Pauli distorted double folded potential

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A new method to incorporate the Pauli principle into the double folding approach to the nucleus-nucleus potential is proposed. The description of the exchange terms at the level of the quasiclassical one-body density matrix is used. It is shown that in order to take into account the Pauli blocking properly, a redefinition of the density matrices of the free isolated nuclei must be done. A solution to the self-consistent incorporation of the Pauli blocking effects in the mean-field nucleus-nucleus potential is obtained in the Thomas-Fermi approximation.

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I. INTRODUCTION

For several decades the microscopic calculations of the nucleus-nucleus potential to describe the scattering phenomena have been the subject of great interest in heavy ion physics [1–7]. A large variety of different theoretical models have been proposed to this aim. The difficulties in solving this problem are caused by the very complicated connection between the intrinsic degrees of freedom of the colliding nuclei and the dynamics of their relative motion. Therefore many assumptions to calculate the nucleus-nucleus potential at the numerical level are needed. However, these simplified approaches have to satisfy the fundamental quantum mechanical principles, and the Pauli principle is the most important one to be considered in the nucleus-nucleus scattering problem. To incorporate the Pauli principle into the standard coupled channel scattering theory, the resonating group method (RGM) [8] was proposed. However, even for the elastic scattering problem (one-channel approximation) the microscopic calculation of the effective Hamiltonian that describes the relative motion of the nuclei is very complicated for two reasons: (i) The antisymmetrization operator leads to very complicated nonlocal matrix elements, and (ii) the RGM equations are not of the Schrödinger type for relative motion due to the nontrivial energy dependence. Thus the numerical applications of the RGM are restricted to cases where the intrinsic wave functions can be based on the harmonic oscillator. However, these harmonic oscillator solutions are not very useful to describe the nucleus-nucleus scattering because of their unrealistic asymptotic behavior.

The double folded model (DFM) [1], which is less fundamental than RGM but starts from realistic nuclear densities, has become one of the most popular methods to calculate the real part of the optical potential. Using the DFM detailed fits to elastic scattering data for many systems are obtained [9–16]. While in earlier publications the one-particle exchange was described by a zero-range pseudopotential, more accurate methods have been developed later [6,7,11,17]. When one uses a finite range effective nucleon-nucleon force in the DFM calculation, the one-body density matrix (DM) for each nucleus is needed. In a simple harmonic oscillator model the DM is known explicitly. In a more realistic case it can be obtained numerically from the solution of the Hartree-Fock (HF) equations [18]. However, this is not suitable for a DFM calculation for two reasons. First, the nucleon-nucleon force and the local densities are used as independent inputs for the DFM. If one wants to calculate the DM within the HF method, the effective nucleon-nucleon force used for calculating the ground states of the colliding nuclei has also to be considered. This force can differ from the one used in the actual DFM calculation. Second, to calculate the DFM potential with a DM that is known numerically is not an easy task. Following the original DFM idea, approximations to express the DM by means of the local density are used. One of the most popular approaches to the DM is given by Campi and Bouyssy (CB) [19]. It consists of a resummation of the Negele-Vautherin expansion [20] and presents the DM in the Slater form with some effective momentum. Recently another approach to the DM based on the extended Thomas-Fermi theory (ETF) has been proposed [21]. It allows a very good description of the exact DFM potential (i.e., the DFM obtained with the exact DM) [22].

In heavy ion scattering the nuclear rainbow phenomena are observed in very precise experiments [9,10,12–15]. In order to explain these phenomena, a strongly attractive nucleus-nucleus potential at small distances (in the interior of nuclei) is of primary importance. In particular the systematics of the elastic scattering in the $^{16}O+^{16}O$ system [12,16,23], which has been measured with high precision over a large region of scattering angles and incident energies ($E_{lab}=75$–1120 MeV with 15 individual energies), has triggered the development of refinements of the DFM. The phenomenon of the nuclear rainbow scattering has been established in this system with the observation of primary Airy maxima in the region of energies between 350 MeV–1120 MeV, and the occurrence of the higher order Airy structures

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has been established down to the lowest energy of 75 MeV. These data have been successfully described by the most recent version of the DFM, which uses density-dependent *N-N* interactions adjusted to reproduce nuclear matter properties [24]. The volume integrals of the corresponding real and imaginary parts of the potentials obtained via fits to the angular distributions have been determined using the DFM or a phenomenological Woods-Saxon squared form factor. A constant rise of the volume integrals towards the lowest energies is observed, giving probably a maximum at a rather small energy. This fact illustrates the persistence of very deep potentials responsible for the observation of the rainbow scattering at low energies. In the description with the DFM of Ref. [24] the main part of the energy dependence of the potential is properly described; it originates from the consistently calculated exchange term. In addition an overall normalization factor that is smaller than unity (0.7–0.9) is needed, which has an additional energy dependence.

There are still questions concerning the theoretical foundations of the DFM. First of all, the DFM potential represents the interaction energy (''energy surface'') [25] of two nuclei that depends on the distance between the mean fields rather than on the dynamical radial variable. Second, the correct treatment of the Pauli principle has to take place.

The DFM is used to describe the potential of elastic scattering, thus it reflects the mean-field effects that occur if the two nuclei overlap in their ground states. In most of the DFM calculations the ''frozen density'' approximation (FDA) is used. It implies that the local densities of the colliding nuclei do not change during the interaction, which is valid at large distances and at high enough energy. This approach touches on the questions of the relative values of collision times and the readjustment times of the nuclear wave functions [4–6]. Selecting the purely elastic channel, also for small impact parameters with large density overlap, we project from the collision those processes in which the ground states of the nuclei are recovered. However, the intrinsic states of the colliding nuclei could nevertheless change during the interaction. Due to the Pauli principle, the occupied states in one nucleus are strictly forbidden for the nucleons of the second nucleus. This process would lead to a ''Pauli excitation,'' provided the momentum distortions are transformed into intrinsic excitations of the two fragments.

In the local nuclear matter approximation the Pauli blocking disturbs the local Fermi distributions of nucleons in the colliding nuclei, an effect that can be considered as a virtual dynamical excitation and that has been discussed in terms of a contribution to the kinetic energy term in the heavy ion potential [4,5]. If this virtual excitation is transformed into a real excitation of one of the nuclei, this will lead to a loss of flux in the elastic channel and consequently to a contribution to the absorption. Such processes are known in atomic physics as Pauli excitations. We also note that nuclear rainbow scattering is only observed in strongly bound systems involving α particles and α-cluster nuclei. This fact implies that the intrinsic excitations of the participating nuclei are suppressed due to the high energy levels of such excitations. Thus the backward scattering of α particles on α-cluster nuclei is related to scattering without energy transfer.

The paper is organized as follows. In the first section we derive the Pauli distorted double folding model (PDDFM) starting from momentum-dependent determinant wave functions using the orthogonalization procedure first considered by Fliessbach [3]. In the second section we discuss the semiclassical content of PDDFM that simplifies the problem significantly and explain the procedure to calculate it. We compare the PDDFM with the standard DFM and discuss the Pauli distortion effects on the nucleus-nucleus potential as applied to the $^{16}O-^{16}O$ elastic scattering in the third section. The summary is given in the last section.

II. PAULI DISTORTED DOUBLE FOLDING MODEL

The DFM potential for two nuclei consisting of N_1 and N_2 nucleons contains the direct and exchange terms and is defined as follows (see for example [16]):

$$V(\boldsymbol{D},\boldsymbol{p}) = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_{10}(\boldsymbol{r}_1)\rho_{20}(\boldsymbol{r}_2-\boldsymbol{D})v_d(s) + \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_{10}(\boldsymbol{r}_1,\boldsymbol{r}_2)\rho_{20} \times(\boldsymbol{r}_2-\boldsymbol{D},\boldsymbol{r}_1-\boldsymbol{D})v_e(s)e^{i\boldsymbol{p}\boldsymbol{s}/\hbar}. \quad (1)$$

Here ρ_{10} and ρ_{20} are the ground-state local densities (direct part) and DM (exchange part) of each nucleus, $\boldsymbol{p}$ is the relative momentum between two nucleons of different nuclei due to their relative motion [we will use $\boldsymbol{P}$ for the relative momenta of nuclei, while $\boldsymbol{p}$ refers to the relative momenta of the corresponding nucleons $\boldsymbol{P}=\mu\boldsymbol{p}$, $\mu=N_1N_2/(N_1+N_2)$], and $\boldsymbol{D}$ is the separation distance between the two centers that define the nuclear densities. Note that here μ corresponds to the reduced mass number. The form factors $v_d(s)$ and $v_e(s)$ of the direct and exchange effective nucleon-nucleon force depend on the nucleon-nucleon distance ($\boldsymbol{s}=\boldsymbol{r}_1-\boldsymbol{r}_2$). The DFM in the form (1) corresponds to the case when spin-isospin states are degenerated and each orbital state is occupied by four nucleons. Throughout the paper we will consider this case because it simplifies the presentation. The formula (1) can easily be generalized for asymmetric nuclei. In this latter case both proton and neutron densities for each nucleus are needed (see for example Refs. [28,33]). However, because only the proton density is available from the electron scattering, the symmetrical formula (1) is widely used.

The direct and exchange parts of the nucleon-nucleon force in general are defined as follows (see for example Refs. [26,27]): $\hat{v}_d(x_1,x_2)=\hat{v}(x_1,x_2)$ and $\hat{v}_e(x_1,x_2)=\hat{v}(x_1,x_2)\hat{P}^r_{12}$, where x consists of spatial $\boldsymbol{r}$ and spin-isospin s,t variables of the nucleons, while $\hat{P}^r_{12}$ stands for the exchange operator of the spatial coordinates. As usual we use the hat to define the operators. Let us consider the central force of the standard form that will be used in the following:

$$\hat{v}(x_1,x_2)=\sum_i v_i(s)(w_i+b_i\hat{P}^\sigma_{12}-h_i\hat{P}^\tau_{12}-m_i\hat{P}^\sigma_{12}\hat{P}^\tau_{12}), \quad (2)$$

where $\hat{P}_{12}^{\sigma(\tau)}$ is the spin (isospin) exchange operator and $v_i(s)$ is the common radial form factor of the force. One can also use a more general central force with different form factors for each exchange term, and the results presented here are easily generalized for this latter force. However, in this case the number of parameters that define the effective force will be much larger. One can recast (1) in terms of Eq. (2) by substituting

$$v_d(s) \to \sum_i X_{d,i} v_i(s),$$

$$v_e(s) \to \sum_i X_{e,i} v_i(s), \tag{3}$$

where $X_{d,i} = w_i + b_i/2 - h_i/2 - m_i/4$, and $X_{e,i} = m_i + h_i/2 - b_i/2 - w_i/4$ are the standard combinations of the exchange parameters that enter into the central nucleon-nucleon force of Eq. (2). In practice the effective nucleon-nucleon force consists of the sum of several terms that represent the short and long range components of the force. For the sake of simplicity we will drop the index *i* in the following and will consider only one term in Eq. (2).

In Eq. (1) the direct term depends on the local densities of each isolated nucleus, while the exchange contribution depends on the corresponding density matrices. Using the CB expansion of the DM or the ETF DM (see Ref. [21]) the exchange term can also be rewritten in terms of the local densities. These methods give the rather accurate expression for the DM $\rho_I(\boldsymbol{r}_1, \boldsymbol{r}_2)$ averaged over the direction $\boldsymbol{s} = \boldsymbol{r}_1 - \boldsymbol{r}_2$. Thus the DFM potential (1) becomes dependent on the modulus *D* and *p*. In order to be used in Eq. (1) there are two possible definitions [22] of the relative momentum of the nucleons $\boldsymbol{p}$: the local value $p^2 = 2m[E_{\text{c.m.}} - V(D)]/\mu$ and the ''global,'' or asymptotic value with $p^2 = 2mE_{\text{c.m.}}/\mu$. Here $E_{\text{c.m.}}$ is the energy of relative motion in the center-of-mass (c.m.) system. In the first case the system of coupled equations with $p = p(V)$ and $V = V(D,p)$ must be solved self-consistently for each separation distance *D*. Subscripts ''0'' for the local densities and the DM indicate that these correspond to the ground states of the isolated nuclei.

The formal foundation of the DFM can be found in the generalized Born-Oppenheimer method [3], where the potential between two nuclei is defined as follows:

$$V(\boldsymbol{D}) = E_0(\boldsymbol{D}) - E_0(D = \infty). \tag{4}$$

In this equation $E_0(\boldsymbol{D})$ is the energy of the two nuclei separated by the distance $\boldsymbol{D}$ without their relative kinetic energy: $E_0(\boldsymbol{D}) = E(\boldsymbol{D}) - P^2(\boldsymbol{D})/2\mu m$, which is the expectation value of the energy operator $\hat{H} - \hat{T}_R$ where $\hat{H}$ and $\hat{T}_R$ are the total microscopic Hamiltonian and the relative motion kinetic energy operator, respectively.

Equation (4) defines the ''energy surface,'' which cannot be strictly identified with the microscopic nucleus-nucleus potential [25] and depends on the parameter $\boldsymbol{D}$ rather than on the dynamical variable $\boldsymbol{R}$. The DFM potential of Eq. (1) also depends on the parameter $\boldsymbol{D}$. Therefore to derive the DFM, we start from anzatz (4). In order to calculate the energy $E(\boldsymbol{D}) = \langle \Phi | \hat{H} | \Phi \rangle$, one has to define the normalized wave function Φ, which describes two nuclei separated by distance $\boldsymbol{D}$ in the c.m. system. At infinite separation, $E(D = \infty)$ has to be equal to the sum of the intrinsic energies of the two isolated nuclei and their relative motion in the c.m. system: $E(D = \infty) = \epsilon_{10} + \epsilon_{20} + E_{\text{c.m.}}$.

In order to take into account the Pauli principle we start from the normalized many-particle wave function (Refs. [3,29])

$$\Phi = n(D)\hat{A}[\Phi_1(\boldsymbol{D}_1)\Phi_2(\boldsymbol{D}_2)e^{i(\boldsymbol{P}_1\boldsymbol{R}_1 + \boldsymbol{P}_2\boldsymbol{R}_2)/\hbar}], \tag{5}$$

where $\boldsymbol{D}_I (I = 1,2)$ are the centers of the nucleon coordinates; $\hat{A} = \Sigma_{\hat{P}} \delta_{\hat{P}} \hat{P}$ is the antisymmetrization operator; $\delta_{\hat{P}}$ the sign of the permutation $\hat{P}$; $\Phi_I(\boldsymbol{D}_I)$ are the wave functions of the interacting nuclei centered around $\boldsymbol{D}_I$; $\boldsymbol{P}_I, \boldsymbol{R}_I (I = 1,2)$ are the momenta and coordinates of the centers of the *I*th nucleus; $\boldsymbol{D} = \boldsymbol{D}_1 - \boldsymbol{D}_2$, and $n(D)$ is the normalization.

Now *E*, E_0, and *V* become functions of $\boldsymbol{D}$ and $\boldsymbol{P}$. If one uses for $P(\boldsymbol{D})$ its asymptotic value $P_{as} = \sqrt{2m\mu E_{\text{c.m.}}}$ (the global definition of the relative momenta) the formula (4) becomes $V(\boldsymbol{D}, \boldsymbol{P}_{as}) = E(\boldsymbol{D}, \boldsymbol{P}_{as}) - E(D = \infty, \boldsymbol{P}_{as})$. At infinite separation we have $E(D = \infty) = E_1 + E_2$, where E_I contains the nucleus center of mass motion. Neglecting the spurious c.m. motion (e.g., the energies of center mass motion in the single-particle potential of the shell model [29]), one can write $E_I \approx \epsilon_{I0} + P_I^2/(2N_I m)$ and $E(D = \infty)$ becomes $\epsilon_{10} + \epsilon_{20} + E_{\text{c.m.}}$.

Assuming that the center-of-mass momenta $\boldsymbol{P}_I$ depend on $\boldsymbol{D}$ and tend to their asymptotic values (in the c.m. system $P_I^2 = 2m\mu E_{\text{c.m.}}$) at infinite separation, the wave function (5) describes two nuclei moving freely with their relative motion perturbed by the nucleus-nucleus potential. In the c.m. system the wave function Φ depends on the parameters $\boldsymbol{D}$ and $\boldsymbol{P}$.

If $\boldsymbol{P}$ depends on $\boldsymbol{D}$ it could be chosen so as to ensure the energy conservation [3]: $E(\boldsymbol{D}, \boldsymbol{P}) = E(D = \infty, \boldsymbol{P}_{as})$, which gives $P(\boldsymbol{D}) = \sqrt{2m\mu[E_{\text{c.m.}} - V(\boldsymbol{D}, \boldsymbol{P})]}$ and corresponds to the local definition of the relative momenta. This potential in turn is used to obtain the scattering wave function of the two nuclei. Thus an iterative self-consistent procedure is used to calculate the scattering solution using the potential obtained with the plane wave relative motion as the first step.

The calculation of $E(\boldsymbol{D}, \boldsymbol{P})$ with arbitrary intrinsic wave function Φ_I is not an easy task. It becomes simpler if one uses single-particle shell model wave functions:

$$\Phi_I = \frac{1}{\sqrt{N_I!}} \hat{A}_I \prod_{\alpha \in I} \phi'_\alpha(x'_\alpha) = \frac{1}{\sqrt{N_I!}} \hat{A}_I \prod_{\alpha \in I} \phi_\alpha(x_\alpha), \tag{6}$$

where $I = 1,2$; *x* contains spatial $\boldsymbol{r}$ and spin-isospin variables *s*,*t*; $x = (\boldsymbol{r}, s, t)$, and $\phi'_\alpha(x')$ stands for the wave function of the shifted spatial argument $x' = (\boldsymbol{r} - \boldsymbol{D}_I, s, t)$: $\phi'_{\alpha \in I}(x') = \phi_\alpha(x)$. The index of each state α contains orbital and spin-isospin quantum numbers. It is the standard coordinate system that is used in the two-center shell models and in the DFM [26].

Using this approximation one can write the total wave function (5) in the form of a Slater determinant whose matrix elements can be calculated easily. To this aim one can introduce momentum-dependent single-particle states in the following way [3]:

$$\tilde{\phi}_{\alpha\in I}(x)\equiv\phi_{\alpha\in I}(x)\exp(i\boldsymbol{p}_I\boldsymbol{r}/\hbar);\quad \left(\boldsymbol{p}_I=\frac{\boldsymbol{P}_I}{N_I}\right). \tag{7}$$

Now the wave function (5) can be written as a Slater determinant,

$$\Phi=\frac{1}{\sqrt{N!\Gamma}}\hat{A}\prod_{\alpha\in 1,2}\tilde{\phi}_\alpha(x_\alpha)=\frac{1}{\sqrt{N!\Gamma}}\det\Lambda_{\alpha,\beta}, \tag{8}$$

where $N=N_1+N_2$, $\Lambda_{\alpha,\beta}=\tilde{\phi}_\alpha(x_\beta)$, $G_{\alpha,\beta}=\{\langle\tilde{\phi}_\alpha|\tilde{\phi}_\beta\rangle\}$ and $\Gamma=\det G$ is the Gram determinant (as defined for example in Ref. [30]) of the set $\{|\tilde{\phi}_\alpha\rangle\}$. The functions $\{\tilde{\phi}_\alpha(\boldsymbol{r})\}$ are square integrable and depend on the position vectors $\boldsymbol{D}_{I=1,2}$ and on the momenta $\boldsymbol{p}_{I=1,2}$. Note that the vectors $|\tilde{\phi}_\alpha\rangle$ are ordered in the sense that one can distinguish to which nucleus they belong to: $\alpha\in I;I=1,2$.

These functions are orthonormal, if they belong to the same nucleus, $G_{\alpha,\beta}\equiv\langle\tilde{\phi}_\alpha|\tilde{\phi}_\beta\rangle=\delta_{\alpha,\beta}(\alpha,\beta\in I)$, but they are not if α and β are states of different nuclei with $G_{\alpha,\beta}\neq\delta_{\alpha,\beta}$.

Due to the fact that the functions $\{\phi_\alpha\}$ are finite with respect to $\boldsymbol{r}$, the function $G_{\alpha,\beta}(D,p)\to 0$ if $D\to\infty$ and (or) $p\to\infty$. Thus the matrix G is strictly diagonal for infinite radial separation and for infinite separation in momentum space (p). This statement can also be approximately valid in the region of small radial overlap or at high enough energies.

If we have overlap with $D\neq 0$ or $p\neq 0$, the Gram determinant does not vanish $\Gamma=\det\{\langle\tilde{\phi}_\alpha|\tilde{\phi}_\beta\rangle\}\neq 0$. In this case the single-particle states of both nuclei are linearly independent and the vectors $\{|\tilde{\phi}_\alpha\rangle\}$ form a basis in the N dimensional subspace of the Hilbert space. This is due to their separation in r space as well as in momentum space. In the case of complete overlap, for $D\to 0$ and $p\to 0$, we have $\Gamma\to 0$, however, the wave function Φ remains well defined and tends towards the ground-state shell model configuration of the composite system [31]. We will not consider this case in the present paper because the values of the potential at zero separation play a negligible role in the elastic scattering problem. Using the well-known technique of Ref. [31], one can calculate the potential of Eq. (4) using the nonorthogonal basis $\{\tilde{\phi}_i\}$ which coincides with the DFM potential at large separation distances where the nonorthogonality vanishes,

$$V(\boldsymbol{D},\boldsymbol{P})=\langle T\rangle+\langle V\rangle-\frac{P^2(\boldsymbol{D})}{2\mu m}-\epsilon_1-\epsilon_2, \tag{9}$$

where

$$\langle T\rangle=\sum_{\alpha,\beta}\langle\tilde{\phi}_\alpha|t|\tilde{\phi}_\beta\rangle(G^{-1})_{\alpha,\beta} \tag{10}$$

and

$$\langle V\rangle=\frac{1}{2}\sum_{\alpha,\beta,\gamma,\delta}\langle\tilde{\phi}_\alpha\tilde{\phi}_\beta|\hat{v}|\tilde{\phi}_\gamma\tilde{\phi}_\delta\rangle[(G^{-1})_{\alpha,\gamma}(G^{-1})_{\beta,\delta}-(G^{-1})_{\alpha,\delta}(G^{-1})_{\beta,\gamma}]. \tag{11}$$

Here $t=-\hbar^2/2m\boldsymbol{\nabla}^2$ is the one-body kinetic energy operator and $\hat{v}$ the central effective nucleon-nucleon force. The matrix G^{-1} is the inverse matrix of G defined previously. If the states $\tilde{\phi}_\alpha$ are orthogonal, the matrix G becomes diagonal and one immediately obtains the DFM from Eq. (9).

At intermediate distances and energies the orthogonality of the single-particle states from different nuclei is violated in the overlap region, where the potential is quite important for the description of the experimentally observed nuclear rainbow scattering. If the nonorthogonality of the single-particle wave functions is significant enough, the usual DFM is expected to fail and one should use the full expressions given in Eqs. (9)–(11). Note that these equations are defined in the momentum-dependent basis $\tilde{\phi}_\alpha$ and are thus difficult to compare directly with the DFM expression.

We will therefore use another option [3]. If the set of states $\{\tilde{\phi}_\alpha\}$ is linearly independent ($\Gamma\neq 0$), it can be orthogonalized and one can consider the corresponding orthonormal set $\{\tilde{\psi}_\alpha\}$. The orthogonalization can be done by means of the Gram-Schmidt procedure (see for example Ref. [30]). One can write down the wave function (5) with the help of this new orthonormal basis. Expanding $\tilde{\phi}_\alpha=\Sigma_\beta C_{\alpha,\beta}\tilde{\psi}_\beta$, where $\det C\neq 0$ and using properties of the determinants, one will get $\Phi=\exp[i\sigma]\det\{\tilde{\psi}_\alpha(x_\beta)\}$, where $\sigma=\arg(\det C)$ (see also Ref. [31]).

Using this wave function, the kinetic energy reads

$$\langle T\rangle=\sum_\alpha\langle\tilde{\psi}_\alpha|t|\tilde{\psi}_\alpha\rangle=\frac{\hbar^2}{2m}\sum_I\int d\boldsymbol{r}\tilde{\tau}_I(\boldsymbol{r}), \tag{12}$$

where $\tilde{\tau}_I=\Sigma_{\alpha\in I}|(\boldsymbol{\nabla}\tilde{\psi}_\alpha)|^2$ is the kinetic energy density corresponding to the momentum-dependent basis $\tilde{\psi}_\alpha$. The potential energy is given by

$$\langle V\rangle=\frac{1}{2}\sum_{\alpha,\beta}[\langle\tilde{\psi}_\alpha\tilde{\psi}_\beta|\hat{v}|\tilde{\psi}_\alpha\tilde{\psi}_\beta\rangle-\langle\tilde{\psi}_\alpha\tilde{\psi}_\beta|\hat{v}|\tilde{\psi}_\beta\tilde{\psi}_\alpha\rangle]. \tag{13}$$

The set $\{\tilde{\psi}_\alpha\}$ is also ordered in the sense that one can distinguish to which nucleus each state belongs by considering its asymptotic behavior $\tilde{\psi}_\alpha\to\tilde{\phi}_\alpha$, if $D\to\infty$ (at finite p). To obtain an expression close to the DFM expression, let us introduce wave functions $\psi_\alpha\equiv\tilde{\psi}_\alpha\exp[-i\boldsymbol{p}_I\boldsymbol{r}/\hbar](I=1,2,\alpha\in I)$, which correspond to the nucleus rest frame.

However, contrary to ϕ_α, these wave functions ψ_α depend on the relative momentum $\boldsymbol{p}$. Introducing again $\psi'_\alpha(x')\equiv\psi_\alpha(x)$ one finally finds another definition of the DFM potential that we call the Pauli distorted double folding model (PDDFM). Assuming spin-isospin degeneracy (e.g., each orbital state is occupied by four nucleons [31]) and using Eq. (2) the nucleus-nucleus potential reads

$$V(\boldsymbol{D},\boldsymbol{P}) = X_d \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2) v(|\boldsymbol{r}_1 - \boldsymbol{r}_2 + \boldsymbol{D}|) + X_e \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1,\boldsymbol{r}_2)\rho_2 \times (\boldsymbol{r}_2 - \boldsymbol{D}, \boldsymbol{r}_1 - \boldsymbol{D}) v(s) e^{i\boldsymbol{p}\boldsymbol{s}/\hbar} + \varepsilon(D), \quad (14)$$

where the densities $\rho_I(\boldsymbol{r}_1,\boldsymbol{r}_2) = \Sigma_{st}\rho_I(x_1,x_2)$ are obtained with the wave functions ψ_α, and we use the definitions $x_1 = \boldsymbol{r}_1, s, t$, $x_2 = \boldsymbol{r}_2, s, t$, and $\rho_I(\boldsymbol{r}) = \rho_I(\boldsymbol{r},\boldsymbol{r})$. Notice that $\rho_I(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and $\rho_I(\boldsymbol{r})$ are related to the DM and the local densities are calculated with the orthogonal wave functions $\tilde{\psi}_\alpha$ through $\tilde{\rho}_I(\boldsymbol{r}_1,\boldsymbol{r}_2) = \rho_I(\boldsymbol{r}_1,\boldsymbol{r}_2)e^{i\boldsymbol{p}_I\boldsymbol{s}}$ and $\tilde{\rho}_I(\boldsymbol{r}) = \rho_I(\boldsymbol{r})$. The last term in Eq. (14) represents the excitation energy of the nuclei during the interaction and is given by $\varepsilon(D) = \varepsilon_1(D) + \varepsilon_2(D)$, with

$$\begin{aligned}\varepsilon_I &= \sum_\alpha \langle \tilde{\psi}_\alpha | t | \tilde{\psi}_\alpha \rangle + \frac{1}{2} \sum_{\alpha,\beta \in I} [\langle \tilde{\psi}_\alpha \tilde{\psi}_\beta | \hat{v} | \tilde{\psi}_\alpha \tilde{\psi}_\beta \rangle \\ &\quad - \langle \tilde{\psi}_\alpha \tilde{\psi}_\beta | \hat{v} | \tilde{\psi}_\beta \tilde{\psi}_\alpha \rangle] - \frac{P_I^2(\boldsymbol{D})}{2N_I m} - \boldsymbol{\epsilon}_I \\ &= \frac{\hbar^2}{2m} \int d\boldsymbol{r}[\tau_I(\boldsymbol{r}) - \tau_{I0}(\boldsymbol{r})]) \\ &\quad + \frac{X_d}{2} \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 [\rho_I(\boldsymbol{r}_1)\rho_I(\boldsymbol{r}_2) - \rho_{I0}(\boldsymbol{r}_1)\rho_{I0}(\boldsymbol{r}_2)] \\ &\quad \times v(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \\ &\quad + \frac{X_e}{2} \int d\boldsymbol{R} d\boldsymbol{s} [\rho_I^2(\boldsymbol{R},s) - \rho_{I0}^2(\boldsymbol{R},s)] v(s), \end{aligned} \quad (15)$$

where τ_{I0} and τ_I are the kinetic energy densities of the ground and excited states, respectively. We have used the spin-isospin degeneracy and the fact that $\tilde{\tau}_I(\boldsymbol{r}) = \tau_I(\boldsymbol{r}) + k_I^2 \rho_I(\boldsymbol{r})$, where $\tau_I = \Sigma_{\alpha \in I} |(\boldsymbol{\nabla}\psi_\alpha)|^2$ refers to the rest frame of each nucleus.

The nucleus-nucleus potential $V(D)$ given by Eq. (14) is formally equivalent to those of Eqs. (9)–(11) but differs from DFM for three reasons. First, the Pauli distorted DM $[\rho_I(x_1,x_2) = \Sigma_{\alpha \in I} \psi_\alpha^*(x_2)\psi_\alpha(x_1)]$ enters into Eq. (14) instead of those of the ground state for each isolated nucleus $[\rho_{I,0}(x_1,x_2) = \Sigma_{\alpha \in I} \phi_\alpha^*(x_2)\phi_\alpha(x_1)]$, which are used in the usual DFM as in Ref. [1]. Second, the direct term in Eq. (14) depends on the incident energy because the orthogonalization is performed at a given relative momentum p, which defines the momentum-dependent functions ψ_α. Finally, an intrinsic excitation energy term appears in the PDDFM.

In order to calculate the DM ρ_I one needs to know the relative momentum of the nucleons $\boldsymbol{p}(\boldsymbol{D})$ explicitly. In the DFM it is assumed that $p^2(D) = 2m[E_{\text{c.m.}} - V(D,p)]/\mu$. Thus, the problem of determining the potential taking into account the dependence of $\boldsymbol{p}(\boldsymbol{D})$ on the ''final'' potential has to be solved self-consistently. Using Eq. (5) as an anzatz, one can calculate a model nucleus-nucleus potential by self-consistently orthogonalizing the single-particle states (SPS) at each separation $\boldsymbol{D}$. It is necessary to emphasize that the considered excitations due to the Pauli principle are not of a dynamical origin. They are rather kinematic and contribute to the ''total kinetic energy'' in the potential (see also [4,5]) and will act as a repulsive potential term. In order to describe the actual excitations of the nuclei involved in the scattering, one should solve the true dynamical problem, which is an extremely difficult task.

In order to approach this self-consistent solution (see the next section), one can also define a momentum-dependent density matrix for each nucleus as follows: $\tilde{\rho}_I(x_1,x_2) = \Sigma_{\alpha \in I} \tilde{\psi}_\alpha^*(x_2)\tilde{\psi}_\alpha(x_1)$. The orthogonality of the single-particle states in different nuclei means that we have $\hat{\tilde{\rho}}_1 \hat{\tilde{\rho}}_2 = 0$ or $\hat{\tilde{\rho}}^2 = \hat{\tilde{\rho}}$, where $\hat{\tilde{\rho}} = \hat{\tilde{\rho}}_1 + \hat{\tilde{\rho}}_2$ is the sum of the two DM's.

Thus the PDDFM potential can be obtained starting from SPS ϕ_i of the isolated nuclei and using the orthogonalization procedure. Note that nothing is implied about the choice of these single-particle states. In fact, one can use the single-particle states of the isolated nuclei that correspond to the frozen density approximation (FDA). This procedure was used in Ref. [3] using harmonic single-particle states with the density-independent Brink-Boeker force. The shallow nucleus-nucleus potential was obtained within this approach.

However, the SPS and the mean field of one nucleus can change in the presence of the second nucleus and this complicates the problem significantly. Another problem in the application of the described procedure is the use of explicit single-particle states while the main advantage of the DFM is to employ the local densities only. It will be shown in the next section that these problems can be solved at a semiclassical level.

Due to the rotational invariance the nucleus-nucleus potential depends on the scalar product $(\boldsymbol{DP})$: $V = V[(\boldsymbol{DP})]$. This dependence is also contained in the DM entering into PDDFM. It was pointed out [3] that the dependence on the angle between $\boldsymbol{D}$ and $\boldsymbol{P}$ is very weak and the potential depends mainly on the modulus D and P. Thus in Eq. (14) one can use the DM $\rho_I(\boldsymbol{r}_1,\boldsymbol{r}_2)$ averaged over the direction $\boldsymbol{s}$.

III. SEMICLASSICAL APPROXIMATION

It is possible to simplify the calculations of the potential in Eq. (14) by using semiclassical approaches based on the Thomas-Fermi (TF) method and its extension. In fact, in most of the recent work on the DFM potential such kinds of approaches are applied. For example, the CB approximation [19] to the DM is used in many cases. The CB-DM is taken in the Slater form with an effective momentum that depends on the quantal kinetic energy density τ and the local density ρ. Thus, the CB-DM corresponds to a truncation of the full quantal DM. However, τ and ρ at a quantum level are unknown and therefore their semiclassical counterparts, which can be written in terms of the local density only, are used. In this case one obtains the semiclassical CB-DM, which corresponds to a truncation of the semiclassical DM in the extended Thomas-Fermi (ETF) approximation [21]. Thus, a semiclassical picture is actually included in the DFM.

In coordinate space the semiclassical density matrix is

given by the inverse Wigner transformation (see for example Ref. [32]) of the distribution function $f(\boldsymbol{R},\boldsymbol{p})$, which for a moving nucleus, reads

$$\tilde{\rho}_I(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{p}_I)=\frac{g}{(2\pi\hbar)^3}\int d\boldsymbol{p}'\tilde{f}_I(\boldsymbol{R},\boldsymbol{p}')e^{i\boldsymbol{p}'\boldsymbol{s}/\hbar}$$

$$=\frac{g}{(2\pi\hbar)^3}\int d\boldsymbol{p}' f_I(\boldsymbol{R},\boldsymbol{p}')e^{i(\boldsymbol{p}'+\boldsymbol{p}_I)\boldsymbol{s}/\hbar}$$

$$=\frac{g}{(2\pi\hbar)^3}\int d\boldsymbol{p}' f_I(\boldsymbol{R},\boldsymbol{p}'-\boldsymbol{p}_I)e^{i\boldsymbol{p}'\boldsymbol{s}/\hbar}, \quad (16)$$

where $\boldsymbol{R}=\frac{1}{2}(\boldsymbol{r}+\boldsymbol{r}')$ and $\boldsymbol{s}=\boldsymbol{r}-\boldsymbol{r}'$ while g stands for the spin and isospin degeneracy. For the ground state at the Thomas-Fermi level (the $\hbar^0$ order term in the Wigner-Kirkwood expansion [32]) we have $f_I(\boldsymbol{R},\boldsymbol{p}')=\Theta[p_{F_I}(\boldsymbol{R})-p']$, where $\Theta(x)$ is the unit step function $\Theta(x)=1$ at $x\geqslant 0$ and $\Theta(x)=0$ otherwise. Thus, $\tilde{f}_I(\boldsymbol{R},\boldsymbol{p}')=f_I(\boldsymbol{R},\boldsymbol{p}'-\boldsymbol{p}_I)=\Theta[p_{F_I}(\boldsymbol{R})-|\boldsymbol{p}'-\boldsymbol{p}_I|]$ is just the distribution function related to the DM $\tilde{\rho}_I$, as defined in the previous section.

We suggest that during the interaction these Fermi spheres can deform so that at each point in coordinate space one can define for each nucleus an effective Fermi volume Ω_{F_I} and a distribution function $f_I(\boldsymbol{R},\boldsymbol{p}')=\Theta[p_{F_I}(\omega_{p'},\boldsymbol{R})-p']$, where the momentum p_{F_I} is related to the local density of the nucleus I at the considered point in coordinate space and depends on its orientation $\omega_{p'}$ in momentum space. At a semiclassical level it is not possible to introduce the single-particle states explicitly and the orthogonality condition should be formulated in terms of the semiclassical DM. We assume that the quantum orthogonality condition $\hat{\tilde{\rho}}_1\hat{\tilde{\rho}}_2=0$ has to be fulfilled at the semiclassical level as follows: $(\hat{\tilde{\rho}}_1\hat{\tilde{\rho}}_2)_W=0$, where the subscript W stands for the Wigner transformation of the quantal operator. At the TF level (considering only $\hbar^0$ terms in the Wigner-Kirkwood expansion) one will get

$$(\hat{\tilde{\rho}}_1\hat{\tilde{\rho}}_2)_W=(\hat{\tilde{\rho}}_1)_W(\hat{\tilde{\rho}}_2)_W$$
$$=f_1(\boldsymbol{R}-\boldsymbol{D}_1,\boldsymbol{p}'-\boldsymbol{p}_1)f_2(\boldsymbol{R}-\boldsymbol{D}_2,\boldsymbol{p}'-\boldsymbol{p}_2)=0. \quad (17)$$

By using translational invariance in the c.m. system ($\boldsymbol{P}_1+\boldsymbol{P}_2=0$) we will get the relation

$$\Theta[p_{F_1}(\omega_{p'},\boldsymbol{R}-\boldsymbol{D}_1)-p']\Theta[p_{F_2}(\omega_{p'},\boldsymbol{R}-\boldsymbol{D}_2)-|\boldsymbol{p}'+\boldsymbol{p}|]=0. \quad (18)$$

This means that the Fermi volume of the two interacting nuclei should not overlap in momentum space. Returning to the case of the standard DFM potential, one can see that at a semiclassical level the nonorthogonality of the single-particle states from different nuclei at finite values of distance D and relative momentum p means that their Fermi spheres overlap in momentum space, as shown in the upper part of Fig. 1; this overlap region is forbidden by the Pauli principle. If $p\to\infty$, these Fermi spheres are separated and no overlap occurs. At a given value of $\boldsymbol{R}$, the Fermi momenta of one of the nuclei is $p_{F,1}=p_{F,1}[\rho_1(\boldsymbol{R})]$, while for the second nucleus it stands as $p_{F,2}=p_{F,2}[\rho_2(\boldsymbol{R}-\boldsymbol{D})]$. If $D\to\infty$ and $\boldsymbol{R}$ is finite, the values of $\rho_2(\boldsymbol{R}-\boldsymbol{D})$ and $p_{F_2}\to 0$ and their overlap become zero too. At finite p and D the overlap will appear, implying that the DFM cannot be applied and the full orthogonalization procedure has to be used. There is a significant difference between the quantal and the semiclassical orthogonality conditions. In the first case the orthogonalization procedure defines a distorted density matrix of the interacting nuclei (up to a unitary transformation of the orthonormalized basis). In the semiclassical approximation the single-particle states are not defined and this orthogonalization procedure is not applicable. In order to solve this situation, we use the following geometrical anzatz. If there is no overlap of the initial Fermi spheres, the states of the isolated nuclei are not perturbed and the Pauli principle will not affect the DFM potential. If there is an overlap, we assume that the distribution functions of the interacting nuclei are just the Fermi spheres truncated by the plane going through the curve along the connection line of the initial Fermi spheres. This anzatz is displayed in the lower part of Fig. 1. This is not a unique solution, but is probably the simplest assumption that has already been used previously to calculate the adiabatic nucleus-nucleus potential in the nuclear matter approach such as in Ref. [2].

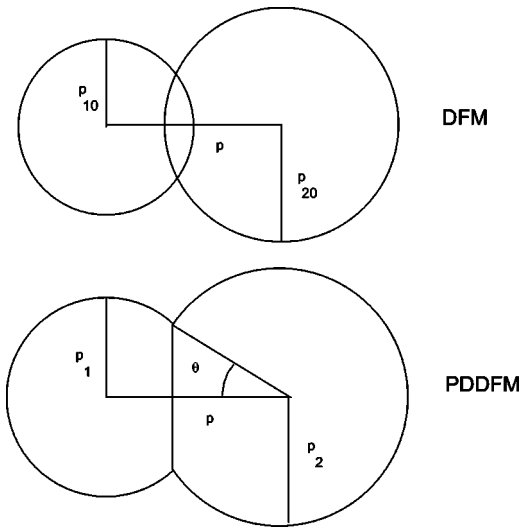

FIG. 1. The Fermi spheres that correspond to the different points in coordinate space of the interacting nuclei separated by the local momentum $p(D)$ in momentum space. The upper part corresponds to the usual double folding model (DFM), where the Fermi spheres with momenta p_{I0} overlap. In the lower part the truncated Fermi spheres with momenta p_I corresponding to the PDDFM are shown.

In fact the deformation of the Fermi spheres for two interacting nuclei can be very complicated. However, the nucleus-nucleus potential reflects the global properties of the colliding nuclei. Therefore, to use truncated Fermi spheres in the present approach can be considered as an average over their different excitations and seems to be reasonable for our aim.

Actually, the Thomas-Fermi approximation corresponds to a local nuclear matter approach. The truncated spheres are determined by two parameters: the radius p_{F_I} and the angle θ as defined in Fig. 1. The angle θ depends on the relative momenta p and the Fermi momenta p_{F_I}. For the case of distortion we have to calculate new values of p_{F_I}. To this aim one can note that the value of the local density of each nucleus is determined by the distribution function as follows:

$$\rho_I(\boldsymbol{R}) = \frac{g}{(2\pi\hbar)^3}\int d\boldsymbol{p}' f_I(\boldsymbol{R},\boldsymbol{p}') = \frac{g}{(2\pi\hbar)^3} V_{F_I}(p_{F_I},\boldsymbol{p}), \tag{19}$$

where we assume a sharp border distribution function f_I and V_{F_I} is the volume in momentum space occupied by nucleons of a given nucleus after the distortion. To determine the value of p_{F_I} we have to know the volume V_{F_I} or the value of the local density ρ_I. There are at least three options to determine ρ_I that correspond to three different approximations.

(i) For fast (large p) or peripheral (large D) collisions the overlap of the initial Fermi spheres from different nuclei is rather small and the total configuration in momentum space has a well-developed two-piece picture. In this case the FDA is often used. In our semiclassical consideration the FDA simply means that the local densities of the nuclei do not change during the interaction $\rho_I = \rho_{I,0}$. However, to satisfy the Pauli principle one has to deform the Fermi distributions of the colliding nuclei, which means that the corresponding DMs change during the interaction $\rho_I(\boldsymbol{r},\boldsymbol{r}') \neq \rho_{I,0}(\boldsymbol{r},\boldsymbol{r}')$. Note, that in our approach the FDA only means that the local densities are fixed while usually the FDA consists of fixing the single-particle states (i.e., the DM). In this case the values of $p_{F,I}$ are simply determined by the conservation of the volume in momentum space $V_{F_I} = V_{F_{I0}}$.

(ii) At lower energies (or deeper penetration) the adiabatic process starts to give contributions. In this case the interacting nuclei still keep their individuality but the intrinsic degrees of freedom of each nucleus start to change to a new equilibrium configuration due to the presence of the second nucleus [quasiadiabatic approximation (QAA)]. In this case at each separation distance D one can find the DM (and local densities) of each nucleus, which minimize their energies under the assumption that the phase space available for the nucleons in one nucleus is restricted by the presence of the second due to the Pauli-blocking effect. Continuing along these lines, one will get new values of the nuclear local densities ρ_I that determine the volume V_{F_I} in momentum space and consequently the value of p_{F_I}.

(iii) Finally, in a very slow collision (or a total overlap in D space), the fully adiabatic process has to be considered. In this case the total density of the composite system tends to its equilibrium value to give the minimum energy of the total system. In this case there is only one Fermi sphere in momentum space that corresponds to the total density of the compound system.

At finite energy all these considered cases occur at different points in coordinate space. It means that the FDA used in the present version for the PDDFM potential is still valid in some external region, but cannot be used to describe the potential at the smallest distances. In this situation a comment is needed. The effective nucleon-nucleon forces that are used in the DFM also depend on the total density of the system, which reflects the in-medium properties of the force. In our semiclassical approach (FDA or QAA) the total distribution function is just the sum of distribution functions of the interacting nuclei and, therefore, the total local density is simply given by the sum of the local nuclear densities $\rho = \rho_1 + \rho_2$. In the FDA the local densities of the nuclei do not change during the interaction and the ''sudden approximation'' for the total density is used. In contrast, in the QAA case the densities ρ_I change due to the minimization of the intrinsic energies.

Now we are ready to derive the necessary formulas for the truncated Fermi spheres at the TF approximation level. The DM of the ground state at the TF level is given by a step function in momentum space. This latter case corresponds to the full Fermi sphere in momentum space. If there is overlap we define the truncated Fermi spheres (see Fig. 1). Some words of caution must be added. The truncated Fermi sphere corresponds to an excited state of the nucleus, because the states that correspond to the forbidden overlap region are depopulated and new states with another Fermi momentum p_F are occupied. In this case the distribution function for each nucleus is given by

$$f_I(\boldsymbol{R},\boldsymbol{p}') = \Theta[p_{F_I}(\omega_{p'},\boldsymbol{R}) - p'], \tag{20}$$

where the new Fermi momenta $p_{F_I} = p_{F_I}(\omega_{p'},\boldsymbol{R})$ depend on their orientation in p space. The DM's for the truncated Fermi-spheres averaged over the direction of $\boldsymbol{s}$ are obtained as

$$\rho_I(\boldsymbol{R},s) = \frac{g p_{F_I}^3}{12\pi^2\hbar^3}\left[\hat{j}_1(p_{F_I}s/\hbar)(1-x_0) + \frac{3\hbar^3}{(p_{F_I}s)^3}[x_0\sin(p_{F_I}s/\hbar) - \sin(x_0 p_{F_I}s/\hbar)]\right], \tag{21}$$

where $\hat{j}_1(x) = (3/x)j_1(x)$ is normalized to unity at $x = 0$, $j_1(x)$ is the spherical Bessel function of order 1, $x_0 = \cos(\theta)$ is the cosine of the angle that is determined by the point where the new Fermi spheres cross (see Fig. 1), and g stands for the degeneracy in spin and isospin. At $s = 0$ we obtain the local densities that correspond to the truncated Fermi spheres

$$\rho_I(\boldsymbol{R}) = \frac{g p_{F_I}^3}{24\pi^2\hbar^3}(2 - 3x_0 + x_0^3). \tag{22}$$

One can see that if there is no overlap we have $x_0 = -1$. Then Eqs. (21) and (22) give the usual formulas for the full Fermi spheres. By changing the DM (and to some extent the local density) we will change the intrinsic energies of the

nuclei for a given distance D. The kinetic energy densities enter into the intrinsic energy term (15) of the PDDFM potential (14). For the ground state at the TF level the kinetic energy density reads

$$\tau_{I0}(\boldsymbol{R}) = \frac{g p_{F_{I0}}^5}{10\pi^2\hbar^5}.$$

To calculate the kinetic energy density of the excited state τ_I, the integration of the TF distribution function (20) is taken over the truncated Fermi sphere of radius p_{F_I}:

$$\tau_I(\boldsymbol{R}) = \frac{g}{(2\pi)^3\hbar^5}\int d\boldsymbol{p} p^2 f_I(\boldsymbol{R},\boldsymbol{p}), \tag{23}$$

and we obtain

$$\tau_I(\boldsymbol{R}) = \frac{g p_{F_I}^5}{80\pi^2\hbar^5}(4-5x_0+x_0^5), \tag{24}$$

where again the sign of x_0 is chosen to be $x_0=-1$ if there is no overlap.

It is interesting to note that in both the QAA or the FDA cases the proposed model needs no new parameters: all the quantities that enter into the final formulas are determined within the framework of the present formalism. The difference between the FDA and the QAA approaches appears only in the definition of the Fermi momenta p_{F_I}: in FDA they are taken from the condition that the densities of nuclei do not change (conservation of the Fermi volume in momentum space), while in QAA they are determined self-consistently. In both cases the formalism in calculating the distorted Fermi spheres can be used.

To this end we would like emphasize the difference between the method proposed in Ref. [3] and our semiclassical PDDFM. First, no explicit shell model is used in our approach in contrast to the harmonic shell model of Ref. [3]. This allows us to use realistic nucleon densities and effective forces, which is important for the calculation of nucleus-nucleus potentials. Second, the FDA in Ref. [3] means that the mean fields of the nuclei do not change during the interaction, while in our approach the FDA implies that the local densities of nuclei are frozen and the densities overlap. Third, our semiclassical PDDFM is defined only in the classically allowed region where $P^2(D)>0$ and one can separate Fermi spheres in momentum space. Thus it can be used at high enough energies and with effective forces, which give attractive potentials consistent with the semiclassical content of PDDFM. Hence a direct comparison of the results obtained in Ref. [3] and here is not possible. The relation between these two approaches will be discussed elsewhere.

IV. DISCUSSION OF THE ^{16}O + ^{16}O SYSTEM

In order to judge the reliability of the present approach we apply our PDDFM in the FDA to the ^{16}O+^{16}O system. In these reactions the nuclear rainbow phenomena in the elastic scattering are observed in a wide energy range. This case has been successfully described with the DFM potential using the BDM3Y1 (Paris) effective nucleon-nucleon force. The main conclusion from this analysis is that deep potentials are needed over the full energy range from $E_{lab}=1150$ MeV to 75 MeV. Actually, the depth of DFM potentials has to be renormalized by a factor $N_R(E)$ that tends to unity at high energies and approaches a value of 0.7 at lower energies. This behavior of the $N_R(E)$ factor can be considered as an ''experimental'' fact. One part of this renormalization is considered to occur due to the ''polarization'' potential introduced by inelastic couplings, which are rather weak in the present case. It appears now that these renormalizations can be understood if we consider the present solution to the problem of the Pauli distortion in the DFM. Actually, we expect that this distortion becomes significant (Fermi spheres overlap in momentum space) at low energies and consequently reduces the depth of the DFM potential.

In the present analysis we use the density-dependent BDM3Y1 force with both the Paris and the Reid-Elliott form factors. The parameters of these interactions are taken from Ref. [33]. In Fig. 2 the intrinsic excitation of the ^{16}O nuclei due to Pauli distortion at $E_{lab}=75$ MeV (index a) and $E_{lab}=750$ MeV (index b) is plotted as a function of the separation distance D. We use the two possible definitions of the relative momenta as discussed in the main text: the global asymptotic (index 1 in Fig. 2) and the local (index 2 in Fig. 2) values. In the second case the problem of the self-consistency of the equations for the final potential has been solved. One can see that the effect of the Pauli distortion in the global case is stronger then in the local case. This is due to the increase of the relative momenta in the inner region, where the depth of the potential is more than 100 MeV. One can see that at high enough energies (750 MeV) the distortion of the intrinsic state is rather weak and gives a small contribution to the total energy for all distances D. At small energy (75 MeV) the situation is different and the intrinsic excitations in the local case reach up to 18 MeV at zero separation. However, at these small distances the FDA may not be valid and one must consider the QAA approach.

In Figs. 3 and 4 the changes of the PDDFM potential relative to the DFM potential are plotted at different energies for the Paris and Reid-Elliott M3Y forces. The effect of the Pauli distortion would be very strong (see Fig. 3) if one uses the asymptotic value of the momenta (i.e., global definition of the relative momentum) resulting in up to a 50% difference in the potentials at $E_{lab}=75$ MeV. This difference has a rather monotonic dependence that decreases at large separation distances. Figure 3 also illustrates the result obtained with the local (self-consistent) definition of the relative momenta $p(D)$: the large difference between the DFM and PDDFM is now suppressed by the increase of $p(D)$ in the interior. Apart from the trivial result that the Pauli distortion decreases when the incident energy increases, one can see that a prominent maximum for the contribution of the Pauli blocking appears at some distance R_{PB} (Fig. 4). The position of this maximum tends to smaller radial distances in the interior for higher energies. In order to understand this behavior, we look into the local definition of the relative momentum $p(D)$. If the potential is deep enough, it will

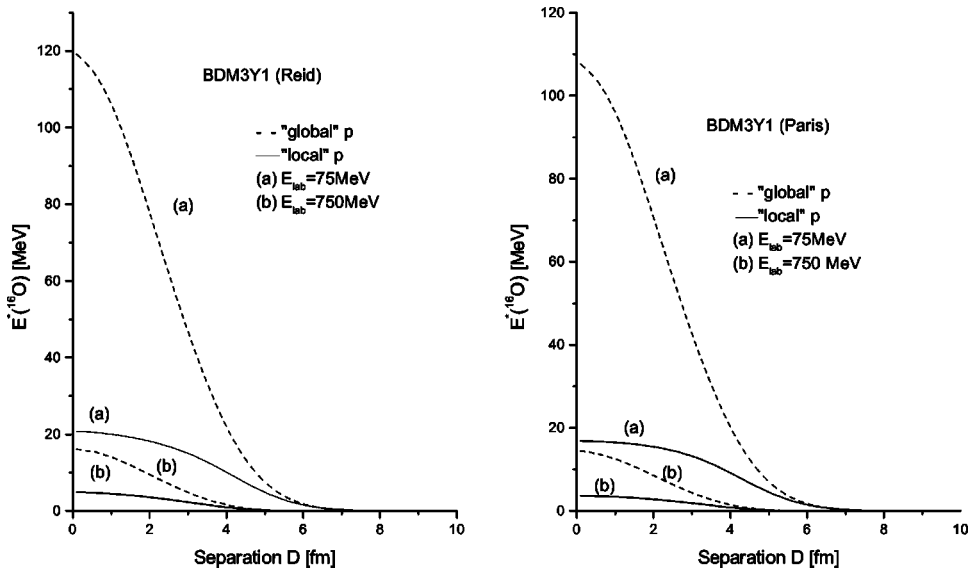

FIG. 2. The intrinsic excitation of the ^{16}O nuclei due to the Pauli effect, calculated for two energies $E_{lab}=75$ MeV (a) and 750 MeV (b), with the ''global'' definition (1) and with the local definition (2) of the relative momenta of the nucleons with the BDM3Y1 (Reid) and BDM3Y1 (Paris) nucleon-nucleon forces.

strongly increase the momentum $p(D)$. With these large relative momenta we will have effectively a ''repulsion'' of the Fermi spheres at small distances. The smaller overlap in momentum space will reduce the Pauli distortion. Still it is interesting to note that the maximum of the Pauli distortion appears in the range of distances between 3 and 6 fm, where the deep potential determines the occurrence of nuclear rainbow scattering.

Another important quantity associated with the nucleus-nucleus potential is its volume integral J_V. We have calculated the volume integrals for both DFM and PDDFM potentials at different energies using the BDM3Y1 (Paris) as well as the BDM3Y1 (Reid) versions of the nucleon-nucleon force. The results are presented in Fig. 5. The ''experimental'' values of J_V have been obtained by fits of experimental angular distributions with the optical potential, whose real part was taken to be of the Woods-Saxon square type or from the DFM with the proper renormalization [12,16] as mentioned before. One can see that at high enough energy the two approaches, PDDFM and DFM, give similar values of J_V, and that they reasonably agree with the ''experimental'' values. At lower energies a significant difference appears: the DFM gives increasing values of J_V, while the PDDFM gives a smooth maximum at an energy around 100 MeV. In fact the data from Ref. [12] exhibit almost constant values of J_V in the energy range of 75 MeV$<E_{lab}<$124 MeV, which can be considered as in agreement with our PDDFM results. At small energies they still overestimate the ''experimental'' values, a fact that can be a consequence of the FDA violation. Two comments may be added here: (a) the ''experimental'' absolute values of J_V may depend on the minimization procedure (the shape of potentials, the imaginary parts, and other details; see for example J_V for SW2 and DFM in Ref. [16]) and (b) our DFM potential slightly differs from the one used in Refs. [12,16], where the authors have used the CB-DM. This last DFM also contains some uncer-

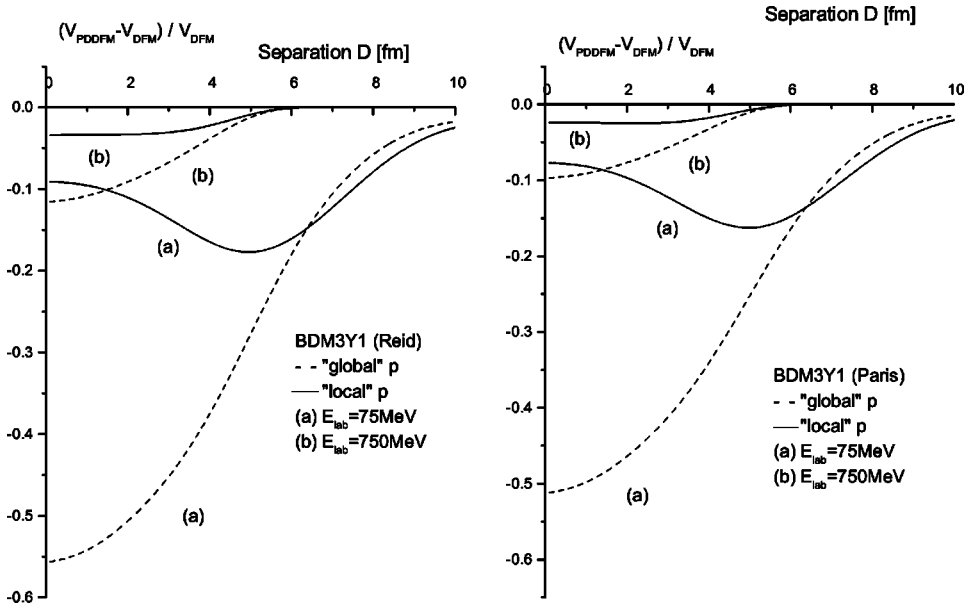

FIG. 3. The relative deviation of the PDDFM potential with respect to the DFM potential for the ^{16}O$-^{16}$O system calculated at two energies $E_{lab}=75$ MeV (a) and 750 MeV (b), with the ''global'' (1) and local definitions (2) of relative momentum of the nucleons calculated with BDM3Y1 (Reid) and BDM3Y1 (Paris) nucleon-nucleon forces.

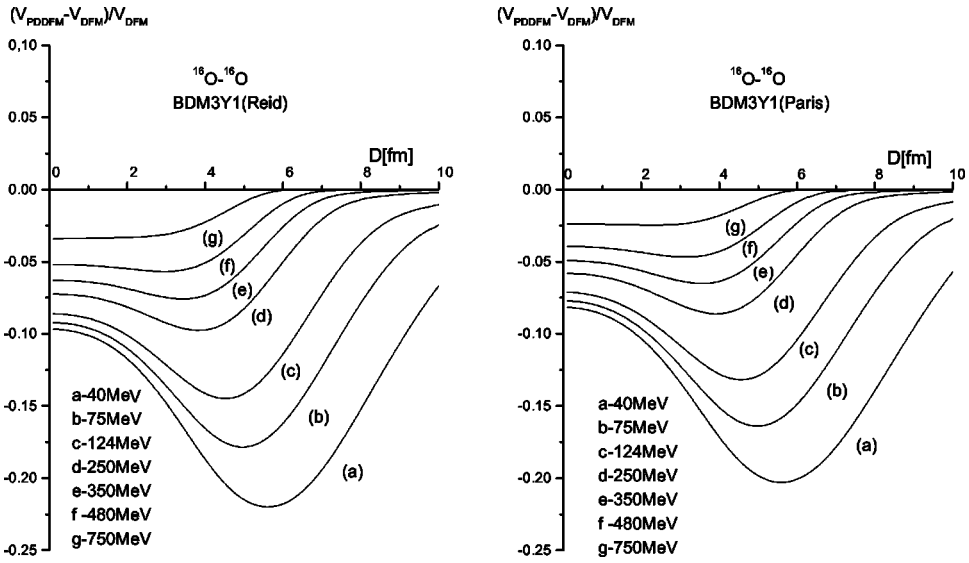

FIG. 4. The relative deviation of the PDDFM potential with respect to the DFM potential for the $^{16}O-^{16}O$ system with the local definition of the relative momentum calculated with BDM3Y1 (Reid) and BDM3Y1 (Paris) nucleon-nucleon forces at different energies in the laboratory.

tainties because CB-DM depends on the quantal kinetic energy density that is unknown. Therefore the empirical expression for the kinetic energy density was used in Refs. [12,16]. Our analysis shows that the DFM potential with the semiclassical kinetic energy density in CB-DM is systematically deeper and can differ locally within 10% from those of Refs. [12,16]. This difference can be found in the values of volume integrals, where it amounts to approximately 5% (see below). In contrast we have used in the present work the semiclassical Thomas-Fermi approximation, which is self-consistent with the semiclassical result for the nucleus-nucleus potential.

The experimental systematics for the volume integral J_V confirm our result obtained with the microscopic DFM potential. To describe the experimental data with the DFM a normalization factor smaller than unity $N_R(E)<1$ has been introduced in Refs. [12,16]. In the PDDFM this reduction is understood as a consequence of Pauli blocking.

To illustrate the validity of the approach we present here some preliminary results for the analysis of $^{16}O+^{16}O$ elastic scattering data. A systematic analysis of the experimental data in a wide energy region within the PDDFM approach will be given in forthcoming publications. Our aim here is to illustrate the Pauli distortion effect in the nucleus-nucleus potential. It was already shown that the difference between the two approaches, DFM and PDDFM, become larger at smaller energies. At the same time one can expect that the FDA will not be valid at very small energies. Thus we present here the analysis of data at ''intermediate'' energies of $E_{lab}=124$ MeV and 145 MeV. The imaginary part of the optical potential was taken in the standard way as the sum of the Woods-Saxon (WS) volume shapes and the derivative for the WS surface term [12]. The parameters of the imaginary part were fitted to minimize the χ^2 value calculated with a uniform 10% error for the data points. The result is shown in Fig. 6. The solid lines represent the best fit with the optical model using the PDDFM potential for the real part together with the renormalization constant N_R as indicated in the fig-

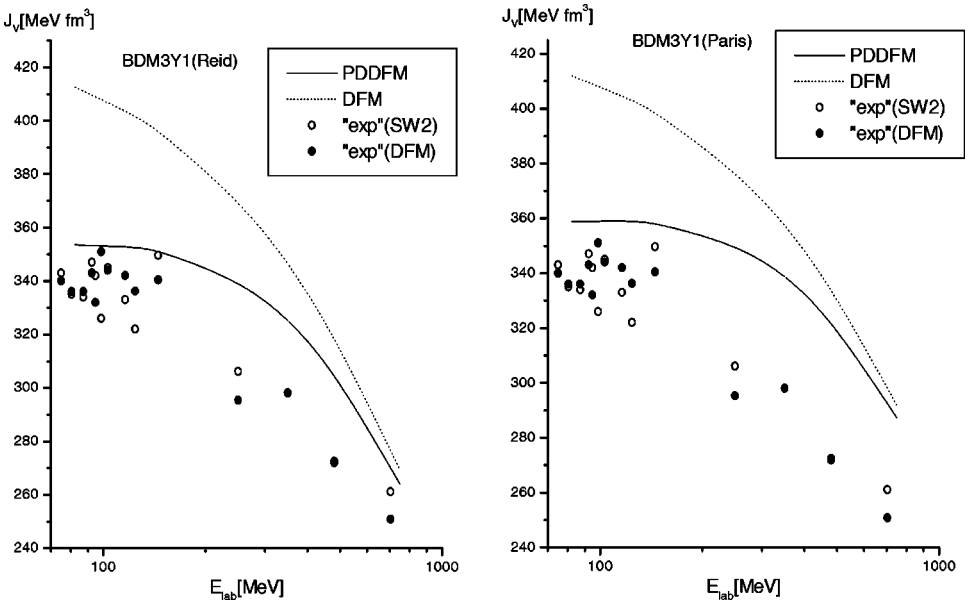

FIG. 5. The volume integrals of the PDDFM and DFM potentials calculated with BDM3Y1 (Reid) and BDM3Y1 (Paris) forces at different energies together with the ''experimental'' results. The open circles correspond to the results obtained with the real part of optical potential of the squared Woods-Saxon form, the closed ones with the renormalized DFM potential. The data at energies 124–704 MeV are taken from Ref. [16] and at energies 75–124 MeV from Ref. [12].

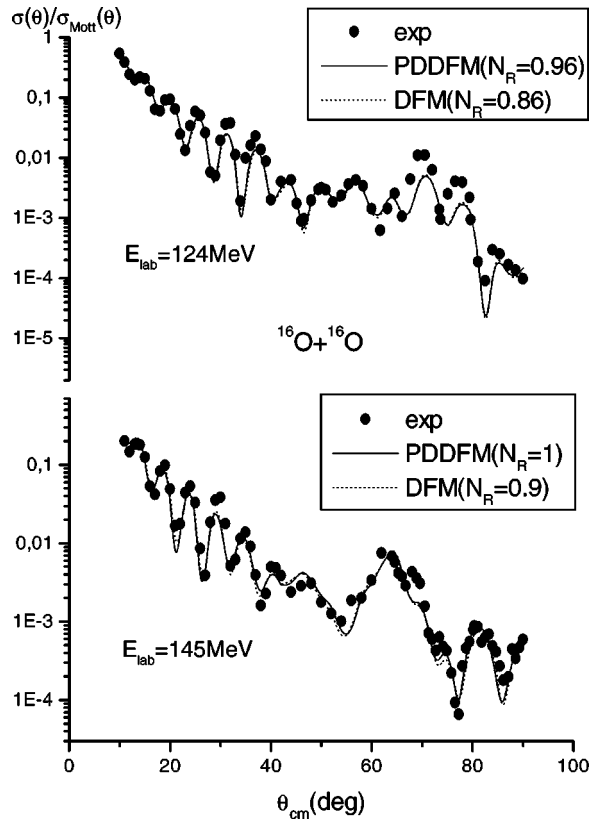

FIG. 6. Angular distributions of the elastic $^{16}O-^{16}O$ scattering at energies $E_{lab}=124$ MeV and 145 MeV, calculated within the optical model with renormalized potentials for the real part of nucleus-nucleus potential with the PDDFM (solid lines) and DFM (dotted lines) approaches, respectively, together with the experimental data from Ref. [34].

ure, while dotted lines represent the result for the DFM. There are still ambiguities within the parameters of the imaginary part, but the factor N_R is well determined, and can be determined with high accuracy. We note that at 124 MeV it is necessary to reduce the PDDFM potential by a factor 0.96, while for the DFM potential the reduction factor needed is 0.86. At 145 MeV the best fit was obtained with the unrenormalized PDDFM potential, while the corresponding DFM potential has to be reduced by a factor $N_R=0.9$. These results completely agree with our discussion of the volume integrals presented in the Fig. 5. Comparing our values with the those reported in Ref. [16] one can also see that the present version of the DFM differs from the DFM of Ref. [16] by approximately 5% with respect to the volume integrals (both with renormalization or without) and gives practically the same angular distributions (with the proper renormalization). We note that the values for the volume integrals J_V corresponding to the fit with the PDDFM shown in Fig. 6 are 344 and 360 MeV fm^3, respectively. These values differ slightly from the values of Ref. [16] (which are 336 and 340 MeV fm^3, respectively). This illustrates the range of ambiguities in the determination of the ''experimental'' values of J_V.

At this point the following remark has to be made. The calculated nucleus-nucleus potentials may strongly depend on the choice of the density-dependent effective nucleon-nucleon force. The DFM as a mean-field approach to the nucleus-nucleus potential needs a realistic nucleon-nucleon interaction, which is able to describe nuclear matter properties (e.g., the saturation point). It is known that M3Y forces produce DFM potentials with two parts, the purely repulsive direct part and the strong attractive exchange part, and the total DFM potential is very deep and thus reduces the Pauli distortion effects. In contrast, the Brink-Boeker force gives a shallower potential. In this case the Pauli distortion is strong. However, it is deeper in our approach than the one obtained in Ref. [3], due to differences in the methods (see comments above). This latter force has no density-dependence and does not reproduce the saturation properties of nuclear matter. The density dependent Gogny force gives a deeper potential, but it is still shallower than that obtained with the M3Y force. The Pauli distortion in this case is also stronger than in the M3Y case. The concept of deep local potentials is confirmed also by the semiclassical RGM analysis of Ref. [35], where it has to be deep enough to account for the Pauli forbidden states into the discrete spectrum of the effective Hamiltonian.

V. SUMMARY

In the present paper we have proposed the Pauli distorted double folding model (PDDFM) for the nucleus-nucleus potential. It coincides with the usual DFM asymptotically, i.e., at high energies and (or) at large distances. In order to comply with the Pauli principle at lower energies and for larger density overlap, one has to modify the Fermi spheres of the interacting nuclei in order to prevent their overlap in momentum space. The corresponding density matrices of the nuclei are defined at the semiclassical Thomas-Fermi level. The parameters of the new truncated Fermi spheres can be determined uniquely within the framework of the frozen density or the quasiadiabatic approximations. Thus, no new parameters are introduced in the PDDFM relative to the original version of the DFM.

The local definition of the relative momenta of the nucleons that are used in the DFM implies that the Pauli effects in the nucleus-nucleus potential have to be calculated self-consistently in the same way as the exchange term entering in the DFM potential as in Ref. [11]. The potential created by the mean-field increases the relative momentum of the nucleons in the region of density overlap, which suppresses the Pauli distortion significantly. In fact, the mean field energy of the two overlapping nuclei obtained in the DFM approach produces a very deep potential already at moderate overlap. In the self-consistent approach the contribution from this potential is comparable to the Fermi momenta of the nucleons, resulting in a ''repulsion'' of the two Fermi spheres. Such a repulsion of the momentum spheres has also been discussed in a mean-field approach for collisions at much higher energies in Ref. [36]. It has been shown in the present approach that the nucleus-nucleus potential remains rather deep down to the lowest energies of 6–10 MeV/nucleon giving rise to refractive scattering, and higher order Airy structures are thus observed down to these low energies. This result can be considered as an explanation for the success of the widely

used DFM. We note that in the calculations of potentials in DFM or PDDFM no free parameters (except for the imaginary part) are needed to reproduce the data.

The Pauli distortion discussed in the present approach can lead to excitations of the two fragments, inducing a loss of flux from the elastic channel (absorption). However, the transformation of the distortion in momentum space into real energy excitations of the nuclei depends on their structure and excitation energy spectrum. If no energy can be transferred the scattering process may remain elastic. Actually the partial wave *S*-matrix elements contributing to the rainbow angles are of the order of 10^{-3} or even below for the $^{16}O + ^{16}O$ case. Scattering systems with nuclei with closed shells or with alpha clustering have large energy gaps for particle-hole excitations, and are therefore particularly suited to observe refractive scattering, because of the reduced absorption. The PDDFM approach thus also gives insight into the observation that refractive scattering (with reduced absorption in the interior) is mostly observed for heavy ion systems consisting of strongly bound nuclei.

The Pauli distortion effect gives a maximum contribution at some intermediate distance, which tends to the interior and decreases when the energy increases. The analysis of the volume integrals of the real part of the nucleus-nucleus potentials shows that the PDDFM gives flat maxima at low energy. In order to check this behavior one has to compare it with the values obtained from the phenomenological optical model analysis of data at lower energies. However, low energy data on rainbow scattering are scarce and their analysis shows some ambiguities. Therefore, new measurements should be done in order to get a more detailed test of the present model at the lower energies.

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