

Long-range charge fluctuations and search for a quark-gluon plasma signal

E. V. Shuryak¹ and M. A. Stephanov²

¹*Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794-3800*

²*Department of Physics, University of Illinois, Chicago, Illinois 60607-7059*

and RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973

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We critically discuss a recent suggestion to use long-range modes of charge (electric or baryon) fluctuations as a signal for the presence of quark-gluon plasma at the early stages of a heavy ion collision. We evaluate the rate of diffusion in rapidity for different secondaries, and argue that for conditions of the Super Proton Synchrotron (SPS) experiments, it is strong enough to relax the magnitude of those fluctuations almost to its equilibrium values, given by hadronic “resonance gas.” We evaluate the detector acceptance needed to measure such “primordial” long-range fluctuations at Relativistic Heavy Ion Collider (RHIC) conditions. We conclude with an application of the charge fluctuation analysis to the search for the QCD critical point.

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I. INTRODUCTION

During the last few years significant attention has been attracted to the issue of *event-by-event fluctuations* in high energy heavy ion collisions. The original goal of such studies formulated in [1,2] has been focused on *equilibrium* thermodynamical fluctuations *at freeze-out*. It was argued that a long and intense final state interaction of secondaries makes heavy ion collision very different from hadronic collisions, in which dynamical fluctuations of quantum origin produce quite different “intermittent” behavior. Experimental data, such as obtained by the NA49 Collaboration [3], have indeed revealed only very small *Gaussian* fluctuations of different quantities incompatible with earlier predictions of large non-equilibrium fluctuations, e.g., due to bubble formation during the phase transition. As discussed in [4–6] in detail, extensive quantities like the total multiplicity N obtain contributions from the *final state* interaction of secondaries due to resonances and the *initial state* fluctuations related to the number of participant and/or spectator nucleons. Furthermore, experimentally, for central collisions $\langle(\Delta N)^2\rangle/\langle N\rangle \approx 2$, and both effects share about equally the responsibility for moving this number away from the Poissonian value $\langle(\Delta N)^2\rangle/\langle N\rangle = 1$. As emphasized in [4,7], the intensive quantities, such as mean p_T , particles ratios, etc., are determined entirely by the final state effects, and are well represented by the resonance gas.

Other goals of the event-by-event fluctuation analysis have also been discussed. In particular, one of us [8], using the analogy between big bang phenomena and the “little bang” in heavy ion collisions, suggested to look for primordial “frozen QGP oscillations,” as is done for anisotropic components of microwave background in the Universe. The idea was inspired by an observation that large-amplitude oscillations can be excited due to instabilities of parton counterflows at the initial stage [9]. On the other hand, two simultaneous papers, by Asakawa, Heinz, and Muller [10] and Jeon and Koch [11] suggested a different set of primordial signals: *equilibrium* charge fluctuations in quark-gluon plasma (QGP), which happen to be a factor 2–3 *smaller* than in the hadron gas.

The idea [10,11] is based on the well known phenomenon of *kinetic slowdown* of fluctuation relaxation, provided *sufficiently long-range harmonics* of a *conserved* charge density are considered. If the relaxation time happens to be shorter than the lifetime of the hadronic stage of the collisions, the authors argue, then the values of such fluctuations should deviate from their equilibrium (resonance gas) values towards their earlier, primordial values, typical for QGP.

Although this idea should work for parametrically long-range effects, whether a number of necessary conditions can indeed be fulfilled in realistic heavy ion collisions depends on the answers to the following questions. (i) How fast is the relaxation due to final-state re-scattering in hadronic gas? (ii) How large should the detector acceptance really be in order to counter the relaxation? (iii) Is there a window of parameters forbidding resonance gas relaxation but allowing for it at the QGP stage? (iv) If not, what fluctuations should follow from nonequilibrium parton kinetics, at very early pre-QGP stages of the collisions?

In this paper we attempt to answer the first two of these questions.

II. RELAXATION OF THE LONG-RANGE FLUCTUATIONS

A. The setting

In this section we develop an analytical description of the time evolution of fluctuations due to the rescattering of charged particles in the hadronic gas at the late stages of the heavy ion collision.¹ The following physical picture underlines our approach.

We consider distributions in rapidity space of a *conserved* net charge (e.g., electric or baryon). We start our description

¹Our approach is different from that of [10–15]. We provide a quantitative description of the time evolution of long-range fluctuations based on a stochastic diffusion equation, which we solve analytically. The value of the diffusion coefficient is our only numerical input and we determine it using realistic hadronic gas properties.

from the beginning of the hadronic phase. In each event this phase “inherits” a certain distribution in rapidity space of the charge from the primordial quark-gluon plasma phase. In the spirit of the statistical approach, we do not consider a specific charge distribution, but an ensemble of distributions corresponding to a set of events. This ensemble of charge distributions is characterized by the mean and the magnitude of fluctuations. The initial magnitude of those fluctuations is the input of our calculation.

The fluctuations of different length, or range, in rapidity space (i.e., different harmonics of the charge distribution) relax on different time scales. The fact that the net charge is conserved is crucial here. Because the relaxation can only proceed via diffusion of the charge, the *longer* range fluctuations relax *slower*. The relaxation time grows as a square of the range. Our goal is to provide a quantitative description of this process.

In particular, we shall derive the equation which solves the following problem: Given a certain detector rapidity acceptance window, and given the initial magnitude of the charge fluctuations, to find the time evolution of that magnitude. From the fact that longer range charge fluctuations relax slower, it is evident that fluctuations of total charge in a wider rapidity window relax slower. Using typical numbers for the lifetime of the hadronic phase and rescattering properties of the hadronic gas we shall determine how wide the rapidity window must be chosen in order to preserve, to a sufficient extent, the primordial (small) fluctuation magnitude.

B. Diffusion in rapidity space

We introduce the density of a conserved *net* charge Q per unit rapidity, $n(y, \tau)$, as a function of time τ in a given event. The objects we are really interested in are the *fluctuations* of the density,

$$f(y, \tau) = n(y, \tau) - n_0. \quad (1)$$

In the Bjorken boost invariant scenario n_0 is proportional to the total charge in the central region, $n_0 = Q_{\text{total}}/y_{\text{total}}$, which is a constant within each event.

We begin by considering evolution of f in a given event. The event-by-event fluctuations will be the subject of Sec. II D. There we shall also address fluctuations of the total charge (Q , or n_0), which we ignore for now. Since f is a density of a *conserved* quantity it should obey a diffusion equation in rapidity space,

$$\partial_\tau f = \gamma \partial_y^2 f, \quad (2)$$

where $\gamma = \gamma(\tau)$ is the diffusion coefficient. This diffusion equation (2) applies, provided the following conditions are satisfied.

(i) We must assume that fluctuations are sufficiently small to justify linear approximation in Eq. (2). This is reasonable if the number of particles carrying the charge is large, which is fulfilled in sufficiently central heavy ion collisions at the Super Proton Synchrotron (SPS) and higher energies.

(ii) We must assume separation of scales—the minimal interval of rapidity we can consider must be much larger than the mean rapidity change of a charged particle in a collision, δy_{coll} . In other words, f must be averaged over a sufficiently wide interval Δy . As we shall see later, the typical δy_{coll} for the electrical charge is of order 0.8, while for the baryon charge it is 0.2. Even in such a large acceptance detector as STAR at the Relativistic Heavy Ion Collider (RHIC), the coverage does not exceed few units of rapidity. However, we can still consider a somewhat idealized limit of rapidity windows much wider than δy_{coll} . This can certainly be reasonable for the diffusion of the baryon charge. Moreover, in order for the ideas we are evaluating to work, this limit is a necessary condition [10,11]. Indeed, if the rapidity window is not large compared to δy_{coll} , the fluctuations will relax by ballistic transport of the charge on the time scale of the mean free time τ_{free} towards their thermodynamic values, too fast for those ideas to work.²

It will not be essential for the analysis of this section how the coefficient $\gamma(\tau)$ is calculated. It may be (ideally) determined from first principles of QCD. In the late hadronic stage of the collision, which is our concern in this paper, one can also use a simple formula to estimate γ . Since, on the microscopic level, γ is due to particle collisions:³

$$\gamma = \frac{(\delta y_{\text{coll}})^2}{2 \tau_{\text{free}}}. \quad (3)$$

Since we shall be using Eq. (3) in Sec. III to estimate γ (or, rather, its time integral, see below), we shall discuss it here. It is clear that in the collisionless limit, $\tau_{\text{free}} \rightarrow \infty$, the diffusion in rapidity is absent, which is in trivial agreement with Eq. (3). It is instructive to consider also the limit of $\tau_{\text{free}} \rightarrow 0$ —the ideal hydrodynamic limit. In this limit diffusion is also frozen—particles have no time to propagate. Equation (3) can still be applied, but one must realize that, effectively, δy_{coll} also vanishes in this limit. The reason is that successive rapidity shifts (each of order unity) are not independent but strongly anticorrelated. This happens because a colliding particle has no time to move out of the region of particles of its initial rapidity, and is more likely to scatter back in than further out. This is also related to the fact that space-time rapidity $(1/2) \ln[(t+z)/(t-z)]$ and momentum rapidity can differ significantly in this limit.

In this paper we need to describe the late not-so-dense hadronic stage of the collision with τ_{free} only a few times smaller than evolution time, and we shall apply Eq. (3) without worrying about complications of the ideal hydro limit.

²In the same spirit, we shall not consider also the effects of finiteness of y_{max} , the total collision rapidity range. These have been studied already [12,13] and correspond to certain boundary conditions in Eq. (2).

³One can think of this as a random walk in rapidity space with Gaussian random steps of mean square length $(\delta y_{\text{coll}})^2$ and compare the equation for the mean square distance from origin: $(\Delta y)^2 = (\tau/\tau_{\text{free}})(\delta y_{\text{coll}})^2$ with the Gaussian solution of the diffusion equation (2), which gives $(\Delta y)^2 = 2 \gamma \tau$.

One can also use cascade codes to estimate γ , or just the diffusion length squared $2\int\gamma d\tau$, directly, with possible anticorrelations already included, bypassing the evaluation of δy_{coll} .

C. Langevin equation

Equation (2) describes relaxation of the density n to its equilibrium value. It implies that we average over time scales longer compared to the characteristic time of fluctuation of f . If, however, we are interested in *fluctuations* of n around its equilibrium value, Eq. (2) is not enough. We must add a noise term

$$\partial_\tau f = \gamma \partial_y^2 f + \xi(y, \tau). \quad (4)$$

The characteristic autocorrelation time of the noise is of the same order as the time scale of the variation of f . This time is very short since the rapidity windows we consider contain many particles: f changes each time any of the particles collides.

The rapidity correlation of the noise can be determined by requiring that equilibrium fluctuations of f are given by thermodynamics. Thus, they are Gaussian with the probability distribution

$$P_{\text{therm}}[f] \sim \exp\left\{-\frac{1}{2}\int dy dy' \frac{f(y)f(y')}{2\chi(y-y')}\right\}, \quad (5)$$

where χ is the equilibrium density-density correlation function in rapidity, which we need not know explicitly at this point,⁴ and we used boost invariance of the Bjorken scenario. In other words,

$$\langle f(y)f(y') \rangle = \chi(y-y'), \quad (6)$$

where the average is over time. Writing the functional Fokker-Planck equation for the probability density $P[f]$ following from the Langevin equation (4), we find that Eq. (5) is a stationary solution when the noise ξ is Gaussian and obeys

$$\langle \xi(y, \tau)\xi(y', \tau') \rangle = -2\gamma\partial_y^2\chi(y-y')\delta(\tau-\tau'). \quad (7)$$

D. Fokker-Planck equation

Before we proceed to solve the Langevin equation (4), we should relate the time average in Eq. (6) to the event-by-event average. They are the same by ergodicity. However, one should also keep in mind that changes in initial conditions will change the equilibrium value n_0 . Such initial state fluctuations, as discussed in the Introduction, can contribute significantly to the observed event-by-event fluctuations of the *extensive* quantities, such as the total charge Q . Such fluctuations are statistically independent from the thermodynamic fluctuations we consider and can be calculated separately

⁴The relaxation time of small linear fluctuations we study will not depend on their absolute magnitude.

and added in quadratures, as is done in [6]. In contrast, for *intensive* quantities, such as, for example, the ratio of positively to negatively charged particles considered in [7], such initial state fluctuations cancel and need not be considered.

Our goal now is to use Eq. (4) to determine how quickly fluctuations approach their thermodynamic distribution (5). In other words, we need to solve the functional Fokker-Planck equation with a given initial distribution $P[f]$, and see how this *functional* evolves with time.

The Fokker-Planck equation corresponding to Eqs. (4) and (7) is given by

$$\partial_\tau P = \int dy dy' \gamma \frac{\delta}{\delta f(y)} \times \left[-\partial_y^2 f(y) P - \partial_y^2 \chi(y-y') \frac{\delta}{\delta f(y')} P \right], \quad (8)$$

and is not very easy to study. Boost invariance helps, however. We shall do a Fourier transform with respect to the variable y . Let us denote harmonics of $f(y)$ by f_k . If we start with the factorized probability distribution $P = \prod_k P_k[f_k]$, then each harmonic evolves independently following the partial differential equation

$$\partial_\tau P_k = \gamma k^2 \frac{\partial}{\partial f_k} \left[f_k P_k + \chi_k \frac{\partial}{\partial f_k} P_k \right]. \quad (9)$$

It is easier to derive this equation directly from the Fourier transformed Langevin equation⁵

$$\partial_\tau f_k = -\gamma k^2 f_k + \xi_k,$$

$$\langle \xi_k(\tau)\xi_{k'}(\tau') \rangle = 2\gamma k^2 \chi_k \delta_{kk'} \delta(\tau-\tau'). \quad (10)$$

E. Gaussian solution

We can now pose the problem mathematically and solve it using Eq. (9). Let us imagine that at some initial time $\tau = 0$ (in our analysis, the beginning of the hadronic phase) the probability distribution for each harmonic f_k is given by a Gaussian with a mean square variance $\langle f_k^2 \rangle = \sigma_k^2$. What happens to this probability distribution with time? Substituting the Gaussian ansatz

$$P_k[f_k] = [2\pi\sigma_k^2(\tau)]^{-1/2} \exp\left\{-\frac{f_k^2}{2\sigma_k^2(\tau)}\right\}, \quad (11)$$

one sees that Eq. (9) preserves the Gaussian shape of the probability distribution. The evolution of the distribution can

⁵The condition of applicability of the diffusion approximation, already discussed above, can be also stated as the condition that we are describing sufficiently long range, i.e., small k harmonics: $k < 1/\delta y_{\text{coll}}$. For high harmonics the relaxation rate is $1/\tau_{\text{free}}$ instead of γk^2 .

therefore be described by the evolution of $\sigma_k(\tau)$. The equation for $\sigma_k(\tau)$ following from Eq. (9) is given by

$$\partial_\tau \sigma_k^2 = -2\gamma k^2 (\sigma_k^2 - \chi_k), \quad (12)$$

with the solution

$$\sigma_k^2(\tau) = \chi_k + [\sigma_k^2(0) - \chi_k] \exp\left\{-2k^2 \int_0^\tau \gamma d\tau\right\}. \quad (13)$$

For simplicity, we neglected the dependence of χ_k on τ in the hadronic phase. Obviously, Eq. (12) can be solved for arbitrary $\chi_k(\tau)$, if one is given. In this paper we apply Eq. (12) to an idealized scenario, in which χ_k changes abruptly at the QGP-to-hadron-gas transition, and we see how the charge fluctuation magnitude relaxes to its hadron gas equilibrium value. Equation (13) demonstrates explicitly that, due to the local conservation of charge, harmonics with small k relax very slowly. Using Eq. (3), the exponent in Eq. (13) can be also written as $k^2(\Delta y_{\text{diff}})^2$, where

$$(\Delta y_{\text{diff}})^2 \equiv 2 \int_0^\tau \gamma d\tau = \int_0^\tau (\delta y_{\text{coll}})^2 \frac{d\tau}{\tau_{\text{free}}} \quad (14)$$

is the mean distance on which a charged particle diffuses during the time τ .

F. Relaxation in a rapidity window

In experiment one is measuring the event-by-event fluctuation of charge,

$$Q = \int_{y_1}^{y_2} n dy, \quad (15)$$

in an interval of rapidity, $[y_1, y_2]$. This fluctuation, $\Delta Q = Q - \langle Q \rangle$, can be written as a weighted sum of the fluctuations of the Fourier components f_k ,

$$\langle (\Delta Q)^2 \rangle = \int \frac{dk}{2\pi} |c_k|^2 \langle f_k^2 \rangle = \int \frac{dk}{2\pi} c_k^2 \sigma_k^2(\tau). \quad (16)$$

The weight is determined by the Fourier harmonics, c_k , of the function equal to unity for $y \in [y_1, y_2]$ and zero otherwise, i.e.,

$$|c_k|^2 = \frac{\sin^2(k\Delta y/2)}{(k/2)^2} \quad (\Delta y \equiv y_2 - y_1). \quad (17)$$

The main contribution is from harmonics with $k \sim 1/\Delta y$ and smaller. Equations (13) and (16) determine the evolution of the magnitude of the fluctuation $\langle (\Delta Q)^2 \rangle$.

For example, consider the case when the hadron gas ‘‘inherits’’ from the QGP phase practically zero fluctuations of the charge (or fluctuations much smaller than required by hadron gas thermodynamics—an idealization of [10,11]), i.e., $\sigma_k^2(0) \ll \chi_k$. Then, using Eqs. (13), (14), (16), and (17), we can calculate the magnitude of the fluctuation of the charge Q in the rapidity window Δy after time τ :

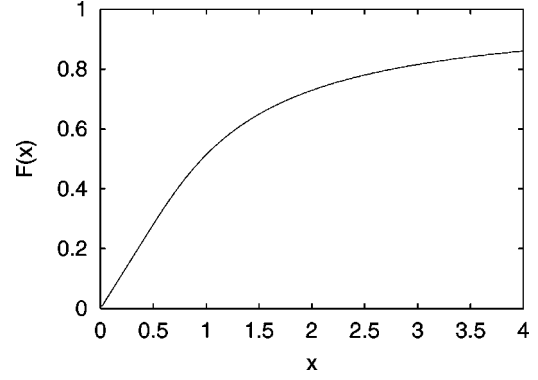


FIG. 1. The function describing the onset and saturation of the equilibrium fluctuation magnitude in a rapidity window with time: Eqs. (18) and (19).

$$\begin{aligned} \langle (\Delta Q)^2 \rangle &= \int \frac{dk}{2\pi} \frac{\sin^2(k\Delta y/2)}{(k/2)^2} \chi_k [1 - \exp\{-k^2(\Delta y_{\text{diff}})^2\}] \\ &= \chi_0 \Delta y F(x), \\ x &\equiv \frac{\Delta y_{\text{diff}}}{(\Delta y/2)}. \end{aligned} \quad (18)$$

The dependence on time τ and the rapidity window Δy enters through the ratio x of Δy_{diff} given by Eq. (14) and the half width of the window $\Delta y/2$.⁶ The function F is given by

$$F(x) = 1 + \frac{x}{\sqrt{\pi}} (1 - e^{-1/x^2}) - \text{erf}\left(\frac{1}{x}\right). \quad (19)$$

It rises as $x/\sqrt{\pi}$ at small x and saturates as $1 - 1/(x\sqrt{\pi})$ at very large x (Fig. 1). $\chi_0 \Delta y$ is the equilibrium thermodynamic size of the fluctuation $\langle (\Delta Q)^2 \rangle$ (it scales linearly with the size Δy as it should). The reason that F saturates only as a power of time, and not exponentially, is the fact that the characteristic relaxation time of the long-range harmonics diverges.

III. REALISTIC RESCATTERING IN HADRONIC GAS

A. Cascade estimates of the diffusion rates

Our estimates for Δy_{diff} in the hadronic gas phase can be simplified by noting that the dependence of γ on τ in Eq. (14) is mostly due to τ_{free} . The rapidity change per collision, δy_{coll} , is approximately independent of τ , since the decrease of the temperature is relatively weak in the whole hadronic stage, $T = 160 - 110$ MeV, and kinematics of low-energy scattering changes little. In other words, Δy_{diff} is given by the random walk formula

⁶We assumed, for simplicity, that $\chi_k = \text{const}$ in the interval $k < 1/\Delta y$, which is true when the density-density correlation length is much smaller than Δy —a good approximation for the large rapidity windows we consider: $\Delta y \gg \delta y_{\text{coll}}$.

$$\Delta y_{\text{diff}} = \delta y_{\text{coll}} \sqrt{N_{\text{coll}}}, \quad (20)$$

where

$$N_{\text{coll}} = \int_0^{\tau} \frac{d\tau}{\tau_{\text{free}}} \quad (21)$$

is the total number of collisions per particle during the time τ , from the beginning of the hadronic phase, $\tau=0$, until freeze-out.

At comparatively low relative energies of rescattering in question, all relevant processes are due to certain resonances, such as Δ for $\pi\pi$ collisions, ρ for $\pi\pi$ collisions, etc. So, our first task is to determine what is the mean change of rapidity in such collisions. Taking pions and nucleons with thermal distribution, we have found probabilities of different resonances and have determined the following values for the mean rapidity shifts per collision

$$\delta y_{\text{coll}}^{\pi} \approx 0.8, \quad \delta y_{\text{coll}}^N \approx 0.2. \quad (22)$$

The second (and much more involved) task is to evaluate the average number of rescatterings, for each type of particles. The average value can be evaluated from standard rates

$$N_{\text{coll}}^i = \sum_j \int dt \sigma_{ij} v_{ij}(t) n_j(t) \quad (23)$$

and even a very simple estimate shows that those numbers are not small. The nucleons suffer especially multiple collisions, of the order of 10–20, since the Δ -induced cross section at its peak is as large as 200 mb, and there are many pions around. (This phenomenon is sometimes referred to as *pion wind*.) However, more accurate estimates depend on many details, like the geometrical location of the particle in question, expansion of the fireball, etc. Those can only be obtained from either realistic hydro simulations or from hadronic cascades.

We have chosen the latter alternative, using two popular hadronic cascade codes. First, we have generated AuAu RHIC events (100+100 GeV) with RQMD [16], and looked at the total rescattering number per particle located at midrapidity $y \approx 0$. The results are shown in Fig. 2, as a distribution over the number of rescatterings suffered by kaons and nucleons. One can see that for the nucleons the distribution has a very wide tail, reaching large values and with an average being of the order of 20. Kaons, on the other hand, suffer a much smaller number of collisions, about five on average, and there is about 20% of kaons without rescattering at all.

Unfortunately standard RQMD output does not allow us to trace an individual pion, which appears and disappears, and we do not have the mean rescattering pion number directly from that simulation. We can only evaluate the total number of π - N rescatterings

$$N_{\text{coll}}^{\pi} = N_{\text{coll}}^N \frac{\langle dN_N/dy \rangle}{\langle dN_{\pi}/dy \rangle} \approx 2, \quad (24)$$

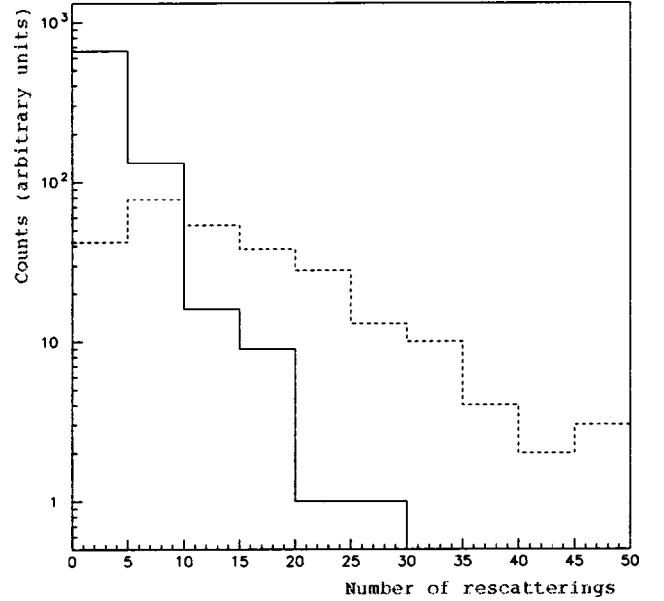


FIG. 2. Distribution over the number of rescatterings in central AuAu 100+100 A GeV collisions, according to the RQMD code. The solid line is for kaons, the dashed one for nucleons, and both are taken in the midrapidity region ($|y| < 2$).

but those are far from being dominant.

Fortunately, a similar code, UrQMD [17], has been extensively studied, and in the talk by Bleicher one finds a graph showing dN_{coll}/ds for baryon-baryon, baryon-meson, and meson-meson collisions, under the same conditions. Dividing by the number of mesons and integrating over subcollision invariant energy s , we obtained the total number of collisions from meson-meson scattering to be $N_{\text{coll}}^{\text{meson}} \approx 8$ per meson. Most of the mesons are pions, of course, and this number fits well between the N and K ones mentioned above.

Adopting these numbers, we can combine it with rapidity change per scattering, and get our final estimates for the diffusion distance at RHIC,

$$\Delta y_{\text{diff}}^{\pi} \approx 2.2, \quad \Delta y_{\text{diff}}^N \approx 0.9. \quad (25)$$

At SPS the pion multiplicity is about a factor of 2 smaller, and the rescattering number is reduced to $N_{\text{coll}}^N \approx 15$ and $N_{\text{coll}}^{\pi} \approx 5$, so that at SPS

$$\Delta y_{\text{diff}}^{\pi} \approx 1.8, \quad \Delta y_{\text{diff}}^N \approx 0.77. \quad (26)$$

Let us now compare it to some typical experimental conditions at SPS. For the accepted interval of rapidity of $\Delta y = 2$, the argument of $F(x)$ plotted in Fig. 1 is directly these values of Δy_{diff} , and one can read F from it: the result is 0.7 and 0.42 for pions and nucleons.

In the preceding section we described an idealized case $\sigma_k^2(0) \ll \chi_k$. For our estimate of the effect we shall use a more general expression, which gives the time dependence of $\langle (\Delta Q)^2 \rangle$ given its initial value at the beginning of the

hadronic phase, $\tau=0$, $\langle(\Delta Q)^2\rangle_0=\sigma_0^2(0)\Delta y$,⁷ and the hadronic gas equilibrium value $\langle(\Delta Q)^2\rangle_{\text{eq}}=\chi_0\Delta y$,

$$\langle(\Delta Q)^2\rangle=\langle(\Delta Q)^2\rangle_{\text{eq}}F(x)+\langle(\Delta Q)^2\rangle_0[1-F(x)],$$

$$x=\frac{\Delta y_{\text{diff}}}{(\Delta y/2)}. \quad (27)$$

For example, starting from the fluctuations suppressed by a factor of about 3 [10,11], i.e., $\langle(\Delta Q)^2\rangle_0/\langle(\Delta Q)^2\rangle_{\text{eq}}\approx 1/3$, and using $F\approx 0.70$ for the fluctuations of the electric charge we find $\langle(\Delta Q)^2\rangle/\langle(\Delta Q)^2\rangle_{\text{eq}}\approx 0.80$. For example, only a 20% suppression survives until freeze-out after a threefold suppression at the QGP–hadron-gas transition. A similar estimate for the baryon number with $F\approx 0.42$ is more encouraging: $\langle(\Delta B)^2\rangle/\langle(\Delta B)^2\rangle_{\text{eq}}\approx 0.62$.

B. Charge fluctuations at SPS

We have shown above, that the pion diffusion in rapidity in the hadronic phase is sufficiently strong. For a typical detector with the acceptance $\Delta y=2$ it reduces the initial deviation from the equilibrium value of an event-by-event charge fluctuation to nearly its equilibrium value in the hadronic gas given by [7]

$$\frac{\langle(\Delta Q)^2\rangle}{\langle N_{\text{ch}}\rangle}\approx 0.7, \quad (28)$$

which is very far from the QGP value. The deviation from the Poissonian value of 1 is due to the π^+ , π^- multiplicity correlation induced by resonance decays.

Furthermore, a specific *centrality dependence* of charge fluctuations in PbPb collisions follows from the resonance gas freeze-out scenario. The magnitude of charge fluctuations (28) should *increase* towards most central collisions. This is opposite to what the QGP signal is supposed to do, if one expects it to show up in more central collisions.

This opposite trend is due to the fact that more central collisions correspond to a *lower* freeze-out temperature [18]: the larger multiplicity is, the later freeze-out occurs. Significant centrality dependence of the radial flow observed at SPS confirms this idea. It follows that for most central collisions one expects less resonances, and a correspondingly decreased deviation of charge fluctuation from the Poissonian value 1.

A simple estimate of this effect can be made using the fact that the contribution of the resonances is controlled by the Boltzmann factor. Thus, for example, a decrease of temperature by $\Delta T\approx 20$ MeV leads to a decrease of the contribution of the ρ_0 resonance by a factor of 2. A similar decrease should occur in the contribution of other resonances, such as ω . Therefore the variation with centrality of the freeze-out temperature of order 20 MeV would correspond to

⁷As with χ_k , we neglect the k dependence in σ_k in the interval of $k\sim 1/\Delta y$.

the centrality dependence of Eq. (28) of the order of 10%, rising towards more central events.

C. Charge fluctuations at RHIC

Finally, let us briefly discuss the prospects of observation of QGP-like charge fluctuations at RHIC. The STAR detector has significantly larger rapidity acceptance, and although diffusion of pions is slightly stronger at RHIC, one can hope to see deviations from the resonance gas value. Indeed, for the rapidity window $\Delta y=4$, Eqs. (25) and (27) predict that the primordial QGP factor 1/3 suppression of the electric charge fluctuation will survive as factor

$$\frac{\langle(\Delta Q)^2\rangle}{\langle(\Delta Q)^2\rangle_{\text{eq}}}\approx 0.7 \quad (29)$$

suppression. Another important prediction is that this suppression must strengthen, i.e., the ratio (29) *decreases*, as the width Δy is *increased*. In other words, opening a wider rapidity ‘‘aperture’’ allows us to see deeper back into the history of the collision.⁸

The baryon number fluctuations have slower diffusion, but rapid proton-neutron conversion together with virtual invisibility of neutrons basically undermines the very idea: the relaxation slowdown can only occur if the current under consideration is a conserved one. A similar problem arises if one considers strangeness fluctuations.

IV. π^+/π^- FLUCTUATIONS NEAR THE CRITICAL POINT E

In this section we discuss a new signature of the critical end point E [19] based on the π^+/π^- fluctuations. As discussed in [19,4], the main feature of thermodynamics near the critical point is the presence of long-wavelength fluctuations of the magnitude of the chiral condensate, or, the σ field. One of the signatures proposed in [19,4] was based on the fact that such light σ quanta cannot decay at freeze-out, if it occurs near point E , and their large population survives until the time after freeze-out when their rising mass exceeds the 2π threshold. The signature proposed in [19,4] used the fact that produced pions had a necessarily nonthermal spectrum with low mean p_T .

The contribution of σ decays to the pion spectrum is, to some extent, similar to the contributions of other, standard, resonances in the resonance gas model, such as ρ^0 . In particular, the produced $\pi^+\pi^-$ pairs introduce positive correlation between the numbers of positive and negative pions, therefore reducing the fluctuation of π^+/π^- ratio. The most significant difference is that the contribution of σ decays is only present near the critical point.

⁸As emphasized in [10–13], Δy should be sufficiently smaller than the total collision rapidity range y_{max} . Otherwise the fluctuations will be trivially constrained by the total charge conservation. In a sense, we would be looking beyond the QGP stage of the collision.

In order to estimate this contribution, we assume that the thermal population of σ 's is approximately half of the so-called "direct" charged pions (since the mass of the σ is comparable to m_π), as is done in [4]. Thus the decaying σ 's produce the number of *charged* pions of order $2 \times 2/3 \times 1/2 = 2/3$ of the number of "direct" charged pions. The latter is about 1/3 of the total number, the rest are produced by resonance decays. Therefore, the contribution of σ 's to the charge pion multiplicity is about $2/3 \times 1/3 \approx 20\%$. Thus, the contribution of σ 's can be estimated to result in a reduction of fluctuations by an amount of order 20%.⁹

Even though the actual contribution will also depend on the detector acceptance of the soft part of the pion spectrum, it is reasonable to expect a noticeable suppression, comparable to the suppression of about 30% due to standard resonances (ρ^0 , ω , etc.) in Eq. (28). What is important is that such an additional suppression of the π^+/π^- fluctuations can only appear if the freeze-out occurs near the critical

point. As a function of the collision energy, this effect will be seen as a dip in the magnitude of the π^+/π^- fluctuation, thus providing a signature for the discovery of E . Away from the critical point the fluctuations will be compatible with the ordinary resonance gas result calculated in [7].

NA49 Collaboration [3] have now acquired data from collisions at 40, 80, and 160 A GeV of initial energy at SPS. We are looking forward to the analysis of these data. If they show a monotonic dependence of the π^+/π^- ratio fluctuation on collision energy as well as on centrality, consistent with the resonance gas prediction, then one should be able to rule out the presence of the critical point in regions of the QCD phase diagram, the location of which can be determined by the analysis similar to [20]. Collisions at other energies, in particular those of RHIC, will be needed to scan other regions of the phase diagram. If a signal described here is observed, other signatures discussed in [4] must also be seen to confirm the discovery of the critical point.

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⁹This can be compared to the reduction due to ρ^0 resonance, which is about 9% [7]. Another 20% in Eq. (28) is contributed by ω and other heavier resonances.

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