Microscopic study of fission fragment angular distributions

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The statistical scission model is applied to fission fragment angular distributions from heavy-ion induced reactions. It is shown that the statistical scission model predicts angular distributions in good agreement with those measured from heavy-ion induced fission of some reaction systems where the fission barrier has vanished or is very small relative to the nuclear temperature. The statistical variances extracted from these model calculations are compared with their corresponding values from a microscopic theory, which includes nuclear pairing interaction. It is found that the values of statistical variance S_0^2 are very sensitive to the pairing energy. The effects of pairing interaction on the fragment angular distributions are illustrated and discussed.

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I. INTRODUCTION

Recent publications [1–5] have focused on extensions of the transition state model to the domain of fusionlike heavy-ion reactions in which the fission barrier is small or absent. For heavy reaction systems where the angular momentum and excitation energy are large, fission fragment angular distribution is analyzed with the statistical scission model (SSM). Versions of this model have been published by various authors [6–9]. Furthermore, the superconductivity theory and the BCS Hamiltonian, the success of which in dealing with the pairing effects of ground state nuclei is well recognized [9], have also been applied in the evaluation of the statistical variances and the corresponding fission fragment angular distributions [10].

The variances S_0^2 have been evaluated by employing the microscopic theory of interacting fermions, using the realistic set of single particle eigenvalues. Our microscopic calculation of the statistical variances and their related angular distributions are in very good agreement with experiment specially at lower spin and moderate excitation energies. In Sec. II the basic theoretical formulas are presented, together with the model calculation of the statistical variances based on the statistical scission model. The theory of superconductivity is outlined in Sec. III. In Sec. IV the resulting fragment angular distributions are presented and are compared with those obtained from the statistical scission model.

II. FORMALISM OF THE STATISTICAL SCISSION MODEL

According to the statistical scission model the relative cross section $W(\theta)$ for fission fragments to be emitted in the direction n forming an angle θ with the beam axis, when the target and projectile spins are zero is given by [11]

$$W(\theta)\propto\sum_{I_{\min}}^{I_{\max}}(2I+1)T_I\Sigma_{K=-I}\times\frac{(2I+1)\exp\left(-\frac{K^2}{2S_0^2}\right)}{\Sigma_K\exp\left(-\frac{K^2}{2S_0^2}\right)}|d^I_{M=0,K}(\theta)|^2. \tag{1}$$

The normalized $d^I_{M,K}(\theta)$ functions are defined by [12]

$$d^I_{M,K}(\theta)=[(I+M)!(I-M)!(I+K)!(I-K)!]^{1/2}\sum_X\times\frac{(-1)^X\left[\sin\left(\frac{\theta}{2}\right)\right]^{K-M+2X}\left[\cos\left(\frac{\theta}{2}\right)\right]^{2I-K+M-2X}}{(I-K-X)!(I+M-X)!(X+K-M)!X!}. \tag{2}$$

Here the direction of spin projectile m (m is the projection of total angular momentum I along n) is taken to be a Gaussian with variance S_0^2. Where S_0^2 for spherical fission fragments is given by either of the two following equations [13]:

$$S_0^2=2\Im_{sph}\frac{T}{\hbar^2}[(2\Im_{sph}+\mu R_c^2)/\mu R_c^2] \tag{3}$$

or

$$S_0^2=2\sigma^2\left\{\left[2\sigma^2+\left(\mu R_c^2\frac{T}{\hbar^2}\right)\right]\Big/\left(\mu R_c^2\frac{T}{\hbar^2}\right)\right\}, \tag{4}$$

with $\sigma^2=\Im_{sph}T/\hbar^2=\frac{2}{5}MR^2T/\hbar^2$. The quantities $\Im_{sph}$, T, M, and R are the moment of inertia, nuclear temperature, mass, and radius of one of the symmetric fission fragments. R_c is the distance between the centers of fragments at scission configuration and is equal to $1.225(A_1^{1/3}+A_2^{1/3})\times(c/a)^{2/3}$ (A_1 and A_2 are mass numbers of fission fragments).

For a scission configuration of two unattached deformed fragments, the variance S_0^2 is given by either of two equations:

$$S_0^2=\left(2\,\Im_{\parallel}\frac{T}{\hbar^2}\right)/[(2\Im_{\perp}+\mu R_c^2)/(\mu R_c^2+2\Im_{\perp}-2\Im_{\parallel})] \tag{5}$$

or

$$S_0^2=2\sigma_{\parallel}^2\left\{\left[2\sigma_{\perp}^2+\mu R_c^2\frac{T}{\hbar^2}\right]\Big/\left[\mu R_c^2\frac{T}{\hbar^2}+2\sigma_{\perp}^2-2\sigma_{\parallel}^2\right]\right\}, \tag{6}$$

where $\Im_{\parallel}$, $\Im_{\perp}$, $\sigma_{\parallel}^2$, and $\sigma_{\perp}^2$ are moments of inertia and spin cutoff parameters of a single fission fragment rotating about an axis parallel and perpendicular to the symmetry axis, respectively.

The primary fission fragments are assumed to have spheroidal shapes with the principal one-half axes of magnitude in terms of their ratio c/a, namely, $a = 1.225(A_c/2)^{1/3}(c/a)^{-1/3}$ and $c = 1.225(A_c/2)^{1/3}(c/a)^{2/3}$ where A_c is the mass number of composite system. The total intrinsic excitation energy in the two fission fragments at scission is given by [10]

$$E^* = E_{\rm c.m.} + Q - E_{Coul} - E_{def} - E_{rot}, \tag{7}$$

where Q represents the difference in energy between the entrance channel nuclei and the ground state of the two fission fragments. $E_{coulb} + E_{def}$ is the sum of the Coulomb and deformation energies stored in potential energy at the instant of scission. The Coulomb energy is estimated by use of the expression [13]

$$E_{Coul}(MeV) = 1.44\frac{Z^2}{2c}, \tag{8}$$

where Z is one-half of the charge of the composite system. The rotational energy E_{rot} of the system at the scission configuration for spin I and projection m on the scission axis is [14]

$$E_{rot} = \frac{\left[\left(I + \frac{1}{2}\right)^2 - m^2\right]\hbar^2}{2\mu c^2 + 4\Im_{\perp}}, \tag{9}$$

where μ is the reduced mass of the fission fragments. The temperature of each fission fragment was assumed to be given by

$$T = \left[\frac{E^*/2}{LDP}\right]^{1/2}, \tag{10}$$

where the liquid drop parameter LDP$=A/8$, A is the mass number of one fragment, and the total excitation energy E^* is divided equally between the two symmetric fission fragments.

For the calculation of angular anisotropies, Eq. (1) was used with the values of S_0^2 estimated with a macroscopic model. We have estimated S_0^2 by considering the largest principal moment of inertia, $S_0^2 = 2(\frac{2}{5})M(c^2 + a^2)T/\hbar^2$. Angular distributions have been calculated using these values of S_0^2 and they are compared with their corresponding measured values. The results for several reactions will be discussed in the next section.

III. MICROSCOPIC CALCULATIONS

The variance S_0^2 entering the statistical scission model can be estimated also with a microscopic theory of interacting fermions using a realistic set of single particle levels. For deformed fission fragments with axial symmetry the single particle states arise from the motion of a nucleon in the deformed average potential. They are characterized by the projection Ω of the angular momentum on the nuclear symmetry axis.

Employing the microscopic theory with nuclear pairing, the spin cutoff parameter $\sigma_{\parallel}^2(E)$ is defined by

$$\sigma_{\parallel}^2(E) = \Im_{\parallel}\frac{T}{\hbar^2} = \frac{1}{2}\left\{\sum \Omega_{ip}^2 \sinh^2\left(\frac{1}{2}\beta E_{ip}\right) + \sum \Omega_{in}^2 \sinh^2\left(\frac{1}{2}\beta E_{in}\right)\right\}, \tag{11}$$

where $\beta = 1/T$, E_{ip} is the proton quasiparticle energy, and E_{in} is the neutron quasiparticle energies, E_i are related to the single particle energies ε_i by

$$E_i = [(\varepsilon_i - \lambda)^2 + \Delta^2]^{1/2}, \tag{12}$$

where λ is the chemical potential and Δ is the ground state gap parameter.

The quantity $\Im_{\parallel}$ is the moment of inertia about an axis parallel to the symmetry axis. The spin cutoff parameter $\sigma_{\parallel}^2(E)$ is determined by the properties of the intrinsic states. Hence Eq. (11) is a definition of the moment of inertia $\Im$.

Since the interaction between the neutron and proton is neglected, the value of the moment of inertia is the sum of the proton and neutron moments of inertia,

$$\Im = \Im_p + \Im_n. \tag{13}$$

The temperature dependence of $\Im$ is investigated by examin-

TABLE I. Variances obtained from theoretical calculations.

Reaction	E_{lab}	E^*	c/a	T	S_0^2 [a]	S_0^2 [b]	$\frac{W(\theta)}{W(90)}$ [c]
^{208}Pb+^{19}F	190	145.9	1.54	2.27	203.4	147.6	
^{208}Pb+^{19}F	110	71.9	1.54	1.60	140.8	104.8	
^{208}Pb+^{24}Mg	210	141.8	1.56	2.21	195.2	147.2	
^{208}Pb+^{24}Mg	140	79.1	1.56	1.65	144.0	110.9	
^{209}Bi+^{16}O	94	61.8	1.54	1.48	103.5	95.9	2.0
^{142}Nd+^{16}O	250	172.3	1.32	2.95	180.3	121.0	8.0 ± 0.6
^{170}Er+^{16}O	250	193.7	1.41	2.89	206.9	146.6	6.4 ± 0.6
^{170}Er+^{16}O	140	93.2	1.41	2.00	135.7	101.1	6.3 ± 0.6
^{197}Au+^{14}N	102	74.1	1.50	1.68	129.6	99.5	2.65
^{197}Au+^{14}N	110	81.5	1.50	1.76	136.8	204.3	3.2
^{232}Th+^{16}O	140	137.9	1.59	2.11	207.9	155.5	
^{238}U+^{16}O	250	252.9	1.60	2.82	294.8	214.9	3.2 ± 0.2
^{192}Os+^{16}O	250	201.4	1.49	2.78	225.8	162.8	4.7 ± 0.3
^{209}Bi+^{12}C	81	55.8	1.53	1.42	117.5	90.0	
^{174}Yb+^{12}C	95	61.9	1.41	1.63	107.4	82.4	6.3 ± 0.4

[a]Theoretical variances from BCS calculations.
[b]Theoretical variances from SSM calculations.
[c]Experimental anisotropy, Refs. [16,17].

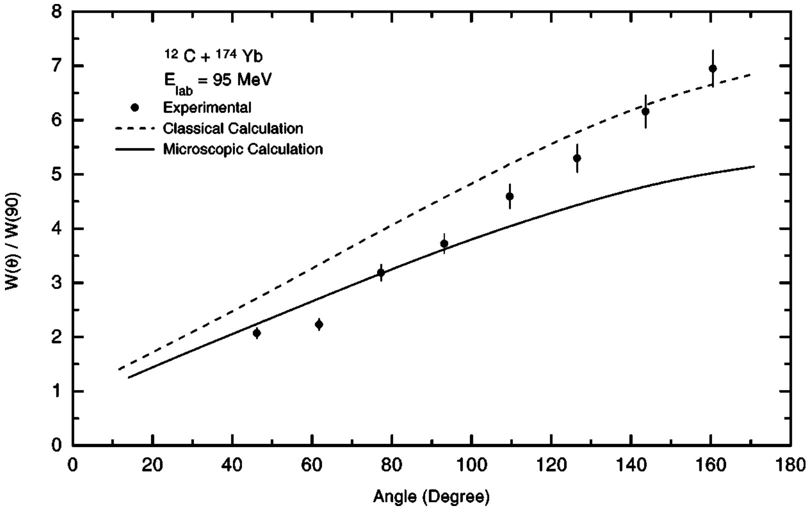

FIG. 1. Comparison of the experimental fission fragment angular distributions for $^{12}C + ^{174}Yb$ with calculated results based on the SSM. The results of the microscopic calculations are also displayed for comparison.

ing the data on angular distribution of fission fragments. The results will be given in the next section.

IV. RESULTS AND DISCUSSION

The statistical variances S_0^2 have been calculated assuming spheroidal fragments in the fission reactions ($^{12}C + ^{174}Yb$), ($^{14}N + ^{197}Au$), ($^{16}O + ^{170}Er$), ($^{19}F + ^{208}Pb$), and ($^{24}Mg + ^{208}Pb$) at typical energies. The theoretical values of S_0^2 are computed using Eq. (5) by assuming the liquid drop value L of the radius constant $r_0 = 1.225$ F, $L = A/8$, and $E_{def} = 10$ MeV. The simulation of the effect of prescission particle emission on the fission fragment angular distributions is more complex due to the reduction of the nuclear temperature at the scission configuration [15]. Variances S_0^2 calculated with SSM including postscission neutron emission show that the change in the over all variance S_0^2 is quite small. In the present calculations the effects of prescission and postscission particle emission on the theoretical angular distributions of fission fragments were neglected. Our theoretical values of S_0^2 for deformed fragments with energies in MeV are listed in Table I.

Theoretical fission fragment angular distributions have been calculated by means of Eq. (1) and by using the values of the variances S_0^2 from Table I. They are compared with their corresponding experimental values taken from Refs. [16,17] and are plotted in Figs. 1–4. From these figures we see that the statistical scission model gives generally a good agreement with experiment. Fission fragment angular distributions are also evaluated by means of the superconductivity theory. For the details of this theory see our previous publications [18,19]. The important parameter to be calculated in this model is $\Im_{\parallel} T/\hbar^2$. This quantity is directly related to the average of the variance S_0^2 over the particle spectrum and is given by the spin cutoff parameter, $\sigma_{\parallel}^2(E) = \Im_{\parallel} T/\hbar^2$.

The spin cutoff parameters were calculated for the shapes corresponding to the saddle point configuration. Assuming that the transition nucleus has a spheroidal shape, the ratio of the major to minor axis, $\chi = c/a$, is calculated from the relation

$$\frac{\chi^{2/3}(\chi^2 - 1)}{\chi^2 + 1} = \frac{\Im_{sph}}{\Im_{eff}} = \Im_{sph}\frac{T}{\hbar^2}. \tag{14}$$

The corresponding deformation then is $\delta = (c-a)/a$. The set

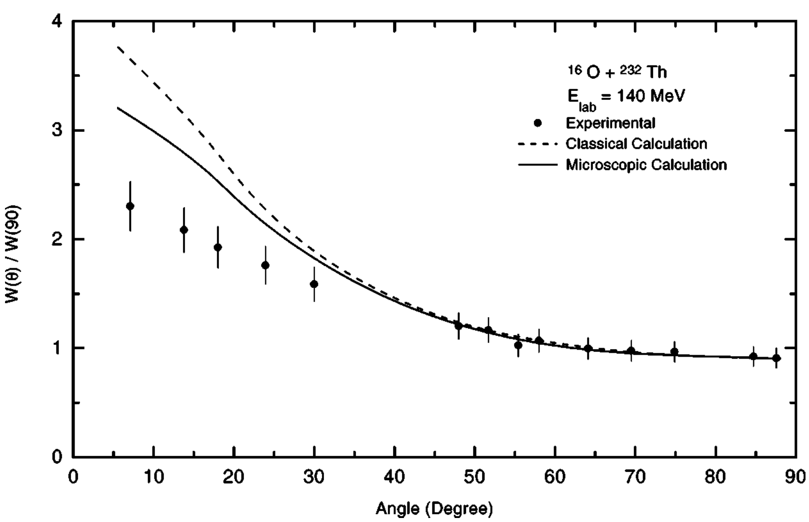

FIG. 2. Same as Fig. 1 for $^{16}O + ^{232}Th$ reaction.

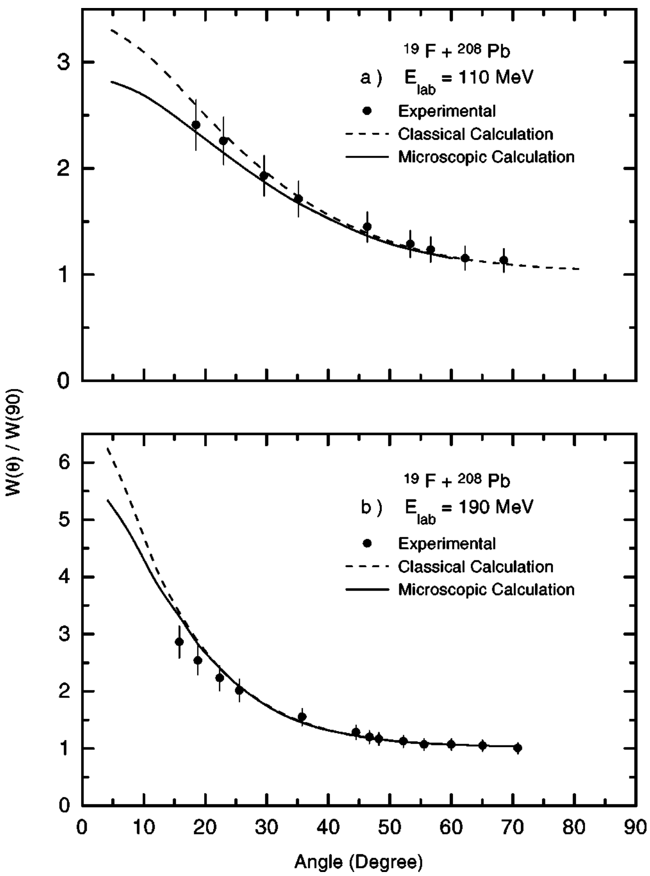

FIG. 3. Experimental fission fragment angular distributions of ^{19}F+^{208}Pb reaction for (a) E_{lab}=110 MeV and (b) E_{lab}=190 MeV. The solid curves are SSM calculations. The dashed curves are the results from microscopic calculations.

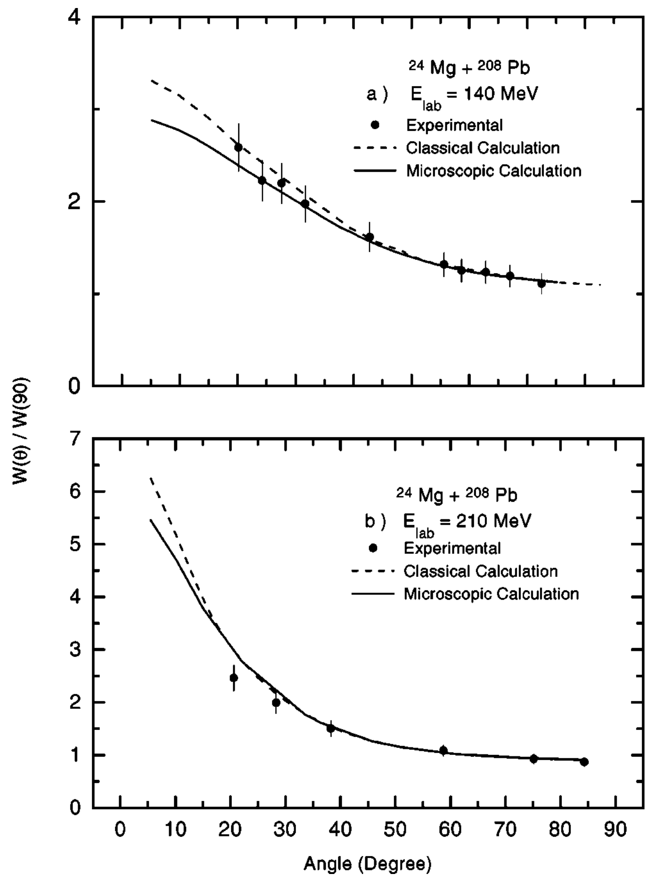

FIG. 4. Same as Fig. 3 for ^{24}Mg+^{208}Pb reaction for (a) E_{lab}=140 MeV and (b) E_{lab}=210 MeV. (All microscopic calculations reported here correspond to a prolate deformation.)

of Nilsson single particle states were generated at this deformation. The spin cutoff parameter is then calculated from Eq. (11) using the set of Nilsson single particle levels. The results obtained from such calculations are listed in Table I. The full fragment angular distributions computed from the deduced variances are also plotted in Figs. 1–4 for comparison. From the examination of these figures, we conclude that the variance S_0^2 from SSM calculations generally results in fission fragment angular distributions that are in reasonable agreement with experiment. However, variances obtained from the superconductivity theory give a rather superior fit to the data and this agreement is even better at lower excitation energies. It is interesting to note that, as is apparent from Table I, the values of S_0^2 obtained from microscopic calculations are generally higher by about 20% than their corresponding values obtained from macroscopic calculations. This is probably due to the strong dependence of the quantity $\sigma_{\parallel}^2(E)$ on the single particle energies near the Fermi energy. It is worth noting that in the microscopic calculations, the pairing diminishes at higher excitation energies [20]. So, it is expected that the variance S_0^2 will approach its macroscopic values at these energies. We hope that in time the superconductivity theory presented in this paper will serve as a means of calculating the statistical variances as well as the angular distribution of fission fragments at higher excitation energies and angular momentum.

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[1] M. Govil, J.R. Huizenga, W.U. Schröder, and J. Toke, Phys. Lett. B **197**, 515 (1987).
[2] H. Rossner, D. Hilscher, E. Holub, G. Ingold, U. Jahnke, and H. Orf, Phys. Rev. C **27**, 2666 (1983).
[3] M.B. Tsang, D. Ardouin, C.K. Gelbbbke, W.G. Lynch, and R.Z. Xu, Phys. Rev. C **28**, 747 (1983).
[4] D. Bond, Phys. Rev. Lett. **52**, 414 (1984); Phys. Rev. C **32**, 471 (1985); **32**, 483 (1985).
[5] H.H. Rossner, T.R. Huizenga, and W.U. Schröder, Phys. Rev. Lett. **53**, 38 (1984).
[6] T.R. Huizenga, H.H. Rossner, and W.U. Schröder, Phys. Rev. C **33**, 560 (1986).

[7] P. Schmitt, G. Mouchaty, and O.R. Haenni, Nucl. Phys. **A427**, 614 (1984).
[8] H. Rossner, D. Hilscher, E. Holb, and H. Grf, Phys. Rev. C **27**, 2666 (1983).
[9] A.N. Behkami and P. Nazarzadeh, J. Sci., Islamic Repub. Iran **9**, 193 (1998).
[10] A.N. Behkami and P. Nazarzadeh, Nucl. Sci. J. **36**, 100 (1999).
[11] J.R. Huizenga, W.U. Schröder, and H. Rossner, in *Proceedings of the International Symposium on Heavy-Ion Physics*, edited by S. Kubano, M. Ishihara, and M. Ichimura [J. Phys. Soc. Jpn. Suppl. **54**, 257 (1985)].
[12] A.N. Behkami, Nucl. Data Tables **10**, 1 (1971).
[13] A.N. Behkami and P. Nazarzadeh, Comm. Nucl. Data Prog. **20**, 24 (1998).
[14] P. Nazarzadeh, Ph.D. thesis, Shiraz University, 1997.
[15] D.J. Hinde, in *Proceedings of the Second International Conference on Nucleus-Nucleus Collisions*, Visby, Sweden, 1985, edited by H. a. Gustafsson (North-Holland, Amsterdam, 1986).
[16] M.B. Tsang, H. Utsunomiya, C.K. Gelbke, W.G. Lynch, Z.R. Ru, B.B. Back, S. Saini, P.B. Baisden, and M.A. McMahan, Phys. Lett. **129B**, 18 (1983).
[17] P.D. Bond, Phys. Rev. C **32**, 471 (1985).
[18] A.N. Behkami and Z. Kargar, J. Phys. G **18**, 1023 (1992).
[19] A.N. Behkami, Proceedings of the International Conference on Clustering Phenomena in Nuclear Physics, St. Petersburg, 2000 (to be published).
[20] M.G. Mustafa, M. Blann, A.V. Ignatyuk, and S.M. Grimes, Phys. Rev. C **45**, 1078 (1992).