Polarization phenomena in the reaction $p+\alpha\rightarrow p+\pi^0+\alpha$

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We present a general analysis of polarization phenomena for three-body processes (in noncoplanar kinematics), in terms of a *P*- and *T*-odd acoplanarity parameter. The spin structure of the matrix element and the polarization phenomena contain new contributions, with respect to binary processes, which can be conveniently expressed as functions of this parameter. We apply this formalism to the reaction $p+\alpha\rightarrow p+\pi^0+\alpha$, in view of characterizing the different mechanisms involved and studying the excitation of the Roper resonance. We find that the polarization transfer coefficient D_{nn}, where $\vec{n}$ is normal to the proton scattering plane, is especially sensitive to the spin of the exchanged particle.

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I. INTRODUCTION

Experimental studies at the Saturne National Laboratory [1] have shown that the reaction $p+\alpha\rightarrow p+\pi^0+\alpha$ may be very selective for the excitation of the Roper resonance. A general description of this reaction must deal with three particles in the final state and, therefore, with noncoplanar kinematics. In order to disentangle the reaction mechanisms, polarization phenomena are very useful, and in this case, relatively simple, as only protons have nonzero spin.

This process has been considered in Ref. [2], where the polarization transfer coefficient D_{nn}, from the initial to the final proton, has been calculated in a particular kinematics, related to the emitted α particle. The authors showed that this polarization observable is especially sensitive to the relative role of two main mechanisms for $p+\alpha\rightarrow p+\pi^0+\alpha$ (Fig. 1): the excitation of Roper resonance (through σ exchange [2]) and the Deck mechanism [3] (Δ excitation of the α particle, through π and ρ exchanges).

Here we give a general formalism for the study of polarization phenomena in three body reactions, which apply to any kinematical conditions. Our aim is to find general properties, independently from the reaction mechanism. We will then apply our calculation to $p+\alpha\rightarrow p+\pi^0+\alpha$ and compare polarization phenomena for different meson exchanges in Roper excitation, in particular for ω exchange, which is, in our opinion, the most probable mechanism in particular at high energies [4].

II. NONCOPLANAR KINEMATICS

The main feature of a process with three particles in final state $1+2\rightarrow 3+4+5$ is the noncoplanarity of the kinematics. This can be expressed, for the case of $p+\alpha\rightarrow p+\pi^0+\alpha$, introducing the following combination of three-momenta:

$$a=\frac{\vec{q}\cdot\vec{p}_1\times\vec{p}_2}{E_1E_2E_\pi},$$

where $\vec{p}_1$ and $\vec{p}_2$ are the three-momenta of the initial and final proton, $\vec{q}$ is the three-momentum of the produced pion, and E_1, E_2, E_π are the corresponding energies. This expression enters in the definition of all five independent kinematical variables which are necessary for the complete description of a process $1+2\rightarrow 3+4+5$. These variables can be chosen in the following way (using notations of four-momenta as illustrated in Fig. 1): $s=(k_1+p_1)^2$ is the square of the total energy W of the colliding particles, $s=W^2$, in the center-of-mass system; $w_1^2=(q+p_2)^2$ is the square of the effective mass of the produced $\pi+p$ system. $w_2^2=(q+k_2)^2$ is the square of the effective mass of the produced $\alpha+\pi$ system. $t=(p_1-p_2)^2$ is the four-momentum transfer square (from the initial to the final proton); $\mathcal{P}=\epsilon_{\mu\nu\alpha\beta}p_{1\mu}p_{2\nu}k_{1\alpha}k_{2\beta}$ is the relativistically invariant generalization of the acoplanarity a, which was previously defined.

The variable a is connected with the azymuthal angle ϕ, between the two reaction planes which characterize the process $p+\alpha\rightarrow p+\pi^0+\alpha$: one plane is the scattering plane of

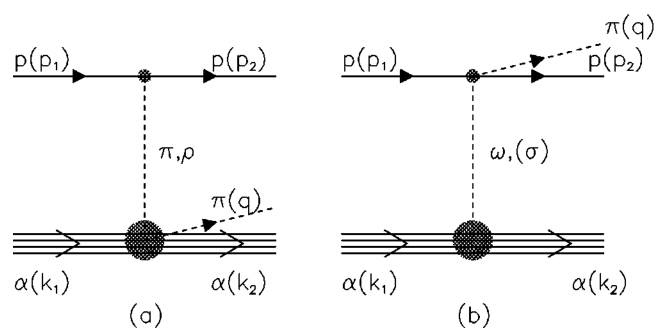

FIG. 1. Feynman diagrams corresponding to the mechanisms discussed in the text, for $p+\alpha\rightarrow p+\pi^0+\alpha$: Deck mechanism (a), nucleon excitation (b).

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the proton (i.e., the plane defined by the three-momenta $\vec{p}_1$ and $\vec{p}_2$) and the other is the plane defined by the pion three-momentum $\vec{q}$ and the transferred momentum $\vec{p}=\vec{p}_1-\vec{p}_2$. The angle ϕ can be identified with the Treiman-Yang angle [5], which is currently used in the description of the properties of one-meson exchange in high-energy collisions. This angle is also convenient for the description of the possible mechanisms for the Roper excitation, in $p+\alpha\to p+\pi^0+\alpha$. It is important to mention that the parameter a is not only a pseudoscalar quantity, but it is a T-odd variable, as it is the product of 3 three-momenta.

The noncoplanarity of the general kinematics for $1+2\to 3+4+5$ results in specific properties of the helicity amplitudes and consequently in the polarization phenomena, different from the binary collisions, as $1+2\to 3+4$. To illustrate this, let us note, as an example, that in $p+\alpha\to p+\pi^0+\alpha$, the vector of polarization of the final proton can have, in the general case, all nonzero components. On the opposite, for any $1+2\to 3+4$ process, the proton polarization (for a P-invariant interaction) has only one nonzero component along the normal to the scattering plane, due to the presence of only one reaction plane.

Therefore the polarization of the final proton can be parametrized in the following general form:

$$\vec{P}=\vec{n}P_n+a(\vec{m}P_m+\vec{k}P_k), \tag{1}$$

where the unit vectors $\vec{m}$, $\vec{n}$, and $\vec{k}$ are defined as $\vec{n}=\vec{p}_1\times\vec{p}_2/|\vec{p}_1\times\vec{p}_2|$, $\vec{k}=\vec{p}_1/|\vec{p}_1|$, $\vec{m}=\vec{n}\times\vec{k}$, and P_n, P_m, and P_k are the three independent and nonzero components of the final proton polarization vector. The components P_m and P_k appear multiplied by the parameter a, therefore noncontributing in the case of coplanar kinematics.

III. GENERAL FORMALISM

The presence of the noncoplanarity $(a\neq 0)$ has to be taken into account in establishing the spin structure of the matrix element for $p+\alpha\to p+\pi^0+\alpha$. If the P invariance of the strong interaction holds, the matrix element is described by the following general parametrization (in the c.m.s. of the considered reaction):

$$\mathcal{M}=\chi_2^\dagger[\vec{\sigma}\cdot\vec{m}f_1+\vec{\sigma}\cdot\vec{k}f_2+a(i\tilde{f}_1+\vec{\sigma}\cdot\vec{n}\tilde{f}_2)]\chi_1, \tag{2}$$

where χ_1 and χ_2 are the two-component spinors of the protons in the initial and final states; f_1, f_2, $\tilde{f}_1$, and $\tilde{f}_2$ are the scalar independent amplitudes for $p+\alpha\to p+\pi^0+\alpha$, which are functions of the five kinematical variables, defined above. The P invariance of the strong interaction requires that all these amplitudes are even functions of the variable a. Such construction of the matrix element results in specific properties of the helicity amplitudes $F_{\lambda\lambda'}$ (where λ and λ' are the helicities of the initial and the final proton) for noncoplanar collisions: in case of binary processes $1+2\to 3+4$ [6], the P invariance implies that $|F_{-\lambda-\lambda'}|=|F_{\lambda\lambda'}|$, while for the process $p+\alpha\to p+\pi^0+\alpha$, amplitudes with opposite sign of helicities are different. Therefore the number n of independent helicity amplitudes for $1+2\to 3+4+5$ is given, in general, by $n=(2s_1+1)(2s_2+1)(2s_3+1)(2s_4+1)(2s_5+1)$, where s_i is the spin of the ith particle. This number is, as a rule, twice smaller for coplanar kinematics.

Using the parametrization (2) we can calculate any polarization observable in terms of the scalar amplitudes and of the parameter a. For example, the dependence of the differential cross section on the polarization $\vec{P}$ of the proton beam, in the general case of noncoplanar kinematics, is characterized by three independent analyzing powers, i.e.,

$$\frac{d\sigma}{d\omega}(\vec{p}\alpha\to p\pi^0\alpha)=\left(\frac{d\sigma}{d\omega}\right)_0 \times[1+\vec{n}\cdot\vec{P}A_n+a(\vec{m}\cdot\vec{P}A_m+\vec{k}\cdot\vec{P}A_k)], \tag{3}$$

where $(d\sigma/d\omega)_0$ is the differential cross section (with unpolarized proton beam) and $d\omega$ is the element of the phase space for the three-particle final state.

For the analyzing powers A_n, A_m, and A_k the following expressions can be found (after summing over the polarization states of scattered protons):

$$A_n\left(\frac{d\sigma}{d\omega}\right)_0=-2\,\mathrm{Im}(f_1f_2^*+a^2\tilde{f}_1\tilde{f}_2^*),$$

$$A_m\left(\frac{d\sigma}{d\omega}\right)_0=2\,\mathrm{Im}(f_1\tilde{f}_1^*-f_2\tilde{f}_2^*), \tag{4}$$

$$A_k\left(\frac{d\sigma}{d\omega}\right)_0=2\,\mathrm{Im}(f_1\tilde{f}_2^*+f_2\tilde{f}_1^*),$$

$$\left(\frac{d\sigma}{d\omega}\right)_0=|f_1|^2+|f_2|^2+a^2(|\tilde{f}_1|^2+|\tilde{f}_2|^2).$$

The dependence of the components of the final proton polarization $\vec{P}_f$ on the initial polarization $\vec{P}$ can be parametrized in the following way:

$$\vec{m}\cdot\vec{P}_f=D_{mm}\vec{m}\cdot\vec{P}+D_{mk}\vec{k}\cdot\vec{P}+aD_{mn}\vec{n}\cdot\vec{P},$$

$$\vec{n}\cdot\vec{P}_f=aD_{nm}\vec{m}\cdot\vec{P}+aD_{nk}\vec{k}\cdot\vec{P}+D_{nn}\vec{n}\cdot\vec{P}, \tag{5}$$

$$\vec{k}\cdot\vec{P}_f=D_{km}\vec{m}\cdot\vec{P}+D_{kk}\vec{k}\cdot\vec{P}+aD_{kn}\vec{n}\cdot\vec{P},$$

where D_{ij}, $i,j=m,n,k$, are the coefficients of polarization transfer from the initial to the final proton. All contributions in Eq. (5), which are proportional to the noncoplanarity parameter a cannot be present in the corresponding formulas for the binary process $1+2\to 3+4$. Equations (5) can be applied to binary processes, after setting $a=0$, but in noncoplanar kinematics the number of independent coefficients of polarization transfer is larger.

Using the suggested parametrization of the spin structure of the noncoplanar matrix element for $p+\alpha\to p+\pi^0+\alpha$, Eq. (2), the following expressions for the coefficients D_{ij} can be given, in terms of the scalar amplitudes f_i, $\tilde{f}_i$, and the parameter a:

$$D_{nn}\left(\frac{d\sigma}{d\omega}\right)_0=-|f_1|^2-|f_2|^2+a^2(|\tilde{f}_1|^2+|\tilde{f}_2|^2),$$

$$D_{mm}\left(\frac{d\sigma}{d\omega}\right)_0=|f_1|^2-|f_2|^2+a^2(|\tilde{f}_1|^2-|\tilde{f}_2|^2),$$

$$D_{kk}\left(\frac{d\sigma}{d\omega}\right)_0=-|f_1|^2+|f_2|^2+a^2(|\tilde{f}_1|^2-|\tilde{f}_2|^2),$$

$$D_{mk}\left(\frac{d\sigma}{d\omega}\right)_0=2\ \mathrm{Re}(f_1f_2^*+a^2f_1f_2),$$

$$D_{km}\left(\frac{d\sigma}{d\omega}\right)_0=2\ \mathrm{Re}(f_1f_2^*-a^2\tilde{f}_1\tilde{f}_2^*),$$

$$D_{mn}\left(\frac{d\sigma}{d\omega}\right)_0=2\ \mathrm{Re}(f_1\tilde{f}_2^*-f_2\tilde{f}_1^*),$$

$$D_{nm}\left(\frac{d\sigma}{d\omega}\right)_0=2\ \mathrm{Re}(f_1\tilde{f}_2^*+f_2\tilde{f}_1^*),$$

$$D_{nk}\left(\frac{d\sigma}{d\omega}\right)_0=2\ \mathrm{Re}(-f_1\tilde{f}_1^*+f_2\tilde{f}_2^*),$$

$$D_{kn}\left(\frac{d\sigma}{d\omega}\right)_0=2\ \mathrm{Re}(f_1\tilde{f}_1^*+f_2\tilde{f}_2^*).$$

From the expression for the coefficient D_{nn}, one can see that $D_{nn}=-1$, for coplanar kinematics ($a=0$). This is a known result [7], which follows from the P invariance of strong interaction and this result is valid for any amplitudes f_1 and f_2 and for any model of the considered process and for any kinematical conditions, provided $a=0$. In noncoplanar kinematics, in general, the presence of noncoplanar amplitudes gives $D_{nn}\geqslant-1$, therefore the quantity $1+D_{nn}$ characterizes the relative role of noncoplanar amplitudes $\tilde{f}_1$ and $\tilde{f}_2$. We will show later that different mechanisms for $p+\alpha\rightarrow p+\pi^0+\alpha$ are characterized by a different relative role of noncoplanar amplitudes and will give different values of D_{nn}.

Note that our choice of coordinate frame differs from Ref. [2], because it is more convenient for a generalized treatment of polarization phenomena in the noncoplanar regime. The situation is essentially simplified in the case of collinear kinematics for the considered process, when all three particles move along the initial three-momentum of the proton beam. In this case the matrix element reduces to

$$\mathcal{M}_{\mathrm{col}}=\chi_2^+\vec{\sigma}\cdot\vec{k}\chi_1 f(E_1,E_2,E_\pi),$$

where f is the single collinear amplitude, which is, in the general case, a function of three energies: the energies of the initial and final proton and the energy of the produced pion.

The presence of a single and specific spin structure of the matrix element in collinear kinematics means that all polarization phenomena can be predicted exactly, in a model independent form

$$D_{mm}=D_{nn}=-D_{kk}=-1,$$

and all other polarization observables have to be identically zero, due to the azimuthal symmetry of the collinear kinematics. This result, which is derived in a straightforward way in our formalism, definitely shows that measurements of polarization phenomena in collinear kinematics are not interesting, as they are insensitive to the reaction mechanism. In other words, all possible reaction mechanisms give the same spin structure, so their relative role cannot be disentangled by measuring polarization observables. Different mechanisms give different contributions to the amplitude $f(E_1,E_2,E_\pi)$, but only the cross section is sensitive to this amplitude.

IV. COMPARISON OF DIFFERENT MECHANISMS

Let us consider now the properties of the scalar amplitudes and the polarization phenomena for the process $p+\alpha\rightarrow p+\pi^0+\alpha$, for both mechanisms illustrated in Fig. 1. Following Ref. [2], the Deck mechanism results from π and ρ exchanges, but the Roper excitation is induced by σ exchange. From the general properties of $\pi\alpha$ scattering (for the Deck mechanism) and of the process $\sigma+N\rightarrow N+\pi$ (for the Roper excitation), one can easily show that both noncoplanar amplitudes $\tilde{f}_1$ and $\tilde{f}_2$ are zero for σ as well as π, independently on their parametrizations. Therefore, the full matrix element (in such approximation) for $p+\alpha\rightarrow p+\pi^0+\alpha$, has the same structure as for coplanar kinematics, with two independent amplitudes f_1 and f_2, only. The numerical values of these amplitudes and their dependence on the kinematical variables have to be different for σ and π exchanges. But for any amplitudes f_1 and f_2 the polarization phenomena have following general properties.

$D_{nn}=1$, in the whole region of kinematical variables (for coplanar and noncoplanar kinematics). Let us stress once more, that because our choice of coordinate frame is different than Ref. [2], the coefficient D_{nn} differs from the D_{nn} defined in Ref. [2].

The polarization of the final proton has only one nonzero component, in the $\vec{n}$ direction, i.e., along the normal to the proton scattering plane.

The sign and absolute value of this component depend on the relative role of the considered mechanisms, and this dependence is very sensitive to the details of the corresponding amplitudes.

This ''coplanarlike'' behavior of σ and π exchanges in $p+\alpha\rightarrow p+\pi^0+\alpha$ is related to the fact that these mediators are spinless particles. Such mechanisms cannot connect different reaction planes. This conclusion does not depend on details, approximations, values of the constants or shape of form factors which are typically taken in the numerical applications, because it is based only on the value of the spin of the exchanged particles.

Earlier we suggested that the ω exchange is the most probable mechanism for the Roper excitation, in this energy range [4]. The most important difference with respect to σ exchange is due to the spin and has evident implications for the polarization phenomena: a vector particle exchange induces all four amplitudes different from zero, in the general case.

Let us consider, as an illiustration, the spin structure of ω exchange, taking into account, for simplicity, in the ωNN^* vertex only the transverse, i.e., $M1$ form factor. In this case the matrix element for $p+\alpha\to p+\pi^0+\alpha$ can be written in the form

$$\mathcal{M}_\omega=\chi_2^\dagger\vec{\sigma}\cdot\vec{q}\,\vec{\sigma}\cdot\vec{k}_1 \times\vec{k}_2\chi_1 F_\alpha(t)F_{NN*}(t)\frac{1}{t-m_\omega^2}\frac{f_{N*}}{w_1-m^*+i(\Gamma/2)}, \tag{6}$$

where $\vec{k}_1$ and $\vec{k}_2$ are the three-momenta of the initial and final α particles in c.m.s. of the considered reaction $F_\alpha(t)$ and $F_{NN*}(t)$ are the form factors of the $\omega\alpha\alpha$ and ωNN^* vertexes, f_{N*} is the constant for the decay $N^*\to N+\pi$, m^* and Γ are the mass and the width of the Roper resonance N^*. After evident transformations of the spin structure in Eq. (6) one can see that for ω exchange, all four scalar amplitudes, coplanar and noncoplanar, are present. This is due to the exchange by vector particles, which connects strongly the different planes of the considered reaction and it does not depend on approximations in writing the matrix element (6) or on the choices of the form factors, but only on the spin 1 nature of the exchanged particles.

This shows the important role characterized by acoplanarity, which induces, in general, large deviations from the relation $D_{nn}+1=0$. The spin transfer coefficient D_{nn} contains also a strong dependence on the kinematical variables of the considered process.

Another result which holds for the ω exchange is the presence of all three nonzero analyzing powers for $\vec{p}+\alpha\to p+\alpha+\pi^0$, induced by the different components of the target polarization $\vec{P}$. More exactly, this is correct for the interference of the ω-exchange mechanism of the Roper excitation and the π exchange for the Deck mechanism.

Figure 2 shows the sensitivity of D_{nn} to the ratio $r=(|\tilde{f}_1|^2+|\tilde{f}_2|^2)/(|f_1|^2+|f_2|^2)$, which characterizes the relative role of noncoplanar and coplanar scalar amplitudes, for the different values of the parameter a^2,

$$D_{nn}=\frac{-1+a^2r}{1+a^2r}.$$

The point where the coefficient D_{nn} vanishes, $a^2r=1$, depends on the ratio r. The polarization phenomena for the ω exchange differ essentially from σ exchange. In this sense,

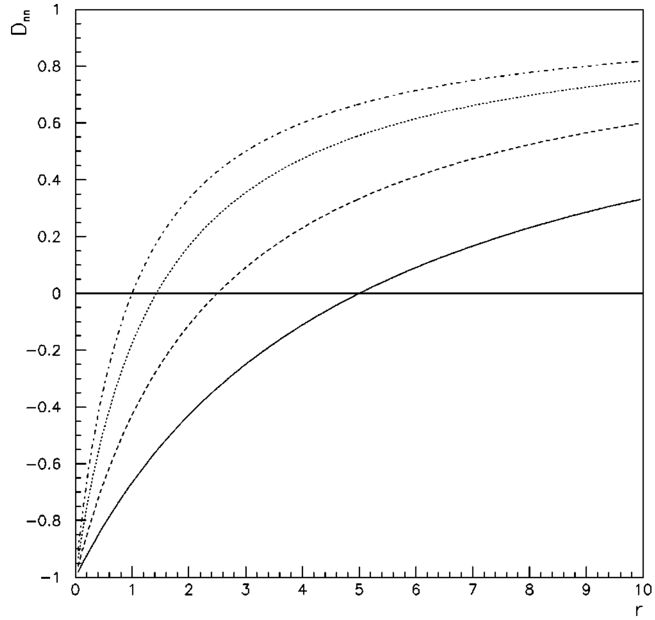

FIG. 2. Dependence of D_{nn} on the ratio r (sum of the square of noncoplanar amplitudes over the sum of the square of coplanar amplitudes) for different values of the noncoplanarity parameter a: $a=0.2$ (solid line), $a=0.4$ (dashed line), $a=0.7$ (dotted line), $a=1$ (dashed-dotted line).

the σ exchange can never be generalized to an effective $\sigma+\omega$ exchange [8], for any choices of constants, form factors, or amplitudes.

V. CONCLUSIONS

We established the spin structure of the matrix element for the process $p+\alpha\to p+\pi^0+\alpha$, for the general case of noncoplanar kinematics and analyzed the polarization phenomena. The acoplanarity parameter a, $a=\vec{q}\cdot\vec{p}_1\times\vec{p}_2/E_1E_2E_\pi$, plays an essential role in the analysis of all the observables. The T- and P-odd nature of the parameter a is the source of specific features of the spin structure and the polarization effects, such as, for example, the presence of three independent analyzing powers in $\vec{p}+\alpha\to p+\alpha+\pi^0$, which are induced by all components of the target polarization (even for the P-invariant strong interaction).

The suggested general parametrization of the noncoplanar matrix element allowed us to derive the most general properties of the different possible mechanisms, which are believed to play a role in $p+\alpha\to p+\pi^0+\alpha$. We showed that the matrix element for σ exchange, often advocated to describe the Roper excitation and for the π exchange (Deck mechanism), has an evident ''coplanarlike'' form, with vanishing noncoplanar amplitudes $\tilde{f}_1$ and $\tilde{f}_2$. But the ω exchange (which seems the most probable physical candidate for the Roper excitation) induces a very rich spin structure of the corresponding contribution to the matrix element (with all four nonzero amplitudes), and specific polarization phenomena, which differ essentially from the case of σ exchange. For Roper excitation, only ω exchange can induce noncoplanar polarization phenomena. However the ρ exchange for the Deck mechanism is also characterized by noncoplanar contribution to the matrix element, but different from ω exchange. For the description of ρ exchange another set of unit vectors $\vec{m}$, $\vec{n}$, and $\vec{k}$ is more preferable, where $\vec{n}$ is normal to the α scattering plane. A single ''magneticlike'' spin structure for process $\rho+\alpha \rightarrow \pi+\alpha$ is present, inducing new noncoplanar amplitudes. The flexibility in the choice of the coordinate basis for different mechanisms is also an advantage related to the properties of the suggested general noncoplanar analysis.

Future experimental data on polarization observables for $p+\alpha \rightarrow p+\pi^0+\alpha$, which require a detection system in noncoplanar kinematics, will constitute a crucial test in order to disentangle the mechanisms involved. Of course, initial and final state interactions would strongly affect the quantitative predictions of specific mesonic models, especially concerning cross section predictions. Polarization observables are generally less affected, in particular, the spin transfer coefficients. Our previous experience shows that one can reproduce the tensor analyzing power for inclusive $\vec{d}+p$ scattering (in a parameter free model), without need of distortion effects [4]. A well-known method in order to extract the nuclear spin response, is the measurement of the spin-flip probability in $(\vec{d},\vec{d}\,')$ and $(\vec{p},\vec{p}\,')$ scattering on various nuclear targets. A systematical study showed that this observable reflects the nuclear structure and not the reaction mechanism [9].

The present work is, to the best of our knowledge, the first attempt of a general analysis of polarization effects in noncoplanar kinematics, which can be done in a model independent way. Simple (traditional) mesonic exchanges have been used above only for illustration of our general considerations. The suggested formalism applies to any $1+2 \rightarrow 3+4+5$ processes, in the general (i.e., noncoplanar) kinematical conditions.

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