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Schematic model for narrow $\Delta(1232)$ resonances bound in a nucleus

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A schematic model explaining the recent evidence for bound states of the $\Delta(1232)$ resonance in ^{12}C with a width of about 5 MeV found in the $^{12}\mathrm{C}(e,e'p\pi^-)^{11}\mathrm{C}_{\mathrm{g.s.}}$ is suggested. It interprets the observed narrow resonances as coherent bound states enforcing a fixed phase relation for the rescattering of the decay π's in the nucleus. The coherent summing of the rescattering diagrams results in a cancellation of the tails of the resonance and a constructive interference in the maximum of the resonance leading to the observed narrow states at the maximum of the $\Delta(1232)$ shifted by the binding energy. The same mechanism may contribute to narrow Σ hypernuclear states and explains their successful and failed observation, respectively.

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I. INTRODUCTION

In a recent paper [1], evidence for narrow states of bound Δ^0 resonance in ^{12}C with a width of about 5 MeV produced in the $^{12}\mathrm{C}(e,e'p\pi^-)^{11}\mathrm{C}_{\mathrm{g.s.}}$ reaction was reported. The evidence is based on two peaks with about five and four standard deviations statistical significance at 282 MeV and 296 MeV excitation energy above the ^{12}C ground state, respectively. These energies could be understood in the framework of a simple single-particle potential model. In this model, all spin and isospin dependence of the Δ-nucleus potential was neglected. It showed that the observed energies of the two peaks and the general shape of the spectrum were in accord with the assumption of a bound Δ. It should, however, be noted that the evidence of Ref. [1] is based on two spectra. In the meantime, a third spectrum with somewhat fewer statistics has been measured showing no clear signals.

An explanation of the unexpected narrowness in the framework of conventional models of the nucleus could not be given. However, before one rejects the idea of narrow baryon resonances in nuclei, one should consider that the lifetime of a resonance is influenced by the propagation of the field to which it couples. A striking example is a Rydberg atom in a parallel plate resonator [2]. Its spontaneous electromagnetic radiation is inhibited by the suppression of the vacuum modes of the free transition. If the cutoff frequency of the resonator is larger than the free transition frequency, the lifetime is increased by at least a factor of 20.

Since the question of narrow states of the unstable baryon resonances Σ and Δ has been studied extensively [3–6] (for a summary, see [7]) in the framework of recognized many-body expansions of the nucleus, the claim of narrow states seems to be answered. In the nucleus, the Σ can decay strongly via the $\Sigma N\to\Lambda N$ reaction resulting in a width of about 30 MeV [4]. In a more refined theory, a quenching of the width to 5 MeV was found and traced back to Pauli blocking and the propagation of mesons in the nucleus [3].

Narrow Δ's are even less probable since besides the strong $\Delta N\to NN$ transition, the intrinsic strong decay $\Delta\to N\pi$ with a width $\Gamma\approx110$ MeV seems to forbid such an idea. The kinetic energy of the emitted nucleons is $T_N=147$ MeV for the $\Delta N\to NN$ transition, making the Pauli blocking effect negligible. For the $\Delta\to N\pi$ decay, $T_N=28$ MeV, which is comparable to the $T_N=39$ MeV for the $\Sigma N\to\Lambda N$ transition. Nevertheless, no quenching of the Δ width was found in the calculations using the same model as for the Σ [5].

However, a closer study of the theory of pionic modes of excitation in nuclei [8] gives a hint where something might have been overlooked so far. It is mostly assumed that the production of the Δ resonance happens ''quasifreely,'' that is to say, that the Δ takes all the momentum of the exciting π or γ. Consequently, the initial state of the Δ is described always as a wave packet with the Fermi momentum distribution of the initial nucleon and expanded in a particle-hole basis. This paper, in contrast, describes a schematic model that assumes an initial bound state and follows its coherent propagation in a finite nucleus.

II. BASIC MODEL IDEAS

The basic idea is already explained in Ref. [1] and represents the motivation for the experiment. In that paper, the discussion is restricted to the production of a Δ^0 in the $^{12}\mathrm{C}(e,e'p\pi^-)^{11}\mathrm{C}$ reaction. It is evident that the same final state can be reached by the production of a Δ^+ and a subsequent charge exchange. Therefore, no distinction between the Δ^0 and Δ^+ is made in this paper.

The two reaction mechanisms distinguished in Ref. [1] were the ''quasifree Δ'' production and the production of a ''bound Δ'' where the whole nucleus takes the momentum transfer. The first reaction mechanism favors an emission of the decay π and p in the forward direction with respect to the three-momentum transfer $\vec{q}$ in the laboratory system, whereas for the second reaction they are almost equally distributed over 180°. Therefore, the first mechanism can be suppressed by putting two magnetic spectrometers under large π and p emission angles. A Monte Carlo simulation of the two phase spaces belonging to the two reaction mechanisms has been presented in Ref. [1].

The amplitudes corresponding to the two reaction mechanisms are depicted in the graphs of Fig. 1 and Fig. 2. It is essential to realize that they represent two distinctly different initial quantum states. The ''quasifree Δ'' is a wave packet traveling through the residual nucleus ^{11}C. It decays statisti cally and is not coherent with the ^{11}C residue. The ''bound

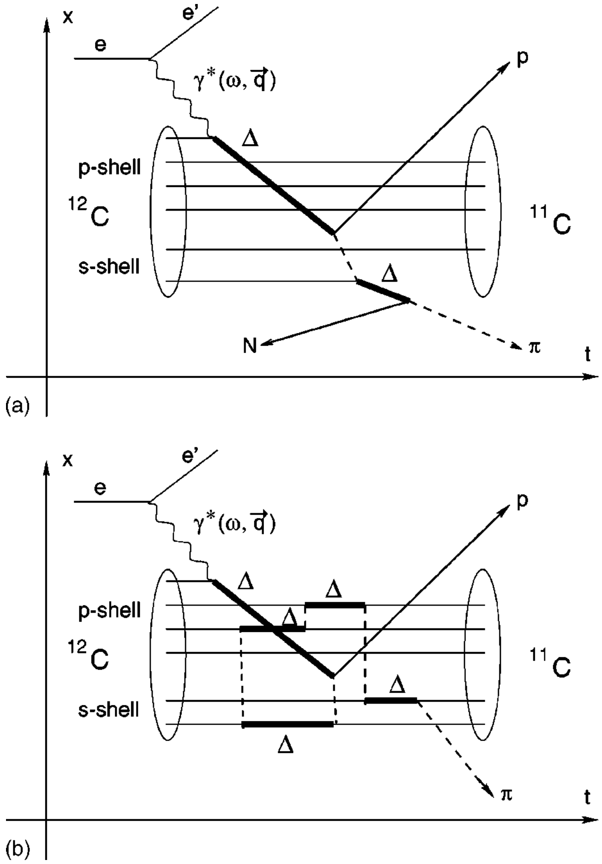

FIG. 1. Amplitude for ''quasifree Δ'' production. (a) Quasifree Δ decay with quasifree rescattering. (b) Quasifree Δ decay with rescattering through bound Δ's.

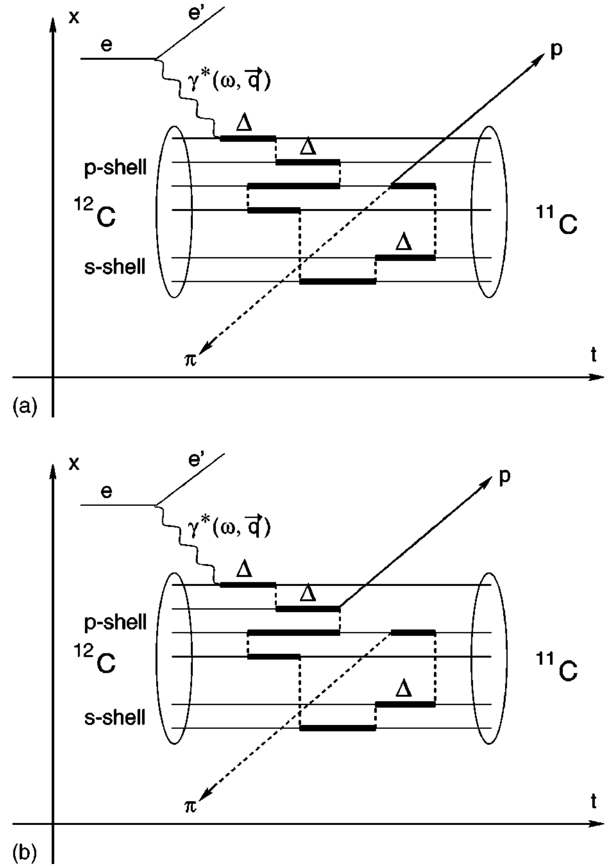

FIG. 2. Amplitude for the production of a ''bound Δ.'' (a) Back-to-back decay as a free Δ at rest (mode I). (b) Decay separated by rescatters in the nucleus (mode II).

Δ'' is assumed to form together with the ^{11}C core a coherent bound state in a mean-field potential. It decays with a fixed phase relation to the other nucleons. The existence of a mean-field potential for the Δ in analogy to that for a nucleon is the first schematic assumption of the model.

The Δ will decay by its intrinsic decay $\Delta \rightarrow N\pi$. Figure 1 symbolizes that the decay particles of the quasifree Δ are emitted forward against the nucleus system in the direction of the momentum transfer $\vec{q}$. Figure 2 shows that, in contrast, the decay particles of the bound Δ are in good approximation back to back since the hole nucleus has taken the momentum transfer $\vec{q}$ and is moving only slowly in its direction. The decay π is rescattered $\mathcal{N}$ times through resonant absorption and reemission before it leaves the nucleus. Three cases finishing in the same final state can be distinguished. In the first case, the Δ decays in the nucleus system as in the free decay back to back at the end of its rescatters [Fig. 2(a), mode I]. In the second case, the Δ decays after k rescatters and emits a p; the accompanying π continues with $(\mathcal{N}-k)$ rescatters until it leaves the nucleus [Fig. 2(b), mode II]. The third case corresponds to the second, only now the π knocks out directly a second $\mathcal{N}$ at the end of the scattering chain (mode III). This is the already mentioned $\Delta N \rightarrow NN$ decay mode in the nucleus. One might consider a mode IV corresponding to mode II with the NN final state. However, as will be discussed later, this mode is greatly suppressed for the coherent, bound Δ state.

For both reaction mechanisms, one can assume that the photon produces the Δ in the whole nuclear volume with equal probability. The mean free path of the π is

$$\langle l_{\pi}(\omega)\rangle = \frac{1}{\rho\sigma_{\mathrm{abs}}(\omega)}, \tag{1}$$

where the density of protons and neutrons $\rho_{p,n} = \frac{1}{2}\ 0.17\ \mathrm{fm}^{-3}$. Observing the Pauli blocking strictly, the rescattering is given by all possible charge states for the π and the nucleon N in the $\pi N \rightarrow \Delta^{0,+} \rightarrow \pi N$ reactions. Their cross sections can be estimated by taking the experimental data for the total cross sections of the $\pi^{-}p \rightarrow \pi^{-}p, \pi^{0}n$ reactions [12] and using the proper isospin coupling coefficients. The cross sections on the proton and neutron are equal and their sum is $\sigma_{\mathrm{abs}}(\omega_0) = \sigma_0 = 21\ \mathrm{fm}^2$, where $\omega_0 = 297$ MeV for the maximum of the resonance. The Pauli blocking can, however, be relaxed for the following reasons. The mass differences between the $\Delta^{-,0,+,++}$ should be of the same order as those between the $\Sigma^{-,0,+}$ and are of the same order as the differences between the $1p_{3/2}$ and the $1p_{1/2}$ shells. They are small against the total energy of the rescattered π and, therefore, the reactions $\pi^{+}p \rightarrow \Delta^{++} \rightarrow \pi^{+}p$ leaving an n and $\pi^{-}n \rightarrow \Delta^{-} \rightarrow \pi^{-}n$ leaving a p in the $1p_{1/2}$

shell should be considered too. Then a value of $\sigma_0 = 28\ \mathrm{fm}^2$ follows for which a mean free path of $\langle l_\pi(\omega_0)\rangle = 0.42\ \mathrm{fm}$ results. If the Δ is produced in the center of the nucleus, the π will escape after a random walk with

$$\mathcal{N}(\omega) = g_{\mathrm{corr}}\left(\frac{R}{\langle l_\pi(\omega)\rangle}\right)^2 = g_{\mathrm{corr}}[R\rho\sigma_{\mathrm{abs}}(\omega)]^2 \tag{2}$$

rescatters with $R=3\ \mathrm{fm}$, the effective radius of ^{12}C. Due to the quadratic dependence of $\mathcal{N}$ from the traveled distance, a geometric factor $g_{\mathrm{corr}} = 1.33$ has to be applied if it is assumed that the Δ is homogeneously produced over the nucleus. If the π was produced at the surface, it escapes between a few rescatters if it flies to the ''thin side'' and the maximal rescatters if it flies to the ''thick side.'' Of course, for a Δ produced with a mass above or below the resonance mass of $m_\Delta = m_N + \omega_0 = 1232\ \mathrm{MeV}$, the mean free path is longer and $\mathcal{N}$ is smaller. As a consequence of these considerations it is assumed in this schematic model that all rescattering probabilities for a π escaping after 1 to $\mathcal{N}$ rescatters are equal.

III. CALCULATION OF WIDTH

A. Bound Δ

In order to calculate the width of the Δ state, one has to sum up all amplitudes and consider their phases in the rescatters. The starting point is the total wave function consisting of the internal wave function $e^{-i/\hbar[\omega_0 t - i(\Gamma/2)t]}$ of the decaying excited state of the nucleon, i.e., the Δ resonance, and the external wave function $\psi(t,\vec{r})$ describing the motion of the Δ in time and space:

$$\Psi(t,\vec{r}) = \Psi_0 e^{-i/\hbar[\omega_0 t - i(\Gamma/2)t]}\cdot\psi(t,\vec{r}). \tag{3}$$

In the case of the quasifree initial state, the external wave function is given by

$$\psi(t,\vec{r}) = e^{-i/\hbar(\epsilon t - \vec{k}\vec{r})}, \tag{4}$$

approximating the wave packet of Fig. 1 by a plane wave. ϵ is the sum of the kinetic energy $\epsilon_T = k^2/2m_\Delta$ and the Δ-nucleus interaction energy ϵ_V. Since the decay of the Δ is dealt with explicitly, it is assumed that ϵ is real. We turn first to the case of the proposed bound Δ since it is simpler than that of the quasifree Δ.

In the case of the bound Δ, the external wave function is given by

$$\psi(t,\vec{r}) = e^{-(i/\hbar)\epsilon_\alpha t}\cdot\psi_\alpha^\Delta(\vec{r}). \tag{5}$$

Here ϵ_α is the single-particle eigenvalue of the bound Δ and $\psi_\alpha^\Delta(\vec{r})$ is its spatial wave function with a set of quantum numbers α.

The Fourier transform of the time-dependent part of the total wave function represents the amplitude as a function of the nucleon excitation energy:

$$G(\omega) = G_0\frac{\hbar}{\sqrt{2\pi}}\frac{-i}{(\omega-\omega_0-\epsilon_\alpha)+i(\Gamma/2)}. \tag{6}$$

Taking the absolute square gives the standard Breit-Wigner form for the decay curve of the Δ:

$$\sigma(\omega) = \sigma_0\frac{(\Gamma/2)^2}{(\omega-\omega_0-\epsilon_\alpha)^2+(\Gamma/2)^2}. \tag{7}$$

However, the decay π is absorbed and emitted many times, the π propagation experiencing each time a phase advance of

$$\phi(\omega) = \arctan\left(\frac{\omega_0+\epsilon_\alpha-\omega}{\Gamma/2}\right). \tag{8}$$

Here it is assumed that the π does not propagate as a plane wave between the rescatters because the two characteristic wavelengths $\lambda_{\mathrm{Compton}} = 2\pi\hbar/mc = 8.8\ \mathrm{fm}$ and $\lambda_{\mathrm{de\ Broglie}} = 2\pi\hbar/k_\pi = 5.7\ \mathrm{fm}$ have to be compared to the mean absorption length $\langle l_\pi(\omega_0)\rangle \approx 0.4\ \mathrm{fm}$. This means that the usually assumed π phase shifts do not apply since in the given situation the π initial and final states are not asymptotically free plane waves. The propagation of the π with a ''zero range'' is the second schematic assumption of the model. It will be discussed in more detail in connection with the quasifree mechanism in Sec. III B.

In Fig. 2, the leading modes of the rescattering of the π before it leaves the nucleus are sketched. The $\mathcal{N}$ rescattering amplitudes have to be summed coherently according to the rules of Feynman diagrams in the many-body problem [9],

$$G(\omega) = G_0\frac{\hbar}{\sqrt{2\pi}}\frac{-i}{(\omega-\omega_0)+i(\Gamma/2)}[1+a_1(\omega)+a_2(\omega) + \cdots + a_k(\omega)+a_{k+1}(\omega')+\cdots+a_{\mathcal{N}}(\omega')], \tag{9}$$

where the shift of the resonance maximum due to the binding energy ϵ_α has been omitted for convenience. The a_l's represent the π absorption/emission amplitudes. For mode II in Fig. 2(b), the decay proton takes away its kinetic energy T_p and the π rescattering has to be taken at energy $\omega' = \omega - T_p$, where the prime indicates that one-nucleon is ejected. It suffices to consider the one-nucleon knockout since amplitudes for knocking out more than one nucleon are small, as will become clear later.

The $a_l(\omega)$ are given by

$$a_l(\omega) = \left[\sum_{\{n\}\in\{A\}}\frac{\hbar}{\sqrt{2\pi}}\frac{e^{i\phi(\omega)}}{\sqrt{(\omega-\omega_0)^2+(\Gamma/2)^2}}\,\zeta_n\right]^l, \tag{10}$$

where the sum runs over a subset $\{n\}$ of nucleons of the nucleus $\{A\}$, l is the number of π absorptions/emissions, and $\zeta_n = 1 - V_{\alpha_n}(\vec{k}_{\pi,p})$ is the probability that no nucleon is knocked out by the π emission/absorption. The transition matrix element $V_{\alpha_n}(\vec{k}_{\pi,p})$ for the knockout of a nucleon or π from a state with quantum numbers α_n to a plane wave with momentum $\vec{k}_p$ or $\vec{k}_\pi$ is calculated later. As will be shown,

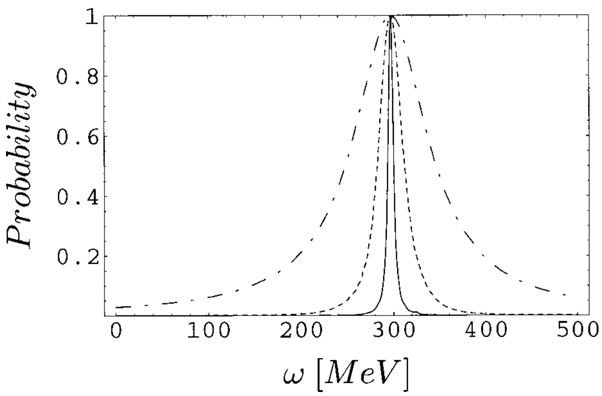

FIG. 3. The normalized energy dependence of the resonance curve for $\sigma_0=1\ \mathrm{fm}^2$ (dash-dotted curve), $10\ \mathrm{fm}^2$ (dashed curve), and $28\ \mathrm{fm}^2$ (solid curve), where σ_0 is the absorption cross section in the maximum of the Breit-Wigner resonance, i.e., for $\omega=\omega_0=297$ MeV. $\zeta=0.96$ as explained in the text.

$\zeta_n \lesssim 1$ and is approximately independent of the quantum numbers α_n at the momenta to be considered here. The neglect of the differences between the wave functions is the third schematic assumption of the model. Therefore, the phase factor and $\zeta=\zeta_n$ can be put in front of the sum, and the absorption probability of the π independent of the order l of rescatters is

$$P_\pi=\sum_{\{n\}\in\{A\}}\frac{\hbar}{\sqrt{2\pi}}\frac{1}{\sqrt{(\omega-\omega_0)^2+(\Gamma/2)^2}} \tag{11}$$

with, of course, $P_\pi \lesssim 1$. This sum has been dealt with already implicitly by calculating the number of π rescatters from all nucleons in the nucleus. The π is rescattered up to $\mathcal{N}$ times before it leaves the nucleus. At a given order l the π is coherently absorbed by any of the A nucleons and, therefore, $P_\pi=1$ for sufficiently large nuclei. Consequently, one gets for the a_l,

$$a_l=\zeta^l e^{il\phi(\omega)}. \tag{12}$$

Considering that their final states of the amplitudes for the initial quantum state of a bound Δ as depicted in Fig. 2 for mode I and mode II are coherent and that of mode III is incoherent, their sum is

$$\begin{aligned}G(\omega)=&G_0\frac{\hbar}{\sqrt{2\pi}}\frac{-i}{(\omega-\omega_0)+i(\Gamma/2)}\\&\times[w_1|\zeta b_{\mathrm{I}}(\omega)+(1-\zeta)b_{\mathrm{II}}(\omega)|^2\\&+w_2|b_{\mathrm{III}}(\omega)|^2]^{1/2},\end{aligned} \tag{13}$$

where $b_{\mathrm{I}},b_{\mathrm{II}},b_{\mathrm{III}}$ are the amplitudes for the three modes distinguished above. ζ can be understood as a damping factor that accounts for the damping of mode I into mode II. w_1 and w_2 are the relative weights of the πp and NN final states. From the ratio of $\sigma(\Delta\to\pi^+p)/\sigma(\Delta\to NN)\approx 2$ in the quasifree reaction, one takes $w_1=0.66$ and $w_2=0.33$. However, the result does not depend on the weights because modes I and III experience the same rescatter chain and consequently the same quenching of the width.

For b_{I}, one gets

$$\begin{aligned}b_{\mathrm{I}}&=[1+\zeta e^{i\phi(\omega)}+\zeta^2e^{i2\phi(\omega)}+\cdots+\zeta^{\mathcal{N}(\omega)}e^{i\mathcal{N}(\omega)\phi(\omega)}]\\&=\frac{1-(\zeta e^{i\phi(\omega)})^{\mathcal{N}(\omega)+1}}{1-\zeta e^{i\phi(\omega)}}.\end{aligned} \tag{14}$$

For b_{II} one gets

$$\begin{aligned}b_{\mathrm{II}}=\Bigg[\sum_{k=0}^{\mathcal{N}(\omega)}&\Bigg(\frac{1-(\zeta e^{i\phi(\omega)})^k}{1-\zeta e^{i\phi(\omega)}}\\&+\zeta^k e^{ik\phi(\omega)}\frac{1-(\zeta e^{i\phi(\omega-28\ \mathrm{MeV})})^{[\mathcal{N}'(\omega)+1]}}{1-\zeta e^{i\phi(\omega-28\ \mathrm{MeV})}}\Bigg)\Bigg].\end{aligned} \tag{15}$$

Since the Δ decays practically at rest in the nuclear system, $\omega'=\omega-28$ MeV. For the number of rescatters after the proton emission, one obtains $\mathcal{N}'(\omega)=\max(0,\{\mathcal{N}(\omega-28\ \mathrm{MeV})-[\sigma^2(\omega-28\ \mathrm{MeV})/\sigma^2(\omega)]k\})$ using Eq. (2).

For b_{III}, one gets the same expression as for b_{I} since it is also a back to back decay. The sum of these amplitudes can be easily calculated using MATHEMATICA [10]. Figure 3 shows the energy dependence of $|G(\omega)|^2$ normalized to $|G(\omega_0)|^2=1$ for different σ_0, which is equivalent to different $\mathcal{N}$'s. In order to get some insight into the mechanism of quenching, the amplitudes b_{II} and b_{III} are neglected for a moment. After absolute squaring, one obtains

$$|G(\omega)|^2\overset{\zeta=1}{=}G_0^2\frac{\hbar^2}{2\pi}\frac{1}{(\omega-\omega_0)^2+(\Gamma/2)^2}\frac{\sin^2(\mathcal{N}(\omega)\phi(\omega)/2)}{\sin^2(\phi(\omega)/2)}. \tag{16}$$

The first factor is the Breit-Wigner curve and the second is the well-known diffraction function of a grating if $\zeta=1$. This second factor reduces effectively the overall width. The basic mechanism for the quenching of the width is the destructive

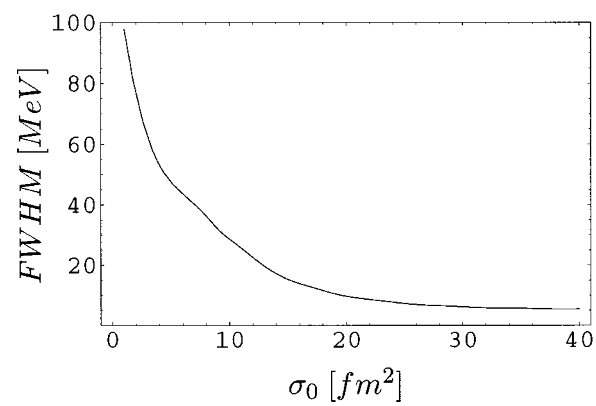

FIG. 4. The full width at half maximum of the resonance curve as a function of the absorption cross section σ_0. $\zeta=0.96$ as explained in the text.

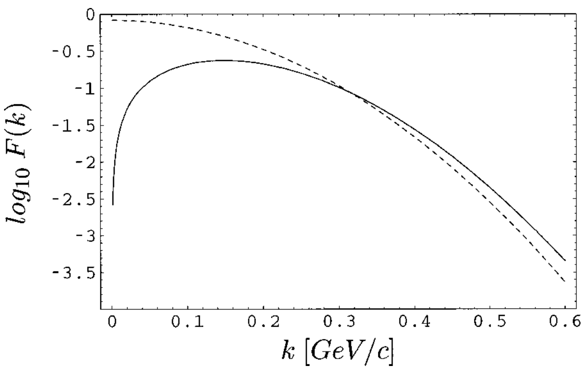

FIG. 5. The logarithm to the base 10 of the emission form factors for the s shell (dashed curve) and the p shell (solid curve) as a function of the three momentum transfer.

interference of the amplitudes for $|\phi(\omega)|>0$, i.e., left and right of the resonance maximum.

Figure 4 shows the dependence of the width from the sum of the absorption cross sections on the protons and neutrons. The relevant cross section has been estimated above to be $\sigma_0 \approx 28\ \mathrm{fm}^2$ resulting in a width of $\Gamma = 6.6$ MeV. This means even a quantitative agreement of the width in this schematic model with the width of the observed peaks.

It remains to consider the damping of mode I into mode II. The amplitudes of mode I contain a finite probability $1-\zeta$ to go to the mode II. For this transition, the decay momentum has to be transferred to the whole nucleus, first after $\mathcal{N}(\omega)$ rescatters by the emitted proton and then after $\mathcal{N}'(\omega)$ by the π leaving the nucleus. The emission probability is given by the matrix element

$$\mathcal{M} = \mathcal{M}_{\mathrm{free}} V_{s,p}(\vec{k}_{\pi,p}) = \mathcal{M}_{\mathrm{free}} \frac{1}{\sqrt{V}} \langle \psi_\alpha^\Delta | U_{\pi N \Delta} | e^{i\vec{k}_{\pi,p}\vec{r}} \rangle, \tag{17}$$

where ψ_α^Δ is the wave function of the bound Δ, $e^{i\vec{k}_{\pi,p}\vec{r}}$ is the plane wave of the proton or π normalized in the volume V, and $U_{\pi N\Delta}$ is the radial dependence of the $\pi N \Delta$ interaction potential describing the free decay, which can be approximated by $U_{\pi N \Delta} = \delta(\vec{r}_\Delta - \vec{r}_N)$ [11]. A simple calculation gives for the transition form factor from the s shell,

$$F_s(k) = \frac{2\pi}{\sqrt{V}} \frac{1}{r_0} \sqrt{\frac{4}{\sqrt{\pi}}} \frac{1}{\sqrt{4\pi}} \int_0^\infty r^2 R_s(r,r_0) 2 \frac{\sin(kr)}{kr} dr, \tag{18}$$

and from the p shell,

$$F_p(k) = \frac{2\pi}{\sqrt{V}} \frac{1}{r_0} \sqrt{\frac{8}{3\sqrt{\pi}}} \frac{1}{3} \sqrt{\frac{3}{4\pi}} \int_0^\infty r^2 R_p(r,r_0) 2i \times \left[\frac{\sin(kr)}{(kr)^2} - \frac{\cos(kr)}{kr} \right] dr, \tag{19}$$

where

$$R_s(r,r_0) = \frac{1}{r_0} e^{-1/2(r/r_0)^2}, \quad R_p(r,r_0) = \frac{r}{r_0^2} e^{-1/2(r/r_0)^2} \tag{20}$$

are the harmonic-oscillator radial wave functions. Figure 5 shows these two form factors for $r_0 = \hbar c / \sqrt{m_\Delta \hbar\omega}$ with $\hbar\omega = 41 A^{-1/3}$ MeV and $V = \lambda^3 = (h/k)^3$. The Δ decay gives a three-momentum transfer $k_{\pi,p} = 0.23$ GeV/c to the proton and the π so that $F_s(k=0.23\ \mathrm{GeV}/c) \approx F_p(k=0.23\ \mathrm{GeV}/c) \approx 0.2$. With this follows $V_\alpha(k) = F^p(k) F^\pi(k) \approx 0.04$ and $\zeta \approx 0.96$.

A systematic study of the contributions to the b_i's shows that the propagation of the π after the first emission of a nucleon, i.e., the second term in Eq. (15), contributes little. The interference of the amplitudes before the first emission dominates and produces the quenching. The coherent bound state can also decay by two or three nucleons or α-particle emission with a subsequent π propagation and emission. It is evident that these modes are even more suppressed by the mechanisms just discussed.

B. Quasifree Δ

It still has to be shown that the model proposed here is in accord with the observation of broad and shifted Δ's in the quasifree production as depicted in Fig. 1. In contrast to the case of the bound Δ, the π originates from the decay of the quasifree Δ described as a plane wave. Consequently, it will also be a plane wave and preferentially be scattered quasifreely and will knock out the next nucleon [Fig. 1(a)]. Since the quasifree process is much more likely than the capture of a Δ in a bound state as discussed in Sec. III A, this quasifree rescattering will go until the π leaves the nucleus. This will happen quickly since with each rescatter the π will lose about $\varepsilon_\pi = T_N - \epsilon_s \approx 50$ MeV, where T_N is the kinetic energy and ϵ_s is the one-particle energy of the knocked-out nucleon N. At this low energy away from the resonance energy, the π will propagate with a long mean free path.

The difference of the π propagation between the quasifree and the bound Δ can be intuitively understood. In the first case, the π is a free wave packet approximated by a plane wave, whereas in the second case, the bound Δ will fluctuate virtually into a πN state before the π absorption with both particles having the frequency composition of the wave function of the bound Δ. Such a virtual π has a high probability to be absorbed and propagate through the bound Δ's. It should be distinguished from the usual plane-wave propagator.

The Fourier transform of Eqs. (4) and (3) for the quasifree case becomes

$$G(\omega) = G_0 \frac{\hbar}{\sqrt{2\pi}} \frac{-i}{(\omega - \omega_0 - \Pi(\vec{k},\vec{k}_i)) + i(\Gamma/2)} e^{-\vec{k}^2/m_\Delta \Gamma}, \tag{21}$$

where $\Pi(\vec{k},\vec{k}_i)$ represents the "self-energy of the Δ," $\vec{k}_i$ is the initial momentum of the struck nucleon, and $\vec{k} = \vec{q} + \vec{k}_i$.

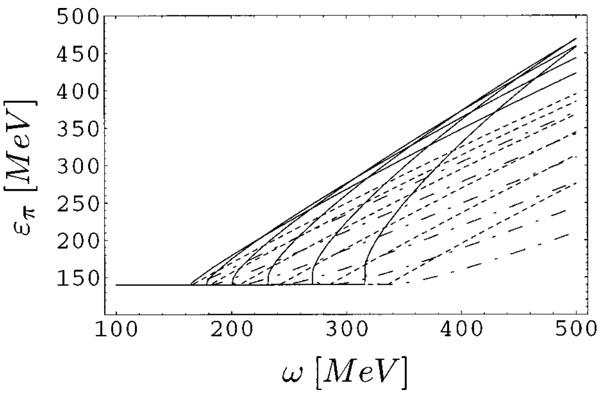

FIG. 6. The total energy of the decay π as a function of the energy transfer of the virtual photon for the following values of $|\vec{q}|=350$ MeV/c: $|\vec{k}_i|=250,100,0$ MeV/c; $\cos \measuredangle(\vec{k}_i,\vec{k})=1,0,-1$; $\cos \measuredangle(\vec{k},\vec{p}_\pi)=1$ (full line),0 (dashed),-1 (dashed pointed).

The factor $e^{-\vec{k}^2/m_\Delta \Gamma}$ stems from the averaging of the plane wave of the quasifree Δ over the nuclear volume.

In the Fermi gas model, which was successful in describing the experiments (see, e.g., [11] and references therein), one gets

$$\Pi(\vec{k},\vec{k}_i)=\frac{\vec{k}^2}{2m_\Delta}-\epsilon_s. \tag{22}$$

In order to calculate the quasifree Δ resonance shape, one has to average the amplitude in Eq. (21) over the kinematically allowed ranges of $\vec{k}_i$ and $\vec{p}_\pi$, the momentum of the π after the decay of the Δ. The final πN state in Fig. 1(a) will cover a range of total energy $\varepsilon_\pi=\sqrt{m_\pi^2+p_\pi^2}$ and, consequently, the respective amplitudes have to be added incoherently. ε_π is a function $\varepsilon_\pi=\varepsilon_\pi(\omega,\vec{k}_i,\measuredangle(\vec{q},\vec{k}_i),\measuredangle(\vec{k},\vec{p}_\pi))$, which can again be easily calculated using MATHEMATICA. In the Fermi gas model with $|\vec{k}_i|\leq k_F=250$ MeV/c and $|\vec{q}|=350$ MeV/c, the three-momentum transfer of the experiment of Ref. [1], one gets the range of ε_π depicted in Fig. 6. The result of the averaging for the probability distribution according to Eq. (21) is shown in Fig. 7. The dashed line represents this largely dominant quasi free Δ propagation as depicted in Fig. 1(a).

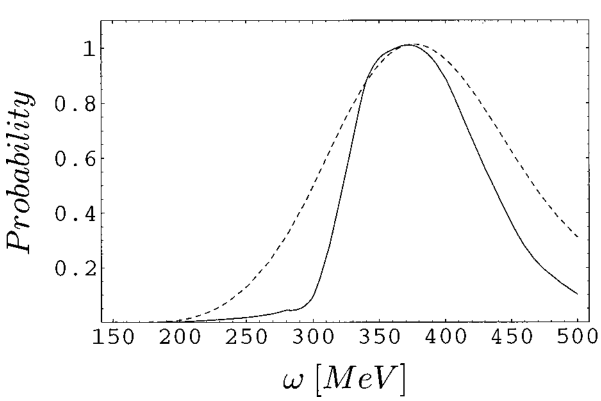

FIG. 7. Dashed line: the normalized probability according to Eq. (21) after averaging over $|\vec{k}_i|\leq k_F=250$ MeV/c and $|\vec{q}|=350$ MeV/c with $\epsilon_s=-20$ MeV. Full line: the normalized probability according to Eq. (23) after averaging as described in the text.

However, with a small probability, the decay π may produce a bound Δ in the $(A-1)$ nucleus as depicted in Fig. 1(b). In this case, the same rescattering chain as for the bound Δ described in the preceding subsection will occur. The sum of the amplitudes then reads, in complete analogy to Eq. (9),

$$\begin{aligned}G(\omega)=G_0\frac{\hbar}{\sqrt{2\pi}}\frac{-i}{[\omega-\omega_0-\Pi(\vec{k},\vec{k}_i)]+i(\Gamma/2)}e^{-\vec{k}^2/m_\Delta\Gamma}\\ \times[1+a_1(\varepsilon_\pi)+a_2(\varepsilon_\pi)+\cdots+a_k(\varepsilon_\pi)\\ +\cdots+a_{\mathcal{N}}(\varepsilon_\pi)]\end{aligned} \tag{23}$$

with ε_π being the energy of the π after the decay of the quasifree Δ. The π propagates again until it can escape the nucleus. The full line in Fig. 7 gives the probability for this process after averaging over the momenta $\vec{k}_i$ and the angles $\measuredangle(\vec{q},\vec{k}_i),\measuredangle(\vec{k},\vec{p}_\pi)$ as for the quasifree case in Fig. 1(a). Since the factor due to the rescattering in Eq. (23) cannot be calculated at infinitely many ε_π, the amplitudes have been sorted into 12 20-MeV-wide bins of ε_π. The absolute squares of the amplitudes in these bins have been added incoherently.

From a consideration of the quasifree case, the salient difference from the bound Δ becomes transparent. The momentum distribution of the energy eigenstate of the bound Δ has no influence on the energy ε_π of the decay π. Since the nucleus takes all recoil momentum, $\omega=\varepsilon_\pi$ and the π can propagate coherently at resonance energy. In contrast, the momentum distribution of the momentum eigenstate of the Δ makes the π propagate incoherently with $\varepsilon_\pi=\varepsilon_\pi(\omega,\vec{k}_i,\measuredangle(\vec{q},\vec{k}_i),\measuredangle(\vec{k},\vec{p}_\pi))$.

IV. DISCUSSION

The idea of the schematic model presented here should be the basis of a complete self-consistent many-body calculation including all effects of isospin-spin, spin-spin, and isospin-isospin correlations and a realistic N-N interaction along the lines of models summarized in [8], however in a finite nucleus. A more realistic study required the consideration of the Δ-nucleus potential, the full wave functions, and the Δ-particle-hole propagation at the same time. The calculations mentioned in the Introduction are starting from quasifree nucleons and the Δ's have a momentum transferred by the exciting particle. This means that these particles are not bound Δ eigenstates. In those calculations, the Δ's are propagating as plane waves with sharp momentum. This destroys the fixed phase relations for the π propagation.

It is important to realize that the kinematics of the experiment favors the "bound Δ's," i.e., a Δ stuck in the nucleus, over the "quasifree Δ," i.e., a Δ moving through the nucleus. In elastic π-nucleus scattering, the coherent rescattering in Fig. 2 will of course contribute. In this case, of course, no proton is emitted. However, this is a tiny little contribution to the diffractive scattering produced by the very dominant absorption into the channels with emitted protons. In other words, the π-nucleus scattering is largely dominated by diffractive scattering on a very absorptive disk. Therefore, the narrow Δ's could not be seen in the experiments performed so far. Only the measurement of the decay p and π in the final state discussed here enables us to select a kinematics that favors the ''bound Δ'' over the ''quasifree Δ'' mechanism.

It is further instructive to consider the analogy to the Mössbauer effect [13]. The ''bound Δ'' states here can be identified with the ''recoilless production of Λ hypernuclear states'' in Ref. [13] or the ''recoilless photoabsorption'' in the Mössbauer effect. The Debye-Waller factor corresponds merely to the transition form factors of Fig. 9 of Ref. [1] and is of the order of 5×10^{-3}. This means that the production of the quasifree states is much bigger than the production of the bound states. Since $|\vec{q}|\geqslant\omega$ for virtual photons, this cannot be improved in experiments with the electromagnetic probe.

The quenching mechanism will in principle also be present for the $\Sigma N\to\Lambda N$ reaction through the possible rescattering of the virtual π on the Λ, but it might be a smaller effect due to the smaller number of rescatters $\mathcal{N}$. A complete calculation, which includes the already established effect of Pauli blocking as well as the effect of the coherent initial state, should finally confirm that also narrow Σ hypernuclei are possible as suggested by experiment [14,15]. It is important to note that in the experiments in which Σ hypernuclei have been seen, no cut on the final state is needed in order to enrich the state of the coherent bound baryon over the quasifree state. These experiments were done close to the magic K^- momentum in the strangeness exchange reaction $A(K^-,\pi)A_\Sigma$ at which the three-momentum transfer to the Σ is small compared to the Fermi momentum favoring the population of the bound states over the quasifree states [13]. Additionally, the bound states have the biggest binding energy or smallest mass, i.e., smaller than the quasifree continuum. This feature is clearly seen in the experiment if the mass resolution is good enough to separate the narrow bound Σ states from the quasifree continuum and the background [14,15].

The schematic model proposed here explains the narrow bound Δ states in the framework of a schematic model. No modification of the resonance on the microscopic quark/gluon level is needed. If this mechanism will be confirmed by the required more complete calculations, the speculations in Ref. [1] have to be restricted. The quenching factor is determined here by the amplitude of the baryon resonance and its phase. Beside the quenching of the Δ width, one may speculate about the quenching of the Σ width or that of higher-lying Δ's and N's.

It is amusing to think about the possibility that higher-lying nucleon resonances could be quenched in a nucleus and in this way a narrow excitation spectrum of the nucleon could be produced. A rough estimate of count rates for an optimized experimental setup does not exclude such a speculation. Of course, the phase dependence and the cross sections is less favorable for these resonances. If the evidence for narrow Δ's can be established experimentally in the coming years, a dedicated setup may become worthwhile.

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