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Squeezing mode in nuclear collisions

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The time dependent Schrödinger equation is solved analytically for a simplified model of moving infinite walls. A new knockout mode is described which might occur during heavy ion collisions. The outer shell nucleons are ionized due to the increase of level energy when two nuclei are approaching fast enough. This is analogous to the Mott effect but in contrast occurs only if the reaction time is short enough that no common ionization threshold in the compound system is established. To demonstrate this pure nonequilibrium effect a simulation of realistic heavy ion collision by a nonlocal Boltzmann equation is performed.

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When a system with bound states is exposed to a compression beyond certain values the bound states break off and decompose into their constituents. This pressure ionization known as the Mott transition is well established in different fields of physics [1–4]. Alternatively in a many-body system the density could be increased. The theoretical treatments agree in that the ionization threshold is lowered faster than the binding energy with increasing density which leads to a crossover and ionization at the Mott density. These treatments rely on the fact that one has a certain degree of equilibration in the system, at least that the system has one unique ionization threshold.

This situation is however somewhat different when processes occur far from equilibrium. Then there might not be enough time to establish a common threshold in the system. In particular if two nuclei approach each other in a heavy ion collision it takes a certain time before a compound system is established or decomposition happens at higher energies. One can easily imagine that there will be no common ionization threshold for the two nuclei at the early stage of reaction. Instead we will show that this leads to a new escaping mode by squeezing states which should lead to the nonequilibrium emission of particles. The principle phenomena has already been investigated in the past as light nonequilibrium particle emission [5]. Here we will show that with the help of an exactly solvable model of time dependent Schrödinger equation a new, not-yet described effect arises. The different features are transversal angular distribution and a lower bound of projectile energy that cause this squeezing mode to happen. There are experimental signals for dynamical particle emission [6,7]. In contrast to diabatic emission of particles [8] limited to beam energies much below the Fermi energy we consider here the case of faster processes around the Fermi energy. After presenting the exactly solvable model we will confirm this mode by realistic simulations which will show a transversal distribution and low energy of emitted particles while diabatic emitted particles are longitudinally peaked.

Another intuitive picture is the following. The Pauli principle will forbid overoccupation of states, which should result in a Fermi gap which is closed during dissipation. Before this quasiequilibration happens one essentially has the situation where an outer nucleon feels a rapidly increasing force due to the other nuclei and the Pauli-forbidden areas in phase space. If the speed of nuclei is high there is no many-body equilibration or dissipation but rather a shrinking of phase space for the outer nucleons during the time after first touching of the two nuclei. Therefore it is reasonable to assume a picture where the boundary of the outer nucleons is shrinking with time. We will solve such a time dependent Schrödinger equation to show that indeed the level energy of the outer nucleons goes up and eventually leads to ionization. Since this is opposite to the Mott transition described above where the threshold decreases we call this mode ‘‘squeezing mode’’ in the following. There are similarities to the elevator resonant activation mode [9] where a time dependent potential inside the wall creates resonant levels which trap the particles and lift them above the barrier. Here we merely squeeze the wall.

Let us assume that the outer shell nucleons are bound states which can be parametrized by a simple one-dimensional infinite wall model, i.e., a free particle inside an infinite wall of distance b at the initial time $t=0$. Then the binding energy is $[k_n=(\pi n)/b]$

$$E_n^0=\frac{\hbar^2k_n^2}{2m} \tag{1}$$

and the normalized wave function

$$\Psi(0,x)=\sqrt{\frac{2}{b}}\sin(k_nx)e^{i\phi}, \tag{2}$$

where we note that the physical state is undetermined up to a phase ϕ which will be employed to find a solution of the time dependent Schrödinger equation. We now solve the time dependent Schrödinger equation with the boundary condition where the wall is moving inwards with the speed v. The mathematical problem is

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\Psi(t,x)=0,$$

$$\Psi(t,0)=\Psi(t,b-vt)=0 \tag{3}$$

together with the initial state (2). Of course the moving wall can be equivalently formulated as a *time dependent* δ or Hill-Wheeler potential. This case can be considered as a special case of [10]. The normalized solution reads

$$\Psi(t,x)=\sqrt{\frac{2}{b-vt}}\sin\frac{k_n x b}{b-vt}\exp\left(-i\frac{m^2vx^2+k_n^2\hbar^2bt}{2m\hbar(b-vt)}\right) \tag{4}$$

which determines the phase ϕ, especially it leads to the initial phase at $t=0$,

$$\phi=-\frac{x^2mv}{2b\hbar}. \tag{5}$$

The insertion of Eq. (5) into Eq. (3) verifies the solution. In [10] such classes of solutions have been used to expand any initial condition at $t=0$. Here we want to point out that already the basic solution (4) with the extra phase ϕ at inital time $t=0$ bears a physical meaning in itself. The solution (4) represents a nonseparable solution. Other classes of potentials which admit a separable solution can be found in [11].

The probability density ρ and current s are easily computed

$$\rho(x,t)=|\Psi|^2=\frac{2}{b-vt}\sin^2\frac{\pi n x}{b-vt},$$

$$s(x,t)=2\,\mathrm{Im}\,\Psi^+\partial_x\Psi=\rho(x,t)\frac{xv}{vt-b} \tag{6}$$

with $\dot\rho+\partial_x s=0$, as it should. The kinetic energy becomes

$$\begin{aligned}E_n(t)&=-\int dx\Psi\frac{\hbar^2\nabla^2}{2m}\Psi^+\\&=\frac{mv^2}{12n^2\pi^2}(2n^2\pi^2-3)+\frac{n^2\pi^2\hbar^2}{2m(b-vt)^2},\end{aligned} \tag{7}$$

which shows that even at time $t=0$ the initial binding energy (1), $E_n^0=(\hbar^2\pi^2n^2)/(2mb^2)$, is shifted by

$$E_\phi=\frac{E_{\rm proj}}{3}\left(1-\frac{3}{2n^2\pi^2}\right) \tag{8}$$

with the projectile energy $E_{\rm proj}=mv^2/2$ due to the finite velocity v corresponding to the finite phase ϕ. This means that due to a finite velocity v of the projectile, the target system gets a phase jump ϕ and an energy E_ϕ immediately when they touch each other. The kinetic energy of a level increases with time as

$$E_n(t)=E_\phi+E_n^0\frac{b^2}{(b-vt)^2}. \tag{9}$$

Ionization happens when this energy becomes larger or equal to a now introduced threshold E_c such that the effective initial binding energy would be $E_n^b=E_c-E_n^0$. Consequently we obtain for ionization

$$\frac{b^2}{(b-vt)^2}\geqslant\frac{E_c-E_\phi}{E_n^0}. \tag{10}$$

Therefore we find immediate ionization if the phase kinetic energy E_ϕ exceeds the threshold E_c

$$E_{\rm proj}\geqslant\frac{3}{1-\dfrac{3}{2n^2\pi^2}}E_c. \tag{11}$$

For lower projectile energies we have to wait long enough to reach the threshold which leads to

$$\frac{b}{v}\geqslant t\geqslant\frac{b}{v}\left(1-\sqrt{\frac{E_n^0}{E_c-E_\phi}}\right), \tag{12}$$

where the first inequality comes from the restriction of the model in that there should be some space between the walls.

Therefore we have two cases. If the projectile is fast enough, Eq. (11), roughly larger than Fermi energy, we knock out particles at the first instantce of touching. This can be seen in analogy to the observation of [12] where a model of instantly removed walls was studied. Due to the time dependent solution here we can give the velocity criterion where such effect should happen.

For slower projectiles and fast enough reaction time to prevent many-body equilibration and larger reaction time than the critical time (12) we will have a knockout of the outer shell nuclei as well. Both cases should be considered as a nonequilibrium mode.

The latter condition for projectiles below or around Fermi energy, Eq. (12), can be translated into a geometrical condition. For the model we assume two approaching spherical nuclei with equal radii R. The case of different radii is straightforward. The impact parameter B should then be smaller than the sum of radii of the two nuclei $B<2R$ in order to allow the necessary overlap. Assuming constant projectile velocity v_p, the time between the first touching of the nuclei and the closest approach is

$$t_m=\frac{\sqrt{(2R)^2-B^2}}{v_p}\approx\frac{R-B/2}{v}. \tag{13}$$

Here we approximate the relative velocity v of the nuclei by a constant velocity such that at a time $t=0$ we have the distance R and at the time of maximal overlap we have the distance $B/2$, and the effective velocity $v=v_p(R-B/2)/\sqrt{(2R)^2-B^2}$ which leads to Eq. (13).

The condition for reaction time (12) together with $2R>B$ translates now into a condition for the geometry

$$B \leqslant 2R\sqrt{\frac{E_n^0}{E_c - E_\phi}} \leqslant 2R. \qquad (14)$$

In other words the condition (14) gives the simple restriction on the reaction geometry concerning radii and impact parameter for which an outerbound state characterized by the binding energy E_n^0 is ionized. Rewriting Eq. (14) the condition for the projectile velocity reads

$$\frac{3}{1-\frac{3}{2n^2\pi^2}} E_c \geqslant E_{\mathrm{proj}} \geqslant \frac{3}{1-\frac{3}{2n^2\pi^2}}\left(E_c - E_n^0\left(\frac{2R}{B}\right)^2\right). \qquad (15)$$

Therefore we have two projectile energy ranges. Either Eq. (11) for which ionization happens immediately or for projectile energies according to Eq. (15) we have overlap and possible ionization if the projectile energy is not too low. These lower boundaries in projectile energy clearly distinguish this model from the diabatic model [8].

In order to draw possible predictions to observable effects one would like to have the characteristic angular distribution of such emitted particles due to phase space squeezing. Since we do not have the exact solution of the two-dimensional geometry we associate approximately the emission angle with the ratio of impact parameter to the distance of nuclei $\sin\alpha = B/(b - vt)$ which corresponds to the above geometrical model. The angular distribution will be according to the momentum of the Wigner function. The latter one can be given with Eq. (4):

$$\begin{aligned} f(p,R,t) &= \int_{-(b-vt)}^{b-vt} dr e^{-irp}\Psi^+\left(R-\frac{r}{2},t\right)\Psi\left(R+\frac{r}{2},t\right) \\ &= 2\frac{\sin\xi}{\xi}\left[\xi^2\cos\frac{(n\pi)}{\xi}^2 - n^2\pi^2 - \cos\left(\frac{2\pi nR}{b-vt}\right)\right] \end{aligned} \qquad (16)$$

and $\hbar k = p + mvR/(b - vt)$. Integrating over all allowed spatial coordinates and using the angular relation as described above we obtain the angular distribution which is momentum and impact parameter dependent. In Fig. 1 we see that for small impact parameters indeed there is a longitudinal emission pattern in agreement with the finding of the diabatic model [13,5]. But for specific impact parameters around 6–8 fm for a projectile energy around the Fermi energy we see a clear transversal distribution.

When such ionization due to phase space squeezing happens, one can expect small energies beyond the threshold. Therefore it should be possible to observe such a mode as emitted particles with small total energy and the angular distribution should be transversal symmetric for the emitted particles. A good plot to observe this mode would be the transverse energy versus the ratio of the transverse to total energy. The usual multifragmentation products are on a clear correlation line between these two observables. The described mode here should be visible as a correlation with small transverse energy but a large ratio between transverse and total energy.

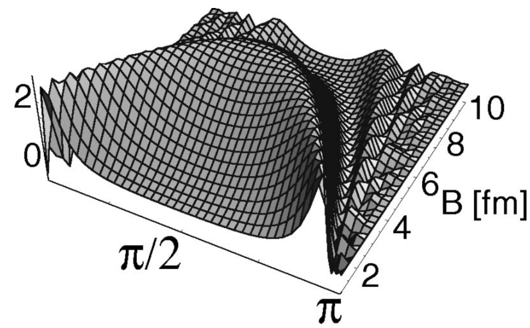

FIG. 1. The Wigner function (16) integrated over the spatial coordinates and zero momentum versus angle and impact parameter according to $b - vt = B/\sin(\alpha)$. The projectile energy is around Fermi energy. For nonzero momenta the transversal maximum is shifted to lower impact parameter.

In order to verify the existence of such a mode we solved the nonlocal kinetic equation (nonlocal BUU) [14–16] which leads to Fig. 2. We have used a soft parametrization of the mean-field and realistic nuclear potentials, for details see [17]. In the second panel we plotted the total kinetic energy of free particles including their Fermi motion. Calculating the current one could get rid of the latter motion [16] in order to come nearer to the experiments which leads of course to a main almost linear correlation starting from zero. An alternative way would be to use coalescence models which we do not want to use here in order to maintain theoretical consistency. The particles are considered as free if their kinetic energy overcomes the binding mean-field energy.

We see a separation of the energy distribution of emitted particles into two branches at 60 fm/c. The main lines almost at $E_{trans}/E = 2/3$ are the emitted particles due to thermal emission and multifragmentation. Besides this line we recognize some events in the right lower corner of this plot. These 0.2% of emitted particles are due to the squeezing mode since they have distinctly smaller energy than the rest of the emitted particles and are clearly transversal. Interestingly this mode is seen at the time of closest approach and again later at 120 fm/c where the neck structure appears. This can be understood since during the time in between this mode is shadowed and screened by the two nuclei and other emission channels.

From the right panels one sees that these emitted particles originate really from the surface and especially from the touching point. The number of indicated trajectory points is not representative of the amount of emitted particles since it is only a cut in the (x,z) plane. The total number is given in the middle panel as plot label. From this we see that the homogeneous surface emitted particles at the beginning are very small and not due to the squeezing mode. At the time of a remarkable amount of squeezed particles one sees that they come mainly from the region of overlap. Moreover the potential surface plot shows the deepening and squeezing of the

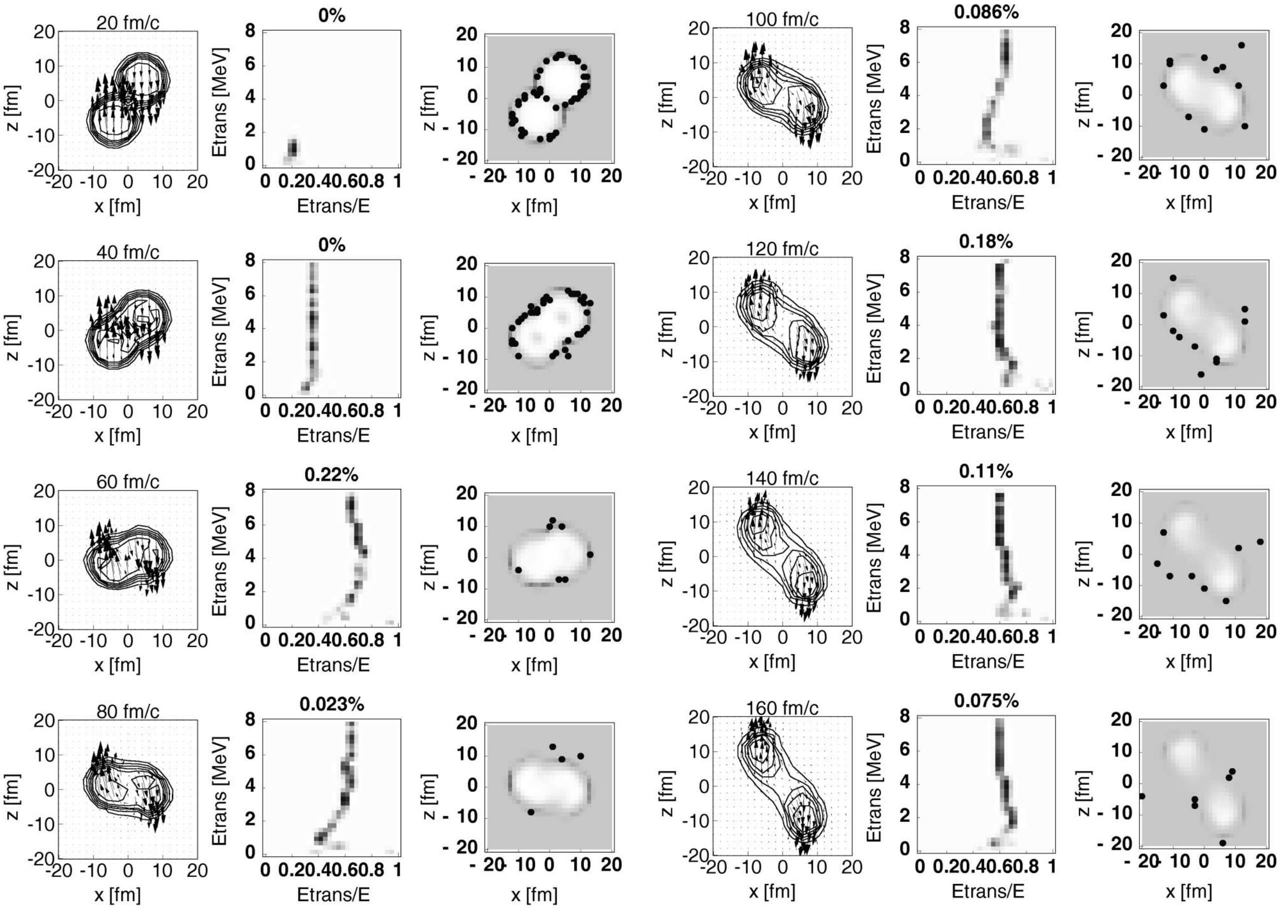

FIG. 2. The evolution of a Ta+Au collision at 33 MeV lab energy and 8 fm impact parameter. The left panels give the spatial density contour-plot in the $(x,0,z)$ plane and the local currents as arrows. The middle panels show the transversal energy distribution versus the ratio of transversal to total energy. The amount of emitted particles due to the squeezed mode which is located in the right lower corner is given as plot label. The right panels represent the contour-plots of the total energy of particles. The dark areas give the positive total energy indicating areas where particles can escape and the lighter color scales the deepness of negative total energy indicating bound states. The black dots are the position of test particles in the $(x,0,z)$ plane which contribute to the new squeezing mode.

potential at the touching point. Both observations suggest that these emitted particles are probably due to the squeezing mode described above.

Let us comment that the local Boltzmann equation (BUU) leads to an even more pronounced effect. Since we consider the nonlocal extension of the Boltzmann equation as more realistic we give here the smallest estimate of the effect. Moreover, the simplified picture given above neglects completely the rebinding mechanisms, e.g., by mean fields which will limit the ionization. This was neglected for analytical solvability. However the realistic simulation shows that a small percentage of events might show this behavior.

To summarize a new mode is predicted due to fast shrinking of available phase space for outer particles when two nuclei collide. The time in such collisions is too short to establish a common ionization threshold. Instead the energy of the level increases and ionization can occur. In opposition to the Mott transition, here the levels increase which gives rise to the name ''squeezing mode.'' The characteristics of such a mode are that the emitted particles have very small total energy and are transversal. This feature and the fact that the mode appears at the beginning of the collision distinguishes it from the ''towing mode'' [18]. In a corresponding plot we could identify such events in nonlocal BUU simulations. In order to demonstrate this mode an analytical solution of the time-dependent Schrödinger equation has been given.

The described mode is not restricted to nuclear collisions. Anytime two clusters of particles collide fast enough to prevent the formation of a common threshold and long enough to squeeze the outer-bound states this squeezing mode should be possible to observe.

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