) breaking effects for the vector couplings $\delta_V(B\to B')$ a reanalysis of strangeness-changing semileptonic hyperon decays [15] would be most useful.

[1] J. Reich *et al.*, Nucl. Instrum. Methods Phys. Res. A **440**, 535 (2000).
[2] H. Abele, Nucl. Instrum. Methods Phys. Res. A **440**, 499 (2000).
[3] I. S. Towner and J. C. Hardy, in *Symmetries and Fundamental Interactions in Nuclei*, edited by E. M. Henley and W. C. Haxton (World-Scientific, Singapore, 1995).
[4] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[5] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995).
[6] M. M. Pavan *et al.*, Phys. Scr. **T87**, 65 (2000).
[7] W-Y.P. Hwang *et al.*, Phys. Rev. C **57**, 61 (1998).
[8] R. Machleidt, Phys. Scr. **T87**, 47 (2000).
[9] R. E. Behrends and A. Sirlin, Phys. Rev. Lett. **4**, 186 (1960).
[10] A. Sirlin, Rev. Mod. Phys. **50**, 573 (1978).
[11] A. Garcia, J. L. Garcia-Luna, and G. Lopez Castro, Phys. Lett. B **500**, 75 (2001).
[12] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[13] A. Krause, Helv. Phys. Acta **63**, 3 (1990).
[14] E. Jenkins and A. V. Manohar, Phys. Lett. B **259**, 353 (1991).
[15] M. Bourquin *et al.*, Z. Phys. C **21**, 1 (1983); **21**, 17 (1983); **21**, 27 (1983).