Total ^{4}He photoabsorption cross section reexamined: Correlated versus effective interaction hyperspherical harmonics

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Two conceptually different hyperspherical harmonics expansions are used for the calculation of the total ^{4}He photoabsorption cross section. Besides the well-known method of correlated hyperspherical harmonics, the recently introduced effective interaction approach for the hyperspherical formalism is applied. Semirealistic *NN* potentials are employed and final-state interaction is fully taken into account via the Lorentz integral transform method. The results show that the effective interaction leads to a very good convergence, while the correlation method exhibits a less rapid convergence in the giant dipole resonance region. The rather strong discrepancy with the experimental photodisintegration cross sections is confirmed by the present calculations.

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The ^{4}He photodisintegration in the giant dipole resonance region is a particularly interesting reaction. An understanding of this process in terms of a microscopic calculation is not only a challenge in few-body physics, but could also lead to a deeper insight in the physics of more complex nuclei. A few calculations of the process were performed, see [1]. While for four-nucleon ground state and low-energy scattering calculations with realistic interactions are available (see, e.g., recent papers [2,3]), calculations of the ^{4}He photodisintegration with realistic forces are still lacking because of the obvious difficulties in describing correctly the intermediate-energy four-nucleon dynamics. In the most advanced calculation [1], which also covers an energy range above the three-body breakup threshold, semirealistic *NN* potential models were employed and the complicated four-nucleon final state interaction was treated exactly applying the Lorentz integral transform (LIT) method [4]. The semirealistic potential models lead to rather realistic results for the total three-nucleon photoabsorption cross section [5] and the ^{4}He inverse energy weighted sum rule (see below). Hence one might expect a rather realistic description of the ^{4}He giant dipole resonance cross section, which is situated close to the breakup threshold. However, the obtained cross sections show a considerably more pronounced giant dipole resonance than seen in the experimental results published in the last two decades (see [1]). On the other hand, the experimental situation is not yet completely settled. Older photoabsorption data (see discussion in Ref. [6]) and a more recent determination of the photoabsorption cross section via photon scattering [7] show a stronger giant dipole peak. A round of experiments presently being carried out at Lund will hopefully help to clarify this unsatisfying situation.

Besides clarification on the experimental side it is also necessary to check the obtained theoretical result. In a recent calculation of the ^{4}He photoabsorption with the effective interaction hyperspherical harmonics (EIHH) method [8], small deviations from the above-mentioned calculation of Ref. [1] in a correlated hyperspherical harmonics (CHH) approach were found. The slightly different results are most probably due to not fully convergent HH expansions. Therefore, it is the aim of the present work to study the convergence in both cases in greater detail extending the calculations to higher order terms. On the one hand it will allow establishing the correct ^{4}He total photoabsorption cross section with semirealistic *NN* potential models. On the other hand, it will show which is the most efficient HH approach. This is very important in view of calculations of the ^{4}He photoabsorption with realistic forces.

The rate of convergence of an HH expansion is generally rather slow in nuclear physics problems. In particular the short-range repulsion of the *NN* interaction leads to high-momentum components in the nuclear wave function that can be parametrized only by including a very large number of HH basis functions H_n. The convergence can be improved introducing proper short-range two-body correlation functions $f(r_{ij})$,

$$H_n(\rho,\Omega)\rightarrow\prod_{i,j} f(r_{ij})H_n(\rho,\Omega), \tag{1}$$

where ρ and Ω denote hyper-radius and hyperangle, while r_{ij} is the relative distance of particles i and j. The function $f(r)$ can be obtained from the solution of the two-body Schrödinger equation, since at short distances the role of other particles is rather unimportant. Though such correlations lead to a considerable improvement [9], one still needs in general a rather large number of HH terms in order to reach convergence. The convergence can be improved considerably if one introduces state-dependent and/or longer-range correlations [10]. However, the two-body long-range correlations are less under control, because correlations among more particles become more important. In addition, different from short-range correlations, they change considerably from ground to continuum states. In this respect long-

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range ground-state correlations would not be appropriate for the description of LIT states $\tilde{\Psi}$ in electromagnetic disintegrations of nuclei, since these states contain information about the continuum states.

A quite different approach is the HH effective interaction method. The two-body Hilbert space is divided in two subspaces with projection operators P and Q ($P+Q=1$, $P\cdot Q=0$, $\dim P=N_p$). In an HH calculation P and Q spaces are realized via the hyperangular quantum number K (P space: all HH states with $K\leqslant K_{max}$). Applying the Lee-Suzuki similarity transformation [11], one decouples P and Q space interactions in such a way that one obtains an effective interaction in the P space,

$$V_P=P\left[\sum_{i<j} V_{i,j}\right]_{eff} P. \tag{2}$$

The two-body Hamiltonian with this potential has exactly the same eigenvalues as the lower N_p eigenvalues of the bare interaction acting on the full $P+Q$ space. Due to the effective two-body interaction, one yields an enormous improvement of the convergence for the ground-state energies of nuclei with $A=3$–6 [8]. The introduction of such two-body effective interactions in few-body calculations was first made for the harmonic oscillator basis [12]. In comparison the HH basis offers further advantages, e.g., the presence of collective HH coordinates allows to construct a medium-affected two-body effective interaction that leads to a better convergence.

Since one can interpret the effective interaction as a kind of momentum expansion one cannot expect that high-momentum components are included in a correct way in the wave function if one works with a small number of HH functions. On the other hand, one should find a much improved convergence for observables that contain little information on high-momentum components. Thus it is no surprise that also for the nuclear radii an extremely good convergence was observed in Ref. [8]. Similar good convergence results are expected to hold for observables that are governed by not too high momentum components.

Both methods, the CHH and the EIHH, will be used in the following study of the $\gamma+{}^4\text{He}\rightarrow X$ reaction.

We calculate the nuclear ^{4}He total photoabsorption cross section in the dipole approximation

$$\sigma_{tot}(E_\gamma)=4\pi^2(e^2/\hbar c)E_\gamma R(E_\gamma) \tag{3}$$

with

$$R(E_\gamma)=\int df|\langle f|D_z|\Psi_0\rangle|^2\delta(E_f-E_0-E_\gamma), \tag{4}$$

where D_z is the third component of the nuclear dipole operator $\vec{D}$, while E_0 and E_f denote the eigenvalues of the nuclear Hamiltonian H for ground and final states, $|\Psi_0\rangle$ and $|f\rangle$, respectively, and E_γ is the photon energy. The dipole approximation is well established in low-energy photonuclear reactions. Deviations from the exact result are expected to be very small for the total cross section, since also the important meson exchange current contribution is implicitly taken into account within the dipole approximation (Siegert's theorem).

With the LIT method the cross section is calculated indirectly. In a first step the LIT of the response function $R(E_\gamma)$, i.e.,

$$L(\sigma_R,\sigma_I)=\int dE_\gamma\frac{R(E_\gamma)}{(E_\gamma-\sigma_R)^2+\sigma_I^2}=\langle\tilde{\Psi}|\tilde{\Psi}\rangle, \tag{5}$$

is determined via the asymptotically vanishing LIT state $\tilde{\Psi}$, which fulfills the following bound-state-like differential equation:

$$(H-E_0-\sigma_R+i\sigma_I)|\tilde{\Psi}\rangle=D_z|\Psi_0\rangle. \tag{6}$$

The parameters σ_R and σ_I entering here are defined by Eq. (5). The second step of the method consists in the inversion of L in order to obtain $R(E_\gamma)$ (see, e.g., [13]).

Formally, one can rewrite Eq. (5) in the following form:

$$L(\sigma_R,\sigma_I)=\sum_\nu\frac{|\langle\Psi_0|D_z|\tilde{\Psi}_\nu\rangle|^2}{(\tilde{E}_\nu-E_0-\sigma_R)^2+\sigma_I^2}, \tag{7}$$

where $|\tilde{\Psi}_\nu\rangle$ ($\tilde{E}_\nu$) are eigenfunctions (eigenvalues) of H in a truncated space. They are obtained with the same boundary conditions as $|\Psi_0\rangle$. Generally speaking, these states are either bound states or pseudoresonance states. It is clear that the low-energy (small σ_R) behavior of L is dominated by the positions of the lowest eigenvalues $\tilde{E}_\nu$.

In the following we compare results obtained with CHH and EIHH methods for the ^{4}He photodisintegration. Different from the above-mentioned calculation of Ref. [1] correlations are also introduced for the ground-state wave function. Our CHH and EIHH calculations agree very well for ground-state energy [-30.69 MeV (CHH), -30.71 MeV (EIHH)] and rms matter radius [1.421 fm (CHH), 1.422 fm (EIHH)] being a bit different from those of Ref. [1] (-29.24 MeV, 1.43 fm). However, there is no significant change of the photoabsorption cross section due to the more precise bound state. In fact one finds a small reduction of the response function $R(E_\gamma)$, but due to the higher binding energy the decrease is compensated by the increased value for E_γ [see Eq. (3)].

In Fig. 1 we show the convergence patterns of the transform L using the MT-I/III potential [14] as NN interaction. For the CHH case, depicted in Fig. 1(a), one observes a very rapid convergence for K_{max} values from 1 to 7 and $K_{max}=7$ was adopted in Ref. [1] for this reason. However, our present results show that the convergence pattern changes considerably slowing down sharply for higher K_{max} so that $K_{max}=11$ still does not lead to a completely convergent result. The EIHH results of Fig. 1(b) show a much nicer convergence behavior, particularly at low σ. This can be understood in view of the fact that the effective interaction method is, as mentioned, a kind of momentum expansion and thus brings an enormous acceleration to the convergence of the

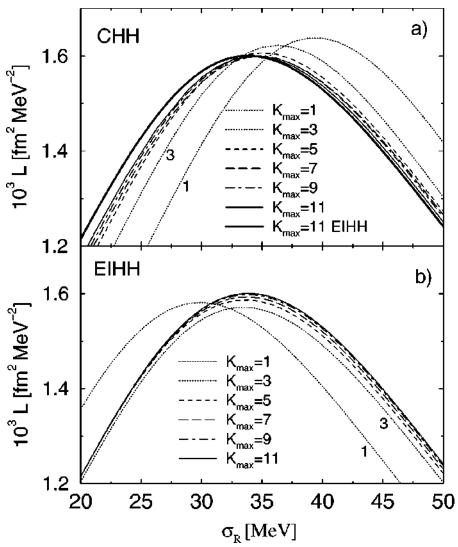

FIG. 1. Convergence pattern of $L(\sigma_R,\sigma_I)$ with CHH (a) and EIHH (b) methods with various maximal values K_{max} of the hyperangular quantum number K (MT-I/III potential, $\sigma_I = 20$ MeV); in (a) also the EIHH result with $K_{max}=11$ is shown.

lowest eigenvalues that dominate the low-energy cross section [see Eq. (7)]. Besides the CHH results in Fig. 1(a) we illustrate the EIHH transform for $K_{max}=11$. One sees that the results are very similar and studying the convergence patterns one might expect that the converged CHH result will come quite close to the EIHH result. In the comparison one should also not forget that small differences might remain even for the converged results, since both calculations are carried out in completely different ways. In particular the CHH calculation is numerically less accurate, since it includes a nine-dimensional Monte Carlo integration.

In Fig. 2 we illustrate the results for the total ^{4}He photoabsorption cross section obtained from the inversion of the LITs of Fig. 1 with $K_{max}=7,9,11$. Again one has a very nice convergence for the EIHH case, while the CHH results are not yet completely convergent. One sees that for increasing K_{max} the peak of the CHH cross section is shifted to lower energies in direction of the EIHH peak.

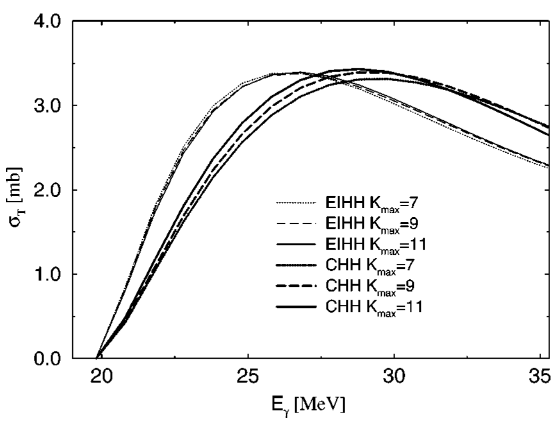

FIG. 2. Convergence pattern of the total ^{4}He photoabsorption cross section for CHH and EIHH methods (MT-I/III potential).

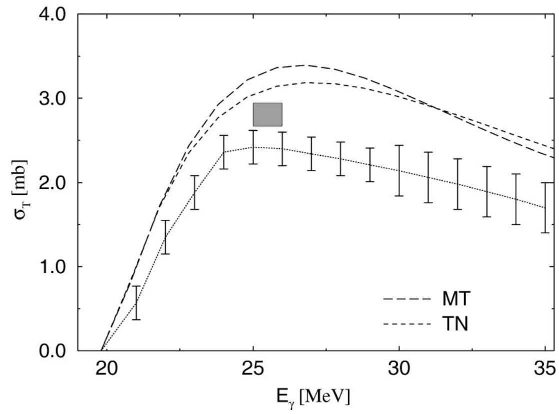

FIG. 3. Total ^{4}He photoabsorption cross section. Theoretical results (EIHH method): MT-I/III (long dashed) and TN potentials (short dashed); experimental results: sum of (γ,n) cross section from [6] and $(\gamma,p)^3$H cross section from [15] (dotted curve with error bars) and indirect determination via Compton scattering from [7] (shaded area).

In Fig. 3 we show the EIHH cross section results with MT-I/III and TN [13] potentials ($K_{max}=11$) in comparison to experimental data. Though the theoretical results are somewhat different from those of Ref. [1], the comparison with experiment is not improved. One still observes a considerably stronger giant dipole peak than that found in photoabsorption experiments. At lower energies the situation has changed a bit, since the obtained results exhibit a stronger deviation from experiment. On the other hand, it is evident that the experimental result of Ref. [7], where the total photoabsorption cross section is extracted from Compton scattering via dispersion relations, agrees much better with the theoretical results similarly to the already mentioned older ^{4}He photoabsorption data.

Our results for the total cross section are at variance with the calculation of Ref. [16] for the MT potential, where the two-body breakup cross sections have been calculated up to the three-body breakup threshold. The cross section for the n^3He channel of Ref. [16] agrees much better with the photoabsorption data of the 1980s (e.g., [6]), while results for the p^3H channel are not shown. If one makes the rather safe assumption that the p^3H cross section has about the same size as the n^3He cross section, one obtains an estimate for the total cross section at the three-body breakup threshold of about 2 mb. Our cross section is about 65% higher, which is a rather large difference. We should mention that we checked that our cross section satisfies the inverse energy weighted sum rule, which, because of the inverse energy weighting, is exhausted in the resonance region. Unfortunately, the authors of Ref. [16] were not able to check the sum rule because of the missing cross section beyond the three-body breakup threshold. It would be very interesting to see whether a cross section with such a low-peak value could fulfill both the inverse energy weighted and the total cross section sum rule for the MT-I/III potential.

In conclusion, the results of our present independent calculation definitely confirm the rather strong discrepancy with the experimental cross section discovered in Ref. [1]. It will be very interesting to get further clarification of the experimental cross section from the experiments at Lund. Since the semirealistic potential models lead to a rather good result for the ^{4}He rms radius, which is the dominant ingredient in the sum rule for the inverse energy weighted cross section, one may think that there is not much space to change the theoretical cross section. In case of the photodisintegration of the three-nucleon systems one finds only a 10% reduction of the peak cross section due to more realistic *NN* interactions and three-nucleon forces [5]. On the other hand it would be extremely interesting if the difference between realistic and semirealistic interactions for the photonuclear cross section is much higher in the four-nucleon case than in the three-nucleon case.

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