Anomalous multiplicity fluctuations from phase transitions in heavy-ion collisions

H. Heiselberg[1] and A. D. Jackson[2]
[1]*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
[2]*NBI, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
(Received 13 June 2000; published 21 May 2001)

Event-by-event fluctuations and correlations between particles produced in relativistic nuclear collisions are studied. The fluctuations in positive, negative, total, and net charge are closely related through correlations. In the event of a phase transition to a quark-gluon plasma, fluctuations in total and net charge can be enhanced and reduced, respectively, which, however, is very sensitive to the acceptance and centrality. If the colliding system experiences strong density fluctuations due, e.g., to droplet formation in a first-order phase transition, all fluctuations can be enhanced substantially. The importance of fluctuations and correlations is exemplified by event-by-event measurement of the multiplicities of J/Ψ's and charged particles since these observables should anticorrelate in the presence of comover or anomalous absorption.

DOI: 10.1103/PhysRevC.63.064904 PACS number(s): 25.75.−q, 12.38.Mh, 24.60.Ky, 24.85.+p

Event-by-event fluctuations have been measured at the SPS [1,2] and, with the higher multiplicities of RHIC and LHC, will become an important tool for studying the anomalous fluctuations and correlations that might remain following the phase transition to a quark-gluon plasma (QGP). Studies of event-by-event fluctuations at SPS energies do not indicate the presence of new physics [3–5]. It has, however, been proposed that large multiplicity fluctuations can arise from density fluctuations or droplets created by a first-order phase transition [4]. Recently, it has also been suggested that fluctuations in net charge should be suppressed in a QGP [6,7,9]. Here, we shall consider multiplicity fluctuations in some generality to see how they are affected by conservation of total charge and strangeness, to understand the correlations between various measured fluctuations, and to show how their measurement can reveal interesting details of the collision process.

Multiplicity fluctuations between various kinds of particles can be strongly correlated. As an example, consider the multiplicities of positive and negative pions, N_+ and N_-, in a rapidity interval Δy for any relativistic heavy-ion experiment. (Similar analyses can be performed for any two kinds of distinguishable particles.) We define the fluctuation in any multiplicity N as

$$\omega_N \equiv \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}. \tag{1}$$

Empirically, the fluctuations of Eq. (1) are typically of the order of unity in relativistic nuclear collisions, which is consistent with the expectations of Poisson statistics.

The net positive charge from the protons in the colliding nuclei is much smaller than the total charge produced in an ultrarelativistic heavy-ion collision. For example, $\langle N_+\rangle$ exceeds $\langle N_-\rangle$ by only $\sim 15\%$ as in Pb+Pb collisions at SPS energies. The fluctuations in the number of positive and negative (or neutral) pions are also very similar, $\omega_{N_+} \simeq \omega_{N_-}$. Charged particle fluctuations have been estimated in thermal as well as participant nucleon models [4] including effects of resonances, acceptance, and impact parameter fluctuations. By varying the acceptance and centrality, the degree of thermalization can actually be determined empirically [9]. Detailed analysis indicates that the fluctuations in central Pb+Pb collisions at the SPS are thermal whereas peripheral collisions are a superposition of pp fluctuations [10].

The fluctuations in the total ($N_{ch} = N_+ + N_-$) and net ($Q = N_+ - N_-$) charge are defined as

$$\frac{\langle (N_+ \pm N_-)^2\rangle - \langle N_+ \pm N_-\rangle^2}{\langle N_+ + N_-\rangle} = \frac{\langle N_+\rangle}{\langle N_{ch}\rangle}\omega_{N_+} + \frac{\langle N_-\rangle}{\langle N_{ch}\rangle}\omega_{N_-} \pm C, \tag{2}$$

where the correlation is given by

$$C = \frac{\langle N_+ N_-\rangle - \langle N_+\rangle\langle N_-\rangle}{\langle N_{ch}\rangle/2}. \tag{3}$$

Fluctuations in positive, negative, total, and net charge can be combined to yield both the intrinsic fluctuations in the numbers of $N_\pm$ and the correlations in their production as well as a consistency check. These quantities can change as a consequence of thermalization and a possible phase transition.

In practice, $\omega_{N_+} \approx \omega_{N_-}$, so that the fluctuation in total charge simplifies to

$$\omega_{N_{ch}} \equiv \frac{\langle N_{ch}^2\rangle - \langle N_{ch}\rangle^2}{\langle N_{ch}\rangle} = \omega_{N_+} + C, \tag{4}$$

and that for the net charge becomes

$$\omega_Q \equiv \frac{\langle Q^2\rangle - \langle Q\rangle^2}{\langle N_{ch}\rangle} = \omega_{N_+} - C. \tag{5}$$

The fluctuation in net charge is related to the fluctuation in the ratio of positive to negative particles, $\omega_Q \simeq \langle N_+/N_-\rangle\langle N_{ch}\rangle\omega_{N_-/N_+}/4$ and volume (or impact parameter) fluctuations [5,6]. The virtue of this expression is that volume fluctuations can, in principle, be extracted empirically. Alternatively one can vary the centrality bin size or the acceptance [9]. In the following we shall assume that such ''trivial'' volume fluctuations have been removed.

The analysis has so far been general and Eqs. (2)–(5) apply to any kind of distinguishable particles, e.g., positive and negative particles, pions, kaons, baryons, etc.—irrespective of what phase the system may be in or whether it is thermal or not. In the following, we shall consider thermal equilibrium, which seems to apply to central collisions between relativistic nuclei, in order to reveal possible effects on fluctuations of the presence of a quark-gluon plasma.

Bosons/fermions have thermal fluctuations, $\omega_N=1\pm\langle n_p^2\rangle/\langle n_p\rangle$ where $n_p=[\exp(\epsilon_p/T)\mp1]^{-1}$ is the boson/fermion distribution function, which are slightly larger/smaller than those of Poisson statistics for a Boltzmann distribution. Massless bosons, e.g., gluons have $\omega_B=\xi(2)/\xi(3)=1.37$ and massless fermions, e.g., quarks have $\omega_F=2\xi(2)/3\xi(3)\simeq0.91$ independent of temperature. Massive bosons have smaller fluctuations with, for example, $\omega_\pi=1.11$ and $\omega_\rho=1.01$ when $T=m_\pi$.

In a *thermal hadron gas* (HG) as created in relativistic in nuclear collisions, pions can be produced either directly or through the decay of heavier resonances, ρ, $\omega,\ldots$. The resulting fluctuation in the measured number of pions is

$$\omega_{N_+}=\omega_{N_-}=f_\pi\omega_\pi+f_\rho\omega_\rho+f_\omega\omega_\omega+\cdots, \tag{6}$$

where f_r is the fraction of measured pions produced from the decay of resonance r, and $\Sigma_r f_r=1$. These mechanisms are assumed to be independent, which is valid in a thermal system.

The heavier resonances such as $\rho^0,\omega,\ldots$ decay into pairs of $\pi^+\pi^-$ and thus lead to a correlation

$$C^{HG}=\frac{1}{3}f_\rho+f_\omega+\cdots. \tag{7}$$

Resonances reduce the fluctuations in net charge in a HG to $\omega_Q=0.70$ [5,3]. In addition, total charge neutrality reduces fluctuations in net charge when the acceptance is large and thus increases correlations as will be discussed below.

A *phase transition* to the QGP can alter both fluctuations and correlations in the production of charged pions. To the extent that these effects are not eliminated by subsequent thermalization of the HG, they may remain as observable remnants of the QGP phase. As shown in Refs. [6,7], net charge fluctuations in a plasma of *u, d* quarks and gluons are reduced partly due to the intrinsically smaller quark charge and partly due to correlations from gluons

$$\omega_Q=\frac{\langle N_q\rangle}{\langle N_{ch}\rangle}\omega_F\frac{1}{N_f}\sum_{f=u,d,\ldots}^{N_f}q_f^2, \tag{8}$$

where N_f is the number of quark flavors, q_f their charges, and N_q the number of quarks. The total number of charged particles (but not the net charge) can be altered by the ultimate hadronization of the QGP. This effect can be estimated by equating the entropy of all pions to the entropy of the quarks and gluons. Since 2/3 of all pions are charged and since the entropy per fermion is 7/6 times the entropy per boson in a QGP

$$\langle N_{ch}\rangle\simeq\frac{2}{3}\left(\langle N_g\rangle+\frac{7}{6}\langle N_q\rangle\right), \tag{9}$$

where the number of gluons is $\langle N_g\rangle=(16/9N_f)\langle N_q\rangle$. Inserting this result in Eq. (8), we see that the resulting fluctuations are $\omega_Q=0.18$ in a two-flavor QGP (and $\omega_Q=0.12$ for three flavors). As pointed out in [6], lattice results give $\omega_Q\simeq0.25$. However, according to [8] a substantial fraction of the pions are decay products from the HG, and the entropy of the HG exceeds that of a pion gas by a factor of $1.75-1.8$. As described in [7] the net charge fluctuations should be increased by this factor in the QGP, i.e., $\omega_Q\simeq0.33$ in a two-flavor QGP, whereas it remains similar in the HG, $\omega_Q\simeq0.6$.

However, if the high density phase is initially dominated by gluons with quarks produced only at a later stage of the expansion by gluon fusion, the production of positively and negatively charged quarks will be strongly correlated on sufficiently small rapidity scales. If, for example, the entropy density increases by an order of magnitude in going from a HG to QGP without additional net charge production, fluctuations in net charge will be reduced significantly,

$$\omega_Q^{QGP}\simeq\frac{s_{HG}}{s_{QGP}}\omega_Q^{HG}. \tag{10}$$

The resulting fluctuation in net charge is necessarily *smaller* than that from thermal quark production as given by Eq. (8). A similar phenomenon occurs in string models where particle production by string breaking and $q\overline{q}$ pair production results in flavor and charge correlations on a small rapidity scale [11,12]. The recently measured charged particle density at midrapidity in central nuclear collisions at RHIC [13] is only $\sim30-40\%$ larger than pp scaled up by the nuclear mass as was also found at SPS energies [1]. If entropy and multiplicities are proportional, the net and total charged particle fluctuations should be the same as at SPS according to Eq. (10) unless anomalous nonthermal fluctuations occur as will be discussed below.

The *strangeness* fluctuation in kaons $K^\pm$ might seem less interesting at first sight since strangeness is not suppressed in the QGP: The strangeness per kaon is unity and the total number of kaons is equal to the number of strange quarks. However, if strange quarks are produced at a late stage in the expansion of a fluid initially dominated by gluons, the net strangeness will again be greatly reduced on sufficiently small rapidity scale. Consequently, fluctuations in net/total strangeness would be reduced/enhanced.

The *baryon number* fluctuations have been estimated in a thermal model [7]. It is, however, not known how the annihilation of baryons and antibaryons in the hadronic phase affect these results. If only charged particles are detected, but not K^0, $\overline{K^0}$, neutrons, and antineutrons, the fluctuations have smaller correlations as compared to the total and net strangeness or baryon number.

Total charge conservation is important when the acceptance Δy is a non-negligible fraction of the total rapidity. It reduces the fluctuations in the net charge as calculated within the canonical ensemble, Eqs. (6)–(9). If the total positive charge (which is exactly equal to the total negative charge plus the incoming nuclear charges) is independently distributed according to the single particle distributions, the resulting fluctuations within the acceptance are

$$\omega_Q = 1 - f_{acc}, \tag{11}$$

$$\omega_{N_{ch}} = 1 - f_{acc} + 2 f_{acc} \omega_{N_+}, \tag{12}$$

where $f_{acc} = N_{tot}^{-1} \int_{\Delta y} (dN_{ch}/dy) dy$ is the acceptance fraction or the probability that a charged particle falls into the acceptance Δy. Since charged particle rapidity distributions are peaked near midrapidity, charge conservation effectively kills fluctuations in the net charge even when Δy is substantially smaller than the laboratory rapidity, $y_{lab} \simeq 6$ (11) at SPS (RHIC) energies. Total charge conservation also has the effect of increasing ω_{ch} towards $2\omega_{N_+}$ according to Eqs. (4) and (12). Similar effects can be seen in photon fluctuations when photons are produced in pairs through $\rho^0 \to 2\gamma$. In the WA98 experiment, $\omega_\gamma \simeq 2$ is found after acceptance corrections and eliminating volume fluctuations [2].

When the acceptance Δy is too small, particles from a thermal ensemble can diffuse in and out of the acceptance during hadronization and freeze-out [7]. Furthermore, correlations due to resonance production will disappear when the average separation in rapidity between decay products exceeds the acceptance. Each of these effects tend towards Poisson statistics when $\Delta y \lesssim \delta y$, where δy denotes the rapidity interval that particles diffuse during hadronization, freeze-out, and decay. If ω_Q is the canonical thermal fluctuation of Eqs. (7)–(9), the resulting fluctuation after correcting for both δy and total charge conservation is approximately

$$\omega_Q^{exp} \simeq \left(\frac{\Delta y}{\Delta y + 2\delta y} \omega_Q + \frac{2\delta y}{\Delta y + 2\delta y} \right) (1 - f_{acc}). \tag{13}$$

Here, the factor $(1 - f_{acc})$ is due to total charge conservation as in Eq. (11). The remainder is fluctuations from two sources: a thermal one with fluctuations ω_Q and a random one with Poisson fluctuations, each weighted with the fraction of the charged particles they contribute.

The resulting fluctuations in total and net charge are shown in Fig. 1 assuming $\omega_{N_+} = \omega_\pi \simeq 1.1$ and $\delta y = 0.5$. As mentioned above, f_{acc} and Δy are related by the measured charge particle rapidity distributions [1]. Preliminary NA49 data [1,5] agree well with the net and total fluctuations in a HG ($C = 0.4$) from Eqs. (13) and (9). Residual volume fluctuations are significant for $\omega_{N_{ch}}$ [4] and have been estimated and subtracted. The curves apply to RHIC energies as well after scaling δy with Δy.

The net charge fluctuations in a thermal HG corrected for finite acceptance and diffusion are slightly below the value without any intrinsic correlations given by Eq. (11). This reduction is due to the correlations in the HG that leads to $\omega_Q < 1$. In high energy pp collisions there are stronger rapidity correlations between unlike than like charged particles [11,12] leading to similar magnitude for ω_Q.[1] Therefore the net charge fluctuations does not vary much by going from peripheral pp-like high energy nuclear collisions to central collisions that are more likely to produce a thermal hadronic gas. The fluctuations in total charge are, however, very different because the total charge fluctuations in pp collisions increase dramatically with collision energy, $\omega_{N_{ch}}^{RHIC} \simeq 6$ and $\omega_{N_{ch}}^{LHC} \simeq 20$ as compared to $\omega_{N_{ch}}^{SPS} \simeq 2.0$ [11,12]. Peripheral collisions will therefore be very different from central ones and the centrality dependence should be studied carefully to assess the degree of thermalization before anomalous fluctuations due to phase transitions can be determined [9].

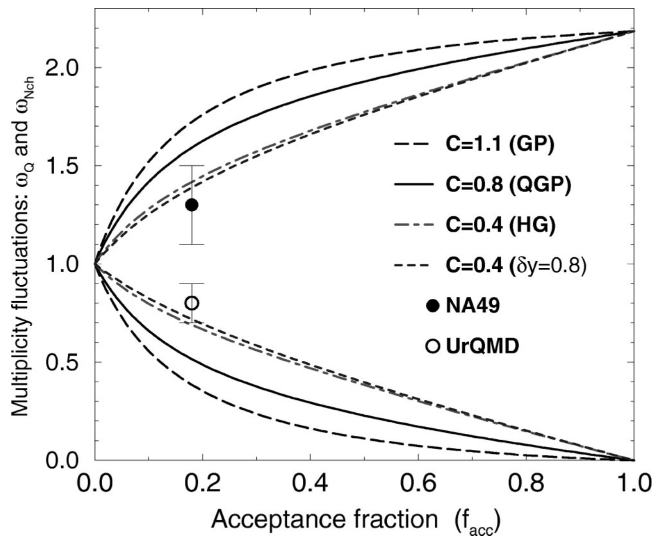

FIG. 1. Acceptance dependence of thermal fluctuations in net charge [ω_Q of Eq. (13), lower curves] and total charge ($\omega_{N_{ch}}$, upper curves). Correlations increase from a hadron gas ($C \simeq 0.4$) to a QGP ($C \simeq 0.8$) and a pure gluon plasma ($C \simeq 1.1$) (see text).

Large nonthermal fluctuations can arise as a consequence of density fluctuations due, e.g., to droplet formation in first-order phase transitions. These could lead to large fluctuations in multiplicities [4,9,14] of charged particles and therefore also in the total and net charge. The above estimates for the fluctuations were of the order of unity. They implicitly assumed a uniform expanding system. Consider a scenario where the total multiplicity within the acceptance arises from a normal hadronic background component (N_{HG}) and from a second component (N_{QGP}) that has undergone a transition,

$$N = N_{HG} + N_{QGP}. \tag{14}$$

Its average is $\langle N \rangle = \langle N_{HG} \rangle + \langle N_{QGP} \rangle$. Assuming that the multiplicity of each of these components is statistically independent, the multiplicity fluctuation becomes

[1]The correlations are related to the two-body density distributions, e.g., $\langle N_+ N_- \rangle = \int \rho_{+-}^{(2)}(y_1, y_2) dy_1 dy_2$, where the integral extends over $y_1, y_2 \in \Delta y$.

$$\omega_N=\omega_{HG}+(\omega_{QGP}-\omega_{HG})\frac{\langle N_{QGP}\rangle}{\langle N\rangle}. \tag{15}$$

Here, ω_{HG} is the standard fluctuation in hadronic matter $\omega_{HG}\simeq 1$. The fluctuations due to the component that had experienced a phase transition, ω_{QGP}, depend on the type and order of the transition, the speed with which the collision zone goes through the transition, the degree of equilibrium, the subsequent hadronization process, the number of rescatterings between hadronization and freeze-out, etc. If thermal and chemical equilibration eliminate all signs of the transition, $\omega_{QGP}\simeq\omega_{HG}$. At the other extreme, the droplet scenario could produce $\omega_{QGP}\sim\langle N\rangle\sim 10^2-10^3$ if most hadrons arrive from a droplet so that either all or none fall into the acceptance [4]. This is a promising signal worth looking for. Since droplets or density fluctuations are expected to be charge neutral, net charge fluctuations should vanish $\omega_Q\simeq 0$ whereas $\omega_{ch}\simeq 2\omega_{N_+}\sim 2\omega_{QGP}$.

General correlators between all particle species should be measured event by event, e.g., the ratios [4]

$$\frac{\langle N_i/N_j\rangle}{\langle N_i\rangle/\langle N_j\rangle}\simeq 1+\frac{\omega_{N_j}}{\langle N_j\rangle}-\frac{\langle N_iN_j\rangle-\langle N_i\rangle\langle N_j\rangle}{\langle N_i\rangle\langle N_j\rangle}, \tag{16}$$

where $N_{i,j}$ are the multiplicities in acceptances i and j of any particle type. (We assume that N_j is so large that it never vanishes.) In the presence of droplets, N_i and N_j would be strongly correlated in nearby rapidity intervals and at all azimuthal angles.

As another example, consider correlations between multiplicities of charmonium particles N_ψ, $\psi=J/\Psi,\psi',\chi,\ldots$, and charged particle multiplicity N in a given rapidity interval Δy. If a ψ is absorbed on comovers or anomalously suppressed by QGP, one would expect *anticorrelations* because the number of comovers and QGP should scale with the multiplicity N. By contrast, direct Glauber absorption should not depend on the multiplicity of particles in Δy for a given centrality since it is the result of collisions with participating nucleons in Glauber trajectories along the beamline.

To quantify this anticorrelation, we model the absorption of ψs by simple Glauber theory

$$\frac{\overline{N}_\psi}{N_\psi^0}=e^{-\sigma_{c\psi}\rho_c l}\equiv e^{-\gamma N/\langle N\rangle}, \tag{17}$$

where $\overline{N}_\psi$ is the average number of ψ's for given charge particle multiplicity N, and N_ψ^0 is the number of ψ's before comover or anomalous absorption sets in but after direct Glauber absorption on participant nucleons. In Glauber theory, the exponent is the product of the absorption cross section ($\sigma_{c\psi}$), the absorber density (ρ_c), and the average path length (l) traversed in matter. The density, therefore also the exponent, is proportional to the multiplicity N with coefficient $\gamma=-d\ln N_\psi/d\ln N$. In a simple comover absorption model for a system with longitudinal Bjorken scaling, γ can be calculated to be approximately [15]

$$\gamma\simeq\sum_c\frac{dN_c}{dy}\frac{\langle v_{c\psi}\sigma_{c\psi}\rangle}{4\pi R^2}\ln(R/\tau_0), \tag{18}$$

where dN_c/dy is the comover rapidity density, $\sigma_{c\psi}$ the absorption cross section, $v_{c\psi}$ the relative velocity, R the transverse size of the overlap zone, and τ_0 the formation time. Comover absorption reduces the number of ψ by a factor $e^{-\gamma}$ where γ increases with centrality up to $\gamma\sim 1$ for typical parameters employed in comover absorption models [16].

Since fluctuations in the exponent are small, $\gamma\sqrt{\omega_N/\langle N\rangle}\ll 1$, the anticorrelation is

$$\frac{\langle NN_\psi\rangle-\langle N\rangle\langle N_\psi\rangle}{\langle N_\psi\rangle}=-\gamma\omega_N. \tag{19}$$

It is negative and proportional to the amount of comover and anomalous absorption. It vanishes when the absorption is independent of multiplicity ($\gamma=0$). The rapidity interval should not be less than the typical relative rapidities between comovers and the ψ or the rapidity range of a droplet.

Even in central heavy-ion collisions ψ's are rarely produced and so $N_\psi=1$ or 0. In both cases one should measure dN_{ch}/dy and average over events with and without a ψ separately. If comovers absorb the ψ we expect that $\langle dN_{ch}/dy\rangle$ is slightly smaller for the events with a ψ than without, leading to the negative correlation in Eq. (19).

A quantitative assessment of the average suppression of ψ's due to comover absorption versus direct ψ-nucleon absorption has been debated ever since the first measurements of J/Ψ suppression. The anticorrelations of Eq. (19) directly quantify the amount of comover or anomalous absorption and can therefore be exploited to distinguish between these and direct Glauber absorption mechanisms. In that respect it is similar to the elliptic flow parameter for ψ [15]. For a sample of N_{events}^{ψ}, the statistical uncertainty in γ as determined by Eq. (19) is $\sim\sqrt{\langle N\rangle/N_{events}^{\psi}}$. If we take a rapidity bin $\Delta y\sim 1$ and consider central heavy-ion collisions, $\langle N\rangle$ will range from $\sim 10^2-10^3$ in going from SPS to RHIC energies. A sample of $(10^4-10^5)\,\psi$'s would be sufficient to determine γ with an accuracy of ± 0.1. The analysis of the kaon to pion ratio by NA49 obtains such an accuracy by comparing to a mixed event analysis that removes systematic errors [1].

The impact parameter fluctuations and correlations may not cancel exactly for the ψ/N_{ch} ratio, as they do for the π^+/π^- ratio [5] because their production mechanisms differ. It is hard for the ψ and soft for most of the charged particles and thus their multiplicities scale approximately with the number of binary NN collisions and the number of participants, respectively. The number of NN collisions increase more rapidly with centrality and nuclear mass number ($\propto A^{4/3}$). As a result the impact parameter correlations between the ψ and N in Eq. (19) will be slightly larger than the impact parameter fluctuations implicit in the second term in Eq. (16). The difference is a finite fraction of the total impact parameter fluctuations and will be of similar magnitude but opposite sign as correlations from comover absorption. The net impact parameter fluctuations will, however, depend on centrality and decrease as bin size of the centrality cut decreases, which should make it possible to separate it from other correlations. It would reveal independent information on the soft versus hard production mechanisms.

In summary, we have given a detailed analysis of total and net charge fluctuations and correlations including total charge conservation and diffusion effects and how they depend on the acceptance. The correlations may increase if a QGP is formed resulting in a reduction/enhancement of net/total charge by up to an order of magnitude depending on the model. It is important to measure fluctuations and correlations for various acceptances as well as versus centrality and/or beam energy. At SPS energies the fluctuations in net charge actually *increase* slightly with centrality [10] due to thermalization, which is opposite to the predicted decrease in net charge fluctuations if a QGP is formed in central heavy-ion collisions. At RHIC and LHC energies the correlations versus centrality will be much larger because the total charge fluctuations in pp collisions are $\omega_{N_{ch}}^{RHIC}\simeq 6$ and $\omega_{N_{ch}}^{LHC}\simeq 20$ as compared to $\omega_{N_{ch}}^{SPS}\simeq 2.0$ [11,12]. Peripheral collisions will therefore be very different from central ones and the centrality dependence should be studied carefully to assess the degree of thermalization before anomalous fluctuations due to phase transitions can be determined [9]. It is important to measure all multiplicity fluctuations and correlations and understand the physical effects discussed above before a possible small increase in correlations can be attributed to the formation of a QGP.

We are grateful to S. Voloshin (NA49) and T. Nayak (WA98) for discussions and for showing us preliminary data.

[1] NA49 Collaboration, G. Roland *et al.*, Nucl. Phys. **A638**, 91c (1998); NA49 Collaboration, H. Appelhäuser *et al.*, Phys. Lett. B **459**, 679 (1999); NA49 Collaboration, S. V. Afanasiev *et al.*, hep-ex/0009053.
[2] WA98 Collaboration, T. K. Nayak (private communication).
[3] M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998); Phys. Rev. D **60**, 114028 (1999); B. Berdnikov and K. Rajagopal, *ibid.* **61**, 105017 (2000).
[4] G. Baym and H. Heiselberg, Phys. Lett. B **469**, 7 (1999).
[5] S. Jeon and V. Koch, Phys. Rev. Lett. **83**, 5435 (1999).
[6] S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000).
[7] M. Asakawa, U. Heinz, and B. Müller, Phys. Rev. Lett. **85**, 2072 (2000).
[8] J. Sollfrank and U. Heinz, Phys. Lett. B **289**, 132 (1992).
[9] H. Heiselberg, nucl-th/0003276.
[10] NA49 Collaboration, S. A. Voloshin (unpublished).
[11] H. Bøgild and T. Ferbel, Annu. Rev. Nucl. Sci. **24**, 451 (1974).
[12] J. Whitmore, Phys. Rep. Phys. Lett. **27C**, 187 (1976).
[13] PHOBOS Collaboration, B. B. Back *et al.*, Phys. Rev. Lett. **85**, 3100 (2000).
[14] H. Heiselberg and A. D. Jackson, in *Proceedings of Continuous Advances in QCD*, Minnesota, 1998, edited by A. V. Smilga (World Scientific, Singapore, 1998), p. 430; nucl-th/9809013.
[15] H. Heiselberg and R. Mattiello, Phys. Rev. C **60**, 044902 (1999).
[16] S. Gavin and R. Vogt, Phys. Rev. Lett. **78**, 1006 (1997).