Jacobi shape transition in *fp* shell nuclei

G. Shanmugam and V. Ramasubramanian
Department of Physics, Manonmaniam Sundaranar University, Tirunelveli-627 002, India

S. N. Chintalapudi
Inter University Consortium for Department of Atomic Energy Facilities, III/LB-8, Bidhan Nagar, Calcutta-700 091, India

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Jacobi shape transition from noncollective oblate to super or hyperdeformed collective prolate or triaxial shape taking place in rotating nuclei as in the case of gravitating rotating stars is studied in *fp* shell nuclei ^{44}Ti, ^{48}Cr, ^{52}Fe, and ^{56}Ni. The cranked Nilsson-Strutinsky method is used to detect such transition. The method of tuning the angular velocity to get fixed spin is utilized in these calculations. Pairing is not taken into account since Jacobi transition occurs only at very high spin where pairing correlations would have already vanished. Our results show that all the four nuclei considered in this work are good candidates for detecting the Jacobi shape transition.

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I. INTRODUCTION

Study of structural changes of nuclei at high excitation energy and large angular momentum has led us to a new phase in nuclear structure physics. The experimental analysis of giant dipole resonance built on excited states has started to yield information about the shape transitions that take place in such nuclei. The combined effect of spin and temperature has created a variety of shape transition phenomena in nuclei. One such shape transition from noncollective oblate to highly deformed collective prolate or nearly prolate (triaxial) shape has been recently predicted and observed. This shape transition which is similar to the Jacobi transition in gravitating rotating stars has generated a lot of interest in recent times. The prediction of such a Jacobi transition in ^{45}Sc by Alhassid [1] and its subsequent experimental confirmation by the Seattle group [2] have further kindled our interest in looking for such interesting shape transitions in *fp* shell nuclei.

The aim of this work is to detect the possibility of the so-called Jacobi transition in even–even $N=Z$ fp shell nuclei namely ^{44}Ti, ^{48}Cr, ^{52}Fe, and ^{56}Ni. For this purpose, we use the cranked Nilsson-Strutinsky method modified suitably to take in large deformations. In order to fix the spin in our calculations we use the method [3] of tuning the angular velocity. To investigate how these Jacobi transitions are evolved we construct the potential energy surfaces to get a clear picture.

The rotating liquid drop model (RLDM) [4] has already predicted that nuclei should experience a shape transition at very high spins from noncollective oblate to collective prolate (or nearly prolate) with the superdeformed major to minor axes ratio of 2:1 or more. The shape evolution of rotating nuclei ultimately produces the above shape transition called the Jacobi shape transition. The Jacobi transition is not only a shape transition but it is a second order phase transition from noncollective to collective phases in nuclei. It is further interesting to note that such a shape transition is analogous to the Jacobi shape instability occurring in gravitating rotating stars [5]. Thus this study is very interesting and most important.

The second section of this theoretical study gives the theoretical framework for obtaining potential energy surfaces of the considered nuclei as a function of deformation (β) and nonaxiality (γ) parameters at different spins by the Strutinsky method. The third section gives a short description of the Jacobi transition and its detection. In the last section the results obtained for even-even $N=Z$ nuclei in the *fp* shell region are presented and discussed in relation to the occurrence of Jacobi shape transition in these nuclei.

II. THEORETICAL FRAMEWORK

The shell energy calculations [6] for the nonrotating case ($I=0$) assumes a single particle field

$$H_0=\sum h_0, \tag{1}$$

where h_0 is the triaxial Nilsson Hamiltonian given by [7]

$$h_0=\frac{p^2}{2m}+\frac{1}{2}m\sum_{i=1}^{3}\omega_i^2x_i^2+Cls+D(l^2-2\langle l^2\rangle). \tag{2}$$

The three oscillator frequencies ω_i are given by Hill-Wheeler parametrization as

$$\omega_x=\omega_0\exp\left(-\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma-\frac{2}{3}\pi\right)\right),$$

$$\omega_y=\omega_0\exp\left(-\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma-\frac{4}{3}\pi\right)\right),$$

$$\omega_z=\omega_0\exp\left(-\sqrt{\frac{5}{4\pi}}\beta\cos\gamma\right)$$

with the constraint of constant volume for equipotentials

$$\omega_x\omega_y\omega_z=\mathring{\omega}_0^3=\text{const.} \tag{3}$$

For the Nilsson parameters κ and μ and $\hbar\mathring{\omega}_0$ the following values [7] are chosen: $\kappa=0.093$ and $\mu=0.15$,

$$\hbar\mathring{\omega}_0=\frac{45.3\,\mathrm{MeV}}{(A^{1/3}+0.77)}. \tag{4}$$

The same values are used for protons as well as neutrons. It may be noted that in h_0 [Eq. (2)] the factor in front of $\langle l^2\rangle$ term has been doubled to obtain better agreement between the Strutinsky-smoothed moment of inertia and the rigid rotor value (here within 10%). Accordingly the parameter D has been redetermined with the help of single-particle levels in the mass region indicated. The Hamiltonian (2) is diagonalized in cylindrical representation [8] up to $N=11$ shells using the matrix elements given in Ref. [9].

For the rotating case ($I\neq 0$) the Hamiltonian becomes

$$H_\omega=H_0-\omega J_z=\sum h_\omega, \tag{5}$$

where

$$h_\omega=h_0-\omega j_{\mathbf{z}} \tag{6}$$

if it is assumed that the rotation takes place around the Z axis.

The single particle energy $e_{\mathbf{i}}^\omega$ and wave function $\phi_{\mathbf{i}}^\omega$ are given by

$$h_\omega\phi_{\mathbf{i}}^\omega=e_{\mathbf{i}}^\omega\phi_{\mathbf{i}}^\omega. \tag{7}$$

The spin projections are obtained as

$$\langle m_{\mathbf{i}}\rangle=\langle\phi_{\mathbf{i}}^\omega|j_{\mathbf{z}}|\phi_{\mathbf{i}}^\omega\rangle. \tag{8}$$

The total shell energy is given by

$$E_{\mathbf{sp}}=\sum\langle\phi_{\mathbf{i}}^\omega|h_0|\phi_{\mathbf{i}}^\omega\rangle=\sum\langle e_{\mathbf{i}}\rangle, \tag{9}$$

where

$$e_{\mathbf{i}}^\omega=\langle e_{\mathbf{i}}\rangle-\hbar\omega\langle m_{\mathbf{i}}\rangle. \tag{10}$$

Thus

$$E_{\mathbf{sp}}=\sum e_{\mathbf{i}}^\omega+\hbar\omega I. \tag{11}$$

The total spin I is given by

$$I=\sum\langle m_{\mathbf{i}}\rangle. \tag{12}$$

Since the difficulties encountered in the evaluation of total energy for large deformations through the summation of single particle energies for $I=0$ case may be present for $I\neq 0$ case also [10], we use the Strutinsky shell correction method adopted to $I\neq 0$ case by suitably tuning the angular velocities to yield fixed spins. For unsmoothened single particle level distribution we have

$$I=\int_{-\infty}^{\lambda}g_2\,de^\omega=\sum_{\mathbf{i}}\langle m_{\mathbf{i}}\rangle \tag{13}$$

and

$$E_{\mathbf{sp}}=\int_{-\infty}^{\lambda}g_1e^\omega\,de^\omega+\hbar\omega I=\sum_i e_{\mathbf{i}}^\omega+\hbar\omega I. \tag{14}$$

For the Strutinsky smeared single particle level distribution Eqs. (13) and (14) transform into

$$\widetilde{I}=\int_{-\infty}^{\lambda}\widetilde{g_2}\,de^\omega=\sum_{\mathbf{i}}\langle\widetilde{m_{\mathbf{i}}}\rangle \tag{15}$$

and

$$\widetilde{E_{\mathbf{sp}}}=\int_{-\infty}^{\lambda}\widetilde{g_1}e^\omega\,de^\omega+\hbar\omega\widetilde{I} \tag{16}$$

$$=\sum_{\mathbf{i}}^{N}\widetilde{e_{\mathbf{i}}^\omega}+\hbar\omega\widetilde{I}. \tag{17}$$

In the tuning method we have adapted [3], the total spin is calculated as

$$I=\widetilde{I_{\mathbf{z}}}=\sum_{\nu=1}^{N}\langle\widetilde{J_{\mathbf{z}}}\rangle_\nu^\omega+\sum_{\pi=1}^{Z}\langle\widetilde{J_{\mathbf{z}}}\rangle_\pi^\omega. \tag{18}$$

The above relation allows us to select numerically the ω values that correspond to the chosen integer or half integer spins. Obviously the corresponding frequency values $\omega(I)$ change from one deformation point to another and the corresponding calculation must be repeated accordingly.

The total energy is now given by

$$E_{\mathbf{T}}=E_{\mathrm{RLDM}}+(E_{\mathbf{sp}}-\widetilde{E_{\mathbf{sp}}}), \tag{19}$$

where the rotating liquid drop energy at constant spin

$$E_{\mathrm{RLDM}}=E_{\mathrm{LDM}}-\tfrac{1}{2}I_{\mathrm{rig}}\omega^2+\hbar\omega\widetilde{I}. \tag{20}$$

Here the liquid drop energy E_{LDM} is given by the sum of Coulomb and surface energies as

$$E_{\mathrm{LDM}}(\beta,\gamma)=[(B_{\mathbf{s}}-1)+2\chi(B_c-1)a_{\mathbf{s}}]A^{2/3}, \tag{21}$$

where $B_{\mathbf{s}}$ and B_c are the relative surface and Coulomb energies of the nucleus. Both $B_{\mathbf{s}}$ and B_c are elliptic integrals which depend on the semiaxes lengths. The values used for the parameters $a_{\mathbf{s}}$ and χ are as follows: $a_{\mathbf{s}}=19.7$ MeV and the fissility parameter $\chi=(Z^2/A)/45$ where Z and A are the charge and mass numbers of the nucleus; I_{rig} is the rigid body moment of inertia defined by β and γ including the surface diffuseness correction can be calculated as follows.

In the case of an ellipsoidal shape described by the deformation parameter β and the shape parameter γ, the semiaxes R_x, R_y, R_z are given by

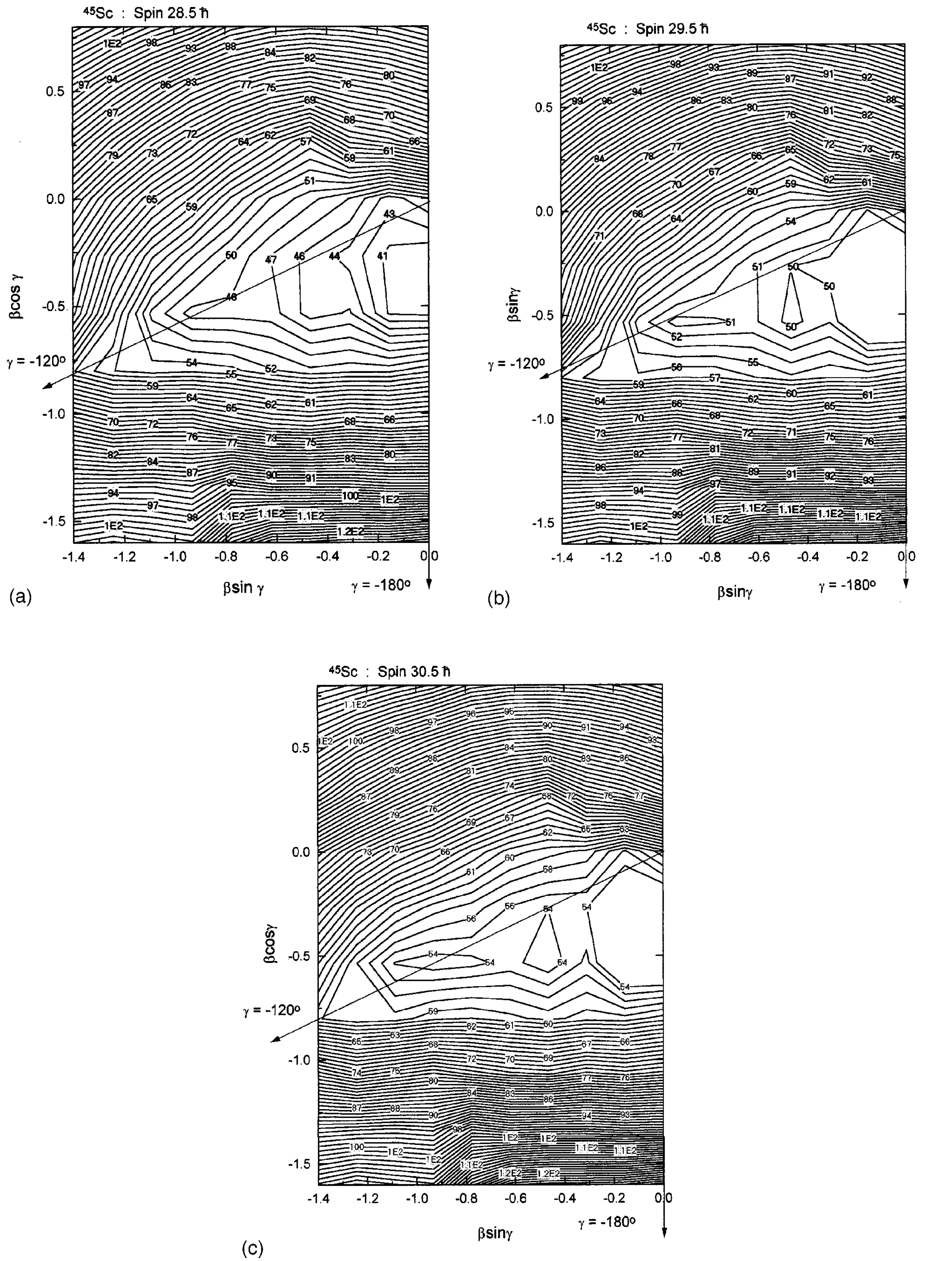

FIG. 1. (a) PES in β-γ plane for ^{45}Sc at spin 28.5$\hbar$, (b) PES in β-γ plane for ^{45}Sc at spin 29.5$\hbar$, (c) PES in β-γ plane for ^{45}Sc at spin 30.5$\hbar$.

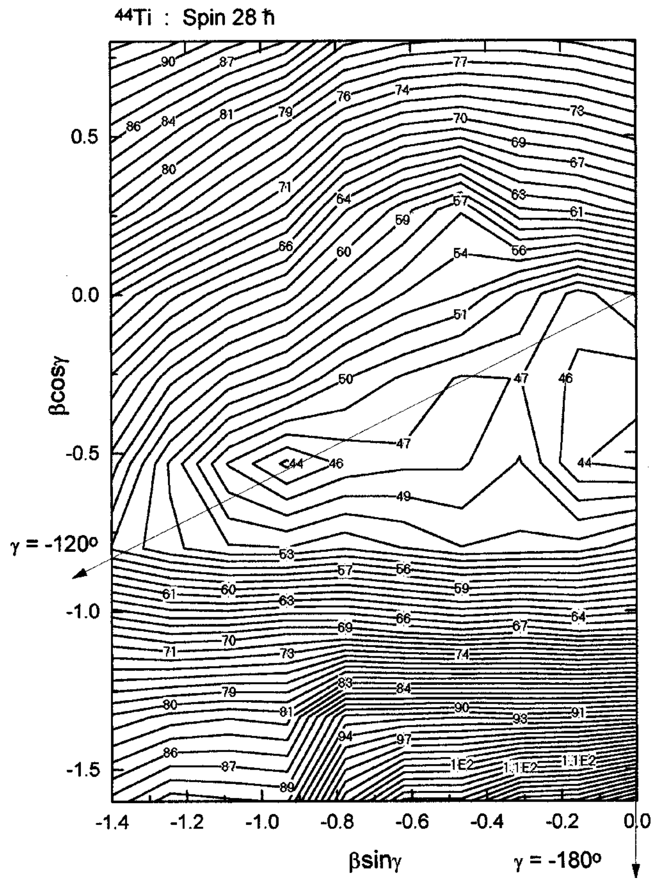

FIG. 2. PES in β-γ plane for ^{44}Ti at spin $28\hbar$.

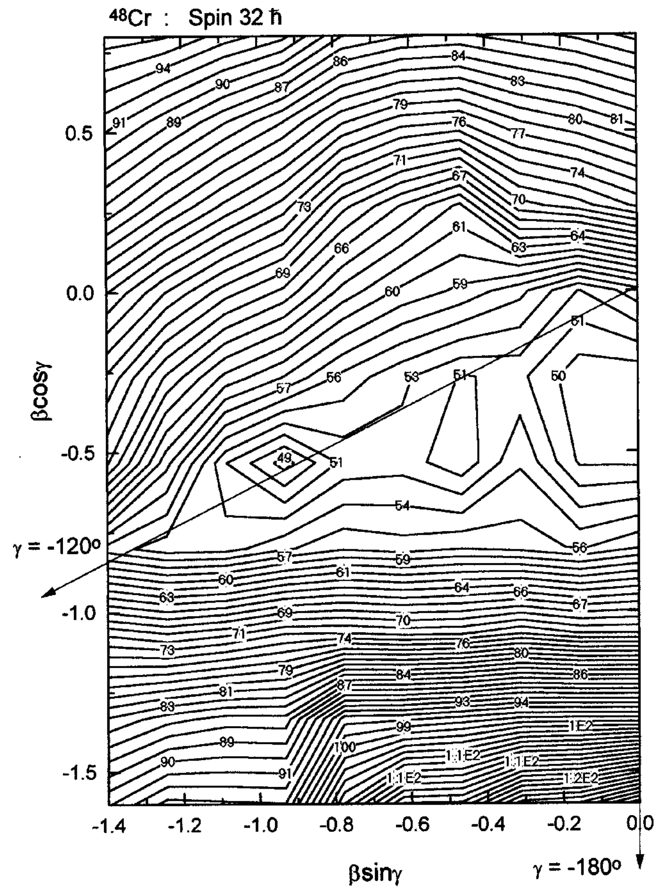

FIG. 3. PES in β-γ plane for ^{48}Cr at spin $32\hbar$.

$$R_x = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\beta \cos\left(\gamma - \frac{2\pi}{3}\right)\right],$$

$$R_y = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\beta \cos\left(\gamma - \frac{4\pi}{3}\right)\right],$$

and

$$R_z = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\beta \cos\gamma\right].$$

By volume conservation we have

$$R_x R_y R_z = R_0^{0\,3}, \tag{22}$$

where R_0^0 is the radius of the spherical nucleus.

The moment of inertia about the z axis is given by

$$\frac{\mathfrak{J}_{\rm rig}(\beta,\gamma) + 2{\rm Mb}^2}{\hbar^2} = \frac{1}{5}\,\frac{AM(R_x^2 + R_y^2)}{\hbar} + \frac{2{\rm Mb}^2}{\hbar^2}, \tag{23}$$

where the diffuseness correction to the moment of inertia is 2 Mb^2 and the diffuseness parameter $b = 0.87$ fm.

Here

$$R_0^0 = r_0 A^{1/3} \quad (r_0 = 1.16\ \mathrm{fm}). \tag{24}$$

The calculations are carried out by varying ω values in steps of $0.03\omega_0$ from $\omega = 0$ to $\omega = 0.45\omega_0$, ω_0 being the oscillator frequency for tuning to fixed spins. Since we are interested mainly in Jacobi shape transition γ is varied from $-180°$ to $-120°$ in steps of $10°$, $\gamma = -180°$ corresponding to noncollective oblate and $\gamma = -120°$ corresponding to collective prolate. Since the Jacobi transition involves large deformation, β values are varied from $\beta = 0.0$ to $\beta = 1.5$ in steps of 0.1.

III. JACOBI TRANSITION AND ITS DETECTION

The study of certain rotating nuclei at high spin is very interesting, since in spite of the presence of the shell effects such nuclei behave very much like a rotating liquid drop. As the spin is increased such a spherical nucleus becomes noncollective oblate whose deformation gradually increases with spin. As it reaches the critical spin the nucleus will undergo another phase transition to nearly prolate or triaxial shape with very large deformation. This behavior is analogous to the Jacobi transition that occurs in gravitating rotating stars. Alhassid [1] has formulated a macroscopic fluctuation theory of the giant dipole resonance (GDR) to study the rotating nucleus ^{45}Sc. The frequency of the giant dipole resonance is inversely proportional to the length of the semiaxis along which the vibration occurs. The deformed nucleus gives the resonance splits whose magnitude can be related to the de-

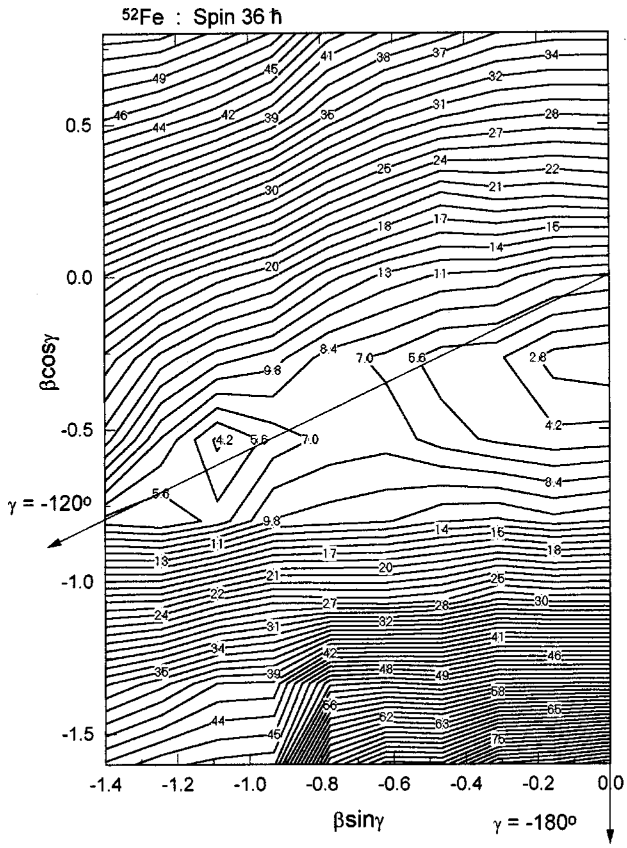

FIG. 4. PES in β-γ plane for ^{52}Fe at spin $36\hbar$.

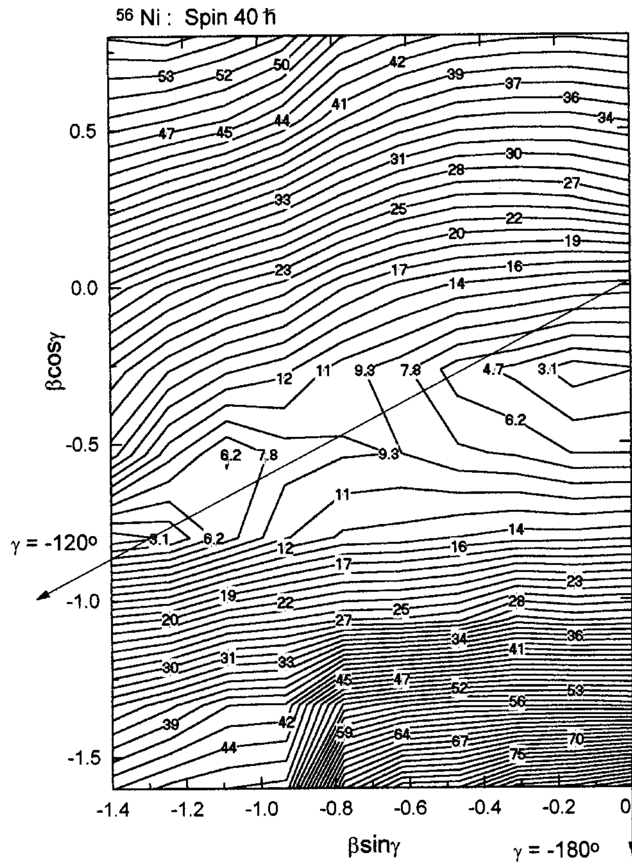

FIG. 5. PES in β-γ plane for ^{56}Ni at spin $40\hbar$.

formation. But the relation between the giant dipole resonance and the equilibrium deformation is quite complex in hot systems because of the macroscopic thermal shape fluctuations. Alhassid [1] using the Landau theory of phase transition has demonstrated that there are three distinct peaks in the GDR curve of ^{45}Sc due to a triaxial shape with large deformation at the spin $I=28\hbar$. He found that such structure of three separate peaks is not seen in the calculations of the giant dipole resonance in heavier nuclei with $A\geqslant 100$ at moderate spins and temperatures. This finding has prompted us to undertake a systematic study of the shape transitions of even-even $N=Z$ nuclei in the *fp* shell region to look for such possible Jacobi transitions. However, our work differs from that of Alhassid in that we are using a straightforward cranked Strutinsky calculation with spin tuning without temperature which is different from the Landau theory used by Alhassid. It is to be noted that while the Alhassid model is built on temperature and thermal fluctuations, our model does not take them into account. This will not pose any problem since the Jacobian transition is a spin-driven effect rather than a temperature-caused one. But the shell effects present have to be taken care of in the study of the Jacobi transition in the considered nuclei.

IV. RESULTS AND DISCUSSIONS

An important question in nuclear structure physics is the nature of shape evolution taking place at critical angular momenta near the limit of stability. The rotating liquid drop model predicts that the nucleus should experience a shape transition at very high spins from an oblate noncollective to a triaxial or near prolate collective shape with superdeformed major to minor axis ratios of 2:1 and larger. This shape change corresponds to a second order phase transition similar to the Jacobi phase transition in gravitating rotating stars (in an infinite system).

The aim of this work is to study and predict such Jacobi phase transitions in even-even $N=Z$ *fp* shell nuclei. We have obtained the potential energy surfaces (PES) of these nuclei as a function of spin to look for such phase transitions. Alhassid first predicted a Jacobi transition in a *fp* shell nucleus ^{45}Sc. We performed potential energy surface calculations using the tuned spin-cranked Nilsson-Strutinsky method for this nucleus to begin with. Figures 1(a)–1(c) show the shape transitions for ^{45}Sc obtained by us at spins $28.5\hbar$, $29.5\hbar$, and $30.5\hbar$. It is shown in these figures [see Fig. 1(b) in particular] that a Jacobi transition takes place at a spin of $29.5\hbar$ where a noncollective oblate shape with $\beta \simeq 0.4$ changes to a superdeformed triaxial shape with $\gamma \simeq -130°$ and $\beta \simeq 0.6$ which is the one that survived up to a high temperature in spite of the thermal fluctuations, which was predicted by Alhassid [1] and subsequently experimentally confirmed by the Seattle group [2].

Next we give the PES of ^{44}Ti at a spin of $28\hbar$ in Fig. 2. It is shown that ^{44}Ti undergoes a Jacobi transition at this spin where the shape is changed from noncollective oblate with $\beta=0.5$ to collective hyperdeformed prolate with $\beta>1.0$.

Figure 3 shows the potential energy surface of ^{48}Cr at a spin of $32\hbar$. When we compare this figure with that shown by Betts [11] the similarity is quite striking. This figure shows that at the spin of $32\hbar$, ^{48}Cr has already undergone a Jacobi transition to hyperdeformed prolate shape. This hyperdeformation has been conjectured by Betts [11] to resemble two prolate ^{24}Mg nuclei touching end on end. In the case of ^{52}Fe, at the spin of $36\hbar$ the trend of a significant Jacobi transition from noncollective oblate to hyperdeformed prolate shape can be seen (Fig. 4). Finally, we give the PES for the doubly magic nucleus ^{56}Ni in Fig. 5. The Jacobi transition to a super-hyperdeformed shape is clearly visible in this figure.

It can be concluded that the study of the Jacobi shape transitions in *fp* shell nuclei will throw a lot of light on the second order phase transitions as well as super, hyper, and super-hyperdeformation occurring in them. It should be interesting to see experimentally whether such phase transitions and large deformations can be detected through giant dipole resonance (GDR) cross sections built on excited states.

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