

Self-energy effects in the superfluidity of neutron matter

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The superfluidity of neutron matter in the channel 1S_0 is studied by taking into account the effect of the ground-state correlations in the self-energy. To this purpose the gap equation has been solved within the generalized Gorkov approach. A sizable suppression of the energy gap is driven by the quasiparticle strength around the Fermi surface.

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It is widely recognized that superfluidity is an extremely subtle process when it is considered on an entirely microscopic level. Most of the calculations in nuclear physics, at least concerning finite nuclei, are therefore based on phenomenological effective interactions. On the other hand a series of nuclear matter and neutron matter calculations exists with the bare NN force used in the gap equation [1]. This is insofar the first step of a systematic microscopic approach as it is well known that in the gap equation the bare particle-particle interaction has to be taken to the lowest order and not a G matrix, i.e., a ladder summation, since the gap equation is in itself a two-body equation [2,3]. The next terms in the effective pairing interaction are screening terms. These are due to the possibility of medium polarization which, when treated within the induced interaction theory [4], has turned out to strongly affect the pairing gap [5–7]. On the other hand the self-energy corrections due to polarization effects have attracted much less attention. Only very recently has it been stressed that the consideration of the quasiparticle strength can have a sizable effect on the gap value [8,9]. Here we want to extend the latter work in investigating correlation effects beyond the ones taken into account in Ref. [8] and we will see that they are indeed quite important. Our study must be considered as an intermediate step towards a fully consistent treatment where self-energy and vertex corrections are taken into account on an absolutely equal footing. The latter aspect may turn out to be decisive because sometimes strong cancellation between the two contributions can occur (see, e.g., discussion in Ref. [10]). All those considerations are of great importance for a more microscopic understanding of superfluidity in nuclei as well as in neutron stars, but also for a precise estimate of the pairing gap. In fact, pairing in exotic nuclei at present is studied with effective density dependent interactions modeled on pairing calculations in neutron matter [10,11]. In the latter systems it is well known that superfluidity drives their rotational dynamics [12] as well as their cooling [13]. In this work we shall study pure neutron matter but we expect that very analogous effects will occur in symmetric nuclear matter.

Let us first discuss some properties of the single-particle self-energy $\Sigma_p(\omega)$ of neutron matter. In the Brueckner approach [14] the perturbative expansion of Σ can be recast according to the number of hole lines as follows:

$$\Sigma_p(\omega) = \Sigma_p^1(\omega) + \Sigma_p^2(\omega) + \dots \quad (1)$$

The on-shell values of Σ^1 represent the Brueckner-Hartree-Fock (BHF) mean field and the on-shell values of Σ^2 give the so-called rearrangement term, which gives the largest contribution to the ground-state correlations. The off-shell values enter several physical properties of neutron matter, including pairing. In terms of the self-energy one may calculate, at a given order of the hole-line expansion, the quasiparticle energy, as the solution ω_p of the equation

$$\omega_p = \frac{p^2}{2m} - \epsilon_F + \Sigma_p(\omega_p), \quad (2)$$

where ϵ_F is the Fermi energy. We neglect here the imaginary part of the self-energy. The quasiparticle energy around the Fermi surface is obtained by expanding the self-energy around $p = p_F$ and $\omega = 0$:

$$\omega = \frac{p^2 - p_F^2}{2m^*} = \frac{p^2 - p_F^2}{2m} \frac{m^2}{m_e m_p}, \quad (3)$$

where m^* , m_e , and m_p are the effective mass, e -mass, and p -mass, respectively, calculated at the Fermi surface. The two masses are defined for any momentum as

$$m_e(p) = m \left[1 - \left(\frac{\partial \Sigma(p, \omega)}{\partial \omega} \right) \right]_{\omega = \omega_p}, \quad (4)$$

$$m_p(p) = m \left[1 + \frac{m}{p} \left(\frac{\partial \Sigma(p, \omega)}{\partial p} \right) \right]_{\omega = \omega_p}^{-1}. \quad (5)$$

Their properties have been extensively studied in Ref. [14]. The k -mass is related to the nonlocality of the mean field and, if the self-energy is ω independent (static limit), it coincides with the effective mass. This quantity is of great interest whenever the momentum dependence of the mean field can give some effects such as transverse flows in heavy-ion collisions [15]. The e -mass is related to the quasiparticle strength. The latter gives the discontinuity of the momentum distribution at the Fermi surface, and measures the amount of correlations included in the considered approximation.

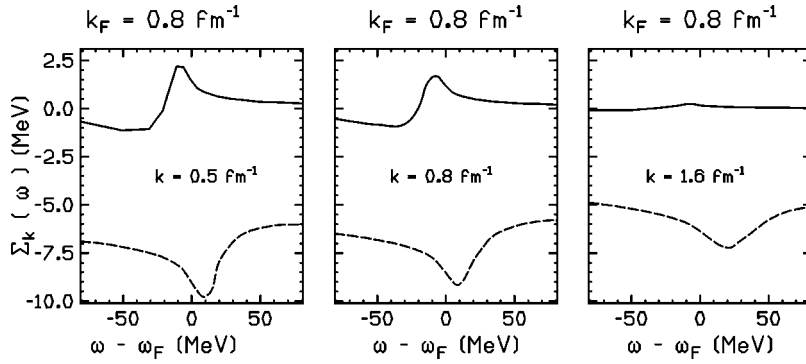


FIG. 1. One-hole-line off-shell $\Sigma_k^1(\omega)$ (dashed line) and two-hole-line off-shell $\Sigma_k^2(\omega)$ (solid line) for three values of the momentum k . The Fermi momentum k_F is fixed at 0.8 fm^{-1} .

The self-energy has been calculated in two approximations: up to the first order (BHF) and up to the second order (EBHF) of the hole-line expansion in the framework of the Brueckner theory adopting the continuous choice [16]. The Argonne V_{14} potential has been used for the bare interaction [17]. The calculations have been performed for a range of Fermi momenta where the energy gap is expected to be the largest, i.e., $0.5 \leq k_F \leq 1.3 \text{ fm}^{-1}$, corresponding to a density range from 0.0042 through 0.074 fm^{-3} . A typical result for the off-shell neutron self-energy $\Sigma_p(\omega)$ is plotted in Fig. 1. The contribution from the rearrangement term shows a pronounced enhancement in the vicinity of the Fermi energy, which is to be traced back to the high probability amplitude for particle-hole excitations near ϵ_F [16]. At high momenta this contribution vanishes. From $\Sigma_p(\omega)$ the effective masses are extracted according to Eqs. (4) and (5). They are depicted in Fig. 2, where the full calculation is compared to that including only the BHF self-energy. We may distinguish two momentum intervals: at $k \approx k_F$ the momentum dependence of the effective mass m^* is characterized by a bump, whose peak value exceeds the value of the bare mass; far above k_F the bare mass limit is approached. One should take into account that in this range of k_F the neutron density is quite small (at the maximum $k_F = 1.3 \text{ fm}^{-1}$, $\rho = 0.074 \text{ fm}^{-3}$). This behavior of the effective mass m^* is due mostly to the e -mass, as shown in the lower panel of Fig. 2. In both panels of Fig. 2 it is also reported, for comparison, the effective

mass in the BHF limit (only Σ^1 included), which exhibits a much less pronounced bump at the Fermi energy. It is precisely the influence of this increased bump structure, which we want to investigate here, since in Ref. [8] the rearrangement term has not been taken into account.

The generalized BCS theory can be found in various textbooks on the many-body problem [2,18,19]. Here we follow closely the formalism developed in [2], where the gap equation is written

$$\Delta_p(\omega) = - \int \frac{d^3 p'}{(2\pi)^3} \int \frac{d\omega'}{2\pi i} \mathcal{V}_{p,p'}(\omega, \omega') \Gamma_{p'}(\omega') \Delta_{p'}(\omega'). \quad (6)$$

The kernel Γ is defined as

$$\begin{aligned} \Gamma_p(\omega) &= G_p(-\omega) G_p^s(\omega) \\ &= [G_p^{-1}(\omega) G_p^{-1}(-\omega) + \Delta_p^2(\omega)]^{-1}. \end{aligned} \quad (7)$$

The functions $G_p(\omega)$ and $G_p^s(\omega)$ are the nucleon propagators of neutron matter in the normal state and in the superfluid state, respectively. The ω symmetry in the two propagators is to be traced back to the time-reversal invariance of the Cooper pairs. The effective interaction \mathcal{V} is the block of all irreducible diagrams of the interaction. The short-range correlations (ladder diagrams) are already taken into account

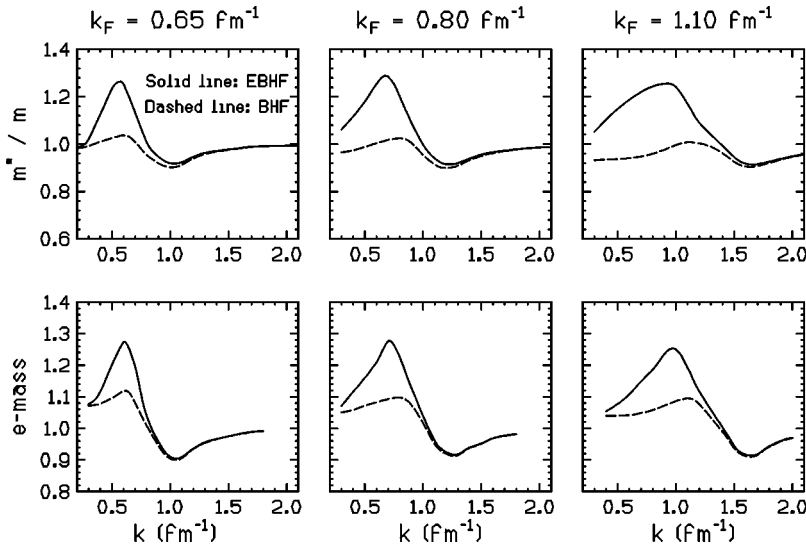


FIG. 2. Effective mass (upper panels) and e -mass (lower panels) for three densities $\rho = k_F^3/3\pi^2$ in the BHF approximation (dashed line) and EBHF approximation (solid line).

by the gap equation and do not appear in the irreducible block \mathcal{V} . The long-range components have been studied in the context of the induced interaction approach [6,7]. In the present Rapid Communication we only consider the first term in the perturbative expansion of \mathcal{V} , namely the bare interaction, because we want to disentangle the influence of the correlations coming only from the self-energy expansion from the ones due to the induced interaction. The complete solution of the generalized gap equation requires further effort.

Assuming the pairing interaction is identified with the bare interaction $V_{p,p'}$, the energy gap does not depend on the energy (static limit), i.e., $\Delta_p(\omega) \equiv \Delta_p$. In this limit the self-energy corrections are not expected to modify the analytical structure of the kernel $\Gamma_p(\omega)$ which is now an even function of energy: at each momentum p there exist two symmetric poles $\pm\Omega_p$ in the complex ω plane. The ω integration can be performed as follows:

$$\int \frac{d\omega}{2\pi i} \Gamma(\omega^2) = -\frac{\mathcal{Z}_p^2}{2\Omega_p}, \quad (8)$$

where we denote by \mathcal{Z}_p^2 the residue of the kernel at the pole Ω_p . Since the largest contribution to the integral is coming from the pole part of the two Green's functions, we expand the single particle propagator to the first order in ω_p [see Eq. (2)], $G_p(\omega) \approx \mathcal{Z}_p \cdot (\omega - \omega_p)^{-1}$, and therefore the denominator in Eq. (7) becomes

$$\Gamma^{-1}(\omega^2) \approx -\mathcal{Z}_p^{-2}(\omega^2 - \omega_p^2) + \Delta_p^2, \quad (9)$$

where \mathcal{Z}_p^{-2} is given by

$$\begin{aligned} \mathcal{Z}_p^{-2} &\approx \left(\frac{\partial G^{-1}(\omega)}{\partial \omega} \right) \Big|_{\omega=\omega_p} \cdot \left(\frac{\partial G^{-1}(-\omega)}{\partial \omega} \right) \Big|_{\omega=-\omega_p} \\ &= \left[1 - \frac{\partial \Sigma_p(\omega)}{\partial \omega} \right] \Big|_{\omega=\omega_p} \end{aligned} \quad (10)$$

and

$$\Omega_p \approx \sqrt{\omega_p^2 + \mathcal{Z}_p^2 \Delta_p^2}. \quad (11)$$

In this approximation the generalized gap equation, Eq. (6), becomes

$$\tilde{\Delta}_p = -\frac{1}{2} \int \frac{d^3 p'}{(2\pi)^3} \frac{\mathcal{Z}_p V_{pp'} \mathcal{Z}_{p'}}{\sqrt{\omega_{p'}^2 + \tilde{\Delta}_{p'}^2}} \tilde{\Delta}_{p'}, \quad (12)$$

where $\tilde{\Delta}_p = \mathcal{Z}_p \Delta_p$ is the real pairing correction to the quasiparticle energy spectrum. The main difference from the BCS limit is the presence of the quasiparticle strength, which is less than one in a small region around the Fermi surface as we saw earlier. The pairing interaction turns out to be reduced in that region, where the Cooper pairs are mainly formed. This is the way self-energy corrections come into play suppressing the pairing gap. As to the self-energy effects, Eq. (12) is quite general because there is no expansion

in the self-energy so far. The self-energy corrections can be taken at any order of approximation.

One may further restrict the ω integration to only the pole part at the Fermi energy, i.e., expanding the self-energy near the Fermi surface according to Eq. (3). In this case the ω integration can also be performed analytically and one easily obtains

$$\tilde{\Delta}_p = -\mathcal{Z}_F^2 \int \frac{d^3 p'}{(2\pi)^3} \frac{V_{p,p'} \tilde{\Delta}_{p'}}{2\sqrt{p_F^2(p' - p_F)^2/m^{*2} + \tilde{\Delta}_{p'}^2}}, \quad (13)$$

where \mathcal{Z}_F is the quasiparticle strength at the Fermi surface and coincides with the inverse of the e mass defined by Eq. (4). It arises from the discontinuity of the momentum distribution at the Fermi surface and measures the amount of correlations included in the model. As is well known the pairing modifies the chemical potential which is calculated self-consistently with the gap equation from the closure equation for the density of neutrons. In our approximation it is given by

$$\begin{aligned} \rho &= 2 \int \frac{d^3 p}{(2\pi)^3} \int \frac{d\omega}{2\pi i} G_p^s(\omega^+) \\ &\approx 2 \int \frac{d^3 p}{(2\pi)^3} \frac{\mathcal{Z}_p}{2} \left(1 - \frac{\omega_p}{\sqrt{\omega_p^2 + \tilde{\Delta}_p^2}} \right). \end{aligned} \quad (14)$$

The prefactor 2 is due to spin degeneracy. Our numerical investigation is based on the solution of the two coupled gap equations, Eqs. (12) and (15), along with the self-energy approximated to the second order of the hole-line expansion.

The Argonne V_{14} potential has been adopted as the pairing interaction which is consistent with the self-energy data where the same force has been used. The gap equation has been solved in the form of Eq. (12), coupled with Eq. (15). This is quite a satisfactory approximation, especially in view of studying the self-energy effects. The results are reported in Fig. 3 for a set of different k_F values. The solid line represents the solution of the gap equation in the standard BCS limit with the free single-particle spectrum. This is very close to the prediction obtained replacing the bare mass by the effective mass calculated in the BHF but still keeping $\mathcal{Z}=1$ (dotted line). This similarity stems from the fact that at the Fermi surface m^*/m from BHF is close to one as shown in Fig. 2. The self-energy effects are estimated in two approximations. In the first one m^* and the \mathcal{Z} factor are calculated from the approximation $\Sigma = \Sigma^1$ in a BHF code. In the considered density domain the \mathcal{Z} factor is ≈ 0.9 . Despite its moderate reduction a strong suppression of the gap is obtained as shown in Fig. 3 (upper long-dashed line). It is due to the exponential dependence of the gap on all quantities. Still a further but more moderate reduction is obtained when the rearrangement term is included in the second approximation $\Sigma \approx \Sigma^1 + \Sigma^2$ (short-dashed line). The smaller \mathcal{Z} factor

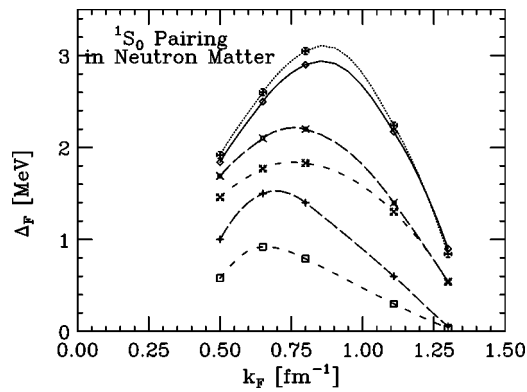


FIG. 3. Energy gap in different approximations ($\tilde{\Delta}$ in the text). Solid line: free single-particle spectrum; dotted line: effective mass from BHF approximation and $Z=1$; upper (lower) long-dashed line: solving the gap equation in the form of Eq. (12) [Eq. (13)] with BHF effective mass and Z ; upper (lower) short-dashed line: solving the gap equation in the form of Eq. (12) [Eq. (13)] with EBHF effective mass and Z .

($Z \approx 0.83$ at $k_F = 0.8 \text{ fm}^{-1}$) is to a certain extent counterbalanced by an increase of the effective mass ($m^*/m \approx 1.2$ at the same k_F).

The self-energy corrections are mostly concentrated around the Fermi surface; therefore it is not appropriate to use Eq. (13) for a quantitative prediction of the gap, since it extends their effect beyond the Fermi surface. The self-energy effect turns out to be overestimated by Eq. (13) as we checked numerically. In Fig. 3 the results are reported for the

two adopted approximations of the self-energy (lower long-dashed and short-dashed lines, respectively).

In Ref. [8] the self-energy effect has also been investigated within the generalized gap equation but the self-energy has been considered only at the level of BHF approximation. However the approximations adopted in Ref. [8] for solving the gap equation underestimate the correlation effects so that only a moderate reduction is obtained. In the present approach, including only Σ^1 , the reduction is more pronounced. Taking into account, in addition, the rearrangement term Σ^2 results in a further non-negligible reduction.

In conclusion we have shown that the superfluidity of a strongly correlated Fermi system requires a description in the context of the generalized Gorkov approach. The fact that the quasiparticle strength can be significantly smaller than one cannot be counterbalanced by a corresponding enhancement of the effective mass. Moreover we have shown that reliable predictions from the generalized gap equation can only be obtained if the correlation effects are fully taken into account. We have treated here neutron matter but we expect that the self-energy effects on the gap are very similar in symmetric nuclear matter. The next important step forward will be to include not only self-energy effects but also, on an equal footing, vertex corrections. This shall be studied in a future work.

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