Are the level densities for *r*- and *rp*-process nuclei different from nearby nuclei in the valley of stability?

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Nuclei reached in *r*- and *rp*-process reactions in nuclear astrophysics are considerably more neutron or proton rich than most nuclei for which level densities have been studied. Two models which predict somewhat different behavior for nuclei away from the valley of stability are examined. It is found that one of the two provides a better description of the available data. A term which lowered the level density parameter as isospin increased at fixed *A* did not produce as much improvement as one which reduced the level density parameter based on the distance from the valley of stability. As a by-product of this study, a formulation of level density systematics near the valley of stability is also tested.

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I. INTRODUCTION

Calculations of neutron and proton capture processes in astrophysics require nuclear level densities as input parameters. Because of a lack of information about the level density for the nuclei of interest, the level density parameter is usually estimated based on information near the bottom of the valley of stability. In this paper we examine the possibility that information already available suggests that this extrapolation should be done differently.

Level density formulas are usually based on the work of Bethe [1], who utilized the assumption that an energy independent density of single particle states g was present. Bethe showed that in terms of this density g the state density has the form

$$\rho(U)=\frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{aU})}{a^{1/4}U^{5/4}}, \tag{1}$$

where $a=(\pi^2/6)g$ and U is the excitation energy. The parameter g is expected to be proportional to A, leading to the result that a is also proportional to the nucleon number of the nucleus.

Various refinements to the model have been added since its introduction. Particularly important was the addition of an energy shift δ to the energy to account for pairing and shell effects. Fits to nuclear level densities using this model are parametrized by a and δ.

More recently, it has been concluded that the basic assumption of the Bethe model breaks down at high energies [2–6]. The finite depth of the nuclear well limits the maximum energy of the hole state density. Furthermore, the single particle states become increasingly broad as their energy exceeds the binding energy; as more particles occupy these states the total width of the state becomes too broad to speak of an equilibrium state. Viewed in another way, it is clearly not possible to form compound states with energies so large that the total binding energy of the nucleus is exceeded. This limit is about $8A$ MeV for a nucleus of mass A in the bottom of the valley of stability.

There are two physical effects, isospin and single particle binding energy, which might give a dependence of the level density parameter a on Z and N rather than simply on A. To the extent that the nuclear force is isospin conserving, levels of a given nucleus will be characterized by well-defined values for isospin. For a nucleus with neutron number N and proton number Z, the isospin values allowed range upward from $T_{\min}=|(N-Z)/2|$. Further, if a fixed single particle basis is assumed, the nucleus with $T_{\min}=0$ will not only have $T=0$ levels but also all of the $T=1$ and higher levels found in the adjacent $T_{\min}=1$ nuclei. Similarly, the nuclei with $T_{\min}=1$ will have all of the levels with $T=2$ and higher present in the neighboring nuclei for which $T_{\min}=2$ plus the levels of $T=1$. As one moves to higher and higher values of $T_{\min}$, the total number of levels would decrease monotonically. This model predicts that for a given A value, the level density parameter would decrease with increasing $|N-Z|$. The typically used form

$$a=\alpha A \tag{2}$$

would then become

$$a=\alpha A/\exp[\beta(N-Z)^2]. \tag{3}$$

The analysis described in Refs. [2–6] suggests that a somewhat different dependence on N and Z is possible. If the limit on the single particle states plays a role in modifying the level density, then profound effects would be expected as the drip lines are approached. As N and Z change to reach the drip line, eventually even the lowest levels available are particle unstable. If this limit is important, a somewhat different form of a would be expected:

$$a=\alpha A/\exp[\gamma(Z-Z_0)^2], \tag{4}$$

where Z_0 is the Z of the beta stable isotope of mass A. Note that for low A Eqs. (3) and (4) predict similar results since $Z_0\approx A/2\approx N$ for $A<30$. As A increases, the results predicted by the two relations will differ substantially.

TABLE I. Fitting forms and parameters for a.

(A) Fit of form: $a=\alpha A$:	$\alpha=0.1097$	χ^2/degrees of freedom$=7.52$
(B) Fit of form: $a=\alpha A/\exp[\beta(N-Z)^2]$:	$\alpha=0.1110$	-5.6% compared to (A)
	$\beta=0.000641$	
(C) Fit of form: $a=\alpha A/\exp[\gamma(Z-Z_0)^2]$	$\alpha=0.1138$	-12.6% compared to (A)
	$\gamma=0.0493$	
	$Z_0=0.5042A/(1+0.0073A^{2/3})$	

The particular forms used in Eqs. (3) and (4) are somewhat arbitrary. Arguments for the specific choice made will be presented in Sec. II.

II. ANALYSIS

Testing these two models is difficult because of the lack of level density data near the drip lines. A recent paper [7] has derived information on level density systematics for $20 \leqslant A \leqslant 41$ based largely on level density information derived from low-lying resolved levels. Because $Z_0 \approx A/2 \approx N$ in this region, a definitive test of the two hypotheses could not be made in this mass region.

Many level density studies [8–12] rely on values inferred from counting resolved levels observed with low energy neutrons. This data set includes only levels of one parity and a limited number of spins if the neutrons are low enough in energy that only S wave neutrons interact with the nucleus. This data set then requires knowledge of both spin cutoff parameters and parity ratios to give a total level density. Two other difficulties dissuaded us from using this data set. The analysis of these levels gives the level density at only one energy. In addition, these resonance counts are not usually available for nuclei more than one unit off of the line of stability. It was therefore decided to focus on the tabulated levels at low energy.

Levels listed in the ENSDF data file [13] for nuclei with $20 \leqslant A \leqslant 70$ were used in the search. These levels were converted to state densities by multiplying each level by $2J+1$ and summing in 1/2-MeV bins. The state densities are quoted per MeV. By inspection, it could be seen that a number of nuclei in this mass range had level schemes that were clearly incomplete below 2.5 MeV; these were discarded. For the remaining nuclei, the interval over which the level scheme was thought to be complete was divided into two parts and the number of levels up to the upper boundary of each was calculated. Thus the fit was done to the level density integral at two energies. An initial effort to fit the density itself was abandoned because of substantial fluctuations. Fits were also carried out with the above nuclei ($20 \leqslant A \leqslant 70$) added to three nuclei (^{100}Mo, ^{100}Ru, and ^{100}Pd) with $A=100$ and two nuclei (^{140}Ba and ^{140}Ce) with $A=140$. This set is called the extended set and contained 133 nuclei.

All of the results discussed in this section are based on the extended set; similar results were obtained with the original set except that the difference between the χ^2 reductions for Eqs. (3) and (4) was smaller. This result is plausible, since for most of the A values below 70, Z_0 is close to $A/2$.

The first fits were done with the form given by Eq. (2). For each value of α, the value of δ was varied for each nucleus to minimize the χ^2 contribution of the two points obtained from the integral described previously. Additional searches were conducted with a constant term added to Eq. (2); these yielded very small changes to χ^2.

The next step was to try the form of Eq. (3). Again, for values of α and β the value of δ for each nucleus was varied to achieve the closest match (minimum χ^2 value) for the summed level density between the standard level density form and the data. Variation of α and β over a range of magnitudes and searching for optimal values of δ for each α-β pair allowed a minimum χ^2 to be determined. Some improvement in χ^2 was noted with the inclusion of the β term. These results are shown in Table I.

The expression for Z_0 is listed in Table I and was derived from a fit of the semiempirical mass formula form to the nuclei in the present data set.

Next, the same sequence was followed with Eq. (4). This produced a reduction in χ^2 which was about 12.6% as shown in Table I. A direct comparison of the results of fitting with Eqs. (3) and (4) indicates a preference for Eq. (4).

The valley of stability is not symmetric with respect to the neutron and proton drip lines, with skewing increasing with A. We tried a number of forms which included terms in $(Z-Z_0)$ which were asymmetric as well as some including $(N-Z)$ and $(Z-Z_0)$. All of these forms showed either negligible improvement in χ^2 or unreasonable behavior as A increased or the drip lines were approached. Further work on refining the parametrization will require more data, particularly with larger values of $(Z-Z_0)$ or A.

As can be seen from Table I, form C produced the best fit. Typical fits are shown in Figs. 1–4. In each of these figures one calculated point is included beyond the region that was fit.

The choice of the forms in Eqs. (3) and (4) is somewhat arbitrary. The exponential form is better than the corresponding Taylor series forms [e.g., $\alpha A+\beta A(N-Z)^2$ for Eq. (2)] because the nonexponential forms can give negative values for a. On the other hand, the exponential forms do reduce to the simpler Taylor series form if $(N-Z)$ or $(Z-Z_0)$ is small. A number of different A dependences for the exponential argument were investigated. The ones shown are slightly better than neighboring powers of A. As better level schemes for nuclei off the stability line become available, other forms could be tested.

Looking at the magnitude and sign of the fitting constants, we see that the sign is as expected, reducing the level density parameter as the drip line is approached. The magnitude is consistent with a being a small fraction (<0.2 to 0.3) of its

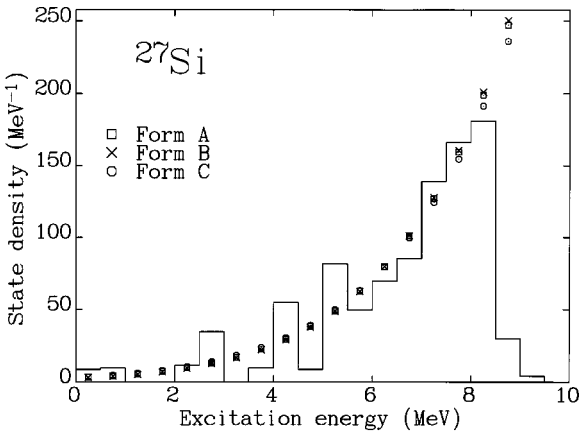

FIG. 1. State density of ^{27}Si. The bars indicate the level density calculated from known levels in 0.5-MeV bins. Symbols marked A–C indicate the fits using forms A through C. The points extend one bin beyond the region fitted. The best fit δ is -0.84 MeV.

value in the region of stable nuclei.

The consequences of including the additional term in the expression for a can be profound. In Table II a values are tabulated for a number of nuclei with $A=24$, 28, 40, 56, 70, 100, 140, 200, and 240. Although the precision of the values of β and γ is not as good as would be the case if information from nuclei farther off the stability line were available, the magnitudes obtained seem reasonable. The extrapolation of a to the drip line has a value of near 0 at or slightly beyond the drip line. For a values that are small relative to those in the valley of stability with the same A value, the Bethe formula should probably be replaced by a Gaussian form.

In Figs. 5 and 6 the effect on the level density of these changes in a is presented. In each case, the state density from fits A, B, and C is plotted. The energy used is 5 MeV added to the δ value for that nucleus. For some nuclei in the plot, factor of 10 changes are predicted in a, which lead to changes in the state density of a number of orders of magnitude.

Nucleosynthesis calculations typically involve (p,γ) or (n,γ) reactions under scenarios such that repeated proton or neutron captures occur. The present results suggest a strong

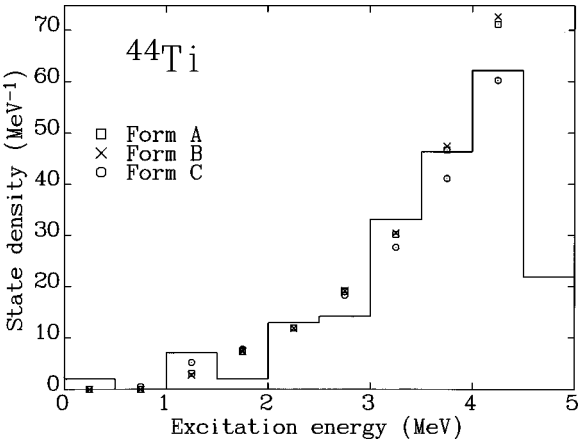

FIG. 2. Same as Fig. 1 for ^{44}Ti. The best fit δ is 0.59 MeV.

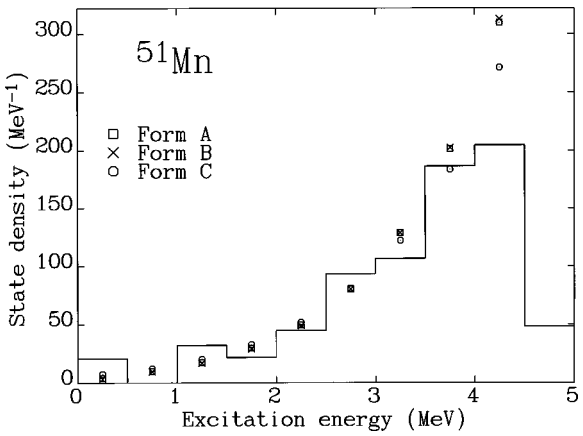

FIG. 3. Same as Fig. 1 for ^{51}Mn. The best fit δ is -0.54 MeV.

inhibition on repeated captures which take the nucleus to the drip line because of the reduction in level density. This would change the balance between beta decay and capture and would push the paths for rp and r process nucleosynthesis closer to the valley of stability. While effects due to isospin [14] should also be considered, the exponential sensitivity of the level density to the level density parameter causes factor of 10 type changes in the most extreme cases examined here. These effects will almost certainly dominate isospin effects.

As found in Ref. [7], the best fit δ values were found to have a systematic behavior. Since δ reflects both pairing and shell effects, the authors of Ref. [7] argued that δ could be predicted by calculating the difference between the actual ground state mass and that predicted by a semiempirical mass formula without pairing or shell effects. The best fit δ values of Ref. [7] were consistent with this expectation.

Similar results were found in the present analysis. Mass values were calculated with the parameters listed in Table III and the difference between the prediction and the actual ground state mass calculated. The mass parameters were derived from a fit to the nuclei in the extended data set. They are in good agreement with other parameter sets derived from fits to nuclei over a broader range in A. This difference Δ is found to be well correlated with the best fit δ value for

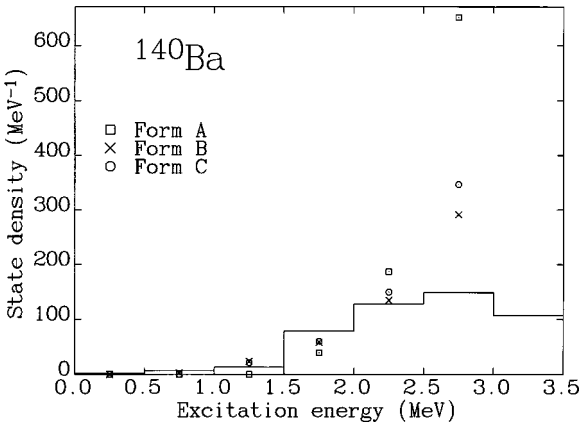

FIG. 4. Same as Fig. 1 for ^{140}Ba. The best fit δ is 0.68 MeV.

TABLE II. State density fitting results.

A	*Z*	Form A	Form C	*A*	*Z*	Form A	Form C
24	7	2.6328	1.0513	100	45	10.9700	10.2648
24	8	2.6328	1.5419	100	46	10.9700	8.4942
24	9	2.6328	2.0503	100	47	10.9700	6.3728
24	10	2.6328	2.4719	100	48	10.9700	4.3349
24	11	2.6328	2.7019	100	49	10.9700	2.6734
24	12	2.6328	2.6776	100	50	10.9700	1.4948
24	13	2.6328	2.4058	140	50	15.3580	0.3057
24	14	2.6328	1.9599	140	51	15.3580	0.7017
24	15	2.6328	1.4475	140	52	15.3580	1.4604
24	16	2.6328	0.9693	140	53	15.3580	2.7558
28	9	3.0716	1.3240	140	54	15.3580	4.7148
28	10	3.0716	1.9077	140	55	15.3580	7.3134
28	11	3.0716	2.4922	140	56	15.3580	10.2851
28	12	3.0716	2.9519	140	57	15.3580	13.1141
28	13	3.0716	3.1700	140	58	15.3580	15.1603
28	14	3.0716	3.0864	140	59	15.3580	15.8897
28	15	3.0716	2.7244	140	60	15.3580	15.0995
28	16	3.0716	2.1804	140	61	15.3580	13.0091
40	14	4.3880	1.6232	140	62	15.3580	10.1619
40	15	4.3880	2.4215	140	63	15.3580	7.1968
40	16	4.3880	3.2752	140	64	15.3580	4.6211
40	17	4.3880	4.0163	140	65	15.3580	2.6902
40	18	4.3880	4.4654	140	66	15.3580	1.4199
40	19	4.3880	4.5012	200	70	21.9400	0.0836
40	20	4.3880	4.1137	200	71	21.9400	0.2271
40	21	4.3880	3.4086	200	72	21.9400	0.5591
40	22	4.3880	2.5608	200	73	21.9400	1.2480
56	22	6.1432	3.4761	200	74	21.9400	2.5259
56	23	6.1432	4.6684	200	75	21.9400	4.6349
56	24	6.1432	5.6846	200	76	21.9400	7.7109
56	25	6.1432	6.2757	200	77	21.9400	11.6309
56	26	6.1432	6.2815	200	78	21.9400	15.9060
56	26	6.1432	6.2815	200	79	21.9400	19.7218
56	27	6.1432	5.7004	200	80	21.9400	22.1703
56	28	6.1432	4.6902	200	81	21.9400	22.5962
56	29	6.1432	3.4987	200	82	21.9400	20.8804
56	30	6.1432	2.3663	200	83	21.9400	17.4937
70	27	7.6790	3.0760	200	84	21.9400	13.2881
70	28	7.6790	4.5082	200	85	21.9400	9.1513
70	29	7.6790	5.9904	200	86	21.9400	5.7140
70	30	7.6790	7.2168	200	87	21.9400	3.2348
70	31	7.6790	7.8828	240	86	26.3280	0.8614
70	32	7.6790	7.8064	240	87	26.3280	1.8674
70	33	7.6790	7.0090	240	88	26.3280	3.6706
70	34	7.6790	5.7057	240	89	26.3280	6.5412
70	35	7.6790	4.2111	240	90	26.3280	10.5688
70	36	7.6790	2.8179	240	91	26.3280	15.4821
100	36	10.9700	0.6858	240	92	26.3280	20.5623
100	37	10.9700	1.3709	240	93	26.3280	24.7602
100	38	10.9700	2.4847	240	94	26.3280	27.0318
100	39	10.9700	4.0828	240	95	26.3280	26.7569
100	40	10.9700	6.0827	240	96	26.3280	24.0124
100	41	10.9700	8.2162	240	97	26.3280	19.5377
100	42	10.9700	10.0619	240	98	26.3280	14.4129
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100	44	10.9700	11.2466	240	100	26.3280	5.8455

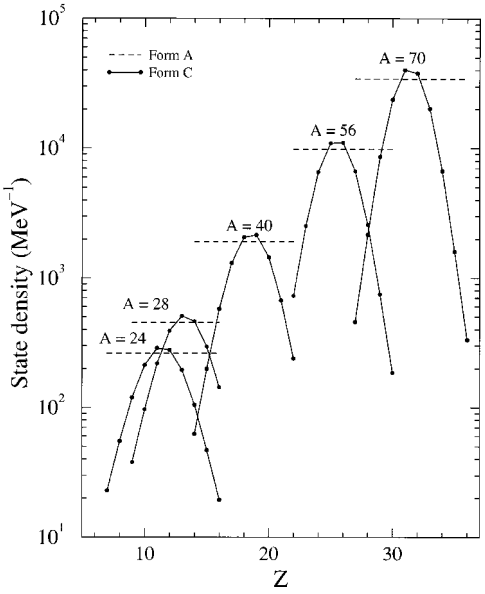

FIG. 5. State densities ($U=5$ MeV) for different fitted forms for $A=24$ to $A=70$. Note the enhancement in the valley of stability, i.e., when $Z \approx Z_0$, for form C.

each nucleus. This comparison is shown in Fig. 7. It is found that the difference is minimized if the δ values are compared with 0.578 times the Δ values.

The presence of this factor is not understood in detail. The present analysis has not included an enhancement factor for collective states. It is possible that the extra factor is serving to model collective enhancements, which could be incorporated into the level density equation by including energy shifts.

Table IV gives the information needed to use the param-

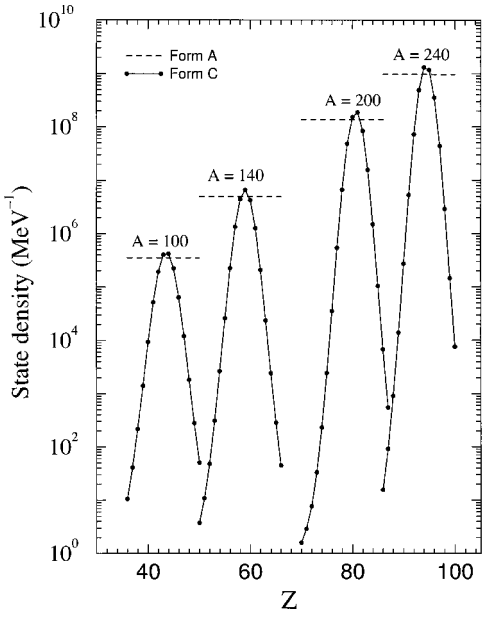

FIG. 6. State densities ($U=5$ MeV) for different fitted forms for $A=100$ to $A=240$. By including nuclei up to mass 240, we show that form C when extrapolated beyond the region used in the fitting gives plausible results.

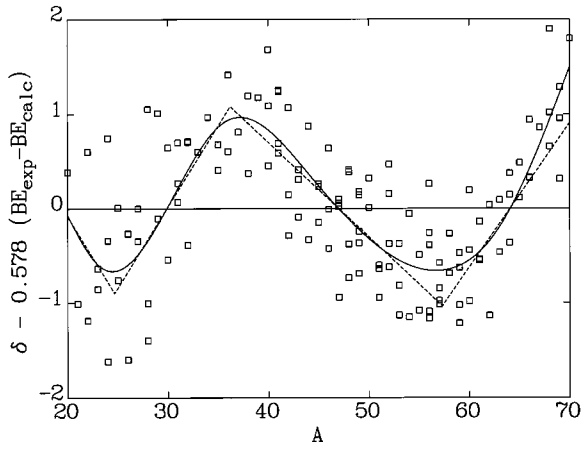

FIG. 7. Comparison of the δ values from the level density fit with 0.578 times the difference between the observed binding energy and that calculated from the formula in Table II.

TABLE III. Mass formula parameters.

$$M = m_p Z + m_n N - a_v A + a_s A^{2/3} + \left(a_c - \frac{a_d}{A^{1/3}} \right) \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{A}$$

$a_v = 14.739$
$a_s = 15.90$
$a_c = 0.6743$
$a_d = 0.4651$
$a_a = 19.37$

TABLE IV. Comparison of binding energy difference and state density energy shift.

(a) Cubic spline parameters

A	Difference
20.00	−0.071
23.19	−0.612
35.52	+0.906
44.14	+0.350
57.64	−0.653
62.67	−0.250
70.00	+1.497

(b) Straight line segment parameters

A	Difference
20.00	−0.082
24.66	−0.898
36.22	+1.082
57.29	−1.036
70.00	+0.909

eters of Table I. If the binding energy difference is multiplied by 0.578, the δ in the state density formula should be this value shifted by the amount obtained from Fig. 7. The parameters in Table IV allow a cubic spline fit (the second derivative with respect to A is set to zero for the two end points) to be calculated; this fit is the smooth line shown in Fig. 7. The second set of parameters in Table IV can be used to generate straight line segment approximations to this value; this result is also shown in Fig. 7 as the broken straight lines.

These values will be determined by the A. As can be seen from the figures, there is some dispersion among the values for fixed A as a function of Z. The use of the spline-fit or straight-line-segment representation will reduce the rms deviation from 1 MeV down to about 0.5 MeV.

III. SUMMARY

An analysis of the systematics of level density parameters for nuclei with $20 \leqslant A \leqslant 70$ has yielded evidence that the level density parameter a has a dependence on N and Z rather than just on A. This analysis indicates that $(Z-Z_0)$ is the appropriate parameter rather than $(N-Z)$. The present results suggest that efforts to study the level densities of nuclei more than two units off the stability line should be undertaken. We expect that complete level schemes up to 2.5 MeV will be difficult to obtain for higher A and for nuclei far off the valley of stability. Thus further tests of this level density approach will likely be based on evaporation spectra or binary reactions.

A relatively comprehensive fit to levels at low excitation gives a level density formula which is useful for $20 \leqslant A \leqslant 70$. It should yield more reliable results off of the stability line, although refinement of the parameters is desirable.

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