Shell model study of neutron-rich nuclei near ^{132}Sn

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The structures of neutron-rich isotones $N=82$–84 of nuclei near ^{132}Sn have been studied within the shell model formalism using available interactions obtained from the well-studied ^{208}Pb region after proper scaling. The theoretically calculated binding energies, excitation energies, transition probabilities, magnetic moments have been compared with the experimental results. For $N=82,83$ isotones, the results agree reasonably well with the experimental data. But for $N=84$ isotones, the agreement between theoretical and experimental energies and transition probabilities is not satisfactory.

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I. INTRODUCTION

It is well known that doubly closed ^{132}Sn exhibits the strongest shell closure and nuclei with few valence nucleons around this show simple structure as in the ^{208}Pb region [1]. These few-valence-particle nuclei around doubly closed shell furnish useful information on single particle excitation energies, nucleon-nucleon interaction, and effective charges in this important part of nuclear chart and is therefore interesting to study both theoretically [2–17] as well as experimentally [18–42]. As in the ^{208}Pb region, the ^{132}Sn region provides an excellent opportunity for application of spherical shell model in truncated spaces and to test available effective nuclear interaction Hamiltonians in this region. As one goes on increasing valence particle number from a doubly closed shell nucleus such as ^{132}Sn, many different features may appear. Nuclei close to magic numbers $Z=50$ and $N=82$, with a few (2–3) neutrons or protons are expected to show predominantly single particle structure. These features are explained reasonably within truncated space shell model calculations [11,13–15,19,22,23,30]. On the other hand, as one departs from $Z=50$ proton and/or $N=82$ neutron closed shells, the increasing number of valence protons and/or neutrons soften the underlying ^{132}Sn core, thereby introducing collective features in the excitation spectra. Where it really happens and how strongly the truncated space shell model description starts failing are important questions for the model. For large number of valence nucleons, calculations may be prohibitively complex due to the extreme large dimensionality of the matrices involved within the same basis set. Another limitation arises due to the restriction in the set of basis states used, which may be inadequate with increase of collectivity.

Neutron-rich nuclei in the ^{132}Sn region lie away from the line of stability and are inaccessible by common techniques of nuclear spectroscopy. Recently, experimental information on the spectroscopic properties of nuclei in this region [18–26] are being available from fission product γ spectroscopy with highly efficient detector arrays. Nuclei with a few valence particles have a reasonably high density of excited states at medium spins and are therefore suitable for γ-spectroscopic measurements. Spectra and lifetimes of some levels as well as beta decay half-lives of some isotonic nuclei with $N=82$ and 83 have been obtained by analyzing fission product γ-ray data acquired at large detector arrays. It may therefore be of interest to calculate theoretically the spectroscopic properties of nuclei in this region and compare with the recent data.

Since long back, the $N=82$ isotones have been extensively studied both theoretically and experimentally [8–17,40–43]. Wildenthal [9,10] pointed out the suitability of shell model calculations to study the structure of these nuclei. With the recent increase in the experimental spectroscopic data of few valence particle nuclei around ^{132}Sn, there have been several attempts to explain the observed features theoretically with renewed interest, especially within the shell model [19,22,23,30]. The simplest method used for $N=82$ isotones [22,23,30] is to extract the two-body proton (π)-proton matrix elements directly from the experimental level spectrum of ^{134}Te. Although this approach only involves the diagonal elements and thus neglects the effect of configuration mixing, it provides a useful guidance in the preliminary interpretation of the observed levels. A detailed analysis of $N=82$ isotones ranging from ^{133}Sb to ^{154}Hf has been done by Wildenthal [44] (referred to in Ref. [23]) in a model space of $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ proton orbitals. A best fit set of single particle energies (SPEs) and two-body-matrix elements (TBMEs) consistent with the spectroscopic observables then available was also suggested. Later, Blomqvist updated the $N=82$ parametrization, mostly the diagonal ones [45] (referred to in Ref. [23]). There are also several detailed studies of $N=82$ isotones in this mass region with realistic effective interaction [4,5]. Andreozzi *et al.* [2,6] have performed shell model calculations for 2–3 valence protons $N=82$ isotones using a realistic effective interaction derived from Bonn *A* nucleon-nucleon potential.

Bhattacharyya *et al.* [30] have performed shell model calculations for $N=83$ isotones by using π-π TBMEs directly from ^{134}Te level spectrum and the π-neutron (ν) TBMEs

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are estimated from ^{210}Bi interactions with proper modifications under certain assumptions. They have also performed similar calculations [28] for $N=84$ nucleus ^{135}Sb by using some of the ν-ν TBMEs directly from ^{134}Sn experimental level spectrum [31]. Properties of $N=84$ isotonic even-even nuclei have been studied [19] using the shell model code OXBASH [46] with realistic interactions obtained with proper scaling of Kuo-Herling interaction [47,48] derived for the ^{208}Pb region.

After going through all the theoretical studies referred above, we observe the following.

(i) None of these calculations has been applied simultaneously, with a single interaction, to all the isotopes near ^{132}Sn (for $Z=50$–55 with $N=82$–85) for which experimental results are now available.

(ii) The electromagnetic properties have been calculated in detail for the $N=82$ isotones (some of the relevant references are [2,4–6,8,11–13,15]). But such calculations for some other isotones near $Z=50$ are certainly needed to compare with the available experimental data, such as transition probabilities, magnetic moments, etc.

(iii) Calculation and comparison of binding energy data are also inadequate.

(iv) No particular isotopic series has been systematically studied to definitely point out the mass or the valence nucleon number for which the truncated space shell model calculations start failing.

Motivation of the present work is to perform a systematic study of this mass region by using shell model code OXBASH [46]. There is an observation by Blomqvist [1] in which he pointed out that there should be many points of similarity between the spectroscopy of the doubly closed shell regions around ^{208}Pb and ^{132}Sn. The single particle orbits above and below the shell gap in the two cases are similarly ordered. Every single particle orbit in the ^{132}Sn region has its counterpart in the ^{208}Pb region, with same radial quantum numbers but one unit larger in angular momentum l and j values. As a consequence, effective interactions in the Sn region can be estimated from the corresponding well studied effective interactions constructed for nuclei in the ^{208}Pb region. With this in view, we found in the literature that two such (1+2)-body nuclear interaction Hamiltonians [7] are available in this mass region which can be used for our purpose. Chou and Warburton [7] extended the study of Warburton and Brown [49] in the ^{208}Pb region to the next lower available doubly magic domain, i.e., in the ^{132}Sn region and constructed two (1+2)-body Hamiltonians KH5082 and CW5082. We have used both these interactions to study the properties of few-valence particle nuclei in this region and compared the results with the available experimental data.

In this work we present the results of our theoretical investigation of the binding energies, excitation energy spectra and electromagnetic properties for nuclei in the range $Z=50$–55 and $N=82$–84. $^{137}_{52}$Te$_{85}$ has also been investigated using one of these interactions (CW5082).

II. FORMALISM: SHELL MODEL SPACE AND INTERACTIONS

Assuming ^{132}Sn as the inert core, the valence model space used in this shell model calculation consists of five proton orbitals [$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$] and six neutron orbitals [$1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, and $1i_{13/2}$]. Two (1+2)-body Hamiltonians available in this model space, as mentioned above, are KH5082 and CW5082. These interactions are described in detail in the work of Chou and Warburton [7]. But for the sake of completeness of our description of the present work, it seems reasonable to mention here briefly the essential features and differences between the two interactions.

TABLE I. Single particle energies used in KH5082 and CW5082. Recent experimental neutron and proton [19] single particle energies are also shown.

	Energy (MeV)		
State	KH5082	CW5082	Expt. [19]
$\pi 1g_{7/2}$	0.0	0.0	0.0
	$(-9.6900)^a$	$(-9.5958)^a$	$(-9.625)^a$
$\pi 2d_{5/2}$	0.9620	0.9203	0.9620
$\pi 2d_{3/2}$	2.7100	2.6406	2.4400
$\pi 3s_{1/2}$	3.0590	2.6680	2.6972[b]
$\pi 1h_{11/2}$	2.7930	2.7579	2.7920
$\nu 1h_{9/2}$	1.4850	1.4850	1.5610
$\nu 2f_{7/2}$	0.0	0.0	0.0
	$(-2.4800)^a$	$(-2.3800)^a$	$(-2.4450)^a$
$\nu 2f_{5/2}$	1.4900	1.4900	2.0050
$\nu 3p_{3/2}$	0.7550	0.7550	0.8540
$\nu 3p_{1/2}$	1.2200	1.2200	1.6560
$\nu 1i_{13/2}$	2.0900	2.0900	2.6950

[a]Absolute value of this SPE in MeV.
[b]Absolute value of this SPE is taken from CW5082.

The KH5082 interaction for the ^{132}Sn region was derived in Ref. [7] from the Kuo-Herling effective interaction [47,48] constructed for the ^{208}Pb region from fundamental nucleon-nucleon potential. This has been done by making two changes in the Kuo-Herling interaction. First, the six neutron-neutron $I=0$ TBMEs of the Kuo-Herling interaction was too attractive. So these six TBMEs were multiplied by a factor of 0.6 for approximate adjustment. Secondly, in order to get the TBMEs of the KH5082 interaction for the ^{132}Sn region from those for the ^{208}Pb region, the variation of TBMEs [50,51] with the sizes in the two regions were taken into account. Thus mass dependence of the TBMEs was approximated [1,7] to be $A^{-1/3}$ and all the 2101 TBMEs of the Kuo-Herling interaction were scaled by a factor of $(132/208)^{-1/3}$.

Proton single particle energies, except for $I^\pi=1/2^+$, were taken from experiment (Table I), assuming yrast states to be the single particle states. Yrast level for $I^\pi=1/2^+$, was not known experimentally at that time [7] and was estimated from local systematics. Because of inadequacy of experimental data at that time neutron single particle energies (except $\nu 2f_{7/2}$ and $\nu 1h_{9/2}$ orbitals) were obtained by comparison of the theoretical and experimental energy centroids for the $Z=54,56$ and $N=83$ isotones.

The CW5082 interaction was obtained from KH5082 by

TABLE II. Comparison of calculated and experimental [26,52] binding energies.

Isotope	Binding energy (MeV) with respect to ^{132}Sn			
	Expt.[a]		Theoretical	
	[52]	[26]	SPE [7]	
			KH5082	CW5082
$^{132}Sn_{82}$	0.0	0.0	0.0	0.0
$^{133}Sb_{82}$	9.628	9.663	9.690	9.596
$^{134}Te_{82}$	20.357	20.560	20.513	20.512
$^{135}I_{82}$	29.034	29.083	29.364	29.102
$^{136}Xe_{82}$	38.959	39.003	39.472	39.103
$^{137}Cs_{82}$	46.375	46.419	47.459	46.582
$^{133}Sn_{83}$	2.420	2.455	2.480	2.380
$^{134}Sb_{83}$	12.749	12.952	12.993	12.768
$^{134}Sb^{m}_{83}$	12.669		12.615	12.420
$^{135}Te_{83}$	23.858	23.902	23.794	23.624
$^{136}I_{83}$	32.817	32.861	32.914	32.505
$^{137}Xe_{83}$	42.985	43.029	43.444	42.864
$^{134}Sn_{84}$	6.162	6.365	6.905	6.705
$^{135}Sb_{84}$	16.521	16.565	17.363	17.017
$^{136}Te_{84}$	28.520	28.564	29.148	28.860
$^{137}Te_{85}$	31.731	31.775		31.762

[a]Error not shown.

replacing the proton-proton TBMEs with those from the effective interaction of Kruse and Wildenthal [7]. Single particle energies of the Kruse-Wildenthal interaction with a reduction of the binding energies of the neutron orbitals by 100 keV as has been done in Ref. [7] are shown in Table I. Moreover, the five neutron-proton TBMEs of Kruse and Wildenthal were also modified as discussed in Ref. [7], in order to reproduce the known $I=0$ and 1 levels of ^{134}Sb. It is necessary to point out here that we take into account, for a particular nuclear energy level $[E(I^{\pi})]$, contributions from all possible partitions (configurations) for a given number of valence particles, i.e., our calculation is unrestricted over the chosen model space.

III. RESULTS AND DISCUSSIONS

We have calculated binding energies (Table II), excitation spectra (Fig. 1 for $N=82,83$ isotones and Fig. 2 for $N=84,85$ isotones) for all the nuclei in the range mentioned before. Magnetic moments (Table III) and transition probabilities $[B(E2),B(E3)$ values] (Tables IV and V) are calculated and compared with the available experimental data.

A. Single particle energies

As already discussed in the previous section, the KH5082 and CW5082 interactions use somewhat different single particle energies. The set of SPEs used for each interaction is shown in Table I. As the neutron single particle energies were not known experimentally at that time [7], recent experimental SPEs [19,25] (for both neutrons and protons) are also shown in the table (column 4) for comparison, except the $\pi 3s_{1/2}$ energy, which is taken from CW5082 SPE data. We took the absolute value of the $\pi 3s_{1/2}$ SPE ($=-6.9278$ MeV) from CW5082 (column 3), and in column 4, Table I, the relative value ($=2.6972$ MeV) of this energy with respect to the experimental SPE of $\pi 1g_{7/2}$ ($=-9.6250$ MeV) is shown.

B. Binding energies

The binding energy information has a particular significance for the far from stability regions where low binding energies may give rise to new phenomena. Precise binding energies, derived from atomic masses, are also important for the modelling of the astrophysical processes, especially in regions around doubly closed shell nuclei of ^{78}Ni and ^{132}Sn [26]. Binding energy or nuclear mass information is a fundamental input for testing nuclear models. The masses of the nuclei around doubly closed shells are especially useful in testing the models due to their simple structure. In Table II, calculated binding energies of nuclei with respect to ^{132}Sn core for the two interactions are compared with two sets of experimental values [26,52]. In the table, columns 2 and 3 give experimental binding energies of Audi *et al.* [52] and those derived from recent measurements of Fogelberg *et al.* [26], respectively. Columns 4 and 5 show theoretical results with the KH5082 and CW5082 interactions, respectively. Theoretical binding energy results (columns 4 and 5) for both the interactions agree reasonably with both sets of experimental data. But the CW5082 interaction (column 5) seems to be a better choice, as expected. This reflects the fact that ^{132}Sn is indeed very stable core for few valence particles and the single particle energies and two-body matrix elements used by Chou and Warburton in CW5082 [7] particularly, are reliable.

The recent experimental binding energies (column 3) agree more closely with the theoretical values (columns 4,5) except for ^{134}Sb in CW5082 results (column 5). This disagreement for ^{134}Sb seems to be due to the fact that the experimental data then available for 1 and 0 states of this nucleus were used for modifying the KH5082 interaction to CW5082 [7]. Moreover, the theoretical results for $N=84$ isotones show larger deviation from the experimental values compared to the $N=82$ and 83 isotones.

C. Excitation energies

The calculated excitation energies are given in Figs. 1 and 2 and compared with the experimental data. Although calculations with both the interactions predict spectra for $N=82$ and 83 isotones (Fig. 1) in reasonably good agreement with the experiment, CW5082 again is undoubtedly a better choice. For $N=84$ isotones (Fig. 2), the agreement is not all satisfactory and it will be discussed in a following section.

1. N=82,83 isotones

The excitation spectra of all these nuclei ($Z=52$–55 for $N=82$ and $Z=52$–54 for $N=83$) agree well with the theo-

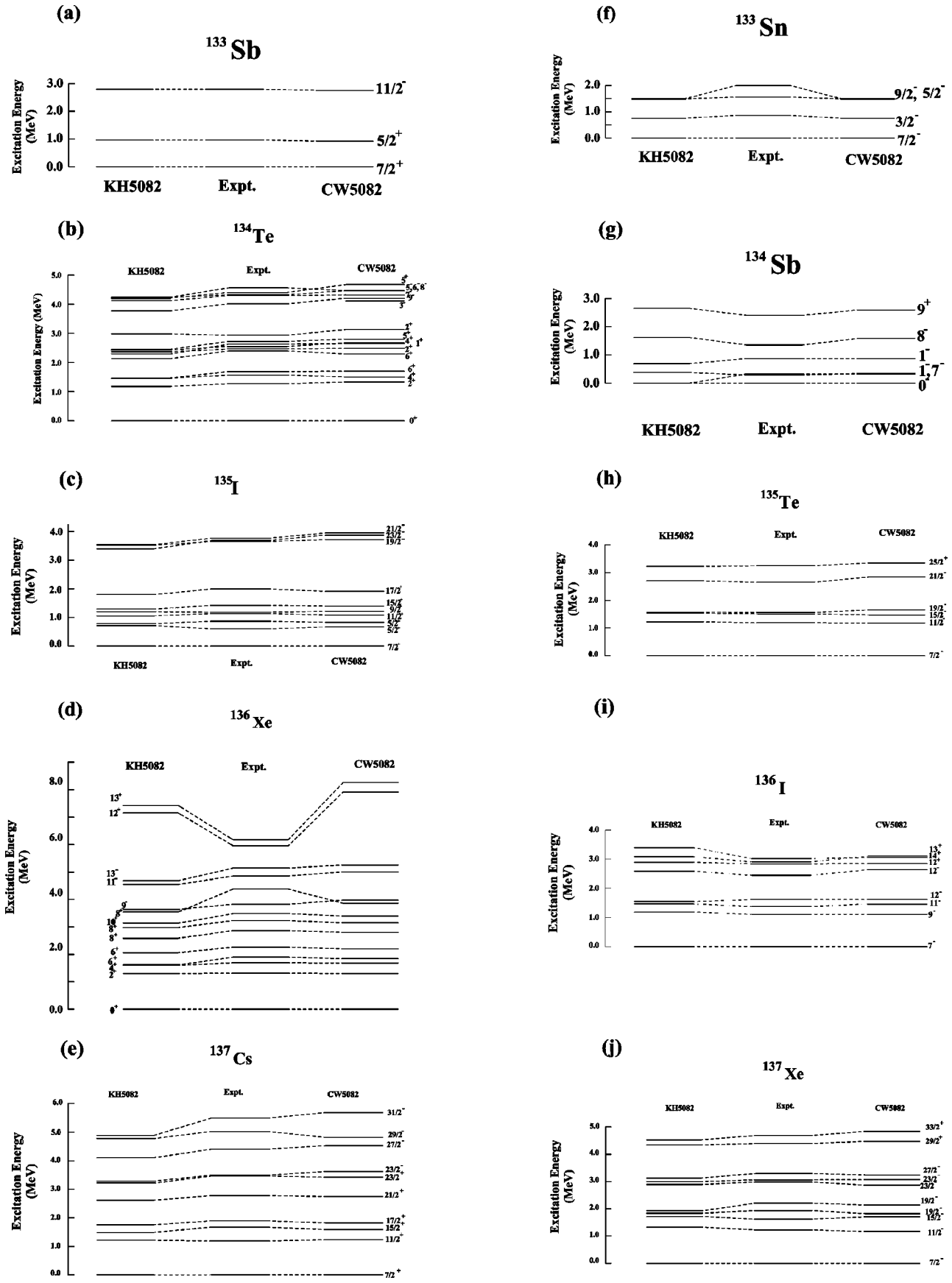

FIG. 1. Comparison of calculated and experimental excitation energies for $N=82,83$ isotones (a) ^{133}Sb, (b) ^{134}Te, (c) ^{135}I, (d) ^{136}Xe, (e) ^{137}Cs, (f) ^{133}Sn, (g) ^{134}Sb, (h) ^{135}Te, (i) ^{136}I, and (j) ^{137}Xe.

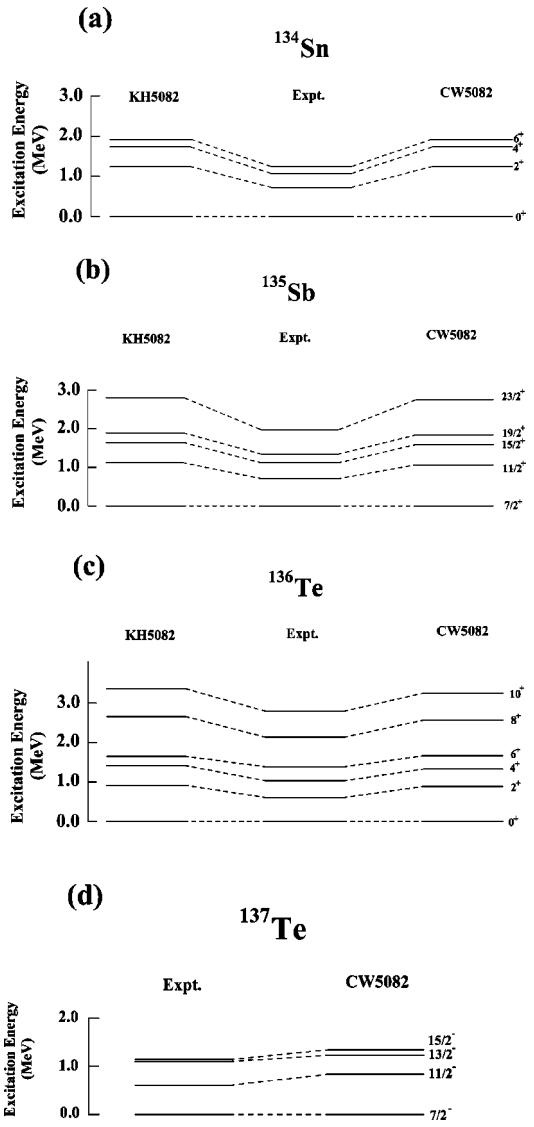

FIG. 2. Comparison of calculated and experimental excitation energies for $N=84,85$ isotones (a) ^{134}Sn, (b) ^{135}Sb, (c) ^{136}Te, and (d) ^{137}Te.

retical results with CW5082 (Fig. 1). The results for ^{136}Xe [Fig. 1(d)] start deviating from the experimental values for higher spin (>12) states reflecting the inadequacy of the model space. Results for ^{133}Sb [Fig. 1(a)] and ^{133}Sn [Fig. 1(f)] show that in the sets of SPEs used by Chou and Warburton [7], although the proton energies are chosen reasonably, the neutron SPEs should be modified using the recent experimental data [19].

2. ^{134}Sb

The levels of such doubly odd, one-neutron–one-proton valence particle nuclei are important as they provide us first-hand information on the details of the ν-π interaction. The lower part of the experimental level scheme of this isotope [Fig. 1(g)] has been obtained from Ref. [37], whereas the relatively higher spin members are from Ref. [25]. The transition of 80 keV [52], which connects the 7^- isomeric state ($T_{1/2}=10$ s) to the 0^- ground state ($T_{1/2}=780$ ms) is not seen in these two references. The lowest states (0^-, 1_1^-, and 7^-) [37] originate predominantly from $\pi 1g_{7/2}\nu 2f_{7/2}$. A recent precise measurement of the binding energy of lowest 0^- by Fogelberg *et al.* in Ref. [26], has modified substantially the value from the previous measurement [52] by about $\simeq 200$ keV. This modification in the experimental value has some implications here. The binding energy of the metastable state has not been remeasured recently; if we assume the previous binding energy value for the metastable state [52] to be correct, the modified energy difference [shown in Fig. 1(g) for experimental data] between the 0^- and the 7^- changes from 80 to 283 keV, more close to the theoretical value [Fig. 1(g), $\simeq 350$ keV for CW5082].

TABLE III. Comparison of calculated and experimental magnetic moments (μ) in nm. The orbital and spin g factors used are given by $g_l^p=1.0$, $g_l^n=0.0$, $g_s^n=-3.826$, and $(g_s^p)_{\rm eff}=2.5$.

Isotope	I^π	μ in (nm)		
		Expt. (error)	Theoretical	
		[40–42]	KH5082	CW5082
$^{134}_{52}Te_{82}$	6^+	+5.08(15)	+5.01	+5.10
$^{136}_{54}Xe_{82}$	2^+	+2.4(5)	+1.69	+1.78
	4^+	+3.2(6)	+3.35	+3.45
$^{137}_{54}Xe_{83}$	$7/2^-$	−0.968(8)	−1.63	−1.64
$^{137}_{55}Cs_{82}$	$7/2^+$	+2.84(1)	+2.92	+2.92

3. N=84,85 isotones

The results for $N=84,85$ isotones are shown in Fig. 2. In our calculation, we note that, for $N=84$ isotones ^{134}Sn, ^{135}Sb, and ^{136}Te [Figs. 2(a)–2(c)], the theoretical energies of the first excited states are about 1200, 1100, and 900 keV, whereas the corresponding experimental values are 725, 707, and 607 keV, respectively. For the $N=85$ isotone, ^{137}Te also, although the agreement is somewhat better, the experimental value for the first excited state is less by about 225 keV than our theoretical result. Thus the theoretical energy values are overestimated compared to the experimental ones for all the $N=84,85$ isotones. But the relative spacings between the excited levels (keeping aside the ground state for each nucleus) match reasonably with the experimental data. We also note that the difference between the theoretical and experimental energies of the first excited state for the $N=84,85$ isotones decreases as the number of valence protons increases, the deviation is maximum for ^{134}Sn.

In a recent study by Korgul *et al.* [19], modification similar to Chou and Warburton [7] has been done in the interaction of Kuo-Herling [47,48]. Along with these modified TBMEs and the recent experimental SPEs [19], they used the

TABLE IV. Comparison of calculated and experimental (errors not shown) $B(E2)$ values in Weisskopf units. Variation of theoretical results with different values of proton and neutron effective charges with CW5082 interaction is also shown in the table.

Isotope	$I_i^\pi \to I_f^\pi$	E_γ (keV)	$T_{1/2}$	$B(E2)$ in (W.u.)					
				Expt.	Theoretical				
				[23,28,30–32,43]	KH5082	CW5082			
					$e_p^{\text{eff}}=1.47$	1.5	1.2	1.2	
					$e_n^{\text{eff}}=1.00$	1.0	1.0	0.8	
$^{134}_{52}Te_{82}$	$4^+ \to 2^+$	297	1.28 ns	4.50	3.70	4.27	4.45	2.86	2.86
	$6^+ \to 4^+$	114	164 ns	2.12	1.69	2.38	2.48	1.59	1.59
$^{135}_{53}I_{82}$	$15/2^+ \to 11/2^+$	288	3 ns	2.21	3.18	4.17	4.34	2.78	2.78
$^{136}_{54}Xe_{82}$	$6^+ \to 4^+$	198	2.95 μs	0.01	0.07	0.00	0.00	0.00	0.00
	$4^+ \to 2^+$	381	1.32 ns	1.26	0.39	1.76	1.84	1.18	1.18
	$2^+ \to 0^+$	1313	0.41 ps	8.48	5.75	7.74	8.06	5.16	5.16
$^{137}_{54}Xe_{83}$	$19/2^- \to 15/2^-$	314	8 ns	0.53	0.10	0.83	0.86	0.61	0.55
$^{136}_{53}I_{83}$	$11^- \to 9^-$	261	4 ns	2.64	5.14	6.48	6.69	4.70	4.28
$^{135}_{52}Te_{83}$	$19/2^- \to 15/2^-$	50	0.51 μs	4.00	3.68	5.00	5.14	3.75	3.28
$^{134}_{50}Sn_{84}$	$6^+ \to 4^+$	174	80 ns	0.88	2.30	2.30	2.30	2.30	1.47
$^{135}_{51}Sb_{84}$	$19/2^+ \to 15/2^+$	225	20 ns	1.08	4.67	4.69	4.72	4.42	3.02

computer code OXBASH [46] and found that the theoretical results reproduce experiment quite well for ^{134}Sn. This is totally different from our results and that obtained by Chou and Warburton [7]. Our results [CW5082 in Fig. 2(a)] for ^{134}Sn agree exactly with Ref. [7] but differ significantly from Ref. [19]. We have noted that the SPE inputs in Refs. [7,19] are different.

In a previous work by Zhang *et al.* [31], the authors interpreted the ^{134}Sn levels observed in their work as complete $(\nu f_{7/2})^2$ level spectrum up to maximally aligned 6^+ state at 1246 keV. Later, while calculating the level scheme of ^{135}Sb, Bhattacharyya *et al.* [28] used the above interpretation and estimated the $\nu f_{7/2}$-$\nu f_{7/2}$ interactions from the experimental spectrum of ^{134}Sn. They used other ν-ν and ν-π interactions from ^{210}Pb and ^{210}Bi, respectively, in the Pb region with proper scaling as discussed in Sec. II above. The calculated level scheme of ^{135}Sb showed good agreement with the experimental sequence [28]. But in our calculations, with CW5082 interaction, the wave functions of 0^+ to 6^+ of ^{134}Sn show different decompositions, having significant contributions from configurations other than $(\nu f_{7/2})^2$, especially for 0^+ and 2^+. The composition of the wave functions of the states are, for the 0^+ state, $\simeq$57% contribution from the $(\nu f_{7/2})^2$ and $\simeq$10% contribution from each $(\nu h_{9/2})^2$, $(\nu i_{13/2})^2$, $(\nu f_{5/2})^2$, and $(\nu p_{3/2})^2$ configurations; for 2^+ state, $\simeq$66% contribution from the $(\nu f_{7/2})^2$, and $\simeq$20% contribution from the $(\nu f_{7/2} p_{3/2})$ configuration; for 4^+ state, $\simeq$82% contribution from $(\nu f_{7/2})^2$ and $\simeq$7% contribution from $(\nu f_{7/2} p_{3/2})$ configuration; finally, for 6^+ state, $\simeq$93% contribution from $(\nu f_{7/2})^2$ and $\simeq$6% contribution from $(\nu f_{7/2} p_{3/2})$ configuration.

TABLE V. Comparison of calculated and experimental $B(E3)$ values in Weisskopf units for ^{134}Te. The proton effective charge is $2.0e$.

$I_i^\pi \to I_f^\pi$	$B(E3)$ in (W.u.) Expt.	Theoretical	
	[34]	KH5082	CW5082
$9_1^- \to 6_1^+$	3.8±0.2	1.12	3.49
$9_1^- \to 6_2^+$	8.0±1.3	11.96	14.32
$3_1^- \to 0_1^+$		0.94	0.89

We shall also see later (Table IV) that the results of transition probability calculations also disagree with the experimental data for these isotones for a standard value of the neutron effective charge. So these limitations of the two interactions in predicting experimental observables in $N=84$ isotones, clearly indicate a necessity of changing particularly the ν-ν TBMEs of the interactions. However, such changes need a consistent modification of the interaction and the single particle energies, which is beyond the scope of the present work. We note that for $N=85$ nucleus ^{137}Te, the binding energy agrees reasonably with the experimental data, and the excitation spectrum with CW5082 interaction also shows better agreement.

D. Electromagnetic properties

1. Magnetic moment

We calculate the g factor of the $I^\pi=6^+$ isomeric state in ^{134}Te whose measured value [38] is 0.846 ± 0.025. With KH5082 interaction, we obtain for the wave function $|(1g_{7/2})^2;6^+\rangle$ a value $g(6^+)=0.495$. But for CW5082 model space, the wave function with 91% of $|(1g_{7/2})^2;6^+\rangle$ and 9% of $|1g_{7/2}2d_{5/2};6^+\rangle$ yields a value of 0.545, without any quenching of g_s^p from its free value (=5.586). This means that the inclusion of $|1g_{7/2}2d_{5/2};6^+\rangle$ configuration increase $g(6^+)$ by $\simeq 10.1\%$.

If we quench g_s^p to 2.5, a value close to effective g_s^p factors used for the $1g_{7/2}$ proton orbital in describing the $N=82$ isotones [8], it changes the results [$g(6^+)_{\rm KH}=0.835$ and $g(6^+)_{\rm CW}=0.851$] and the calculated values using both the interactions lie within the experimental error of the measured value. This quenching was explained by Heyde *et al.* [8] in a two particle shell model calculation with a residual Gaussian two-body interaction within the [$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $3h_{11/2}$] proton single particle states and taking into account core polarization effects, influence from velocity dependent two-nucleon interactions on the expression for the magnetic dipole moment operator and configuration mixed complete wave function in the model space.

In another calculation by Andreozzi *et al.* [6], the magnetic moment of this 6^+ state has been calculated by using the free value of g_s^p and an effective g_l, $g_l^{\rm eff}=1.3g_l^{\rm free}$ giving $g(6^+)=0.835$. In our calculations, with the same effective g_l and the free value of g_s^p as inputs, we obtain values for the g factor as 0.828 and 0.874, for KH5082 and CW5082, respectively.

In Table III, magnetic moments of different states in some nuclei calculated for the two interactions are compared with the experimental values. The results shown in the table have been obtained with a quenched $g_s^p=2.5$ as discussed above for ^{134}Te. The results show reasonable agreement with the experimental values [40–42].

2. Transition probabilities

In Table IV, $B(E2)$ values calculated for the two interactions are compared with those obtained from the measured lifetimes of states in different nuclei. The radial integral involved in the calculation of $E2$ and $E3$ matrix elements are calculated with harmonic oscillator radial wave functions with $\hbar\omega=45A^{-1/3}-25A^{-2/3}$ [46,50].

While calculating the $B(E2,\ 6^+\to4^+)$ for ^{134}Te, under the assumptions of harmonic oscillator single particle wave functions and $\hbar\omega=41A^{-1/3}$, Wildenthal and Larson [10] showed that the effective proton charge in this region should be $1.47e$. Similarly, with radial matrix element taken as 32 fm^2, the $E2$ neutron effective charge for ^{134}Sn is determined to be $1.01e$ in Ref. [31]. In that work, the ^{134}Sn spectrum was interpreted as the complete $(\nu f_{7/2})^2$ level spectrum (discussed earlier in Sec. III C 3). In the present work, to start with, the results obtained with both KH5082 and CW5082 are for proton and neutron effective charges $1.47e$ [10] and $1.0e$, respectively. The theoretical results agree reasonably with the experimental values. This table also gives the variations in the theoretical results with the variation in the choice of neutron and proton effective charges in these isotopes with the interaction CW5082. The attempt to obtain a unique set of effective charges for protons and neutrons, for the entire mass region is not successful as seen from the table. In Table IV, the calculated $B(E2)$ values for the $N=84$ isotones with $e_n^{\rm eff}=1.0e$ indicate necessity of gross reduction of the neutron effective charge for these isotones. We have seen that to reproduce the experimental $B(E2;6^+\to4^+)$ (=0.88 W.u.) in ^{134}Sn, an effective neutron charge of $0.62e$ is needed in our calculation, which is much less than that obtained by Zhang *et al.* [31]. Wave functions obtained in our work (Sec. III C 3) have substantial contributions from configurations other than $(\nu f_{7/2})^2$, which may be a reason for the reduction of the effective charge.

The collective properties of the low-lying states are of particular interest for studying the nature of the double shell closure. It has been already reported by experimental and theoretical studies of $B(E3)$ rates of ^{134}Te by Omtvedt *et al.* [34] that octupole charge and degree of octupole collectivity is substantially lower in the ^{132}Sn region compared to that in ^{208}Pb indicating much stronger shell closure in the Sn region. To study the similar effect within our shell model calculations, $B(E3)$ rates for ^{134}Te have been calculated. The energy [Fig. 1(b)] of the 9^- state agrees with the experimental value quite well. The $B(E3)$ values are shown in Table

V. The result with CW5082 shows that $e_p^{\text{eff}}=2.0e$ is a reasonable choice for effective octupole charge for protons in agreement with the observation by Omtvedt [34] in their RPA calculation.

E. Study of isotopic chain of Te

This isotopic series ($N=82-85$) has been systematically studied with the CW5082 interaction to definitely identify the valence neutron number at which this shell model calculation in the chosen model space starts failing. As we have discussed, the shell model results for binding energies, excitation spectra and the electromagnetic properties show reasonably good agreement with experimental data for $N=82,83$ isotones. For $N=84$ and 85 isotones, the theoretical results indicate that the TBMEs need further modification. For $N=86$, the six valence particle nucleus, ^{138}Te [24], computational limitation on our part prevents us from definitely commenting on this isotope. One can however, consider the experimental R_4 values for these isotopes of Te, to follow the evolution of collectivity. $R_4[=E(4_1^+)/E(2_1^+)]$ values are 1.23, 1.70, and 2.04 for ^{134}Te, ^{136}Te, and ^{138}Te, respectively. The R_4 value for ^{138}Te indicates vibrational nature.

IV. CONCLUSION

In conclusion, we emphasize that the overall reasonably good agreement of the calculated binding energies, excitation spectra, magnetic moments, $B(E2)$ and $B(E3)$ values with the experimental ones, for $N=82,83$, clearly demonstrates that the application of spherical shell model is appropriate and that the effective interactions used are quite capable of explaining the observed behavior of $N=82,83$ nuclei around ^{132}Sn region. But for $N=84$ isotones, the interactions are not as successful as for $N=82,83$ and seem to be inappropriate. To extend the region of applicability of these interactions (CW5082 especially) to $N\geqslant 84$, it may be appropriate to use the new experimental single particle energies for the model space orbitals and to determine all the TBMEs in a consistent way to fit the recently available experimental spectroscopic data of all nuclei in this mass region. The extension of the shell model basis space may also be helpful in studying a particular isotopic series more systematically as well as the onset of collectivity.

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