PHYSICAL REVIEW C, VOLUME 63, 051001(R)

Empirical analysis for differential cross section measurements of $\vec{p}\vec{p}\rightarrow pp\pi^0$

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(Received 23 October 2000; published 9 April 2001)

We show that it is possible to empirically partition the differential cross section for $pp\rightarrow pp\pi^0$ into initial singlet and triplet differential cross sections in a model independent way which can be implemented using the existing technological capabilities at the PINTEX facility. We also show that the differential cross section for $\vec{p}\vec{p}\rightarrow pp\pi^0$ can be expressed as a weighted sum of a set of initial singlet and triplet differential cross sections together with a contribution arising from the initial singlet-triplet correlations for any arbitrary beam polarization $\mathbf{P}(b)$ and target polarization $\mathbf{P}(t)$ either collinear or noncollinear.

DOI: 10.1103/PhysRevC.63.051001 PACS number(s): 13.75.Cs, 25.10.+s, 21.30.Cb, 24.70.+s

Considerable interest has been evinced recently in the experimental study [1–4] of $\vec{p}\vec{p}\rightarrow pp\pi^0$ near threshold leading to model independent partitioning of the total cross section σ, following [5], into cross sections ${}^{2s+1}\sigma_m$ in terms of initial channel spin s and its projection m along the beam direction. The reaction involves only a few lowest-order partial waves in the final state at threshold, which in turn limit the initial partial waves through parity and total angular momentum conservation. The transition is mainly from the initial ${}^3P_0$ state to the final Ss state, in the notation of [1–4], for energies below 300 MeV. In spite of this simplicity, the early total cross section measurements [6–9] differed by a factor of 5 from calculations based on [10]. This catalyzed a variety of theoretical approaches [11–39]. In the introduction to their paper reporting first calculations for spin dependent observables in $NN\rightarrow NN\pi$ near threshold employing the Jülich model, Hanhart *et al.* [39] have observed, ''As far as microscopic model calculations of the reaction $NN\rightarrow NN\pi$ are concerned one has to concede that theory is definitely lagging behind the development on the experimental sector . . . Furthermore they take into account only the lowest partial wave(s). Therefore it is not possible to confront those models with the wealth of experimental information available nowadays, specifically with differential cross sections and with spin-dependent observables.''

More interestingly, the reaction is characterized by large values of momentum transfer, which is expected to reveal facets of short-range spin-dependent interactions involving the nucleons. Clearly, an empirical study of spin dependence of the interaction at short distances necessitates experimental study of the differential cross section for $\vec{p}\vec{p}\rightarrow pp\pi^0$ as a function of the momentum transfer at higher energies. With advances in storage ring technology [40], capabilities have also been reached presently [3] at the PINTEX facility to study pp collisions with beam polarization $\mathbf{P}(b)$ as well as target polarization $\mathbf{P}(t)$ along $\pm x, \pm y, \pm z$ with respect to a right-handed Cartesian coordinate system, with z axis along the beam. Out of the 36 combinations of $\mathbf{P}(b)$ and $\mathbf{P}(t)$ so generated, only 12 are collinear which have been used to measure [1–4] total cross section differences $\Delta\sigma_T$ and $\Delta\sigma_L$. Out of the 12 collinear combinations, six correspond to $\mathbf{P}(b)$ being parallel to $\mathbf{P}(t)$.

The purpose of this Rapid Communication is to explore the possibility of utilizing this advanced PINTEX facility to effect a model independent partitioning of the cross section for $pp\rightarrow pp\pi^0$ at the differential level itself into cross sections $({}^{2s+1}d^2\sigma_m)/(d^3p_f d\Omega)$, where m denotes the projection of the initial channel spin $s=0,1$ along the beam direction, $\mathbf{p}_f$ denotes the relative momentum between the nucleons in the final state and $d\Omega$ denotes the infinitesimal solid angle surrounding the direction of the pion emission in c.m. We also examine here how the measured differential cross section for $\vec{p}\vec{p}\rightarrow pp\pi^0$ with arbitrary $\mathbf{P}(b)$ and $\mathbf{P}(t)$ could be partitioned in terms of a set of singlet and triplet differential cross sections. We demonstrate how a set of suitable measurements of the differential cross sections $(d^2\sigma)/(d^3p_f d\Omega)$ in addition to the unpolarized differential cross section $(d^2\sigma_0)/(d^3p_f d\Omega)$ are sufficient to determine individually the differential cross sections $({}^{2s+1}d^2\sigma_m)/(d^3p_f d\Omega)$. We derive a general result expressing $(d^2\sigma)/(d^3p_f d\Omega)$, for arbitrary $\mathbf{P}(b)$ and $\mathbf{P}(t)$ as a weighted sum which derives contributions not only from the singlet $s=0$ state and three $s=1$ states, but also from an additional term $(d^2\sigma_c)/(d^3p_f d\Omega)$ generated by the initial singlet-triplet correlations. We also outline how these contributions could individually be estimated through appropriate measurements. A brief summary of all the results obtained is presented.

Following [38], the matrix $\mathbf{M}$ in spin space for $pp\rightarrow pp\pi^0$ may be expressed in the form

$$\mathbf{M}=\sum_{s',s=0}^{1}\sum_{\lambda=|s'-s|}^{s'+s}(S^\lambda(s',s)\cdot M^\lambda(s',s)), \tag{1}$$

where s and s' denote, respectively, the initial and final channel spins, the irreducible tensor operators $S^\lambda_\mu(s',s)$ of rank λ are given by

$$S^\lambda_\mu(s',s)=[s']\sum_m (-1)^{s-m}C(s's\lambda;m'-m\mu)|s'm'\rangle\langle sm| \tag{2}$$

following [41] and the irreducible tensor amplitudes $M^{\lambda}_{\mu}(s',s)$ are given by Eqs. (2) and (3) of [38].

The differential cross section for $\vec{p}\vec{p}\rightarrow pp\pi^0$ is then given, using the same notations as in [38], by

$$d^2\sigma=\mathrm{Tr}(\mathbf{M}\rho\mathbf{M}^{\dagger})d^3p_f d\Omega=\mathrm{Tr}(\boldsymbol{B}\rho), \tag{3}$$

where

$$\boldsymbol{B}=\mathbf{M}^{\dagger}\mathbf{M}\, d^3p_f d\Omega \tag{4}$$

is Hermitian and the initial spin density matrix

$$\rho=\frac{1}{4}[1+(\boldsymbol{\sigma}(b)\cdot\mathbf{P}(b))][1+(\boldsymbol{\sigma}(t)\cdot\mathbf{P}(t))] \tag{5}$$

$$=\frac{1}{4}\sum_{\alpha,\beta=0,x,y,z}\sigma_\alpha(b)\sigma_\beta(t)P_\alpha(b)P_\beta(t), \tag{6}$$

where $P_0(b)=P_0(t)=1$ and σ_0 denote 2×2 unit matrices. Using Eq. (6) in Eq. (3), the differential cross section may readily be expressed as

$$d^2\sigma=\sum_{\alpha,\beta}P_\alpha(b)P_\beta(t)B_{\alpha\beta}, \tag{7}$$

where

$$B_{\alpha\beta}=\frac{1}{4}\mathrm{Tr}[\boldsymbol{B}\sigma_\alpha(b)\sigma_\beta(t)]. \tag{8}$$

If both the beam and target are unpolarized, we readily identify the unpolarized differential cross section as

$$d^2\sigma_0=B_{00}=\frac{1}{4}\mathrm{Tr}(\mathbf{M}\mathbf{M}^{\dagger})d^3p_f d\Omega, \tag{9}$$

whereas

$$d^2\sigma=B_{00}+P_z(b)B_{z0} \tag{10}$$

if the beam alone is polarized longitudinally,

$$d^2\sigma=B_{00}+P_z(t)B_{0z} \tag{11}$$

if the target alone is polarized along the z axis, and

$$d^2\Sigma_\alpha=\frac{1}{2}[d^2\sigma_\alpha(\uparrow\uparrow)+d^2\sigma_\alpha(\downarrow\downarrow)]$$
$$=B_{00}+P_\alpha(b)P_\alpha(t)B_{\alpha\alpha},\quad \alpha=x,y,z \tag{12}$$

if $d^2\sigma_\alpha(\uparrow\uparrow)$ and $d^2\sigma_\alpha(\downarrow\downarrow)$ denote, respectively, the differential cross sections when $\mathbf{P}(b)$ and $\mathbf{P}(t)$ are polarized parallel to each other along $\pm\alpha$. Expressing [41]

$$S^{\lambda}_{\mu}(s',s)=\frac{1}{2}[s']^2[s]\sum_{\lambda_1,\lambda_2=0}^{1}[\lambda_1][\lambda_2]\times\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & s'\\ \frac{1}{2} & \frac{1}{2} & s\\ \lambda_1 & \lambda_2 & \lambda\end{Bmatrix}(\sigma^{\lambda_1}(b)\otimes\sigma^{\lambda_2}(t))^{\lambda}_{\mu}, \tag{13}$$

where $\sigma^0_0=\sigma_0$; $\sigma^1_0=\sigma_z$; $\sigma^1_{\pm1}=\mp1/\sqrt{2}(\sigma_x\pm i\sigma_y)$ and inverting (2), using the unitarity of Clebsch-Gordan coefficients, we obtain projection operators $\boldsymbol{\pi}(s,m)=|sm\rangle\langle sm|$ which are given explicitly by

$$\boldsymbol{\pi}(1,1)=\frac{1}{4}[1+\sigma_z(b)+\sigma_z(t)+\sigma_z(b)\sigma_z(t)], \tag{14}$$

$$\boldsymbol{\pi}(1,0)=\frac{1}{4}[1-2\sigma_z(b)\sigma_z(t)+(\boldsymbol{\sigma}(b)\cdot\boldsymbol{\sigma}(t))], \tag{15}$$

$$\boldsymbol{\pi}(1,-1)=\frac{1}{4}[1-\sigma_z(b)-\sigma_z(t)+\sigma_z(b)\sigma_z(t)]. \tag{16}$$

The above add up together to give the well-known triplet projection operator

$$\boldsymbol{T}=\sum_m\boldsymbol{\pi}(1,m)=\frac{1}{4}[3+(\boldsymbol{\sigma}(b)\cdot\boldsymbol{\sigma}(t))], \tag{17}$$

while

$$\boldsymbol{S}=\boldsymbol{\pi}(0,0)=\frac{1}{4}[1-(\boldsymbol{\sigma}(b)\cdot\boldsymbol{\sigma}(t))] \tag{18}$$

denotes the well-known singlet projection operator. Noting that $\Sigma_{s,m}\boldsymbol{\pi}(s,m)=1$ and inserting the same between $\mathbf{M}$ and $\mathbf{M}^{\dagger}$ in Eq. (9), we readily obtain the partitioning of $d^2\sigma_0$ into singlet and triplet differential cross sections ${}^{2s+1}d^2\sigma_m$ which are given explicitly by

$${}^3d^2\sigma_{+1}=\frac{1}{4}(B_{00}+B_{z0}+B_{0z}+B_{zz}), \tag{19}$$

$${}^3d^2\sigma_0=\frac{1}{4}(B_{00}+B_{xx}+B_{yy}-B_{zz}), \tag{20}$$

$$^3d^2\sigma_{-1}=\frac{1}{4}(B_{00}-B_{z0}-B_{0z}+B_{zz}), \tag{21}$$

$$^1d^2\sigma_0=\frac{1}{4}(B_{00}-B_{xx}-B_{yy}-B_{zz}), \tag{22}$$

in terms of the six entities B_{00}, B_{z0}, B_{0z}, B_{xx}, B_{yy}, and B_{zz} which are measurable experimentally using the existing technological capabilities at the PINTEX facility.

Comparison of Eqs. (9) and (7) with Eqs. (20) and (21) of [38] enables us readily to express $B_{\alpha\beta}$ in terms of the irreducible tensors $B^k_\nu(k_1,k_2)$ of rank k which are in turn given explicitly in terms of the irreducible tensor amplitudes $M^\lambda_\mu(s_f,s_i)$ by Eq. (17) of [38]. Thus $B_{00}=\frac{1}{4}B^0_0(0,0)$ and all the other $B_{\alpha\beta}$ are related to the analyzing powers $A^k_\nu(k_1,k_2)=B^k_\nu(k_1,k_2)/B^0_0(0,0)$. In particular, it may then be noted that B_{z0} and B_{0z} represent the longitudinal analyzing powers with respect to the polarized beam and polarized target, respectively. Moreover, B_{z0} goes into B_{0z} if we change $\hat{\mathbf{p}}_i$ to $-\hat{\mathbf{p}}_i$ or equivalently $\hat{\mathbf{p}}_f$, $\hat{\mathbf{q}}$ are changed to $-\hat{\mathbf{p}}_f$, $-\hat{\mathbf{q}}$, keeping $\hat{\mathbf{p}}_i$ unchanged. At energies close to threshold, the leading irreducible tensor amplitude $M^1_0(0,1)$ given by Eq. (35) of [38] alone cannot generate B_{z0} or B_{0z}, but the amplitudes $M^1_\mu(1,0)$ and $M^\lambda_\mu(1,1)$ given by Eqs. (36) and (37) of [38] can obviously generate nonzero B_{z0} and B_{0z}. More importantly, $M^\lambda_\mu(1,1)$ can generate nonzero contributions by interfering with the leading amplitude $M^1_0(0,1)$. It may also be seen that these longitudinal analyzing powers which are not necessarily zero at the double differential level, vanish on integration with respect to d^3p_f due to parity conservation. Integrating further with respect to $d\Omega$ leads to the total cross sections, when Eqs. (19) to (22) relate to formulas already in use [1–5] to effect analysis of the measurements of the total cross sections.

If we replace x,y,z by y,z,x, respectively, in Eqs. (19)–(21), we readily obtain the formulas for $^3d^2\sigma_m$, where m refers to the projection of the initial channel spin s along the x axis. Likewise, replacement of x,y,z by z,x,y in Eqs. (19)–(21) leads to formulas for $^3d^2\sigma_m$, where m refers to the projection of the initial channel spin s along the y axis. These additional formulas involve the transverse analyzing powers B_{x0},B_{0x} or B_{y0},B_{0y} instead of the longitudinal analyzing powers B_{z0},B_{0z}. Thus by measuring $B_{00},B_{x0},B_{0x},B_{y0},B_{0y},B_{z0},B_{0z},B_{xx},B_{yy},B_{zz}$, it is possible to determine the triplet differential cross sections $^3d^2\sigma_m$, where m denotes the projection quantum number either with respect to the beam direction or with respect to the x or y axis. The above measurements involve only the collinear combinations of $\mathbf{P}(b)$ and $\mathbf{P}(t)$.

We have discussed the partitioning of the unpolarized differential cross section into $^{2s+1}d^2\sigma_m$ through

$$d^2\sigma_0=\sum_{s=0}^{1}\sum_{m=-s}^{s}{}^{2s+1}d^2\sigma_m \tag{23}$$

and how the $^{2s+1}d^2\sigma_m$ can be determined individually through appropriate experimental measurements using Eqs. (19)–(22). We next examine here the possibility of factoring the differential cross section $d^2\sigma$ given by Eq. (3) for $\vec{p}\vec{p}\rightarrow pp\pi^0$ for arbitrary $\mathbf{P}(b)$ and $\mathbf{P}(t)$. To do this, we express ρ given by Eq. (5) in channel spin representation. Using the converse of Eq. (13) we may write ρ as

$$\rho=\frac{1}{4}[1-\mathbf{P}(b)\cdot\mathbf{P}(t)]\boldsymbol{S}+\rho^{\boldsymbol{T}}\boldsymbol{T}+\rho^C, \tag{24}$$

where $\boldsymbol{S}$ and $\boldsymbol{T}$ denote, respectively, the singlet and triplet projection operators given by Eqs. (18) and (17). The density matrix $\rho^{\boldsymbol{T}}$, describing the state of triplet polarization, is given by

$$\rho^{\boldsymbol{T}}=\frac{1}{4}[1+\tfrac{1}{3}(\mathbf{P}(b)\cdot\mathbf{P}(t))]\boldsymbol{T}+\frac{1}{8}((\boldsymbol{\sigma}(b)+\boldsymbol{\sigma}(t))\cdot(\mathbf{P}(b)+\mathbf{P}(t)))+\frac{1}{4}((\sigma^1(b)\otimes\sigma^1(t))^2\cdot(P^1(b)\otimes P^1(t))^2) \tag{25}$$

using the notations $P^1_0=P_z$; $P^1_{\pm1}=\mp 1/\sqrt{2}(P_x\pm iP_y)$. Further

$$\rho^C=\frac{1}{8}[((\boldsymbol{\sigma}(b)-\boldsymbol{\sigma}(t))\cdot(\mathbf{P}(b)-\mathbf{P}(t)))+((\boldsymbol{\sigma}(b)\times\boldsymbol{\sigma}(t))\cdot(\mathbf{P}(b)\times\mathbf{P}(t)))] \tag{26}$$

represents singlet-triplet correlations. Clearly, the probabilities for the initial $\vec{p}\vec{p}$ system to exist in the singlet and triplet states are given by

$$w_{\boldsymbol{S}}=\frac{1}{4}[1-\mathbf{P}(b)\cdot\mathbf{P}(t)];\quad w_{\boldsymbol{T}}=\mathrm{Tr}\,\rho^{\boldsymbol{T}}=\frac{1}{4}[3+\mathbf{P}(b)\cdot\mathbf{P}(t)]. \tag{27}$$

Comparison of $\rho^{\boldsymbol{T}}$ with the standard form [42] of the density matrix for spin 1 systems shows that the Fano statistical tensors t^k_q of rank $k=1,2$ representing, respectively, vector and tensor polarizations are given by

$$w_{\boldsymbol{T}}t^1_q=\frac{1}{2}\sqrt{\frac{3}{2}}(P^1_q(b)+P^1_q(t));$$

$$w_{\boldsymbol{T}}t^2_q=\sqrt{\frac{3}{2}}(P^1(b)\otimes P^1(t))^2_q, \tag{28}$$

if we note that the standard [42] irreducible tensor operators τ^k_q for spin 1 are identical to $S^k_q(1,1)$. We next note that the triplet density matrix $\rho^{\boldsymbol{T}}$ given by Eq. (25) is Hermitian. As such one can identify its real eigenvalues w_{e_1},w_{e_2},w_{e_3} together with the corresponding eigenstates $|e_1\rangle,|e_2\rangle,|e_3\rangle$ by solving the characteristic equation so that one may use Eq. (24) in Eq. (3) to lead to the form

$$d^2\sigma = w_S\,{}^1d^2\sigma_0 + \sum_{i=1}^{3} w_{e_i}\,{}^3d^2\sigma_{e_i} + d^2\sigma_c\,, \tag{29}$$

where ${}^1d^2\sigma_0$ is given by Eq. (22) and

$${}^3d^2\sigma_{e_i} = \mathrm{Tr}(\mathbf{M}|e_i\rangle\langle e_i|\mathbf{M}^\dagger); \quad i=1,2,3 \tag{30}$$

represents the cross section for the process when the initial pp system is in the triplet state $|e_i\rangle$. We also note that $d^2\sigma_c$ may be expressed in terms of the $B_{\alpha\beta}$ through

$$\begin{aligned} d^2\sigma_c = \mathrm{Tr}(\boldsymbol{B}\rho^C) = \frac{1}{8}\Bigg[& \sum_{\alpha=x,y,z} (P_\alpha(b)-P_\alpha(t))(B_{\alpha 0}-B_{0\alpha}) \\ & +(P_y(b)P_z(t)-P_z(b)P_y(t))(B_{yz}-B_{zy}) \\ & +(P_z(b)P_x(t)-P_x(b)P_z(t))(B_{zx}-B_{xz}) \\ & +(P_x(b)P_y(t)-P_y(b)P_x(t))(B_{xy}-B_{yx})\Bigg], \end{aligned} \tag{31}$$

using Eq. (26). For example, when $\mathbf{P}(b)$ and $\mathbf{P}(t)$ are collinear with the beam direction, the triplet state is *oriented* [43] and the eigenvalues w_m corresponding to the eigenstates $|1m\rangle$, $m=+1,0,-1$ are given by

$$w_{+1} = \frac{1}{4}[1+P_z(b)\pm P_z(t)\pm P_z(b)P_z(t)];$$

$$w_0 = \frac{1}{4}[1\mp P_z(b)P_z(t)];$$

$$w_{-1} = \frac{1}{4}[1-P_z(b)\mp P_z(t)\pm P_z(b)P_z(t)], \tag{32}$$

where the upper and lower signs in Eq. (32) correspond to $\mathbf{P}(t)$ being parallel or antiparallel to $\mathbf{P}(b)$, if $\mathbf{P}(b)$ is chosen as the axis of quantization. On the other hand, if $\mathbf{P}(b)$ and $\mathbf{P}(t)$ are noncollinear as for example, $\mathbf{P}(b)$ is along x axis and $\mathbf{P}(t)$ is along y axis, the triplet state is *nonoriented* [44] and is characterized by three distinct axes [45], which are along $\mathbf{P}(b),\mathbf{P}(t)$ and $(\mathbf{P}(b)+\mathbf{P}(t))$. In such a case, it is not possible to identify an axis of orientation with respect to which one can define $|1m\rangle$ states or identify all the eigenstates $|e_i\rangle$ of $\rho^{\boldsymbol{T}}$ as $|1m\rangle$ states. However, the eigenstates $|e_i\rangle$ are expressible as

$$|e_i\rangle = \sum_m c_m^{e_i}|1m\rangle \tag{33}$$

in terms of a convenient set of basis states $|1m\rangle$ defined with respect to, say, the beam direction and the complex expansion coefficients $c_m^{e_i}=\langle 1m|e_i\rangle$ which are known. Thus, ${}^3d^2\sigma_{e_i}$ in Eq. (29) may be expressed in terms of $B_{\alpha\beta}$ through

$${}^3d^2\sigma_{e_i} = \sum_m \sum_{m'} c_m^{e_i}\,\mathrm{Tr}(\boldsymbol{B}|1m\rangle\langle 1m'|)c_{m'}^{e_i\,\star}, \tag{34}$$

where $\mathrm{Tr}(\boldsymbol{B}|1m\rangle\langle 1m'|)$ for $m=m'$ are given by Eqs. (19)–(21). For $m\neq m'$ $\mathrm{Tr}(\boldsymbol{B}|1m\rangle\langle 1m'|)=(\mathrm{Tr}(\boldsymbol{B}|1m'\rangle\langle 1m|))^\star$ are given by

$$\begin{aligned} \mathrm{Tr}(\boldsymbol{B}|11\rangle\langle 10|) = \frac{1}{4\sqrt{2}}[& B_{x0}+B_{0x}+\mathrm{i}(B_{y0}+B_{0y})+B_{xz}+B_{zx} \\ & +\mathrm{i}(B_{yz}+B_{zy})] \end{aligned}$$

$$\begin{aligned} \mathrm{Tr}(\boldsymbol{B}|10\rangle\langle 1-1|) = \frac{1}{4\sqrt{2}}[& B_{x0}+B_{0x}+\mathrm{i}(B_{y0}+B_{0y})-B_{xz}-B_{zx} \\ & -\mathrm{i}(B_{yz}+B_{zy})] \end{aligned} \tag{35}$$

$$\mathrm{Tr}(\boldsymbol{B}|11\rangle\langle 1-1|) = \frac{1}{4}[B_{xx}-B_{yy}+\mathrm{i}(B_{xy}+B_{yx})].$$

Thus one can also determine the triplet cross sections ${}^3d^2\sigma_{e_i}$ for an arbitrary triplet state such as given by Eq. (33) using the PINTEX facilities.

We have shown that it is possible, with the existing technological capabilities [3], to determine the singlet and triplet contributions to the cross section at the differential level itself through experimental measurements of the unpolarized differential cross section $(d^2\sigma_0)/(d^3p_f d\Omega)$ together with the measurements of $(d^2\sigma)/(d^3p_f d\Omega)$ when (a) the beam alone is polarized, (b) when the target alone is polarized, and (c) both the beam and the target are polarized parallel to each other. That is, the measurements of the differential cross sections (9)–(12) readily yield $B_{00},B_{z0},B_{0z},B_{xx},B_{yy},B_{zz}$ and hence $({}^{2s+1}d^2\sigma)/(d^3p_f d\Omega)$ through the use of Eqs. (19)–(22). If we replace B_{z0} and B_{0z} in the above set of measurements by the transverse analyzing powers B_{x0} and B_{0x} and change x,y,z by y,z,x in Eqs. (19)–(22), we can determine $({}^{2s+1}d^2\sigma)/(d^3p_f d\Omega)$, where m denotes the magnetic quantum number with respect to the x axis chosen as the axis of quantization. Likewise, measurements of B_{y0} and B_{0y} instead of B_{z0} and B_{0z} and changing x,y,z by z,x,y in Eqs. (19)–(22) allows determination of $({}^{2s+1}d^2\sigma)/(d^3p_f d\Omega)$, where m denotes the magnetic quantum number with respect to the y axis. We have also shown that for any arbitrary geometry of the beam and target polarizations $\mathbf{P}(b)$ and $\mathbf{P}(t)$, either collinear or noncollinear, the differential cross section derives contributions not only from the singlet state and the triplet eigen states $|e_i\rangle$, $i=1,2,3$ of $\rho^{\boldsymbol{T}}$, but also from the singlet-triplet correlations generated in a polarized beam and polarized target experiment. The singlet and triplet correlation contribution $(d^2\sigma_c)/(d^3p_f d\Omega)$ as well as the triplet cross sections $({}^3d^2\sigma_{e_i})/(d^3p_f d\Omega)$, corresponding to any arbitrary initial triple state $|e_i\rangle$ of the form (33), may also be determined using Eqs. (31) and (35). Experimental measurements of the differential cross sections $(d^2\sigma)/(d^3p_f d\Omega)$ readily yield all the $B_{\alpha\beta}$, $\alpha,\beta=0,x,y,z$ by making full use of the PINTEX facility to study $\vec{p}\vec{p}\rightarrow pp\pi^0$, employing the collinear as well as noncollinear combinations of the initial beam and target polarizations $\mathbf{P}(b)$ and $\mathbf{P}(t)$. Although [1–4] are concerned with the study of $\vec{p}\vec{p}\rightarrow pp\pi^0$ at near threshold energies, we may note that all the considerations reported in this Rapid Communication apply equally well at higher energies where measurements can be carried out corresponding to comparatively larger values of momentum transfers. In fact, they may be applied even to analyze elastic NN scattering [where $(d^2\sigma_c)/(d^3p_f d\Omega)$ is, however, zero due to conservation of channel spin, i.e., $s'=s$ in Eq. (1) itself] or any reaction initiated by a polarized beam of nucleons on a polarized nucleon target.

We thank the Council of Scientific and Industrial Research (CSIR), India for support.

[1] H. O. Meyer *et al.*, Phys. Rev. Lett. **81**, 3096 (1998).
[2] P. Thörngren-Engblom *et al.*, Contribution to the Conference ''Mesons and Light Nuclei,'' 1998, Prague-Pruhonice, Czech Republic, nucl-ex/9810013.
[3] H. O. Meyer *et al.*, Phys. Rev. Lett. **83**, 5439 (1999).
[4] P. Thörngren-Engblom *et al.*, Nucl. Phys. **A663&664**, 447 (2000).
[5] S. M. Bilenky and R. M. Ryndin, Phys. Lett. **6**, 217 (1963).
[6] H. O. Meyer *et al.*, Phys. Rev. Lett. **65**, 2846 (1990).
[7] H. O. Meyer *et al.*, Nucl. Phys. **A539**, 633 (1992).
[8] D. A. Hutcheo *et al.*, Nucl. Phys. **A535**, 618 (1991).
[9] A. Bondar *et al.*, Phys. Lett. B **356**, 8 (1995).
[10] D. S. Koltun and A. Reitan, Phys. Rev. **141**, 1413 (1966); Nucl. Phys. **B4**, 629 (1968).
[11] M. E. Schillaci, R. R. Silbar, and J. E. Young, Phys. Rev. Lett. **21**, 711 (1968); Phys. Rev. **179**, 1539 (1969).
[12] T. S. H. Lee and A. Matsuyama, Phys. Rev. C **36**, 1459 (1987).
[13] G. A. Miller and P. U. Sauer, Phys. Rev. C **44**, R1725 (1991).
[14] J. Niskanen, Phys. Lett. B **289**, 227 (1992).
[15] T. S. H. Lee and D. O. Riska, Phys. Rev. Lett. **70**, 2237 (1993).
[16] C. Horowitz, H. O. Meyer, and D. Greiger, Phys. Rev. C **49**, 1337 (1993).
[17] E. Hernandez and E. Oset, Phys. Lett. B **350**, 158 (1995).
[18] B. Y. Park, F. Myhrer, J. R. Morones, T. Meisner, and K. Kubodera, Phys. Rev. C **53**, 1519 (1996).
[19] T. Cohen, J. Friar, G. Miller, and U. van Kolck, Phys. Rev. C **53**, 2661 (1996).
[20] J. A. Niskanen, Phys. Rev. C **53**, 526 (1996).
[21] C. Hanhart, J. Haidenbauer, A. Reuber, C. Shütz, and J. Speth, Phys. Lett. B **358**, 21 (1995).
[22] J. Haidenbauer, C. Hanhart, and J. Speth, Acta Phys. Pol. B **27**, 2893 (1996).
[23] C. Hanhart, Ph.D. thesis, University of Bonn, 1997.
[24] T. Sato, T. S. H. Lee, F. Myhrer, and K. Kubodera, Phys. Rev. C **56**, 1246 (1997).
[25] G. Fäldt and C. Wilkin, Phys. Lett. B **382**, 209 (1996).
[26] G. Fäldt and C. Wilkin, Phys. Rev. C **56**, 2067 (1997).
[27] A. Engel, R. Shyam, V. Mosel, and A. K. Dutt Majumdar, Nucl. Phys. **A603**, 387 (1996).
[28] R. Shyam and U. Mosel, Phys. Lett. B **426**, 1 (1998).
[29] C. Hanhart *et al.*, Phys. Lett. B **424**, 8 (1998).
[30] C. Hanhart, J. Haidenbauer, O. Krehl, and J. Speth, Phys. Lett. B **444**, 25 (1998).
[31] E. Gedalin, A. Moalem, and L. Razdolskaya, nucl-th/9803029.
[32] C. Hanhart and K. Nakayama, Phys. Lett. B **454**, 176 (1999).
[33] M. T. Pena, D. O. Riska, and A. Stadler, Phys. Rev. C **60**, 045201 (1999).
[34] V. Dmitrasinovic *et al.*, Phys. Lett. B **465**, 43 (1999).
[35] C. Hanhart, nucl-th/9911023.
[36] H. Machner and J. Haidenbauer, J. Phys. G **25**, R231 (1999).
[37] C. M. Maekawa, J. C. Pupin, and M. R. Robilotta, Phys. Rev. C **61**, 064002 (2000).
[38] G. Ramachandran, P. N. Deepak, and M. S. Vidya, Phys. Rev. C **62**, 011001(R) (2000).
[39] C. Hanhart, J. Haidenbauer, O. Krehl, and J. Speth, Phys. Rev. C **61**, 064008 (2000).
[40] H. O. Meyer, Annu. Rev. Nucl. Part. Sci. **47**, 235 (1997).
[41] G. Ramachandran and M. S. Vidya, Phys. Rev. C **56**, R12 (1997); G. Ramachandran and M. S. Vidya, in Proceedings of the DAE Symposium on Nuclear Physics, edited by V. M. Datar and Suresh Kumar, Invited talks, 1996, Vol. 39A, p. 47.
[42] G. R. Satchler *et al.*, in *Proceedings of the 3rd International Symposium on Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (University of Wisconsin Press, Madison, WI, 1970).
[43] R. J. Blin-Stoyle and M. A. Grace, in *Hand Buch Der Physik*, Vol. XLII, edited by S. Flugge (Springer-Verlag, Berlin, 1957), p. 557; G. Ramachandran and K. S. Mallesh, Nucl. Phys. **A422**, 327 (1984).
[44] G. Ramachandran and M. V. N. Murthy, Nucl. Phys. **A323**, 403 (1979); **A337**, 301 (1980).
[45] G. Ramachandran and V. Ravishankar, J. Phys. G **12**, L143 (1986).