Analysis of the ¹²C+¹²C reaction using a new type of coupling potential

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A new approach has been used to explain the experimental data for the ${}^{12}C + {}^{12}C$ system over a wide energy range in the laboratory system from 32.0 MeV to 126.7 MeV. This new coupled-channel-based approach involves replacing the usual first-derivative coupling potential by a new, second-derivative coupling potential. This paper first shows and discusses the limitation of the standard coupled-channel theory in the case where one of the nuclei in the reaction is strongly deformed. Then, this new approach is shown to improve consistently the agreement with the experimental data: the elastic scattering and single-2⁺ and mutual-2⁺ excitation inelastic scattering data as well as their 90° elastic and inelastic excitation functions with little energydependent potentials. This new approach makes major improvement on all the previous coupled-channel calculations for this system.

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I. INTRODUCTION

Forty years ago, it was observed that the elastic cross section of the ${}^{12}C + {}^{12}C$ system varies rapidly with bombarding energy. This structure in the excitation functions, which could also be observed in other systems such as ${}^{12}C + {}^{16}O$ and ${}^{16}O + {}^{16}O$, has remained a subject attracting continuous interest from both theoretical and experimental points of views. Consequently, a large body of data over a wide energy range has been accumulated for the ${}^{12}C + {}^{12}C$ system from the systematic studies of this reaction [1–4].

However, there has been no global model that describes consistently the available elastic and inelastic scattering data over a wide energy range and this reaction presents a challenge to the many different theoretical models. Some of the problems can be summarized as follows: (1) no consistent description of the elastic scattering, single- 2^+ and mutual- 2^+ excitation inelastic scattering data as well as their 90° excitation function, (2) the out-of-phase problem between the theoretical predictions and the experimental data for these states, (3) no simultaneous description of the individual angular distributions and resonances, and (4) the magnitude of the mutual- 2^+ excitation inelastic scattering data is unaccounted for.

The elastic scattering data of this system has been studied systematically and progress has been made using the optical model (see the review by Brandan and Satchler [5]). However, the inelastic scattering has received little attention and there is no systematic study over a wide energy range and the above-mentioned problems could not be explained using the standard coupled-channel models (see, for example, [3,6–11]).

Stokstad *et al.* [3] were the first to study the elastic and single- 2^+ excitation inelastic scattering data using the distorted-wave Born approximation (DWBA) and the

coupled-channel methods from E_{lab} = 74.2 MeV to 126.7 MeV. They obtained reasonable agreement with the elastic scattering data. However, they could not reproduce the correct oscillatory structure for the single-2⁺ excitation inelastic scattering data and the magnitude of the data could not be accounted for correctly. They did not study the mutual-2⁺ excitation inelastic scattering data in their calculations. No theoretical calculations have predicted the magnitude of these data correctly over a wide energy range.

Wolf *et al.* [6] studied this system at three different energies. They used a double-folding potential and an angularmomentum-dependent imaginary potential in their coupledchannel calculations. They could not reproduce the experimental data measured at E_{lab} = 74.2, 93.8, and 126.7 MeV. In particular, the theoretical predictions for the mutual-2⁺ excitation inelastic scattering data were very small by factors of 3–10 with respect to the experimental data. The results of the single-2⁺ excitation inelastic scattering calculations were also very oscillatory in comparison with the experimental one. We encountered the same problems in our standard coupled-channel calculations. Varying the parameters and changing the shape of the real and imaginary potentials does not provide a solution, as discussed in Sec. III.

Fry and co-workers [7,8] also worked on this reaction to obtain the integrated cross section (also known as Cormier's resonances) for the single-2⁺ and mutual-2⁺ excitation channels using the coupled-channel method. They made use of a double-folding potential like the one of Stokstad *et al.* [3] and an angular-momentum-dependent imaginary potential. However, this method totally failed and no improvement of the densities in the double-folding potential would solve the magnitude problem of the mutual-2⁺ excitation inelastic scattering data. The same problems are observed in other authors' works such as Sakuragi *et al.* [9] and Ito [10].

Another interesting analysis was made by Ordoñez *et al.* [12]. They showed the necessity of using a real potential that has a minimum in the surface region. They reported a detailed phase-shift analysis of the ${}^{12}C + {}^{12}C$ elastic scattering data in the range of $11.0 \le E_{lab} \le 66.0$ MeV. This analysis revealed a striking sequence of gross structure resonances

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that appeared to form a rotational band from l=0 to at least 16 \hbar . These resonances were simulated by shape resonances in a real potential with a secondary minimum at large radii related to the shape-isomeric doorway states in ²⁴Mg.

The interesting feature of their work is the double-peaked nature of the real potential. It is clear that this potential does explain the resonance data, which other models have failed to reproduce within such a large energy range. As will be argued in Sec. IV, there is a resemblance between this potential and our total nuclear potential ($V_{real} + V_{coupling}$).

Ordoñez *et al.* could not justify this double-peaked potential, other than by asserting it was required to fit the experimental data. This paper and a forthcoming paper [13] shall argue that this deepening at the surface is due to the strongly deformed structure of the ¹²C and may indicate a superdeformed state of the compound nucleus ²⁴Mg. It is also clear that Ordoñez *et al.* took into account the coupling effects in their optical model calculations by introducing such a deepening at the surface without running coupled-channel calculations.

The literature clearly shows that the standard coupledchannel approach can fit neither any of the individual angular distributions nor the 90° elastic scattering excitation function *simultaneously*. For the resonance calculations, the situation is the same. That is, even if one fitted the Cormier's resonances observed for the single-2⁺ and mutual-2⁺ excitation channels, it would be, at the same time, impossible to fit the 90° elastic scattering excitation function. Clearly, the ¹²C +¹²C system has numerous problems to which no consistent global solution has been provided yet.

The overview of previous works indicates that the central potentials are actually quite reasonable since they have given the resonances at the correct energies and with sensible widths. Within the optical model calculations, they have also given very good agreements for the elastic scattering angular distributions or the 90° elastic scattering excitation functions independently. However, the calculations for the mutual- 2^+ excitation inelastic scattering data is in general underpredicted by a large factor and the oscillatory structure of the data can not be reproduced correctly. They have remained unsolved so far.

Therefore, our aim of analyzing the ${}^{12}C + {}^{12}C$ system is to search for a global solution for some of these problems with few energy-dependent potentials within the coupled-channel formalism from 32.0 MeV to 126.7 MeV in the laboratory system.

In the next section, we introduce the model potentials used to analyze the experimental data and the results of these analyses are shown in Sec. III, where we also make a discussion of the limitations of the standard coupled-channel method and highlight the problems. Section IV is devoted to analyses of the experimental data using our new coupling potential and the results are shown in Sec. V. Finally, Sec. VI gives a summary and a discussion of the new and standard coupled-channel calculation.

II. STANDARD COUPLED-CHANNEL CALCULATIONS

A recent critical review by Kondō *et al.* [14] found that a potential with a real depth of ~ 300 MeV was able to account

TABLE I. The parameters of the potentials required to fit the 90° elastic excitation function, displayed in Fig. 2.

V _N (MeV)	R_N (fm)	a_N (fm)	W (MeV)	R_W (fm)	a_W (fm)
345.0	3.62	1.60	Eq. (3)	5.50	0.51

for the 90° elastic excitation function at low energies ($E_{lab} \leq 75.0 \text{ MeV}$). The real potentials proposed in this paper are tested and their parameters have to be readjusted due to the coupling effects in the coupled-channel calculations.

In our coupled-channel calculations, the interaction between the ¹²C nuclei is described by a deformed optical potential. The real potential has the square of a Woods-Saxon shape,

$$V_N(r) = \frac{-V_N}{\left[1 + \exp(r - R_N)/a_N\right]^2},$$
 (1)

and the parameters, as shown in Table I, are fixed to reproduce the 90° elastic scattering excitation function. The Coulomb potential is assumed to be that of a uniformly charged nucleus with a radius of 5.5 fm.

The imaginary potential has the standard Woods-Saxon volume shape,

$$W(r) = -\frac{W}{1 + \exp[(r - R_W)/a_W]},$$
 (2)

and its depth increases quadratically with energy as

$$W = -2.69 + 0.145E_{lab} + 0.00091E_{lab}^2.$$
 (3)

The parameters of the radius and diffuseness are shown in Table I.

Since the ¹²C nucleus is strongly deformed, its collective excitation has been treated in the framework of the coupledchannels formalism. The ¹²C nucleus has a static quadrupole deformation, which is taken into account by deforming the real optical potential with a Taylor expansion about $R = R_0$ in the usual way [15]:

$$U(r-R) = U(r-R_0) - \delta R \frac{\partial}{\partial R_0} U(r-R_0) + \frac{1}{2!} (\delta R)^2 \frac{\partial^2}{\partial R_0^2} U(r-R_0) - \dots$$
(4)

For the projectile P and the target T,

$$\delta R_P = R_P \beta_2 Y_{20}(\theta, \phi),$$

$$\delta R_T = R_T \beta_2 Y_{20}(\theta, \phi).$$
(5)

 R_P and R_T are the radii of the projectile and target. The form factors [15] are



FIG. 1. A comparison of the results of the simultaneous mutual excitation (the dashed line) and the sequential one (the solid line) for the elastic, single-2⁺, and mutual-2⁺ excitations at E_{lab} =93.8 MeV.

$$F_P(r) = R_P \left[\frac{\partial}{\partial R_0} U(r, R_0) \right], \quad F_T(r) = R_T \left[\frac{\partial}{\partial R_0} U(r, R_0) \right], \tag{6}$$

$$H_{P,T}(r) = \frac{1}{(4\pi)^{1/2}} R_P R_T \frac{\partial^2 U(r, R_0)}{\partial R_0^2}.$$
 (7)

 $F_P(r)$ and $F_T(r)$ in Eq. (6) are the first-order form factors that account for the excitations of the projectile and target nuclei, while $H_{P,T}(r)$ in Eq. (7) is the second-order form factor that accounts for their mutual excitation.

In Eq. (5), $\beta_2 = -0.6$ is the deformation parameter of the ¹²C nucleus. This empirical value is derived from its known B(E2) value. The value of B(E2) is 42 e^2 fm⁴ [16]. (A more recent measurement gives an average value of 39 $\pm 4 e^2$ fm⁴ [17].)

In the standard coupled-channel calculations of inelastic scattering involving mutual excitation of the two nuclei, the codes CHUCK [18] and FRESCO [19] are used in such a way that the two nuclei are excited sequentially. However, we think it essential that *simultaneous* mutual excitation of the two nuclei be included in the calculations. To do so, we use the mutual-2⁺ excitation inelastic scattering data that are available. We modify the code CHUCK to take into account the *simultaneous* mutual excitation of the two nuclei does affect the calculations, in particular in the resonance region where the calculations are very sensitive to the small variations of the potential parameters. This is demonstrated in Fig. 1 at $E_{lab}=93.8$ MeV since we have available experimental data for all the states considered in this paper.



FIG. 2. The comparison of the experimental data and the results of the standard coupled-channel calculation for the 90° elastic scattering excitation function.

III. RESULTS

The result of the 90° elastic scattering excitation function obtained using the parameters of Table I is shown in Fig. 2. The theoretical predictions and the experimental data are in very good agreement, but as Kondō *et al.* found, this potential family does not fit the individual elastic scattering and inelastic scattering data as well as their excitation functions *simultaneously*.

We have attempted to obtain reasonable fits to the individual angular distributions by changing the parameters of the real potential, shown in Table I, but without success. Some authors [9,21] also found a potential family that reproduces the individual angular distributions, but does not fit the 90° elastic scattering excitation function.

To overcome this difficulty, we searched for a new potential family by readjusting the parameters of the real potential and letting the imaginary potential change freely. The parameters are shown in Table II. Except in the resonance regions, we obtained satisfactory agreement for the elastic scattering data as shown in Figs. 3 and 4 with dashed lines. However, the theoretical predictions of the magnitudes and the phase of the oscillations are not in good agreement with the experimental data for the single-2⁺ state, as shown in Fig. 5 with dashed lines. The out-of-phase and magnitude problems between the theoretical calculations and the experimental data are clearly seen at many energies. These results for the elastic and single- 2^+ excitation inelastic scattering are almost identical to those obtained by Stokstad et al. [3]. For the mutual- 2^+ excitation inelastic scattering data, as shown in Fig. 6 with dashed lines, there is no agreement and the theoretical predictions of the magnitude of mutual- 2^+ excitation inelastic scattering data are much smaller than the experimental one; they are underpredicted by a factor of 3-10. Nevertheless, our results for the mutual-2⁺ excitation inelastic scattering data are in conformity with the findings of Refs. [6-9,11], a problem mentioned by many authors in a recent international conference on clustering (ICC '99)

TABLE II. The numerical values of the potentials used in the *standard coupled-channel* calculations. V_N , r_N , and a_N stand for the depth, radius, and diffuseness of the real potential, respectively, and W, r_W , and a_W stand for the depth, radius, and diffuseness of the imaginary potential, respectively.

$\overline{E_{lab}}$	V_N (MeV)	r_N (fm)	a_N	W (MeV)	r_W	a_W
		(1111)	(1111)		(1111)	(1111)
32.0	290.0	0.80	1.30	3.0	1.20	0.51
40.0	290.0	0.79	1.28	3.6	1.20	0.51
45.0	290.0	0.80	1.15	3.8	1.20	0.51
49.0	290.0	0.79	1.23	4.2	1.20	0.51
50.0	290.0	0.80	1.21	4.5	1.20	0.51
55.0	290.0	0.80	1.15	5.0	1.20	0.51
57.75	290.0	0.81	1.35	6.3	1.20	0.51
60.0	290.0	0.80	1.30	6.6	1.20	0.51
65.0	290.0	0.79	1.43	7.0	1.20	0.51
70.7	290.0	0.81	1.20	8.5	1.20	0.51
78.8	290.0	0.81	1.30	9.5	1.20	0.51
93.8	290.0	0.82	1.35	12.0	1.20	0.51
98.2	290.0	0.81	1.30	12.5	1.20	0.51
102.1	290.0	0.81	1.30	14.0	1.20	0.51
105.0	290.0	0.81	1.30	14.4	1.20	0.51
112.0	290.0	0.80	1.30	13.0	1.20	0.51
117.1	290.0	0.80	1.35	14.0	1.20	0.51
121.6	290.0	0.80	1.35	14.1	1.20	0.51
126.7	290.0	0.81	1.30	14.2	1.20	0.51

[9,10,22,23]. In order to make a comparison with the new calculations, presented in the next section, some of the results for the single- 2^+ and mutual- 2^+ states are shown in Figs. 5 and 6.

We had anticipated that the inclusion of the simultaneous mutual excitation of two nuclei could solve the magnitude problem of the mutual- 2^+ excitation data. However, although this effect has improved the details of the fits to the experimental data, it failed to provide a solution. The magnitude of the mutual- 2^+ excited-state cross section is still one of the major outstanding problems of this reaction.

In the past, the magnitude problem for the single-2⁺ excitation inelastic scattering calculations was solved for different reactions by changing the empirical β value [24,25]. Thus, the same solution was expected to apply to the ¹²C + ¹²C system for the single-2⁺ and mutual-2⁺ excitations inelastic scattering calculations. For this purpose, we increased the β value to -1.2, which is twice the actual value and has no physical justification. However, although the agreement between theoretical predictions and the experimental data for the magnitudes of the single-2⁺ and mutual-2⁺ excitations inelastic scattering data is improved, the theoretical predictions for the elastic scattering data are very poor; the same holds for the 90° elastic scattering excitation functions.

Within the coupled-channel formalism, the reason for this failure may be understood if the effect of changing the real potential on the inelastic scattering cross section is considered. The method of obtaining the coupling potential that



FIG. 3. Ground-state results: The dashed lines show the predictions of the standard coupled-channel calculations (see Table II for the parameters) while the solid lines show the results of the new coupled-channel calculations, obtained using new coupling potential with the empirical β value ($\beta_2^C = \beta_2^N = -0.6$) (see Table III for the parameters).

describes the inelastic scattering has been based on perturbation theory. Since the coupling potential is connected to the real term by a Taylor expansion around the surface of the nucleus, changing the real potential has a substantial effect on the elastic scattering data, but not on the inelastic scattering one. Therefore, according to this standard procedure, the coupling potential has the same energy-dependence as the central term. Actually, Smithson *et al.* [26] analyzed the inelastic scattering data for the ¹⁶O+²⁰⁸Pb system and asserted that the standard deformation procedure is inadequate for the description of the inelastic scattering data. They also concluded that there is no reason for the coupling potential to have the same energy dependence as the central potential.

IV. NEW COUPLING POTENTIAL

If we consider two ¹²C nuclei approaching each other, the double-folding model will generate an *oblate* potential which is correct at large distance. When these two nuclei come close enough, they create the compound nucleus ²⁴Mg which is a *prolate* nucleus in its ground state, whereas the folding model yields an oblate (attractive) potential in this case. How well the double-folding model describes a prolate nucleus with an oblate potential is unclear and this may be the reason why the earlier calculations using a double-folding model in the coupled-channel method were unable to provide a consistent solution to the problems of this reaction.



FIG. 4. Ground-state results: The dashed lines show the predictions of the standard coupled-channel calculations (see Table II for the parameters) while the solid lines show the results of the new coupled-channel calculations, obtained using new coupling potential with the empirical β value ($\beta_2^C = \beta_2^N = -0.6$) (see Table III for the parameters) (continued from Fig. 3).

The limitations of the standard coupled-channel method, on the one hand, and the *oblate* character of the ¹²C and the *prolate* character of the compound nucleus ²⁴Mg, on the other hand, have motivated us to use a second-derivative coupling potential. In order to describe the above-mentioned configuration, the coupling potential must be *oblate* (attractive) when two ¹²C nuclei are at large distances and must be *prolate* (repulsive) when they are at short distances. The standard and the new coupling potential are shown in Fig. 7.

One possible interpretation of such a second-derivative coupling potential can be made if we express the total potential as a function of the radii for different orientations of the two colliding ¹²C nuclei. If $\theta_{P,T}$ are the angles between the symmetry axes and the axis joining the centers of the projectile and target, then the total potential, as an approximation, can be expressed in the following way:

$$V(r) = V_N + \beta_2 R \frac{dV_C}{dR} [Y_{20}(\theta_P, \phi_P) + Y_{20}(\theta_T, \phi_T)] + \beta_2^2 R^2 \frac{d^2 V_C}{dR^2} [Y_{20}(\theta_P, \phi_P) + Y_{20}(\theta_T, \phi_T)], \quad (8)$$

where V_N is the nuclear potential and V_C is the new secondderivative coupling potential. The final term is due to the mutual excitation.



FIG. 5. Single-2⁺ state results: The dashed lines show the predictions of the standard coupled-channel calculations (see Table II for the parameters) while the solid lines show the results of the new coupled-channel calculations, obtained using new coupling potential with the empirical β value ($\beta_2^C = \beta_2^N = -0.6$) (see Table III for the parameters).

The result for the ${}^{12}C + {}^{12}C$ system is shown in Fig. 8. A second local minimum is observed in the interaction potential for certain orientations. This feature, included only in an *ad hoc* way in the work of Ordoñez *et al.* [12], has not been taken into account in the standard coupled-channel calculations. To investigate this minimum, we looked at the total inverted potential, i.e., the dynamical polarization potential (DPP) plus the bare potential, obtained by the inversion of the *S* matrix [13]. Our analysis suggests that the new coupling potential points to the presence of the superdeformed configurations in the compound nucleus ${}^{24}Mg$, as has been speculated [27,28].

The real and imaginary potentials in these new calculations have the same shapes as in previous calculations [see Eqs. (1) and (2)] and the parameters of the potentials are displayed in Table III. We have analyzed the experimental in the same energy range.

V. RESULTS

The results of the analyses using the new coupling potential are displayed in Figs. 3 and 4 for the ground state, in Fig. 5 for the first excited state (single- 2^+), and in Fig. 6 for the mutual excited state (mutual- 2^+).

The agreement is very good for the elastic scattering and single- 2^+ and mutual- 2^+ excitation inelastic scattering data over the whole energy range studied. The theoretical predic-



FIG. 6. Mutual-2⁺ state results: The dashed lines show the predictions of the standard coupled-channel calculations (see Table II for the parameters) while the solid lines show the results of the new coupled-channel calculations, obtained using new coupling potential with the empirical β value ($\beta_2^C = \beta_2^N = -0.6$) (see Table III for the parameters).

tions of the magnitudes and the phase of the oscillations for the single- 2^+ and mutual- 2^+ excitations inelastic scattering data, which have been the major outstanding problems of this reaction, are in a very good agreement with the empirical values. This new coupling potential has made a substantial improvement at all the energies considered.

The 90° elastic scattering excitation function is also analyzed and the result is shown in Fig. 9. The agreement with the experimental data is excellent over the whole energy range.

Table III indicates that the parameters are almost constant (1%-3% changes) away from the resonance regions. However, at certain energies in the energy range $E_{lab} \sim 90$ -110.0 MeV, the parameters fluctuate. We interpreted the fluctuations at small energies in Table III as the effect of the resonances observed by Cormier *et al.* [29,30], Chappell *et al.* [31–33], and Fulton *et al* [34]. The changes of the potential parameters in the energy range $E_{lab} \sim 90-110.0$ MeV might be related to resonances, which have not yet been observed in the ¹²C+¹²C system. Within such an interpretative scheme, one may infer that these resonances might be associated with the single- and mutual-4⁺ excited states



FIG. 7. The comparison of the standard coupling potential (1) with $\beta = -0.6$, (2) with $\beta = -1.2$ and our new coupling potential for $E_{lab} = 32.0$ MeV. The parameters of the latter are shown in Table III.

of ¹²C, states which are strongly coupled to the ground state.

These predictions motivated us to run an experiment in this energy range. The initial analyses of the experimental data indicate that the variation of these parameters are not actually random since structures relating to the 4^+ state of the 12 C are seen in this energy range. The detailed analyses and the full results will be given in the forthcoming paper [35].

This new, second-order coupling potential, has also been applied successfully to the ${}^{16}O+{}^{28}Si$ and ${}^{12}C+{}^{24}Mg$ systems [20,36]. This model has explained the experimental data successfully.



FIG. 8. The orientation potentials of two nuclei at different angles including the hexadecupole deformation of 12 C.

TABLE III. The numerical values of the potentials used in the *new coupled-channel* calculations. *W* denotes the imaginary potential. V_N , r_N , and a_N stand for the depth, radius, and diffuseness of the real potential, respectively, and r_C and a_C stand for the radius and diffuseness of the coupling potential, respectively (V_C =210.0 MeV).

E _{lab}	V_N	r_N	a_N	W	r _C	a_C
(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
32.0	290.0	0.804	1.19	2.21	0.69	0.70
40.0	288.0	0.806	1.28	2.40	0.69	0.70
45.0	290.0	0.809	1.28	2.97	0.69	0.70
49.0	290.0	0.810	1.28	3.07	0.69	0.70
50.0	290.0	0.813	1.24	3.07	0.69	0.70
55.0	290.0	0.813	1.26	3.17	0.69	0.70
57.75	290.0	0.813	1.26	3.17	0.69	0.70
60.0	290.0	0.813	1.28	3.37	0.69	0.70
65.0	290.0	0.811	1.28	3.57	0.69	0.70
70.7	289.0	0.799	1.29	3.71	0.69	0.70
78.8	287.0	0.785	1.28	5.50	0.68	0.70
93.8	292.0	0.790	1.34	11.9	0.67	0.67
98.2	289.0	0.785	1.27	11.5	0.66	0.65
102.1	289.0	0.810	1.33	11.5	0.65	0.63
105.0	289.0	0.810	1.37	11.5	0.66	0.66
112.0	287.0	0.800	1.28	13.8	0.68	0.67
117.1	290.0	0.810	1.32	14.7	0.69	0.68
121.6	290.0	0.810	1.33	15.3	0.68	0.67
126.7	288.0	0.795	1.30	17.3	0.66	0.67

VI. SUMMARY AND CONCLUSION

We considered the elastic and inelastic scattering of the ${}^{12}C + {}^{12}C$ system from 32.0 MeV to 126.7 MeV in the laboratory system. Although this reaction has been one of the most extensively studied reaction over the last 40 years, there has been no global model that explains consistently the measured experimental data over a wide energy range. In the introduction, we presented the problems that this reaction manifests. We attempted to find a consistent solution to these problems. However, within the standard coupled-channel method, we failed, as others did, to describe certain aspects of the data, in particular, the single-2⁺ and mutual-2⁺ excitation inelastic scattering data, although the optical model and coupled-channel models explain perfectly some aspects of the elastic scattering data.

As discussed in Sec. III, the standard coupled-channel method entails that the coupling potential has the same en-



FIG. 9. 90° elastic scattering excitation function, obtained using new coupling potential with the empirical β value ($\beta_2^C = \beta_2^N = -0.6$).

ergy dependence as the central term. However, our analysis reveals that the coupling potential has a vital importance in explaining the experimental data for the reactions that involve at least one strongly deformed nucleus and that there is no reason for the coupling potential to have the same energy dependence as the central potential. This may explain the failure of the standard coupled-channel calculations.

The comparison of the results obtained using the standard and new coupled-channel calculations indicates that a global solution to the problems of the scattering observables of this reaction over a wide energy range (32.0–126.7 MeV) with little energy dependence on the potentials has been provided by this new coupling potential. The significance of the new approach should be underlined because it does not only fit the present experimental data, but it also leads to other novel and testable predictions. To our knowledge, this has not been yet achieved over such a wide energy range. Studies using this new coupling potential may also lead to new insights into the formalism and a new interpretation of such reactions.

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