# $x$ and $\xi$ scaling of the nuclear structure function at large $x$

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Inclusive electron scattering data are presented for $^2$H, C, Fe, and Au targets at an incident electron energy of 4.045 GeV for a range of momentum transfers from $Q^2=1$ to 7 $(\mathrm{GeV}/c)^2$. Data were taken at Jefferson Laboratory for low values of energy loss, corresponding to values of Bjorken $x \gtrsim 1$. The structure functions do not show scaling in $x$ in this range, where inelastic scattering is not expected to dominate the cross section. The data do show scaling, however, in the Nachtmann variable $\xi$. This scaling appears to be the result of Bloom-Gilman duality in the nucleon structure function combined with the Fermi motion of the nucleons in the nucleus. The resulting extension of scaling to larger values of $\xi$ opens up the possibility of accessing nuclear structure functions in the high-$x$ region at lower values of $Q^2$ than previously believed.

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Deep inelastic electron scattering (DIS) from protons has provided a wealth of information on the parton structure of the nucleon. In general, the nucleon structure functions $W_1$ and $W_2$ depend on both the energy transfer ($\nu$) and the square of the four-momentum transfer ($-Q^2$). In the Bjorken limit of infinite momentum and energy transfer, the structure functions depend only on the ratio of $Q^2/\nu$ (modulo QCD scaling violations). Thus, when taken as a function of Bjorken $x$ ($=Q^2/2M\nu$, where $M$ is the mass of the proton), the structure functions are independent of $Q^2$. In the parton model, $x$ is interpreted as the longitudinal momentum fraction of the struck quark, and the structure function can be related to the quark momentum distributions. This scaling was observed in high energy electron-proton scattering at SLAC, confirming the parton picture of the nucleon. Violations of Bjorken scaling arise at low $Q^2$ due to effects coming from kinematic corrections and higher-twist effects. A better scaling variable for finite $Q^2$ comes from the operator product expansion treatment of DIS, as was shown in Ref. [1]. Using the Nachtmann variable $\xi=2x/(1+\sqrt{1+4M^2x^2/Q^2})$ avoids additional scaling violations arising from finite $Q^2$ corrections to $x$ scaling (which is derived in the infinite momentum limit).

Scaling in $x$ should also be seen in electron-nucleus scattering as both $\nu$ and $Q^2$ approach $\infty$. Because $x$ represents a momentum fraction, it must be between 0 and 1 for scattering from a nucleon. When scattering from a nucleus, $x$ can vary between 0 and $A$, the number of nucleons, due to the nucleon momentum sharing. At finite $Q^2$ and large $x$ ($x \gtrsim 1$), additional scaling violations come from quasielastic (QE) scattering off of a nucleon in the nucleus, rather than scattering off of a single quasifree quark. The quasielastic contribution to the cross section decreases with respect to the inelastic contributions as $Q^2$ increases due to the nucleon elastic form factor, but QE scattering dominates at very low energy loss (corresponding to $x>1$) up to large values of $Q^2$.

Previous measurements of inclusive electron scattering from nuclei for $x \lesssim 3$ and $Q^2 \lesssim 3$ $(\mathrm{GeV}/c)^2$ (SLAC experiment NE3 [2]) showed scaling for $x \leqslant 0.4$, but a significant $Q^2$ dependence for larger $x$ values. For these $x$ values, the momentum transfer is low enough that quasielastic and resonance contributions to the scattering violate the expected scaling in $x$. When the structure function was examined as a function of $\xi$, the behavior was completely different. The data appeared to be approaching a universal curve as $Q^2$ increased, even in regions where the scattering was predominantly quasielastic. This behavior is similar to the local duality observed by Bloom and Gilman [3,4] in the proton structure function. Local duality is basically the observation that the structure function in the resonance region, *when averaged over a range in* $\xi$, has the same behavior as the deep inelastic structure function. It was suggested [2] that in the nucleus, the nucleon momentum distribution would perform this averaging of the structure function, causing the QE and DIS contributions to have the same $Q^2$ behavior, thus leading to scaling for all values of $\xi$. More recent measurements (SLAC experiment NE18 [5]) showed continued scaling behavior up to $Q^2=6.8$ $(\mathrm{GeV}/c)^2$, but the data were limited to values of $x$ very close to 1.

Several calculations were able to reproduce the data fairly well (e.g., Refs. [6–8]) with variations at high $x$ coming from differences in the high momentum components and final state interactions used in the calculations. In most cases the quasielastic and inelastic contributions were calculated seperately, and no attempt was made to give insight into the origin of $\xi$ scaling. One explanation for the origin of $\xi$ scaling was proposed by the Benhar and Liuti [9]. They suggested that the apparent scaling might instead come from an accidental cancellation of $Q^2$ dependent terms, and would occur only for a limited range of momentum transfers [up to $Q^2\sim 7.0$ $(\mathrm{GeV}/c)^2$]. With the new data from Jefferson Lab, we can show that this suggestion is not sufficient to explain the observed scaling.

The present data, from experiment E89-008 at Jefferson Lab, were taken with an electron beam energy of 4.045 GeV for scattering angles between 15 and 74 °, covering a $Q^2$ range from 1 to $7(\mathrm{GeV}/c)^2$. The scattered electrons were measured in the high momentum spectrometer (HMS) and short orbit spectrometer (SOS) in Hall C. Data were taken using cryogenic hydrogen and deuterium targets and solid targets of C, Fe, and Au. Details of the experiment and cross section extraction can be found in Refs. [10,11].

For unpolarized scattering from a nucleus, the inclusive cross section (in the one-photon-exchange approximation) can be written as

$$\frac{d\sigma}{d\Omega dE'}=\sigma_{Mott}[W_2+2W_1\tan^2(\theta/2)], \tag{1}$$

where $\sigma_{Mott}=4\alpha^2E^2\cos^2(\theta/2)/Q^4$, $\theta$ is the scattering angle, and $W_1(\nu,Q^2)$, $W_2(\nu,Q^2)$ are the structure functions. An explicit separation of $W_1$ and $W_2$ requires performing a Rosenbluth separation, which involves measuring the cross section at a fixed $\nu$ and $Q^2$ while varying the incident energy and scattering angle. Because the data is taken at fixed beam energies, we make an assumption about the ratio of the longitudinal to transverse cross section, $R=\sigma_L/\sigma_T=(1+\nu^2/Q^2)W_2/W_1-1$, to extract $W_2$. Given a value for $R$, we can determine the dimensionless structure function $\nu W_2$ directly from the cross section:

$$\nu W_2=\frac{\nu}{1+\beta}\,\frac{d\sigma/d\Omega dE'}{\sigma_{Mott}}, \tag{2}$$

where

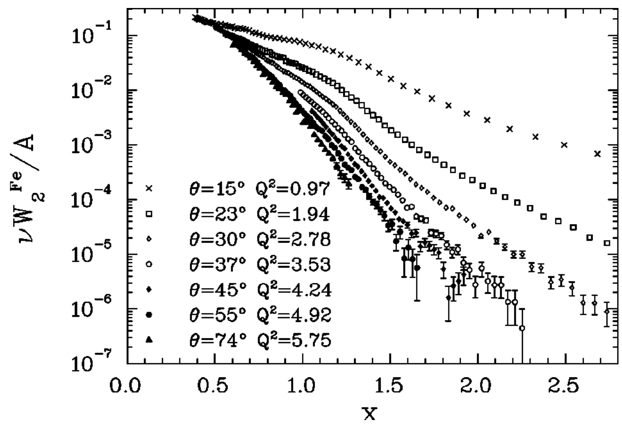


FIG. 1. Structure function per nucleon vs $x$ for iron from the present measurement. The $Q^2$ values given are for $x=1$. Errors shown are statistical only.

$$\beta=2\tan^2(\theta/2)\,\frac{1+\frac{\nu^2}{Q^2}}{1+R}. \tag{3}$$

For our analysis, we use the parametrization $R=0.32/Q^2$ [12], and assign a 100% uncertainty to this value. This parameterization comes from the nonrelativistic plane-wave impulse approximation (PWIA) for quasielastic scattering. It is also consistent with data taken in the DIS region $[0.2<x<0.5$ for $Q^2$ up to 5 $(\mathrm{GeV}/c)^2]$ [13] and a measurement of $R$ near $x=1$ in a $Q^2$ range similar to that of the present experiment [12].

For the HMS ($\theta\leqslant 55°$), the systematic uncertainty in the cross section is typically 3.5–4.5 %, dominated by acceptance, radiative corrections, and bin centering. For the high $x$ points, the systematic uncertainties become larger because of the strong kinematic dependence of the cross section, but are always smaller than the statistical uncertainties. The uncertainty in $R$ causes an additional uncertainty in the extracted structure function of 0.5–5.0 %, which is largest for the largest scattering angles. For the SOS ($\theta=74°$), the total systematic uncertainty in the structure function is typically $\sim$12% (due mostly to large background from pair production), somewhat larger at the highest values of $x$.

Figure 1 shows the extracted structure function for iron as a function of $x$. As in the previous data [2], scaling is seen only for values of $x$ significantly below one, where DIS dominates and resonance and QE contributions are negligible. However, when taken as a function of $\xi$ (Fig. 2), the structure function shows scaling for nearly all values of $\xi$. At low $\xi$, DIS dominates, and scaling behavior is expected from the parton model. For intermediate and high values of $\xi$, where the QE contributions can be significant or even dominate the cross section, the indications of scaling seen in previous data [2] are confirmed.

Figure 3 shows the structure function versus $Q^2$ for iron at several values of $\xi$. At low values of $\xi$, we see a rise in the structure function at low $Q^2$, corresponding to the QE scattering (at fixed $\xi$, low values of $Q^2$ correspond to larger values of $x$). This is followed by a fall to the high-$Q^2$ limit

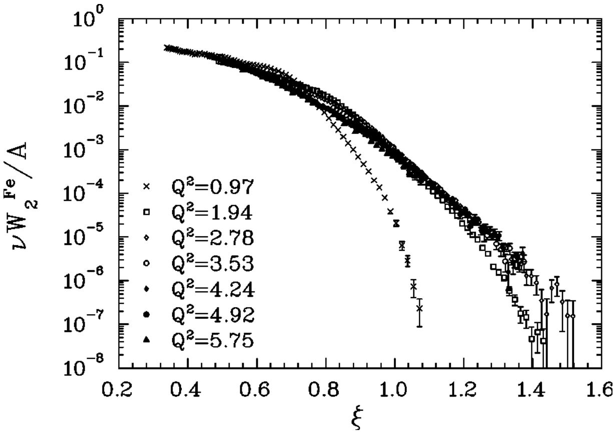


FIG. 2. Structure function per nucleon vs $\xi$ for iron. The $Q^2$ values are given for $x=1$. Errors shown are statistical only.

as the inelastic contributions dominate the scattering. Higher values of $\xi$, corresponding to $x>1$ for all $Q^2$ values measured, contain significant QE contributions. For all values of $\xi$, the structure function is nearly constant, with variations typically less than 10–20%, for $Q^2>2-3$ $(\mathrm{GeV}/c)^2$. Based on structure function evolution observed at high $Q^2$ for fixed (large) values of $\xi$, QCD scaling violations would be expected to cause roughly a 10% decrease in $\nu W_2$ for a factor of two increase in $Q^2$.

The measured structure function is similar for all heavy nuclear targets measured, although the kinematic coverage for the other targets (especially gold) is less than for iron. At values of $\xi$ corresponding to the top of the quasielastic peak, the structure function decreases slightly with $A$, as the increased Fermi momentum broadens and lowers the peak. At extremely high values of $\xi$, the structure function per nucleon is nearly identical for all of the heavy nuclei. Figure 4 shows the structure function for carbon, iron, and gold at $\xi=1.1$ and 1.2.

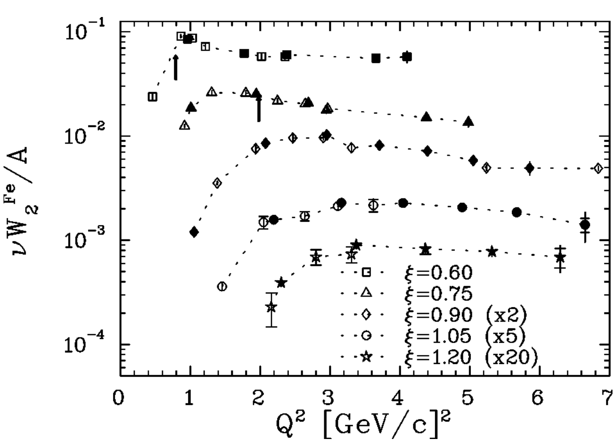


FIG. 3. Structure function per nucleon for Fe as a function of $Q^2$. The hollow points are from the SLAC measurements [2,5]. Dotted lines connect data sets at fixed values of $\xi$. The inner errors shown are statistical, and the outer errors are the total uncertainties. The arrows indicate the position of the QE peak ($x=1$) for $\xi=0.6$ and 0.75.

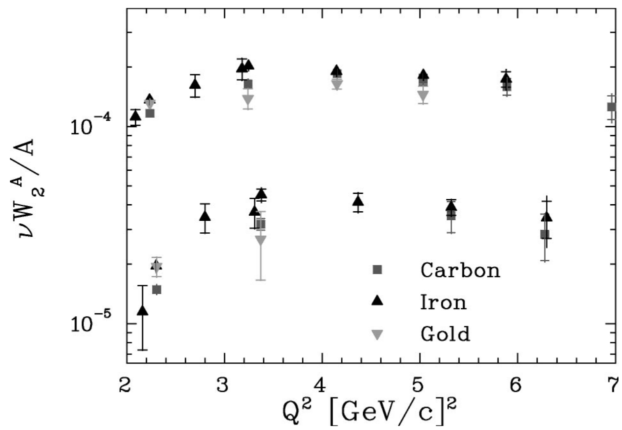


FIG. 4. Structure function per nucleon for C, Fe, and Au as a function of $Q^2$. The upper set of points is for $\xi=1.1$, and the lower set of points corresponds to $\xi=1.2$.

With this new data, it can be shown that the explanation of Ref. [9] is not enough to lead to the observed $\xi$ scaling. It assumes $y$ scaling in the PWIA (where $y$ is the minimum allowed momentum of the struck nucleon along the direction of the virtual photon) [14,15], with scaling violations coming from FSIs and from the transformation from $y$ to $\xi$. For very large values of $Q^2$ ($Q^2>>M_N$), $y$ can be written in terms of $\xi$, with corrections of order $1/Q^2$:

$$F(y)=F[y(\xi,Q^2)]=F\left(y_0(\xi)-\frac{M_N^3\xi}{Q^2}+O(1/Q^4)\right), \quad (4)$$

where $y_0(\xi)\equiv M_N(1-\xi)$. At $y=-0.3$ GeV/$c$ [which corresponds to $\xi\approx1.1$ for $Q^2\gtrsim2$ $(\mathrm{GeV}/c)^2$], the scaling violations from the exact transformation from $y$ to $\xi$ are $>200\%$ between $Q^2=2$ $(\mathrm{GeV}/c)^2$ and $Q^2=4$ $(\mathrm{GeV}/c)^2$, and $\gtrsim50\%$ between $Q^2=4$ $(\mathrm{GeV}/c)^2$ and $Q^2=6$ $(\mathrm{GeV}/c)^2$. This would imply that a $y$-scaling analysis of the data would show similarly large scaling violations. Such an analysis of the new data [10] indicates that final-state interactions produce $\lesssim10\%$ deviations from scaling for these values of momentum transfer, far too small to cancel the transformation induced scaling violations.

In addition, even in a region where the scaling violations from FSIs and the kinematic transformation from $y$ to $\xi$ cancel, this would not lead to $\xi$ scaling. Assuming $y$ scaling in the PWIA and a cancellation between FSIs and the transformation implies only that one would observe scaling in $F(\xi)$, the $y$-scaling function *taken as a function of* $\xi$. This does not explain scaling of the structure function $\nu W_2(\xi,Q^2)$. The additional transformation from $F(\xi,Q^2)$ to $\nu W_2(\xi,Q^2)$ would lead to significant scaling violations, even if there were perfect cancellation between the FSIs and the kinematic transformation.

While the proposed explanation does not lead to the observed scaling, the quality of the scaling indicates that there is some connection between the $y$-scaling picture of quasielastic scattering and the $\xi$-scaling picture of the DIS. While the $\xi$-scaling analysis involves removing only the Mott cross section, and the $y$-scaling analysis also removes the strongly

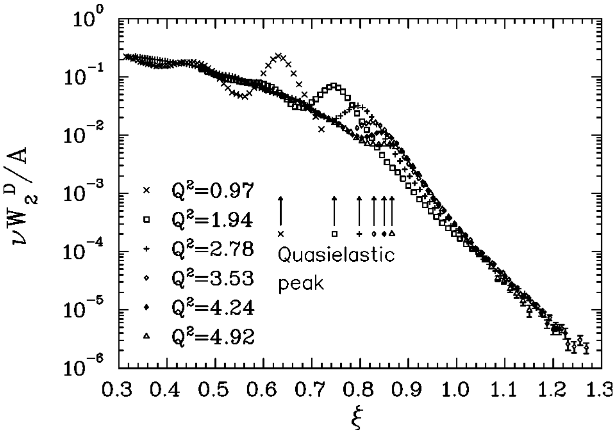


FIG. 5. Structure function per nucleon for deuterium. The $Q^2$ values are given for $x=1$. Statistical errors are shown.

$Q^2$-dependent elastic form factor, both show scaling above $Q^2 \gtrsim 3(\mathrm{GeV}/c)^2$ in the region of low energy loss. In this region, the cross section is dominated by quasielastic scattering and there is no expectation that $\xi$ scaling should be valid. While the connection between $\xi$ scaling and $y$ scaling in nuclei is not fully understood, it is essentially the same behavior as seen by Bloom and Gilman [3] in resonance scattering from a free proton. They measured $\nu W_2^p$ as a function of an improved scaling variable, $x' = Q^2/(2M\nu + M^2)$, and observed that while there was significant resonance scattering at high $x'$ and low $Q^2$, the resonance structure, when averaged over a range in $x'$, agreed with the DIS limit of the structure function. The resonance peaks fall more rapidly with $Q^2$ than the DIS contributions, but at the same time move to larger values of $x'$. The DIS structure function falls with increasing $x'$, at a rate which almost exactly matches the falloff with $Q^2$ of the resonance (and elastic) form factors. This behavior also holds when examining the structure function in terms of $\xi$ instead of $x'$ [16] (note that in the Bjorken limit, $x = x' = \xi$).

In nuclei, this same behavior leads to scaling in $\xi$. When $\nu W_2^A$ is taken as a function of $\xi$, the QE peak falls faster with $Q^2$ than the deep inelastic scattering component, but also moves to larger values of $\xi$. In the case of the proton, the resonance behavior follows the scaling limit on average, but the individual peaks are still visible. In heavy nuclei, the smearing of the peaks due to the Fermi motion of the nucleon washes out the individual resonance and quasielastic peaks, leading to scaling at all values of $\xi$. Figure 5 shows the structure function versus $\xi$ for the deuteron. Because of the smaller Fermi motion in deuterium, the QE peak is still visible for all values of $Q^2$ measured and the scaling seen in iron is not seen in Deuterium near $x=1$ (indicated by the arrows in Fig. 5). Note that for $Q^2 \gtrsim 3(\mathrm{GeV}/c)^2$, the data still show scaling in $\xi$ away from the QE peak.

The success of $\xi$ scaling beyond the deep inelastic region opens up an interesting possibility. In the Bjorken limit, the parton model predicts that the structure functions will scale, and that the scaling curves are directly related to the quark distributions. At finite (but large) $\nu$ and $Q^2$, scaling is observed and it is therefore assumed that the structure functions are sensitive to the quark distributions. It is not clear that this assumption must be correct, but the success of scaling is taken as a strong indication that it is true. In nuclei, we see a continuation of the DIS scaling even where the resonance strength is a significant contribution to the structure function. This opens up the possibility of measuring quark distributions in nuclei at lower $Q^2$ or higher $x$. If one requires that measurements be in the deep inelastic regime [typically defined as $W^2 > 4(\mathrm{GeV}/c)^2$, where $W^2$ is the invariant mass squared of the final hadron state], data at large values of $x$ can only be taken at extremely high values of $Q^2$. Because the quark distributions become small at large $x$, and the cross section drops rapidly with $Q^2$, it can be very difficult to make these high-$x$ measurements in the DIS region. However, the observation of $\xi$ scaling indicates that one might be able to use measurements at moderate values of $Q^2$, where the contributions of the resonances are relatively small compared to the DIS contributions and where these contributions have the same behavior (on average) as the DIS.

A more complete understanding of $\xi$ scaling, through precision measurements of scaling in nuclei and local duality in the proton is required. High precision measurements of duality in the proton have been made recently at Jefferson Lab [16,17], and additional proposals have been approved that will extend these measurements to higher $Q^2$ [18]. There is also an approved experiment to continue $x>1$ measurements at higher beam energies, which will extend the present study of $\xi$ scaling in nuclear structure functions to significantly higher $Q^2$ [19]. Finally, there is an approved experiment that will make a precision measurement of the structure function in nuclei as part of a measurement of the EMC effect [20], which will make a quantitative determination of how far one can extend scaling in nuclei when trying to extract high $x$ nuclear structure.

In conclusion, we have measured nuclear structure functions for $x \gtrsim 1$ up to $Q^2 \approx 7(\mathrm{GeV}/c)^2$. The cross section for $x>1$ is dominated by quasielastic scattering and, as expected, does not exhibit the $x$ scaling predicted for parton scattering at large $Q^2$. However the data do show scaling in $\xi$, hinted at in previous measurements. The $\xi$ scaling in nuclei at large $x$ can be interpreted in terms of local duality of the nucleon structure function, with nucleon motion averaging over the resonances. Measurements of $\xi$ scaling and local duality, combined with a more complete understanding of the theoretical underpinnings of duality and $\xi$ scaling, may allow us to exploit this scaling to access high-$x$ nuclear structure functions, which can be difficult to obtain in the DIS limit.

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