Stochastic one-body transport and coupling to mean-field fluctuations

Sakir Ayik[1] and Yasuhisa Abe[2]
[1]*Tennessee Technological University, Cookeville, Tennessee 38505*
[2]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan*
(Received 17 October 2000; published 3 July 2001)

A stochastic transport description for the single-particle density matrix is briefly discussed. It is shown that the stochastic description contains, in addition to incoherent binary collisions, a coherent damping mechanism due to coupling between mean-field fluctuations and single-particle motion, and an expression for the coherent collision term is derived. In the limit of small fluctuations around equilibrium, the collective and single-particle self-energies due to the coherent mechanism are deduced.

DOI: 10.1103/PhysRevC.64.024609 PACS number(s): 24.30.Cz, 25.70.Ef, 25.70.Lm

I. INTRODUCTION

Dynamical descriptions based on reduced one-body transport theories, both semiclassical and quantal forms, have been very useful for understanding many aspects of nuclear structure and dynamics [1–3]. The simplest form of the one-body description is provided by the time-dependent Hartree-Fock (TDHF) theory, in which dynamics is treated in the mean-field approximation by neglecting coupling to two-body correlations [4]. Over last two decades, much work has been done to improve the TDHF theory beyond the mean field approximation [5–10]. In, so called extended TDHF theory, two-body correlations are incorporated into the equation of motion by truncating the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy at the second level within the Born approximation. The resultant collision term describes the coupling of the single-particle motion to the incoherent $2p$-$2h$ excitations. Such an incoherent damping mechanism is very important at relatively high energy heavy-ion collisions to convert the collective energy of the relative motion into incoherent excitations and thermalize the system. However, at low energies including giant resonance excitations, the incoherent damping mechanism is not effective due to long nucleon mean-free path. Therefore, for a proper description of the damping mechanism at low energies, the coherence between the p-h pairs should be taken into account [11,12]. For this reason, it is highly desirable to improve the TDHF theory by incorporating a coherent collision term into the equation of motion. One possibility for accomplishing this goal is provided by the time-dependent density matrix formalism, in which a truncation of the BBGKY hierarchy is carried out by keeping all the second order terms in the equation for two-body correlations [13,14]. The resulting coupled equations for the one-body density matrix and for the two-body correlations take into account for the coherence effects in particle-particle, hole-hole and particle-hole channels. Here, we follow a different approach, in which the effects of correlations are incorporated into the equation of motion by a stochastic mechanism according to the generalized Langevin description of Mori.

According to the generalized Langevin description of relevant variables developed by Mori [15], the correlations due to coupling with the degrees of freedom, which are not considered explicitly, have two different but intimately related effects: (i) dissipation of energy associated with the relevant variables leading to thermalization of the entire system, which is described by friction or collision terms in the equation of motion, and (ii) dynamical fluctuations of the relevant variables, which are described by the random force term originating from the initial correlations. Consequently, temporal evolution of the reduced one-body density matrix should be governed by a stochastic transport equation, analogous to the generalized Langevin equations for the reduced dynamical variables [15–17]. The associated "random force" in the equation of motion should originate from the initial correlations, with statistical properties specified in accordance with the fluctuation-dissipation relation. Such a stochastic transport description has been developed for the phase-space density in the semiclassical framework, which is usually referred to as the Boltzmann-Langevin approach [18–21,23]. It is also possible to develop a stochastic transport theory for the one-body density matrix in a quantal framework.

The stochastic transport theories, both quantal and semiclassical forms, provide a one-body framework to describe dynamics of density fluctuations in a manner that is consistent with the dissipation-fluctuation relation of nonequilibrium statistical mechanics. Furthermore, the coherent damping mechanism is naturally included in the stochastic transport description. The density fluctuations excited by the stochastic source in the equation of motion are propagated by the mean field, that gives rise to nonlinear fluctuations of the mean field with random amplitudes on the top of its average evolution. The coherent damping mechanism arises from the coupling of the single-particle motion with the mean-field fluctuations, and it provides an efficient mechanism for dissipation and the equilibration of the system, in particular at low energies. In Sec. II, we briefly describe a stochastic one-body transport model in the quantal framework. In Sec. III, we consider the ensemble average evolution of the single-particle density matrix, and show that the coupling to mean-field fluctuations appears as a coherent collision term. In Sec. IV, we investigate small amplitude vibrations around equilibrium and derive expressions for damping widths of the collective and single-particle excitations due to coherent damping mechanism. Finally in Sec. V, summary and conclusions are given. For a brief account of the main results, we refer the reader to [22].

II. STOCHASTIC TRANSPORT EQUATION

Temporal evolution of the single-particle density matrix $\hat{\rho}(t)$ is determined by [5,6,9]

$$i\frac{\partial}{\partial t}\hat{\rho}(t)-[h(\hat{\rho}),\hat{\rho}(t)]=\mathrm{Tr}_2[v,\hat{\sigma}_{12}(t)], \tag{1}$$

where $h(\hat{\rho})$ is the effective mean-field Hamiltonian and v denotes the effective residual interactions. The quantity on the right-hand-side is usually referred to as the collision term, which is determined by the correlated part of the two-particle density matrix,

$$\hat{\sigma}_{12}(t)=\hat{\rho}_{12}(t)-\widetilde{\hat{\rho}_1(t)\hat{\rho}_2}(t), \tag{2}$$

where $\widetilde{\hat{\rho}_1\hat{\rho}_2}$ represents the antisymmetrized product of the single-particle density matrices. The two-body correlations $\hat{\sigma}_{12}(t)$ are determined by the second equation of the BBGKY hierarchy. At sufficiently low energies, the nucleon mean-free path is long, and consequently the BBGKY hierarchy can be truncated at the second level. Retaining only the lowest order terms in the residual interactions, the correlated part of the two-particle density matrix evolves according to

$$i\frac{\partial}{\partial t}\hat{\sigma}_{12}(t)-[h(\hat{\rho}),\hat{\sigma}_{12}(t)]=\hat{F}_{12}(t), \tag{3}$$

where the source term is

$$\hat{F}_{12}(t)=[1-\hat{\rho}_1(t)][1-\hat{\rho}_2(t)]v\widetilde{\hat{\rho}_1(t)\hat{\rho}_2}(t) - \widetilde{\hat{\rho}_1(t)\hat{\rho}_2}(t)v[1-\hat{\rho}_1(t)][1-\hat{\rho}_2(t)]. \tag{4}$$

Solving this equation formally, we can express the development of correlations over a time interval from an initial time t_0 to time t as

$$\hat{\sigma}_{12}(t)=-i\int_{t_0}^{t}dt'\,\hat{G}(t,t')\hat{F}_{12}(t')\hat{G}^{\dagger}(t,t')+\delta\sigma_{12}(t), \tag{5}$$

where

$$\hat{G}(t,t')=T\exp\left[-i\int_{t'}^{t}ds\,h[\hat{\rho}(s)]\right] \tag{6}$$

denotes the mean-field propagator. In this expression, the first term represents the correlations developed by the residual interactions during the time interval, and the second term describes the propagation of the initial correlations $\sigma_{12}(t_0)$ from the initial time t_0 to time t,

$$\delta\sigma_{12}(t)=\hat{G}(t,t_0)\sigma_{12}(t_0)\hat{G}^{\dagger}(t,t_0). \tag{7}$$

The time interval cannot be taken arbitrary large, but should be taken sufficiently small to justify the neglect of the explicit coupling to three-body correlations in Eq. (3) during the time interval. However, the dominant effect of the correlations is still accounted for by the initial correlation term $\sigma_{12}(t_0)$, which, in principle, contains all order correlations that are accumulated up to the time t_0. If we consider an ensemble of identical systems that are prepared with slightly different initial conditions at the remote past, the exact two-body correlations $\sigma_{12}(t_0)$ accumulated until t_0 exhibit nearly random fluctuations. In the extended TDHF theory, the average evolution over such an ensemble is considered, and the ensemble average of the initial correlation term is assumed to vanish, $\overline{\sigma_{12}(t_0)}=0$ [24,25]. This assumption in the semiclassical context is known as the "molecular chaos assumption," and it corresponds to the factorization of the two-particle phase-space density before each binary collisions [26]. In the stochastic transport description, the initial correlation term is retained, but it is treated as a random quantity specified by a Gaussian distribution: each matrix elements has a Gaussian distribution determined with zero mean and a second moment [19]. The second moment of the initial correlation term $\delta\sigma_{12}(t)$ can be determined by following a similar treatment presented in [19]. It is convenient to introduce a shorthand notation for the fluctuating part of the two-body density matrix, suppressing the indices

$$\delta\sigma_{12}(t)=\langle\Phi|A(t)|\Phi\rangle-\overline{\langle\Phi|A(t)|\Phi\rangle_0}=\langle\Phi|\delta A(t)|\Phi\rangle. \tag{8}$$

In this expression, $A(t)=a_i^{\dagger}(t)a_j^{\dagger}(t)a_k(t)a_l(t)$ is the product of the single-particle creation and annihilation operators in the Heisenberg representation, $|\Phi\rangle$ represents a member of the many-body states in the initial ensemble, and $\overline{\langle\Phi|A(t)|\Phi\rangle_0}$ denotes the averaged-uncorrelated part defined by the second term in Eq. (2). In order to calculate the ensemble average $\overline{\delta\sigma_{12}(t)\,\delta\sigma'_{12}(t)}$, first we introduce a closure approximation by making the following replacement:

$$\overline{\delta\sigma_{12}(t)\,\delta\sigma'_{12}(t)}=\overline{\langle\Phi|\delta A(t)|\Phi\rangle\langle\Phi|\delta A^{\dagger}(t)|\Phi\rangle} \approx\overline{\langle\Phi|\delta A(t)\,\delta A^{\dagger}(t)|\Phi\rangle}. \tag{9}$$

The correlation function given by Eq. (14) below is in the second order in the effective interactions. Consequently, to be consistent with the expression of the collision term in Eq. (12), we retain only the uncorrelated contribution in the ensemble average of $\overline{\langle\Phi|\delta A(t)\,\delta A^{\dagger}(t)|\Phi\rangle}$. It is convenient to employ the natural single-particle representation that diagonalizes the average density matrix, $\rho(t)=\Sigma|\phi_i(t)\rangle n_i(t)\langle\phi_i(t)|$, where $n_i(t)$ denotes the occupation numbers. As a result, in the natural representation, the symmetrized second moment of the initial correlation term is determined according to

$$\overline{\langle ij|\delta\sigma_{12}(t)|kl\rangle_t\langle k'l'|\delta\sigma_{12}(t)|i'j'\rangle_t} =\frac{1}{2}S_{ij;i'j'}S_{kl;k'l'}N^{+}_{ijkl}(t), \tag{10}$$

where $S_{ij;i'j'}=\delta_{ii'}\delta_{jj'}-\delta_{ij'}\delta_{ji'}$, $S_{kl;k'l'}=\delta_{kk'}\delta_{ll'}-\delta_{kl'}\delta_{lk'}$, and

$$N^{+}_{ijkl}(t)=[1-n_i(t)][1-n_j(t)]n_k(t)n_l(t) +[1-n_k(t)][1-n_l(t)]n_i(t)n_j(t). \quad (11)$$

In the initial correlation term, the initial time t_0 is not relevant, at any time $\delta\sigma_{12}(t)$ is a Gaussian random quantity with the second moment specified by Eq. (10).

Substituting the expression (5) for the two-particle correlations into Eq. (1) yields a transport equation for the single-particle density matrix

$$i\frac{\partial}{\partial t}\hat{\rho}(t)-[h(\hat{\rho}),\hat{\rho}(t)] =-i\int_{t_0}^{t}dt'\,\mathrm{Tr}_2[v,\hat{G}(t,t')\hat{F}_{12}(t')\hat{G}^{\dagger}(t,t')]+\delta K(t). \quad (12)$$

Here, the first term on the left-hand side is a binary collision term and the second term arises from the initial correlations

$$\delta K(t)=\mathrm{Tr}_2[v,\delta\sigma_{12}(t)] \quad (13)$$

and it describes the stochastic part of the collisions. In analogy with the generalized Langevin description of the reduced dynamical variables, Eq. (12) is regarded as a stochastic transport equation for the fluctuating density matrix in which the stochastic part of the collision term $\delta K(t)$ acts as a random noise [15–17]. According to the stochastic properties of the initial correlations, the random noise also has a Gaussian distribution with zero mean and a second moment determined by the correlation function

$$C_{ij;kl}(t,t')=\overline{\langle i|\delta K(t)|j\rangle\langle k|\delta K(t')|l\rangle}. \quad (14)$$

The collision term essentially involves two different characteristic times: (i) the relaxation time τ_{rel} of the occupation numbers of the natural states, which corresponds to the mean-free time in the semiclassical limit, and (ii) the correlation time τ_{cor} of the matrix elements of the residual interactions, which corresponds to the duration time of binary collisions in the semiclassical limit. Here, we consider the weak-coupling regime specified by $\tau_{cor}\ll\tau_{rel}$ [27], which is valid for a sufficiently dilute system when the binary collisions are not so frequent. In this case, the decay time of the collision kernel in Eq. (12) is determined by the correlation time, and the memory effects associated with the variation of the occupation numbers over this time maybe neglected. As a result, using $\hat{G}(t,t')\hat{\rho}(t')\hat{G}^{\dagger}(t,t')\approx\hat{\rho}(t)$, we can make the following substitution in the collision term:

$$\hat{G}(t,t')\hat{F}_{12}(t')\hat{G}^{\dagger}(t,t')=(1-\hat{\rho}_1)(1-\hat{\rho}_2)v(t,t')\widetilde{\hat{\rho}_1\hat{\rho}_2} -\widetilde{\hat{\rho}_1\hat{\rho}_2}v(t,t')(1-\hat{\rho}_1)(1-\hat{\rho}_2), \quad (15)$$

where $v(t,t')=\hat{G}(t,t')v\hat{G}^{\dagger}(t,t')$, and all density matrices are evaluated at time t. The decay time of the correlation function of the stochastic collision term Eq. (14) is also determined by the correlation time τ_{cor} of the residual interactions. Therefore, for time intervals that are short as compared to the relaxation time $|t-t'|\ll\tau_{rel}$ the stochastic collision term can be propagated by the mean field according to

$$\delta K(t')=\mathrm{Tr}_2[v,\hat{G}(t',t)\,\delta\sigma_{12}(t)\hat{G}^{\dagger}(t',t)]. \quad (16)$$

Then, using the expression (10) for the equal time variance of $\delta\sigma_{12}(t)$, we can easily calculate the correlation function $C_{ij;kl}(t,t')$ in terms of the matrix elements of the residual interactions and the combinations of the occupation factors. Since, the result is rather lengthy, we do not give any expression for the correlation function here, but illustrate the result by considering the projected noise on a collective variable $Q_\lambda(t)$, which maybe time dependent. The projected noise is given by

$$F_\lambda(t)=\mathrm{Tr}\,Q_\lambda(t)\,\delta K(t)=\mathrm{Tr}[Q_\lambda(t),v]\,\delta\sigma_{12}(t) \quad (17)$$

and

$$F^{*}_{\lambda}(t')=\mathrm{Tr}\,Q^{\dagger}_{\lambda}(t')\,\delta K^{\dagger}(t')=\mathrm{Tr}\,\hat{G}(t,t') \times[v,Q^{\dagger}_{\lambda}(t')]\hat{G}^{\dagger}(t,t')\,\delta\sigma_{12}(t), \quad (18)$$

where $\delta K^{\dagger}(t')$ is propagated according to the expression (16). Then, the correlation function of the projected noise becomes

$$\overline{F_\lambda(t)F^{*}_{\lambda}(t')}=\frac{1}{4}\sum\langle kl|[Q_\lambda(t),v]|ij\rangle_t \times\langle kl|[Q_\lambda(t'),v]|ij\rangle^{*}_{t'}\frac{1}{2}N^{+}_{ijkl}(t), \quad (19)$$

where the occupation factors maybe evaluated at time t since they do not change appreciably over the time interval in the weak-coupling regime. Here, the two-body matrix elements $\langle kl|[Q_\lambda(t),v]|ij\rangle_t$, and also in the rest of the paper, denote the antisymmetrized matrix elements.

III. PARTICLE-PHONON COUPLING

In the stochastic transport description, higher order correlations beyond the mean field are incorporated into the dynamical evolution in an approximate manner by a stochastic mechanism. Dynamical evolution is characterized by constructing an ensemble of solutions of the stochastic transport Eq. (12). In this manner, the theory provides a basis for describing the average evolution, as well as, dynamics of density fluctuations. Furthermore, the stochastic evolution involves, in addition to the incoherent damping mechanism due to $2p$-$2h$ excitations, a coherent mechanism arising from the coupling of the single-particle motion with randomly excited nonlinear mean-field fluctuations. When the amplitude of the fluctuations is small, this mechanism appears as a coupling between the single-particle motion and the time-dependent random-phase approximation (RPA) phonons around the mean trajectory. In order to illustrate the origin of this coupling, we consider the average evolution of the density matrix, $\rho(t)=\overline{\hat{\rho}(t)}$, taken over the ensemble generated by the stochastic transport Eq. (12). We calculate the ensemble average of Eq. (12) by expressing the mean-field and the density matrix as $h(\hat{\rho})=h(\rho)+\delta\hat{h}(t)$ and $\hat{\rho}(t)=\rho(t)+\delta\hat{\rho}(t)$, where $\delta\hat{h}(t)=(\partial h/\partial\rho)\delta\hat{\rho}(t)$ and $\delta\hat{\rho}(t)$ represent the fluctuating parts of the mean-field and the density matrix, respectively. Noting that, the ensemble average of the noise $\delta K(t)$ vanishes, the evolution of the average density matrix is governed by the transport equation

$$i\frac{\partial}{\partial t}\rho(t)-[h(\rho),\rho(t)]=K_c(\rho)+K(\rho), \tag{20}$$

where $K(\rho)$ represents the incoherent collision term and the additional term arises from the correlations of the mean-field fluctuations and the density fluctuations

$$K_c(\rho)=\overline{[\delta\hat{h}(t),\delta\hat{\rho}(t)]} \tag{21}$$

and it is referred to as the coherent collision term. This collision term has been investigated in previous publications in quantal [28] and semiclassical frameworks [29] for spatially uniform systems near equilibrium. Here, we carry out a quantal treatment of the collision term in nonequilibrium for finite systems.

In order calculate the coherent collision term, we consider that the fluctuations are small, and can be described by the linearized transport equation around the average evolution $\rho(t)$

$$\begin{aligned} i\frac{\partial}{\partial t}\delta\hat{\rho}-[\delta\hat{h},\rho]-[h(\rho),\delta\hat{\rho}] \\ =-i\int_{t0}^{t}dt'\,\mathrm{Tr}_2[v,\delta\{\hat{G}\hat{F}_{12}\hat{G}^{\dagger}\}]+\delta K(t). \end{aligned} \tag{22}$$

In the collision term, the quantity $\delta\{\hat{G}\hat{F}_{12}\hat{G}^{\dagger}\}$ involves two different contributions arising from the fluctuations of the mean-field propagator $\hat{G}(t,t')$ and from the fluctuations of the density matrix in $\hat{F}_{12}(t')$. According to Appendix A, it can be expressed as

$$\begin{aligned} \delta\{\hat{G}\hat{F}_{12}\hat{G}^{\dagger}\}=G\,\delta\hat{F}_{12}G^{\dagger}+[\delta\hat{\Phi}(t),GF_{12}G^{\dagger}] \\ -G[\delta\hat{\Phi}(t'),F_{12}]G^{\dagger}, \end{aligned} \tag{23}$$

where the quantity $\delta\hat{\Phi}(t)$ represents the density fluctuations without the right-hand side in Eq. (22) according to $\delta\hat{\rho}(t)=[\delta\hat{\Phi}(t),\rho(t)]$. We can approximate the collision term further by assuming that the density fluctuations can be expressed in the same form with the collision term is included, and deduce an effective equation for $\delta\hat{\Phi}(t)$. Then, the fluctuations arising from $\hat{F}_{12}$ in the first term in Eq. (23) can be combined with the fluctuations due to the mean-field propagator in the third term to give

$$\begin{aligned} \delta\{\hat{G}\hat{F}_{12}\hat{G}^{\dagger}\}=(1-\rho_1(t))(1-\rho_2(t))G(t,t') \\ \times[v,\delta\hat{\Phi}(t')]G^{\dagger}(t,t')\rho_1\widetilde{(t)\rho_2}(t)-\text{H.c.} \end{aligned} \tag{24}$$

There is another contribution arising from the second term in the expression (23). Since, the density fluctuations are not correlated in time with the $2p$-$2h$ excitations, this contribution is expected to be small and neglected here.

We analyze transport Eq. (22) in a time-dependent RPA approach and expand the small amplitude density fluctuations in terms of the time-dependent RPA functions

$$\delta\hat{\rho}(t)=\sum\ \delta z_{\lambda}(t)\rho_{\lambda}^{\dagger}(t)+\delta z_{\lambda}^{*}(t)\rho_{\lambda}(t), \tag{25}$$

where $\rho_{\lambda}^{\dagger}(t)$ and $\rho_{\lambda}(t)$ denote the time-dependent RPA functions, and $\delta z_{\lambda}(t)$ and $\delta z_{\lambda}^{*}(t)$ are the stochastic amplitudes associated with these modes. The time-dependent RPA functions describe the correlated p-h excitations around the average trajectory, and their time evolutions are determined by

$$i\frac{\partial}{\partial t}\rho_{\lambda}^{\dagger}(t)-[h(\rho),\rho_{\lambda}^{\dagger}(t)]-[h_{\lambda}^{\dagger}(t),\rho(t)]=-i\,\eta\rho_{\lambda}^{\dagger}(t) \tag{26}$$

and its Hermitian conjugate, where the fluctuating part of the mean field is indicated by $h_{\lambda}^{\dagger}(t)=(\partial h/\partial\rho)\rho_{\lambda}^{\dagger}(t)$ and a small damping term is included in the equation of motion. The initial conditions are supplied by the static RPA functions, and their positive and negative frequency components are evolved according to Eq. (26) and its Hermitian conjugate, respectively. We, also, introduce the dual wave functions, $Q_{\lambda}(t)$ and $Q_{\lambda}^{\dagger}(t)$ associated with the RPA modes [30]

$$i\frac{\partial}{\partial t}Q_{\lambda}(t)-[h(\rho),Q_{\lambda}(t)]+\tilde{h}_{\lambda}=-i\,\eta Q_{\lambda}(t) \tag{27}$$

and its Hermitian conjugate, where $\tilde{h}_{\lambda}=-[Q_{\lambda}(t),\rho(t)](\partial h/\partial\rho)$. As shown in Appendix B, if the RPA functions and their dual functions form a biorthonormal system at the initial instant, they remain orthonormal in time according to the definitions

$$\mathrm{Tr}\,Q_{\lambda}(t)\rho_{\mu}^{\dagger}(t)=\delta_{\lambda\mu},\quad \text{and}\quad \mathrm{Tr}\,Q_{\lambda}(t)\rho_{\mu}(t)=0. \tag{28}$$

When the occupation numbers of the natural states do not change in time, two types of wave functions are related by $\rho_{\lambda}^{\dagger}(t)=[Q_{\lambda}^{\dagger}(t),\rho(t)]$ and $\rho_{\lambda}(t)=-[Q_{\lambda}(t),\rho(t)]$. In the following, we use these relations that provide a good approximation in the collision term even when the occupation numbers are changing in time.

Projecting the transport Eq. (22) on the collective RPA modes and noting that the collision term can be expanded using $\delta\hat{\Phi}(t)=\Sigma\,\delta z_{\lambda}(t)Q_{\lambda}^{\dagger}(t)-\delta z_{\lambda}^{*}(t)Q_{\lambda}(t)$, we can deduce stochastic equations for the random amplitudes [31]

$$i\frac{d}{dt}\delta z_\lambda(t)=\int_{t_0}^{t}dt'\Sigma_\lambda(t,t')\delta z_\lambda(t')+F_\lambda(t), \tag{29}$$

where $F_\lambda(t)$ is the projected noise and $\Sigma_\lambda(t,t')$ denotes the collisional self-energy of the RPA mode

$$\Sigma_\lambda(t,t')=-\frac{i}{4}\sum\langle kl|[Q_\lambda(t),v]|ij\rangle_t \times\langle kl|[Q_\lambda(t'),v]|ij\rangle_{t'}^{*}N_{ijkl}^{-}(t) \tag{30}$$

with

$$N_{ijkl}^{-}(t)=[1-n_i(t)][1-n_j(t)]n_k(t)n_l(t) -[1-n_k(t)][1-n_l(t)]n_i(t)n_j(t). \tag{31}$$

The coupling between different RPA modes through the collision term is neglected in Eq. (29). The expression of the projected noise and its correlation function are given by Eqs. (17), (18), and (19). According to Eq. (29), the variances of the random amplitudes are determined by

$$\frac{d}{dt}\overline{|\delta z_\lambda|^2}=-\Gamma_\lambda(t)\overline{|\delta z_\lambda|^2}+2D_\lambda(t), \tag{32}$$

where $\Gamma_\lambda(t)=-2\,\mathrm{Im}\int^t dt'\Sigma_\lambda(t,t')$ denotes the collisional damping width and $D_\lambda(t)=\mathrm{Re}\int^t dt'\overline{F_\lambda(t)F_\lambda^*(t')}$ is the diffusion coefficient associated with the mode. In the case of small fluctuations around equilibrium, the RPA modes become harmonic $\rho_\lambda^\dagger(t)=\rho_\lambda^\dagger\exp(-i\omega_\lambda t)$ and $\rho_\lambda(t)=\rho_\lambda\exp(+i\omega_\lambda t)$. In this case, time integrals can be carried out to give [32,33]

$$\Gamma_\lambda=\frac{1}{4}\sum|\langle kl|[Q_\lambda,v]|ij\rangle|^2 2\pi\delta(\omega_\lambda-\epsilon_i-\epsilon_j+\epsilon_k+\epsilon_l)N_{ijkl}^{-} \tag{33}$$

and the diffusion coefficient is related to the damping width in accordance with the quantal dissipation-fluctuation relation [34]

$$2D_\lambda=\Gamma_\lambda\frac{1}{2}\coth\frac{\omega_\lambda}{2T}. \tag{34}$$

As a result, the thermal equilibrium value of the variance becomes $\overline{|\delta z_\lambda|_{eq}^2}=N_\lambda^0+1/2$, where $N_\lambda^0=1/[\exp(\omega_\lambda/T)-1]$ is the phonon occupation factor. Following this property, we define

$$\overline{|\delta z_\lambda(t)|^2}=N_\lambda(t)+\frac{1}{2} \tag{35}$$

and regard $N_\lambda(t)$ as the time-dependent occupation factors of the RPA functions. We note that, besides the collisional damping, there are other mechanisms involved in damping of the mean-field fluctuations, that are not incorporated in Eq. (29) for the random amplitudes. In particular, the low-frequency fluctuations are mainly damped by the one-body dissipation mechanism. In order to account for the one-body dissipation mechanisms in an approximate manner, we may evaluate the variances of the random amplitudes directly in terms of the density fluctuations

$$\overline{|\delta z_\lambda(t)|^2}=\overline{\mathrm{Tr}\,Q_\lambda^\dagger\delta\hat{\rho}(t)\cdot\mathrm{Tr}\,Q_\lambda\delta\hat{\rho}(t)}. \tag{36}$$

The density matrix, in general, involves collective fluctuations induced by the correlated p-h excitations and noncollective fluctuations produced by the incoherent p-h excitations, that are described by the collective and the noncollective RPA functions, respectively. As shown in Appendix C, the uncorrelated density fluctuations can be expressed as

$$\overline{\langle i|\delta\hat{\rho}(t)|j\rangle\langle j|\delta\hat{\rho}(t)|i\rangle}=\frac{1}{2}[n_i(t)[1-n_j(t)] +n_j(t)\{1-n_i(t)\}]. \tag{37}$$

Employing this result for the uncorrelated density fluctuations, we can obtain an approximate expression for the phonon occupation factors

$$N_\lambda(t)+\frac{1}{2}=\frac{1}{2}\mathrm{Tr}\{Q_\lambda(t)\rho(t)Q_\lambda^\dagger(t)[1-\rho(t)] +Q_\lambda(t)[1-\rho(t)]Q_\lambda^\dagger(t)\rho(t)\}. \tag{38}$$

In using this expression, damping terms in the RPA Eqs. (26) and (27) should be included with a finite value of η describing the damping of the p-h states into more complex configurations.

We calculate the ensemble average in the coherent collision term (21) by expanding the mean-field fluctuations and the density fluctuations in terms of the time-dependent RPA functions

$$\delta\hat{h}(t)=\sum\delta z_\lambda(t)h_\lambda^\dagger(t)+\delta z_\lambda^*(t)h_\lambda(t) \tag{39}$$

and $\delta\hat{\rho}(t)$ as given by Eq. (25). The collision term involves, in addition to the diagonal terms $\overline{\delta z_\lambda\delta z_\lambda^*}$, the off-diagonal terms $\overline{\delta z_\lambda\delta z_\mu^*}$ arising from the coupling between different RPA modes through the incoherent collision term and its stochastic part. This coupling is neglected in Eq. (29) for the random amplitudes, which may not be very important between collective RPA modes. However, as a result of the collisional coupling, the noncollective RPA modes are strongly mixed up and loose their bosonic character [35]. Therefore, the noncollective modes should be excluded from the coherent collision term, since their effects can be included into the incoherent collision term by renormalizing the residual interactions [29]. The diagonal contributions of the collective modes to the coherent collision term is given by

$$\overline{[\delta\hat{h}(t),\delta\hat{\rho}(t)]}_{coll}=\sum{}' \left\{\left(N_\lambda+\frac{1}{2}\right)[h_\lambda^\dagger,\rho Q_\lambda(1-\rho)] -\left(N_\lambda+\frac{1}{2}\right)[h_\lambda^\dagger,(1-\rho)Q_\lambda\rho]\right\}-\text{H.c.}, \tag{40}$$

where prime indicates the sum over the collective modes. By inspection, it can be seen that the first and second terms in this expression correspond to absorption and excitation of RPA phonons. These rates should be proportional to N_λ and $N_\lambda+1$, respectively, but the average value $N_\lambda+\frac{1}{2}$ appears in both rates. There are other contributions in $K_c(\rho)$ arising from the cross correlations between the collective and the noncollective modes. In schematic models, it is possible to show that these cross correlations give rise to an additional contribution to the collision term, so that the excitation and absorption rates become proportional to $N_\lambda+1$ and N_λ, as it should be. However, in the RPA analysis, it is difficult to extract such a contribution. For the time being, we replace the excitation and absorption factors in Eq. (40) by $N_\lambda+1$ and N_λ, and express the coherent collision term as

$$K_c(\rho)=\sum{}' [h_\lambda^\dagger(t),(N_\lambda(t)+1)(1-\rho(t))Q_\lambda(t)\rho(t) -N_\lambda(t)\rho(t)Q_\lambda(t)(1-\rho(t))]-\text{H.c.} \tag{41}$$

The RPA functions can be determined in terms of the amplitude of the mean-field fluctuations by solving Eq. (27) and its Hermitian conjugate

$$Q_\lambda(t)=-i\int_{t_0}^{t}dt'\,G(t,t')h_\lambda(t')G^\dagger(t,t')e^{-\eta(t-t')}, \tag{42}$$

where the initial condition term is omitted by letting the initial time t_0 to be sufficiently early, so that the time interval $t-t_0$ is much longer than the decay time of the collision kernel, and similarly for $Q_\lambda^\dagger(t)$. At low energies, since the dissipation is dominated by the coherent collision term, we can neglect the incoherent mechanism due to binary collisions. As a result, we obtain an extended TDHF description for the evolution of the single-particle density matrix

$$i\frac{\partial}{\partial t}\rho(t)-[h(\rho),\rho(t)]=K_c(\rho), \tag{43}$$

where the collision term describes the coupling of the single-particle motion with the coherent $2p$-$2h$ excitations. The amplitude of the mean-field fluctuations in the coherent collision term are self-consistently determined by the time-dependent RPA equations. In general, it is difficult to determine the time evolution of the occupation factors $N_\lambda(t)$ of the RPA functions, since they depend on rather complex damping mechanism of the mean-field fluctuations. Since, the high frequency fluctuations are mainly damped by the collisional effects, the corresponding occupation factors are determined by Eq. (32). However, for low frequency fluctuations it is more appropriate to use the expression (38) for an approximate description of these factors.

IV. VIBRATIONS AROUND EQUILIBRIUM

In this section, we consider the nuclear collective vibrations around a finite temperature equilibrium and deduce expressions for the damping widths of collective vibrations and single-particle excitations. Near equilibrium, since the small amplitude density fluctuations are harmonic, the time-dependent RPA equations become [36]

$$(+\omega_\lambda+i\eta)\rho_\lambda^\dagger-[h_0,\rho_\lambda^\dagger]-[h_\lambda^\dagger,\rho_0]=0 \tag{44}$$

and

$$(-\omega_\lambda+i\eta)\rho_\lambda-[h_0,\rho_\lambda]-[h_\lambda,\rho_0]=0. \tag{45}$$

As a result, equilibrium mean-field fluctuations can be expressed in terms of the static RPA modes and the equilibrium occupation factors N_λ^0 of these modes.

For describing the damping of the single-particle excitations, we replace the mean-field Hamiltonian in Eq. (43) by its static value, $h(\rho)\rightarrow h_0$, and express the density matrix in the Hartree-Fock representation according to $\rho(t)=\Sigma|\phi_i\rangle n_i(t)\langle\phi_i|$. Then, from transport Eq. (43), we can deduce a master equation for the occupation numbers

$$\frac{d}{dt}n_i(t)=-\Gamma_i^{>}(t)n_i(t)+\Gamma_i^{<}(t)[1-n_i(t)], \tag{46}$$

where the first and second terms represents loss and gain terms, respectively. The loss term is determined by the imaginary part of the self-energy $\Gamma_i^{>}(t)=-2\,\text{Im}\,\Sigma_i^{>}(t)$ with

$$\Sigma_i^{>}(t)=\sum (1-n_j(t))\left\{\frac{|\langle i|h_\lambda^\dagger|j\rangle|^2}{\epsilon_i-\epsilon_j-\omega_\lambda+i\eta}(N_\lambda^0+1) +\frac{|\langle i|h_\lambda|j\rangle|^2}{\epsilon_i-\epsilon_j+\omega_\lambda+i\eta}N_\lambda^0\right\}, \tag{47}$$

where the first and second contributions describe the decay of the single-particle state $|\phi_i\rangle$ by excitation and absorption of a phonon, respectively. The expression $\Gamma_i^{<}(t)=-2\,\text{Im}\,\Sigma_i^{<}(t)$ in the gain term is obtained from $\Sigma_i^{>}(t)$ by making the substitutions, $N_\lambda^0+1\rightarrow N_\lambda^0$, $N_\lambda^0\rightarrow N_\lambda^0+1$, and $[1-n_j(t)]\rightarrow n_j(t)$. Similar expressions for the single-particle self-energies have been derived in the literature by following different approaches [30,37]. In particular, we note that the retarded single-particle self-energy $\Sigma_i^{ret}(t)=\Sigma_i^{>}(t)+\Sigma_i^{<}(t)$,

$$\Sigma_i^{ret}(t)=\sum \left\{\frac{|\langle i|h_\lambda^\dagger|j\rangle|^2}{\epsilon_i-\epsilon_j-\omega_\lambda+i\eta}[N_\lambda^0+1-n_j(t)] +\frac{|\langle i|h_\lambda|j\rangle|^2}{\epsilon_i-\epsilon_j+\omega_\lambda+i\eta}[N_\lambda^0+n_j(t)]\right\}. \tag{48}$$

has the same form as the one obtained by employing the Matsubara formalism in [39].

In order to describe the collective vibrations, we linearize the transport Eq. (43) around a finite temperature equilibrium state ρ_0. The small amplitude vibrations $\delta\rho(t)=\rho(t)-\rho_0$ are determined by

$$i\frac{\partial}{\partial t}\delta\rho(t)-[h_0,\delta\rho(t)]-[\delta h(t),\rho_0]=\delta K_c(t), \quad (49)$$

where the quantity $\delta K_c(t)$ represents the linearized form of the coherent collision term (41). We can obtain an expression for this collision term by following a treatment similar to the one used in linearizing the collision term in Eq. (22). There are two contributions in $\delta K_c(t)$, coming from the mean-field vibrations through $\delta\{G(t,t')h_\lambda G^\dagger(t,t')\}$ and from the density vibrations $\delta\rho(t)$, which should be combined in a consistent manner. According to Appendix A and Eq. (23), the contributions through the mean-field propagator can be expressed as

$$\delta\{G(t,t')h_\lambda G^\dagger(t,t')\}=[\delta\Phi(t),G_0 h_\lambda G_0^\dagger] - G_0[\delta\Phi(t'),h_\lambda]G_0^\dagger, \quad (50)$$

where $G_0=\exp[-i(t-t')h_0]$, and the quantity $\delta\Phi(t)$ describes the density vibrations without the collision term according to $\delta\rho(t)=[\delta\Phi(t),\rho_0]$. Assuming density vibrations can be represented in the same form with the collision term included, the first term in the expression (50) cancels out with the contributions coming from the density vibrations. As a result, the linearized collision term becomes

$$\delta K_c(t)=\sum[h_\lambda^\dagger,(N_\lambda^0+1)(1-\rho_0)\delta Q_\lambda(t)\rho_0 - N_\lambda^0\rho_0\delta Q_\lambda(t)(1-\rho_0)]-\text{H.c.}, \quad (51)$$

where

$$\delta Q_\lambda(t)=-i\int_{t_0}^{t}dt'\,G_0(t-t')[h_\lambda,\delta\Phi(t')]G_0^\dagger(t-t') \times\exp[-i(\omega_\lambda-i\eta)(t-t')]. \quad (52)$$

We can analyze the density vibrations in the RPA framework similar to the treatment used in the fluctuation analysis, and expand the density vibrations in terms of the static RPA functions as

$$\delta\rho(t)=\sum z_\mu(t)[Q_\mu^\dagger,\rho_0]-z_\mu^*(t)[Q_\mu,\rho_0], \quad (53)$$

where $z_\mu(t)$ and $z_\mu^*(t)$ are the deterministic amplitudes associated with the modes. By projecting the transport Eq. (49) on the RPA modes and substituting the expansion $\delta Q(t)=\Sigma z_\mu(t)Q_\mu^\dagger-z_\mu^*(t)Q_\mu$ into the collision term, we find that the corresponding amplitudes are determined by

$$i\frac{d}{dt}z_\mu(t)-\omega_\mu z_\mu(t)=\int_{t_0}^{t}dt'\,\Sigma_\mu(t-t')z_\mu(t'). \quad (54)$$

Here, $\Sigma_\mu(t-t')$ denotes the self-energy of the mode due to the coherent coupling mechanism, and its Fourier transform is given by [38]

$$\Sigma_\mu(\omega)=\sum\frac{|\langle i|[Q_\mu,h_\lambda^\dagger]|j\rangle|^2}{\omega-\omega_\lambda-\epsilon_j+\epsilon_i+i\eta}\{(N_\lambda^0+1)(1-n_j^0)n_i^0 - N_\lambda^0 n_j^0(1-n_i^0)\}+\sum\frac{|\langle i|[Q_\mu,h_\lambda]|j\rangle|^2}{\omega+\omega_\lambda-\epsilon_j+\epsilon_i+i\eta} \times\{N_\lambda^0(1-n_j^0)n_i^0-(N_\lambda^0+1)n_j^0(1-n_i^0)\}. \quad (55)$$

The first term in the self-energy describes the damping of the collective vibrations by exciting a phonon and a p-h pair. At a finite temperature, the reverse process with a weight $N_\lambda^0 n_j^0(1-n_i^0)$ is also possible, which decreases the damping. There is another contribution to the self-energy represented by the second term in this expression. It describes absorption of a phonon accompanied by p-h excitations that is possible only at finite temperatures. The self-energy of collective modes has been investigated by employing the Matsubara formalism in [39]. The expression (55) of the collective self-energy has essentially the same form as that derived within the Matsubara formalism. The commutator structure in Eq. (55) gives rise to two direct and two cross terms, which correspond to the propagator and the vertex correction terms in the Matsubara treatment, respectively. The expression presented in [39] contains terms that do not involve the propagator $\omega\pm\omega_\lambda-\epsilon_j+\epsilon_i+i\eta$. These terms may be neglected, since they do not lead to the damping of collective modes due to mixing with the particle-hole plus phonon states. Furthermore, it can be shown that in the pole approximation, i.e., $\omega\pm\omega_\lambda-\epsilon_j+\epsilon_i=0$, remaining terms may be combined together to give the same expression for the self-energy as the one given by Eq. (55), except an intermediate summation is missing in the propagation correction terms in [39]. In damping of high-frequency collective modes and damping of single-particle excitations at low energies, the dominant contributions to the self-energies arises from the low-frequency density fluctuations that can be well approximated by surface vibrations. In previous publications, using this approximation, the formulas (48) and (55) have been applied to describe damping of the single-particle and giant resonance excitations [11,39].

V. SUMMARY AND CONCLUSIONS

Development of one-body transport descriptions may lead to novel theoretical tools for understanding the nuclear dynamics at low energies, as well as the complex reaction mechanism in heavy-ion collisions at intermediate energies. For this purpose, much work has been done to deduce an effective one-body transport description by incorporating correlations in an approximate manner so that the applications to the realistic situations may become possible. In the extended TDHF, the effect of two-body correlations are incorporated into the equation of motion in the form of an incoherent binary collision term. Such an incoherent mechanism is very important for damping of collective motion and thermalization of the system at intermediate energies around Fermi energy. However, at low energy nuclear dynamics including giant resonance excitations, the coherence in two-body correlations should be incorporated for a realistic description of the damping mechanism of the collective motion. In the stochastic theory, the transport description is further improved by incorporating higher order correlations in an approximate manner by a stochastic mechanism that is consistent with the fluctuation-dissipation theorem of nonequilibrium statistical mechanics. As a result, in the stochastic theory, it is possible to address the average evolution as well as the dynamics of density fluctuations, in a manner similar to the generalized Langevin description of the reduced variables. Furthermore, the stochastic dynamics contains, in addition to incoherent binary collisions, a coherent damping mechanism resulting from the coupling between the mean-field fluctuations and the single-particle motion, which is not included in the extended TDHF theory. We illustrate this damping mechanism by investigating the average dynamical evolution and derive an expression for the coherent collision term. Considering small amplitude fluctuations around equilibrium, we deduce expressions for the self-energies of single-particle and collective excitations due to the coherent coupling mechanism. While the present results are very encouraging, more work remains to be done for a consistent description of the coherent and the incoherent damping mechanisms in connection with dynamics of nuclear collective motion.

ACKNOWLEDMENTS

One of us (S.A.) gratefully acknowledges the Yukawa Institute for Theoretical Physics and Kyoto University for support and warm hospitality extended to him by the theory group and the supporting staff. This work was supported in part by U.S. DOE Grant No. DE-FG05-89ER40530.

APPENDIX A

The quantity $\delta\hat{G}(t,t')\hat{F}_{12}(t')\hat{G}^{\dagger}(t,t')$ in the collision term in Eq. (22), treating the fluctuating part $\delta\hat{h}(t)$ as a perturbation, is given by

$$\begin{aligned}\delta\hat{G}(t,t')\hat{F}_{12}(t')\hat{G}^{\dagger}(t,t') &= G(t,t')\,\delta\hat{F}_{12}(t')G^{\dagger}(t,t') \\ &\quad -i\int_{t'}^{t} ds[G(t,s)\,\delta\hat{h}(s)G^{\dagger}(t,s), \\ &\quad \times G(t,t')F_{12}(t')G^{\dagger}(t,t')], \end{aligned} \tag{A1}$$

where $G(t,t')$ denotes the average value of the propagator. The evolution of the density fluctuations over short time intervals, by neglecting the right-hand side in Eq. (22), can be expressed as

$$\begin{aligned}\delta\hat{\rho}(t) &= G(t,t')\,\delta\hat{\rho}(t')G^{\dagger}(t,t') \\ &\quad -i\int_{t'}^{t} ds[G(t,s)\,\delta\hat{h}(s)G^{\dagger}(t,s),\rho(t)], \end{aligned} \tag{A2}$$

where the approximation $G(t,s)\rho(s)G^{\dagger}(t,s)\approx\rho(t)$ is employed. This result can be expressed in the form $\delta\hat{\rho}(t) = [\delta\hat{\Phi}(t),\rho(t)]$ with

$$\begin{aligned}\delta\hat{\Phi}(t) &= G(t,t')\,\delta\hat{\Phi}(t')G^{\dagger}(t,t') \\ &\quad -i\int_{t'}^{t} ds G(t,s)\,\delta\hat{h}(s)G^{\dagger}(t,s). \end{aligned} \tag{A3}$$

Combining this result with the expression (A1) leads to Eq. (23).

APPENDIX B

The time-dependent RPA functions and their dual functions maybe orthonormalized according to

$$N_{\lambda\mu} = \mathrm{Tr}\, Q_{\lambda}(t)\rho_{\mu}^{\dagger}(t). \tag{B1}$$

The rate of change of this quantity can be expressed as

$$\frac{d}{dt}N_{\lambda\mu} = \mathrm{Tr}\{\dot{Q}_{\lambda}\rho_{\mu}^{\dagger} + Q_{\lambda}\dot{\rho}_{\mu}^{\dagger}\}. \tag{B2}$$

Using the time-dependent RPA Eqs. (24) and (25), we find

$$i\frac{d}{dt}N_{\lambda\mu} = \mathrm{Tr}\{[h,Q_{\lambda}]\rho_{\mu}^{\dagger} - \tilde{h}_{\lambda}\rho_{\mu}^{\dagger} + Q_{\lambda}[h,\rho_{\mu}^{\dagger}] + Q_{\lambda}[h_{\mu}^{\dagger},\rho]\}, \tag{B3}$$

where $h_{\lambda}^{\dagger} = (\partial h/\partial\rho)\rho_{\mu}^{\dagger}$ and $\tilde{h}_{\lambda} = -[Q_{\lambda},\rho](\partial h/\partial\rho)$. Expanding the commutators, it is easy to see that all the terms on the right-hand side cancel out to give $dN_{\lambda\mu}/dt = 0$.

APPENDIX C

In order to calculate the uncorrelated density fluctuations, we neglect the terms in transport Eq. (22) involving the mean-field fluctuations $\delta h(t)$ and consider density fluctuations in the TDHF representation, $\delta\hat{\rho}(t) = \Sigma|\phi_i(t)\rangle\langle i|\delta\hat{\rho}(t)|j\rangle\langle\phi_j(t)|$. According to Eq. (22), the stochastic evolution of the density matrix is described by

$$i\frac{\partial}{\partial t}\langle i|\delta\hat{\rho}(t)|j\rangle = [\Sigma_i(t) - \Sigma_j^{*}(t)]\langle i|\delta\hat{\rho}(t)|j\rangle + \langle i|\delta K(t)|j\rangle, \tag{C1}$$

where $\Sigma_i(t) = \Sigma_i^{>}(t) + \Sigma_i^{<}(t)$ represents the collisional self-energy of the single-particle states with

$$\begin{aligned}\Sigma_i^{>}(t) &= -i\int^{t} dt' \sum \langle ij|v|kl\rangle_t \langle kl|v|ij\rangle_{t'} \\ &\quad \times[1-n_k(t)][1-n_l(t)]n_j(t) \end{aligned} \tag{C2}$$

and

$$\Sigma_i^<(t)=-i\int^t dt'\sum \langle ij|v|kl\rangle_t\langle kl|v|ij\rangle_{t'}n_k(t)n_l(t)\times[1-n_j(t)]. \tag{C3}$$

In obtaining this result, we neglect the terms in the linearized collision term that arise from the density fluctuations in the intermediate states and assume the average density matrix is diagonal in the TDHF representation. Following Eq. (C1), the variance of elements of the density matrix $\Lambda_{ij}(t)=\overline{|\langle i|\delta\hat{\rho}(t)|j\rangle|^2}$ is determined by

$$\frac{\partial}{\partial t}\Lambda_{ij}(t)=-[\Gamma_i(t)+\Gamma_j(t)]\Lambda_{ij}(t)+D_{ij}(t), \tag{C4}$$

where $\Gamma_i(t)=\Gamma_i^>(t)+\Gamma_i^<(t)$, with $\Gamma_i^>(t)=-2\,\mathrm{Im}\,\Sigma_i^>(t)$ and $\Gamma_i^<(t)=-2\,\mathrm{Im}\,\Sigma_i^<(t)$, describes the collisional damping factor of the single-particle states. The quantity $D_{ij}(t)$ is given in terms of the correlation function $C_{ij;ji}(t,t')$ of the stochastic collision term by

$$D_{ij}(t)=-\int^t dt'[C_{ij;ji}(t,t')+C_{ji;ij}(t,t')], \tag{C5}$$

where a factor $\exp[-\int_{t'}^t ds(\Sigma_i(s)-\Sigma_j^*(s))]$ in the integrand is neglected by assuming that the correlation function is sharply peaked around $t\approx t'$ (Markovian approximation). Using Eqs. (10) and (16) and retaining only the uncorrelated parts in the correlation function, it can be expressed as

$$D_{ij}(t)=\frac{1}{2}[\{1-n_j(t)\}\Gamma_i^<(t)+n_j(t)\Gamma_i^>(t)+\{1-n_i(t)\}\Gamma_j^<(t)+n_i(t)\Gamma_j^>(t)]. \tag{C6}$$

The transport Eq. (20) gives rise to a master equation for the average occupation numbers $n_i(t)$ of the single-particle states

$$\frac{d}{dt}n_i(t)=-n_i(t)\Gamma_i^>(t)+[1-n_i(t)]\Gamma_i^<(t), \tag{C7}$$

where we retain only the incoherent collision term $K(\rho)$. Using this result, we can derive an equation for the quantity $\tilde{\Lambda}_{ij}(t)=[n_i(t)\{1-n_j(t)\}+n_j(t)\{1-n_i(t)\}]/2$, and find that it satisfies the same equation as for the variance $\Lambda_{ij}(t)$ of the stochastic density matrix

$$\frac{\partial}{\partial t}\tilde{\Lambda}_{ij}(t)=-[\Gamma_i(t)+\Gamma_j(t)]\tilde{\Lambda}_{ij}(t)+D_{ij}(t). \tag{C8}$$

As a result, these quantities should be equal $\Lambda_{ij}(t)=\tilde{\Lambda}_{ij}(t)$, and consequently, the uncorrelated density fluctuations can be expressed in terms of the average occupation numbers according to Eq. (37).

[1] G. F. Bertsch and S. Das Gupta, Phys. Rep. **160**, 190 (1988).
[2] W. Casing and U. Mosel, Prog. Part. Nucl. Phys. **25**, 235 (1990).
[3] K. Goeke and P.-G. Reinhard, *Time-Dependent Hartree-Fock and Beyond* (Bad Honnef, Germany, 1982).
[4] K. T. D. Davis, K. R. S. Devi, S. E. Koonin, and M. Strayer, *Treatise in Heavy-Ion Science*, edited by D. A. Bromley, Nuclear Science, Vol. 4 (Plenum, New York, 1984).
[5] C. Y. Wong and H. H. K. Tang, Phys. Rev. Lett. **40**, 1070 (1978); Phys. Rev. C **20**, 1419 (1979).
[6] S. Ayik, Z. Phys. A **298**, 83 (1980).
[7] P. Grange, H. A. Weidenmuller, and G. Wolschin, Ann. Phys. **136**, 190 (1981).
[8] R. Balian and M. Veneroni, Ann. Phys. **135**, 270 (1981).
[9] W. Cassing and S. J. Wang, Z. Phys. A **337**, 1 (1990).
[10] M. Tohoyama, Phys. Rev. C **36**, 187 (1987).
[11] G. F. Bertsch, P. F. Bortignon, and R. A. Broglia, Rev. Mod. Phys. **55**, 287 (1983).
[12] G. F. Bertsch and R. A. Broglia, *Oscillations in Finite Quantum Systems* (Cambridge University Press, Cambridge, England, 1994).
[13] F. DeBlasio *et al.*, Phys. Rev. Lett. **68**, 1663 (1992).
[14] P. F. Bortignon, A. Bracco, and R. A. Broglia, *Giant Resonances and Nuclear Structure at Finite Temperature* (Harwood Academic, Switzerland, 1998).
[15] H. Mori, Theoret. Phys. **33**, 423 (1965).
[16] S. Nakajima, Prog. Theor. Phys. **20**, 948 (1958).
[17] R. W. Zwanzig, in *Quantum Statistical Mechanics*, edited P. H. E. Meijer (Gordon and Breach, New York, 1966).
[18] M. Bixon and R. Zwanzig, Phys. Rev. **187**, 267 (1969).
[19] S. Ayik and C. Gregoire, Phys. Lett. B **212**, 269 (1988); Nucl. Phys. **A513**, 187 (1990).
[20] J. Randrup and B. Remaud, Nucl. Phys. **A514**, 339 (1990).
[21] Y. Abe, S. Ayik, P. -G. Reinhard, and E. Suraud, Phys. Rep. **275**, 49 (1996).
[22] S. Ayik, Yukawa Institute Report No. YITP-00-23; Phys. Lett. B **493**, 47 (2000).
[23] G. F. Burgio, Ph. Chomaz, and J. Randrup, Phys. Rev. Lett. **69**, 885 (1992).
[24] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
[25] P. Danielewicz, Ann. Phys. **152**, 239 (1984).
[26] K. Huang, *Statistical Mechanics* (Wiley, New York, 1962).
[27] L. Von Hove, Physica **21**, 517 (1955).
[28] S. Ayik, Z. Phys. A **350**, 45 (1994).
[29] Y. B. Ivanov and S. Ayik, Nucl. Phys. **A593**, 233 (1995).
[30] P. Danielewicz, Ann. Phys. **A197**, 154 (1990).
[31] S. Ayik, O. Yilmaz, A. Gokalp, and P. Schuck, Phys. Rev. C **58**, 1594 (1998).
[32] D. Lacroix, Ph. Chomaz, and S. Ayik, Phys. Rev. C **58**, 2154 (1998).
[33] S. Ayik, D. Lacroix, and Ph. Chomaz, Phys. Rev. C **61**, 014608 (2000).
[34] C. W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991).

[35] Ph. Chomaz, D. Lacroix, S. Ayik, and M. Colonna, Phys. Rev. C **62**, 024307 (2000).
[36] R. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
[37] P. Danielewicz and P. Schuck, Nucl. Phys. **A567**, 78 (1994).
[38] S. Ayik, Phys. Rev. Lett. **56**, 38 (1986).
[39] P. Bortignon, R. A. Broglia, G. F. Bertsch, and J. Pacheco, Nucl. Phys. **A460**, 149 (1986).