

## Systematics of nuclear deformation in large regions

Y. M. Zhao,<sup>1,4,5,6</sup> A. Arima,<sup>2</sup> and R. F. Casten<sup>3</sup><sup>1</sup>Department of Physics, Southeast University, Nanjing, 210018 People's Republic of China<sup>2</sup>The House of Councilors, 2-1-1 Nagatacho, Chiyodaku, Tokyo 100-8962, Japan<sup>3</sup>Department of Physics, A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520-8120<sup>4</sup>Department of Physics, Saitama University, Urawa-shi, Saitama 338-0825, Japan<sup>5</sup>Bartol Research Institute, University of Delaware, Newark, Delaware 19716-4793<sup>6</sup>Cyclotron Center, Institute of Physical Chemical Research (RIKEN), Hirosawa 2-1, Wako-shi, Saitama 351-0198, Japan

(Received 24 January 2001; published 4 May 2001)

In this paper we present the systematics of nuclear deformation for even-even, even-odd, odd-even, and doubly odd nuclei in four regions: the  $50 < Z \leq 66$  and  $82 < N \leq 104$  region, the  $66 < Z < 82$  and  $82 < N \leq 104$  region, the  $66 < Z < 82$  and  $104 < N < 126$  region, and the  $82 < Z \leq 104$  and  $126 < N \leq 155$  region. Compact trajectories are obtained using the  $P = N_p N_n / (N_p + N_n)$  factor. It is found that there are no apparent shifts in the trajectories between the even-even nuclei previously studied with the  $P$  factor, and the other classes of nuclei included here. This may suggest that the pairing interaction strength of even-even nuclei is very close to that in their odd  $A$  and doubly odd neighbors. Strong anomalies around the  $Z \sim 80$  region are highlighted.

DOI: 10.1103/PhysRevC.63.067302

PACS number(s): 21.10.Re, 23.20.Js

The paramount importance of the residual valence  $p$ - $n$  interaction in the evolution of single-particle structure and in the development of collectivity, phase shape transitions, and deformation has been emphasized by many authors, such as deShalit and Goldhaber [1], Talmi [2], and later Federman and Pittel [3]. This idea was further exploited in Ref. [4]: if the residual valence  $p$ - $n$  interaction is the dominant controlling factor in the development of configuration mixing and collectivity in nuclei, the evolution of structure should in some way scale with the integrated strength of this interaction in the active proton and neutron shells, and the simple valence nucleon product  $N_p N_n$  can be regarded as a variable to gauge this interaction. The usefulness of the  $N_p N_n$  scheme to simplify the phenomenology of nuclear structure observables of even-even nuclei has been well established [5]. A simple modification [6] to  $N_p N_n$ , the  $P$  ( $P = N_p N_n / (N_p + N_n)$ ) factor, can be regarded as the average number of  $p$ - $n$  interactions per valence nucleon. It is also related to the ratio of the integrated valence  $p$ - $n$  interaction strength to the integrated valence pairing strength. A simple argument [6] suggests why deformation of nuclei sets in when  $P \sim 4-5$ , regardless of mass region.

We recently plotted [7] the quadrupole deformation parameters  $e_2$ , which were taken from macroscopic-microscopic calculations [8], in the  $N_p N_n$  scheme.  $e_2$  is defined in Ref. [8]. For the even-even, odd- $A$ , and the doubly odd cases of all known medium-heavy nuclei, no discernible shifts between these different classes of nuclei were visible. The  $N_p N_n$  scheme shows that the  $p$ - $n$  interaction does not depend much on the class of nucleus (even, odd, or odd-odd); it is therefore interesting to see whether the pairing effects embodied in the  $P$  factor modify this result, that is, whether  $P$ -factor plots are likewise independent of class of nucleus. One may also ask whether the  $P$  factor presents any new phenomenology, especially whether it signals any "abnormal" structures in the ground state or low-lying states.

Figure 1 shows the systematics of nuclear deformation parameters,  $e_2$  vs the  $P$  factor for even-even, even-odd, odd-

even, and doubly odd nuclei in four regions: the  $50 < Z \leq 66$  and  $82 < N \leq 104$  region, the  $66 < Z < 82$  and  $82 < N \leq 104$  region, the  $66 < Z < 82$  and  $104 < N < 126$  region, and the  $82 < Z \leq 104$  and  $126 < N < 155$  region. Like the  $N_p N_n$  scheme, the  $P$  factor, generally speaking, presents a striking correlation, that is,  $e_2$  increases with the factor  $P$  very smoothly, then begins to saturate around  $P \sim 6$ . This behavior was discussed in Ref. [6] in the context of the systematics of the ratio  $E_{4^+} / E_{2^+}$  for even-even nuclei. Here it is confirmed using the deformation parameters for all types of nuclei, i.e., even-even, even-odd, odd-even, and doubly odd nuclei. The correlations of the  $82 < Z \leq 104$  and  $126 < N < 155$  nuclei are extremely compact in both the  $N_p N_n$  scheme (Fig. 1(d) in Ref. [7]) and the  $P$ -factor plot [Fig. 1(d) in this paper]. The  $N_p N_n$  scheme shows a strong  $Z = 64$  subshell effect in the  $Z = 50-66$  region [7]. However, in the  $P$ -factor plot, this subshell effect seems to be reduced.

There are strong anomalies in Figs. 1(b) and 1(c) in the  $Z = 76-80$  region. These were also noticed in the  $N_p N_n$  scheme in Ref. [7] but are greatly magnified in the  $P$ -factor plots. These anomalies are correlated according to their  $Z$  values as indicated in Figs. 1(b) and 1(c).

To illustrate this systematics more clearly, Fig. 2 shows the deformation parameter  $e_2$  vs the  $P$  factor for the even-odd and odd-odd nuclei for the  $66 < Z < 82$  and  $82 < N \leq 104$  region, where we focus on the subset of nuclei with  $e_2 < 0.30, P < 6.5$ , and have circled the anomalous points according to their  $Z$  values.

For nuclei with  $Z = 80$  or  $79$ , the anomalies lie along linear trajectories, with the highest  $N$  values farthest from the main correlation and the lowest values closest. For  $Z = 77$  and  $76$  there tend to be a cluster of points far from the main correlation. The  $Z = 78$  case is intermediate.

For the same range of  $Z$  values, but with neutron numbers past midshell ( $104 < N < 126$ ), an almost identical set of anomalies occur, as seen in Fig. 1(c). The fact that, again, the points lying farthest from the main correlation have  $N \sim 104$ ,

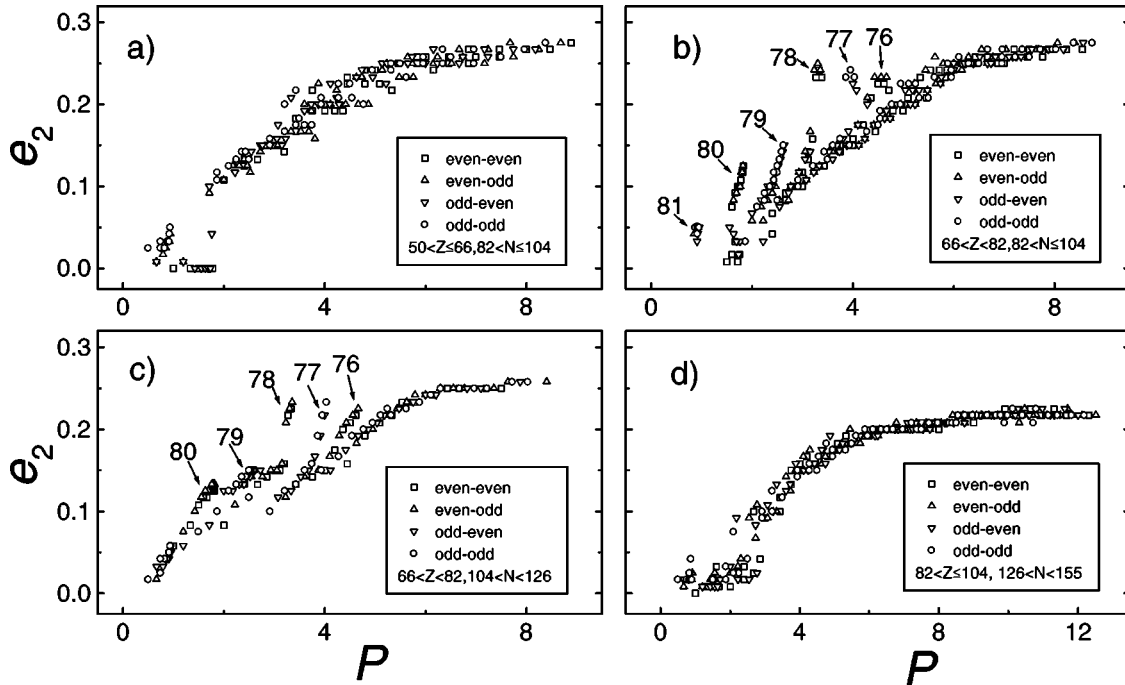


FIG. 1. The deformation parameter  $e_2$  vs the  $P$  factor for nuclei with (a)  $50 < Z \leq 66$  and  $82 < N \leq 104$ ; (b)  $66 < Z < 82$  and  $82 < N \leq 104$ ; (c)  $66 < Z < 82$  and  $104 < N < 126$ ; and (d)  $82 < Z \leq 104$  and  $126 < N < 155$ . In calculating  $P$ , the numbers of valence protons and neutrons,  $N_p$  and  $N_n$ , respectively, are always counted relative to the closest magic number (50, 82, 126). The anomalous clusters of the  $e_2$  data are labeled using their proton numbers  $Z$ . See the text for details.

suggests that the effects are related to some specific feature associated with nuclei near this neutron midshell point. Below, we will speculate on what the origin of this effect might be but, first, it is useful to verify that this effect is not an artifact of using calculated deformations from Ref. [8]; we have inspected empirical  $E_{2^+}$  energies from these nuclei (even-even type) as well. An example of the results is shown in Fig. 3. The same class of nuclei ( $Z$  close to 82,  $N$  close to midshell) show anomalous  $E_{2^+}$  energies, again suggesting larger deformations.

The effect in Figs. 1(b), 1(c), 2, and 3 is a substantial anomaly. One possible explanation is that these nuclei are

near midneutron shell, where coexisting deformed states involving proton excitations across the  $Z = 82$  gap descend into the low-lying spectrum. If this increases the effective valence proton number, then the data points should have been plotted at larger  $P$  values, which would have indeed reduced the anomalies. However, this does not explain why the effect is basically absent in the  $N_p N_n$  plot.

In short, a compact systematics of nuclear deformations for heavy even-even, even-odd, odd-even, and doubly odd nuclei is obtained in the  $P = N_p N_n / (N_p + N_n)$  factor plots. It is found that there are no apparent shifts in comparison to the systematics for the even-even nuclei. This suggests that the pairing interaction strength is almost the same in these dif-

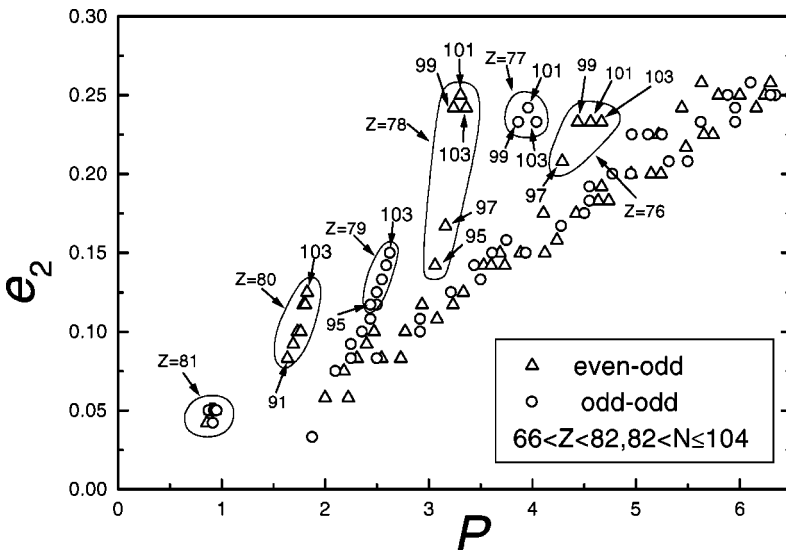


FIG. 2. Similar to Fig. 1(b) except that the  $e_2$  data are limited to the even-odd and odd-odd cases with  $e_2 < 0.30$ ,  $P < 6.5$ .

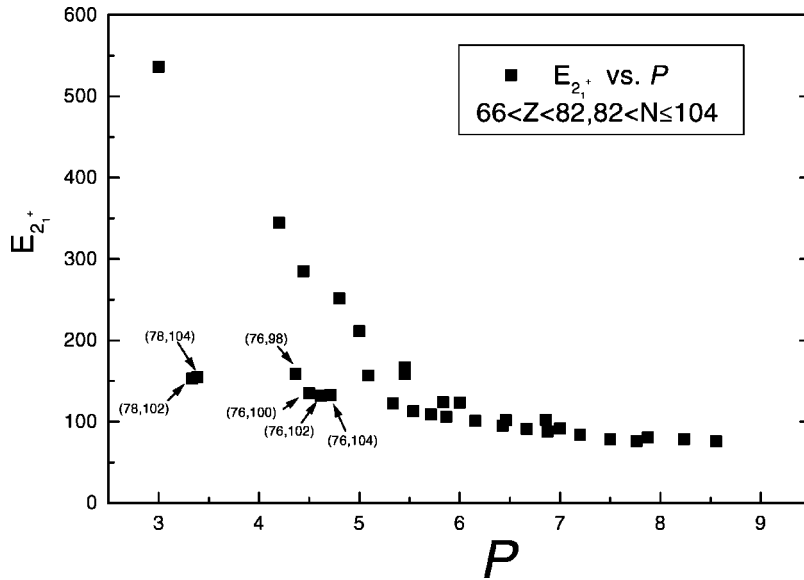


FIG. 3. Similar to Figs. 1(b) and 2 except that empirical  $2_1^+$  energies are plotted against  $P$ . The anomalous points are labeled.

ferent classes of nuclei. Anomalies in the  $Z \sim 76-80$  region are magnified greatly in  $P$ -factor plots compared with those in the  $N_p N_n$  scheme, and they can be classified according to their proton numbers. It seems that nuclei with the largest numbers of valence neutrons in the  $Z=76-82$  region are systematically more deformed than the main correlation of nuclei. No corresponding anomaly occurs in the  $Z=50-66$  and  $Z=82-104$  regions. No real explanation is yet available, although coexisting deformed states related to particle-

hole proton excitations, which appear near the yrast levels for neutron numbers near midshell, may play an important role. The  $N_p N_n$  scheme and the  $P$ -factor scheme thus work in complementary ways to disclose anomalies of structural evolution.

The authors would like to thank Dr. F. Iachello and Dr. S. Pittel for discussions. This work is supported in part by the U.S. DOE under Grant No. DE-FG02-91ER40609 and the U.S. NSF under Grant No. PHY-9970749.

- [1] A. deShalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953).  
 [2] I. Talmi, Rev. Mod. Phys. **34**, 704 (1962).  
 [3] P. Federman and S. Pittel, Phys. Lett. **69B**, 385 (1977); **77B**, 29 (1977); Phys. Rev. C **20**, 820 (1979); P. Federman, S. Pittel, and R. Campas, Phys. Lett. **82B**, 9 (1977).  
 [4] R. F. Casten, Phys. Rev. Lett. **54**, 1991 (1985); Nucl. Phys. **A443**, 1 (1985).  
 [5] For a review, see R. F. Casten and N. V. Zamfir, J. Phys. G **22**,

- 1521 (1996).  
 [6] R. F. Casten, D. S. Brenner, and P.E. Haustein, Phys. Rev. Lett. **58**, 658 (1987).  
 [7] Y. M. Zhao, R. F. Casten, and A. Arima, Phys. Rev. Lett. **85**, 720 (2000).  
 [8] P. Moller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).