Strange particle production from a hadron gas in the chiral mean field formalism

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A mean-field equation of state for a hot and dense hadron gas developed earlier, which incorporates excluded volume repulsive interaction between a pair of baryons and the Kaplan-Nelson chiral Lagrangian term for kaon-baryon interactions is used to study the production of strange particle ratios, e.g., K^+/π^+, K^-/π^-, and K^+/K^-. We find that the strange particle production is considerably enhanced due to the medium modification of their masses in the hadron gas and, therefore, it gives a hint for a partial restoration of chiral symmetry in a dense and hot medium.

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I. INTRODUCTION

Quantum chromodynamics (QCD) predicts a phase transition at large temperature and/or density from the normal color-confined phase of hadron gas (HG) to a deconfined quark gluon plasma (QGP) phase [1,2]. NA50 experiments at CERN have recently claimed the detection of a QGP formation [3]. However, it is believed that the conclusive evidence for such a phase transition will be revealed at the experiments to be carried out shortly on the RHIC machine at the Brookhaven National Laboratory [4]. The diagnostic studies of a QGP are the most difficult part of the phase transition. It is essential to correctly determine the properties of the hot and dense hadronic matter in order to devise some unique signals for a QGP formation in the relativistic heavy-ion collisions [1]. Therefore, the search for a realistic and proper equation of state (EOS) for the hot and dense hadron gas carries an unusual significance. The behavior of the EOS at high energy density depends strongly on which hadrons are actually included in the model and on how these particles are assumed to interact with each other. However, until the advent of relativistic heavy-ion collisions, we did not have any experimental data to directly test the predictions of such an EOS, although these kinds of situations existed either in the early universe or in the core of neutron stars or in both of the above.

The description of hadronic interactions is almost impossible in the QCD field theory due to our limited understanding of its nonperturbative feature. Therefore, we resort to a phenomenological description of such interactions. One important aspect of the hadronic interactions at high densities is a short-range repulsive force existing between a pair of baryons. Two types of approaches, that address this question exist in the literature. One is based on purely geometric hardcore volume of each baryon and is macroscopic in nature. Assigning a hard-core size to baryons will result in a short-range repulsion. This is known as excluded-volume approach [5–7]. The other is based on the mean-field approach formulated in the language of the thermodynamics of extended objects and hence the interactions among hadrons are effectively realized microscopically in terms of the mean fields [8,9] of various hadrons present in the HG. In this approach attractive interactions in a charge symmetric nuclear matter are believed to arise due to the exchange of the σ scalar meson and the repulsive interactions are generated through the exchange of an ω vector meson. However, the Walecka mean-field model fails in describing the interactions present in the hadronic gas existing in the early Universe where the nucleons and antinucleons are present in almost equal but large number and the net baryon density is close to zero. This demonstrates that a major shortcoming of the Walecka model is in not properly handling the repulsive force existing between finite size baryons at high density. In order to surmount this difficulty, Anchishkin and Suhonen extended [10] the mean-field model by considering an extra potential energy term, which exclusively takes care of the hard-core repulsion in the form of the excluded volume effect and thus this term depends on the density as well as on the temperature. Thus they obtained a thermodynamically consistent model for a HG consisting of nucleons-antinucleons and pions. We extended the model [11] for a multicomponent hadron gas and obtained a realistic EOS by including all the particles upto a mass of 1200 MeV in the hadron gas spectrum. We have further included the chiral kaon-baryon interaction term as given by Kaplan and Nelson and a single time derivative interaction arising from the vector current coupling in order to study the effect of kaon condensation in the ultrarelativistic heavy-ion collisions [12,13]. The purpose of the present paper is to use the new EOS involving the mean-field formalism with excluded volume and the chiral terms to find out the ratios of strange particles in order to facilitate its uses in the diagnostic studies of QGP formation.

In high energy nuclear collisions, two beams of several nucleons collide. The quarks and gluons are confined in the colliding nucleons. After the primary collisions, we expect multiple rescatterings and, therefore, entropy rapidly increases and the system quickly achieves thermal equilibrium. Deconfining transition means that the confinement does not survive during thermalization. The presence of a QGP in the fireball thus creates a bigger difference than when we have hadrons in the system only. Chiral symmetry restoring phase transition means quarks in the QGP become almost massless or have the current-quark masses. The resulting hadrons during hadronization will consequently have reduced masses in the dense and hot HG also due to (partial) restoration of chiral symmetry. Such medium modifications of kaon and hyperon masses in a hot and dense medium have been a topic of current interest because these are useful inputs for the study of kaon condensation in a neutron star. Since the pioneering work of Kaplan and Nelson on the possibility of the kaon condensation in nuclear matter [14], a large amount of theoretical effort has been devoted to the study of kaon properties in dense matter. The approaches include chiral perturbation theory [15], the Nambu–Jona-Lasino model [16], and the SU(3) Walecka-type mean-field model [17], etc. Recently we have also undertaken such studies in the framework of a model which combines the mean-field approach with the excluded-volume correction [11,12]. We have witnessed a drastic reduction in the masses of the strange particles in such a model. In this paper, we attempt to calculate the ratios e.g., K^+/π^+, K^-/π^-, and K^+/K^- etc., from this type of EOS for the hadron gas. This study will be of immense help in understanding the strangeness enhancement as a signal for a QGP and/or a chiral symmetry restoring phase transition.

II. EOS FOR HG

The model has been described in our earlier papers [11,12]. The expression for the total pressure of the HG is

$$P=1/3\sum_j d_j\int \frac{d^3k}{(2n)^3}\frac{k^2}{(M_j^{*2}+k^2)^{1/2}}[f_j+f_{j^-}]+P_{\rm VdW}(n,T)+P_B(n_B)+P_\sigma(\sigma)+\sum_m p_m(T), \tag{1}$$

where d_j is the degeneracy factor for the jth species of hadrons. In our calculation we consider the multicomponent hadron gas consisting of mesons $m=\pi$, K, K^*, η, ρ, ω, η, ϕ, and baryons $j=N$, Λ, Σ, and antibaryons $\overline{j}$. We emphasize that the calculation becomes very involved for many particles included in the HG spectrum. Therefore, the contributions of other higher mass particles are not incorporated in our model. We further denote [11]

$$f_{j(\overline{j})}=\left[\exp\left(\frac{(M_j^{*2}+k^2)^{1/2}+U_{\rm VdW}(n,T)\pm U_{Bj}(n_B)\mp\mu_j}{T}\right)+1\right]^{-1}. \tag{2}$$

Here the upper (lower) sign refers to baryons j (antibaryons j), respectively. The expressions for Van der Waals' hard core repulsion terms are [11,12]

$$P_{\rm VdW}(n,T)=nT\frac{V_0n}{1-V_0n}, \tag{3}$$

$$U_{\rm VdW}(n,T)=T\frac{V_0n}{1-V_0n}-T\ln(1-V_0n), \tag{4}$$

with

$$n=\sum_j (n_j+n_j^-), \tag{5}$$

$$P_B=(1/2)m_\omega^2\omega_0^2, \tag{6}$$

$$\omega_0=\frac{1}{m_\omega^2}[g_{\omega NN}n_{BN}+g_{\omega\Lambda\Lambda}n_{B\Lambda}+g_{\omega\Sigma\Sigma}n_{B\Sigma}], \tag{7}$$

$$n_{Bj}=n_j-n_{\overline{j}}=d_j\int\frac{d^3k}{(2\pi)^3}[f_j-f_{\overline{j}}], \tag{8}$$

$$n_B=\sum n_{Bj}(\mu_j,T). \tag{9}$$

The degeneracy factors are given as $d_N=4$, $d_\Lambda=2$, and $d_\Sigma=6$. We have used the following SU(3) relations between the coupling constants $g_{\sigma YY}=(2/3)g_{\sigma NN}$ and $g_{\omega YY}=(2/3)g_{\omega NN}$. We further use [10]

$$g_{\sigma NN}^2/m_\sigma^2=374.84\ \mathrm{GeV}^{-2},\quad g_{\omega NN}^2/m_\omega^2=283.59\ \mathrm{GeV}^{-2}.$$

Furthermore, we get U_{Bj} in terms of the time component of the vector field ω as

$$U_{Bj}(n_B)=g_{\omega jj}\omega_0. \tag{10}$$

The chemical potential of baryons is modified due to the ω-meson interaction:

$$\mu_j^*=\mu_j-U_{Bj}. \tag{11}$$

On the contrary, the attractive interaction of baryons with the σ scalar field modifies their masses as

$$M_j^*=M_j-g_{\sigma jj}\sigma. \tag{12}$$

Similarly the corresponding pressure is

$$P_\sigma(\sigma)=-(1/2)m_\sigma^2\sigma^2, \tag{13}$$

where

$$\sigma=(1/m_\sigma^2)\sum_j g_{\sigma jj}(n_{\sigma j}+n_{\sigma\overline{j}}) \tag{14}$$

and

$$n_{\sigma j(\overline{j})}=d_j\int\frac{d^3k}{(2\pi)^3}\frac{M_j^*}{(M_j^{*2}+k^2)^{1/2}}f_{j(\overline{j})}. \tag{15}$$

In order to make our calculation complete for the kaon-baryon interactions, we further add two more scalar and vector interaction terms as follows:

$$\mathcal{L}_S=\frac{\Sigma_{KN}}{f_K^2}\overline{B}B\overline{K}K, \tag{16}$$

$$\mathcal{L}_V=\frac{-3i}{8f_K^2}\overline{B}\gamma_0B\overline{K}\partial_tK. \tag{17}$$

The first gives the scalar attractive interaction term as given by Kaplan and Nelson who showed that the baryons act on kaons as an effective scalar field because of the explicit breaking of chiral symmetry $SU(3)_R \times SU(3)_L$. The second term gives the repulsive vector interactions between the kaons in the nuclear matter, while it is attractive for antikaons due to G parity. In the above f_k is the kaon decay constant and we take $f_k = 93\,\text{MeV}$ and kN sigma term $\Sigma_{kN} = 350\,\text{MeV}$. The term $\mathcal{L}_S$ obviously reduces the kaon mass as follows:

$$M_K^* = M_K \left(1 - \frac{\Sigma_{KN} \rho_S}{M_K^2 f_K^2} \right)^{1/2}, \tag{18}$$

where ρ_S is the total scalar density of the baryons, i.e., sum of $n_{\sigma j}$ and $n_{\sigma \bar{j}}$. Similarly the presence of $\mathcal{L}_S$ and $\mathcal{L}_V$ modifies the kaon (antikaon) effective energy at zero momentum as

$$E^*_{K(\bar{K})} = M_K^* \pm (3/8 f_K^2) n_B \tag{19}$$

and the number density for kaons (antikaons) is given as

$$n_{K(\bar{K})} = g_K \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left[\dfrac{(M_K^{*2} + k^2)^{1/2} \pm \dfrac{3}{8 f_k^2} n_B \mp \mu_K}{T} \right] - 1}. \tag{20}$$

Furthermore, we impose another constraint equation regarding the strangeness neutrality in the nuclear matter:

$$n_{B\Lambda} + n_{B\bar{\Sigma}} + (n_K - n_{\bar{K}} + n_K^* - n_{\bar{K}}^*) = 0. \tag{21}$$

We can solve these coupled set of equations to calculate the number densities of $K^+, \bar{K}, \Lambda, \bar{\Lambda}, \Sigma, \bar{\Sigma}$, etc.

III. RESULTS AND DISCUSSIONS

In Fig. 1, we have shown the variations of the ratios K^+/π^+ and K^-/π^- with respect to temperature T at various baryon chemical potentials μ_B. At $\mu_B = 500\,\text{MeV}$, we find that the difference between the ratios K^+/π^+ and K^-/π^- increases as T increases. However, at smaller μ_B, the difference becomes much smaller. The difference should vanish as $\mu_B \to 0$. Another important point we find is that the ratio $K^+/\pi^+ = 0.8$ at $T = 160\,\text{MeV}$ and $\mu_B = 500\,\text{MeV}$. This shows that the kaon mass is considerably reduced and it results in an enhanced production of the k meson. Thus the strangeness enhancement occurs due to the medium modification of the K meson properties and, therefore, a very large K^+/π^+ ratio at large T and μ_B demonstrates at least a partial restoration of the chiral symmetry in the hot and dense medium.

In Fig. 2, we have shown the variation of the ratio K^+/K^- with temperature T at different μ_B. As expected, we get a value for the ratio nearly equal to one for smaller values of μ_B (e.g., $\mu_B \leq 300\,\text{MeV}$). However, for larger μ_B (e.g., $\mu_B \leq 600\,\text{MeV}$), we find that the ratio decreases with temperature T. This demonstrates the relative importance of the chiral terms $\mathcal{L}_S$ and $\mathcal{L}_V$ for kaons and antikaons. We find that the $\mathcal{L}_V$ term is more effective for larger μ_B and smaller temperatures (i.e., $T < 160\,\text{MeV}$) and essentially creates a difference between a kaon and an antikaon.

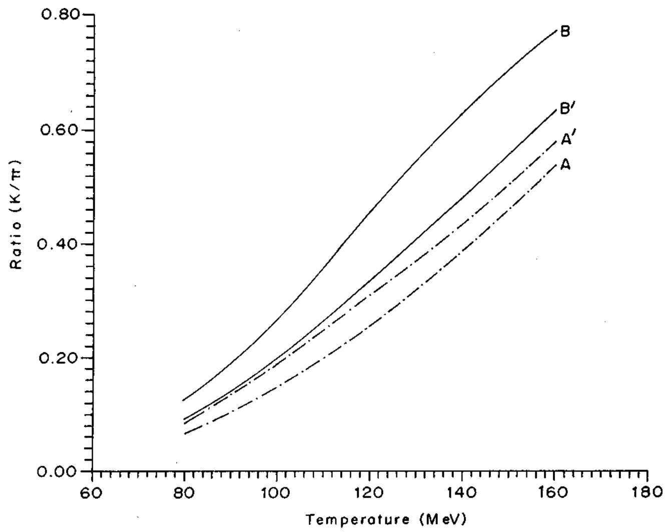

FIG. 1. Variations of the ratios K^-/π^- and K^+/π^+ with respect to temperature T are shown by curves A and B, respectively. A(A′) shows K^-/π^- at $\mu_B = 500(200)$ MeV and B(B′) shows the curves for K^+/π^+ at $\mu_B = 500(200)$ MeV, respectively.

In Fig. 3, we have shown our results for the number densities of nucleons, pions, kaons, Σ and Λ particles, and their antiparticles, respectively, at $\mu_B = 500\,\text{MeV}$. We find that $\bar{N}, \bar{\Sigma}, \bar{\Lambda}$ number densities are much smaller in magnitude in comparison to those of N, Σ, and Λ, respectively, even at very large T ($T \simeq 160\,\text{MeV}$). We are optimistic that these values of the number densities will find their uses in the QGP diagnostic studies in connection with heavy-ion collisions.

In Fig. 4, we have demonstrated the variation of strange

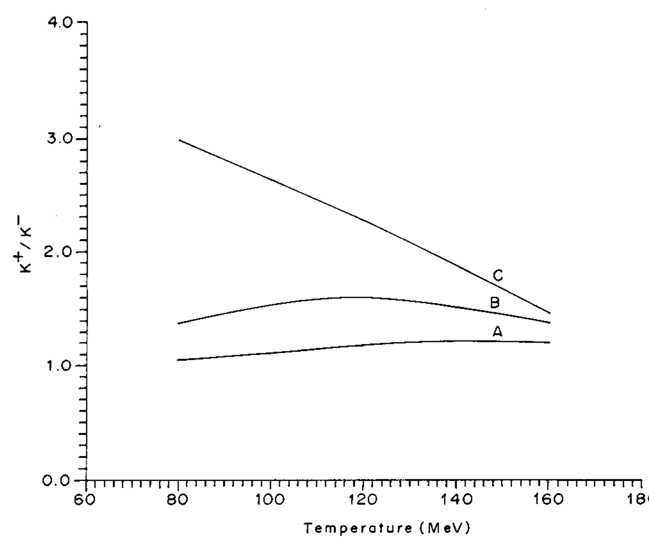

FIG. 2. Variations of the K^+/K^- ratio with respect to temperature T. Curves A, B, and C correspond to μ_B equal to 300, 460, and 600 MeV, respectively.

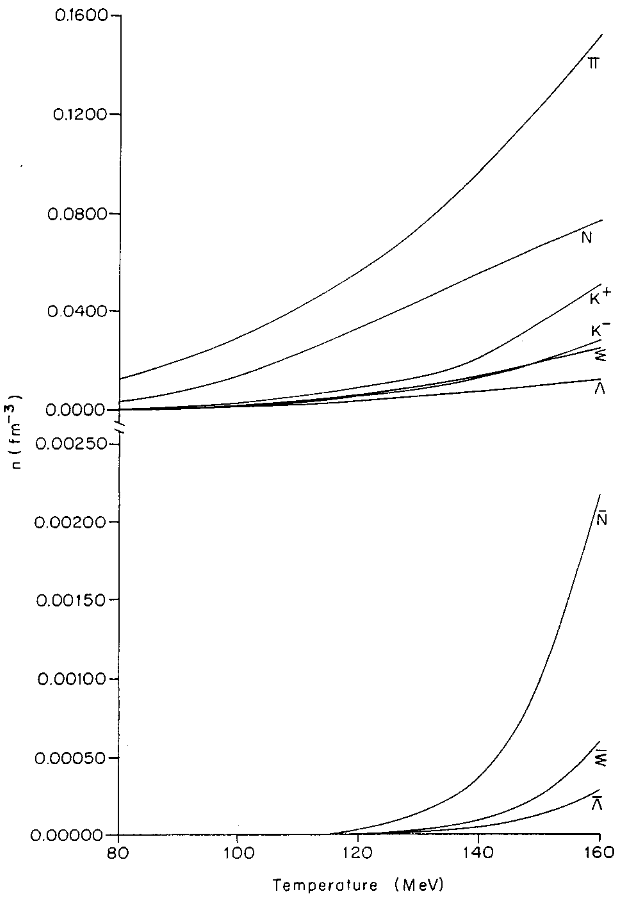

FIG. 3. Abundances of baryons and mesons with respect to temperature T. The curves correspond to the value of $\mu_B = 500$ MeV.

chemical potential μ_k with respect to baryon chemical potential μ_B at different temperatures T. This relationship arises because we put the condition of vanishing total strangeness as given by Eq. (21) for the nuclear matter. We find that for larger T $(T \geqslant 180\,\mathrm{MeV})$, we almost get a linear relationship between μ_k and μ_B. For a strangeness neutral QGP, we get strange quark chemical potential $\mu_s = 0$, which corresponds to a relation $\mu_k = \mu_B/3$. It is unique straight line relation for a HG resulting from the hadronization of a QGP. However, in Fig. 4, the slope of the straight line (for $T \geqslant 180\,\mathrm{MeV}$) is $\simeq 1/8$. This is because the strangeness is distributed among mesons and baryons of much different masses in a HG at a large T and/or μ_B.

In conclusion, the strangeness enhancement has always been regarded as a signal for a QGP formation. It is particularly considered a signal for a chiral symmetry restoration. In this paper, we have attempted to show this by using an EOS for the dense and hot HG where chiral attraction and repulsion terms have been explicitly included besides the excluded volume repulsion as well as σ, ω exchange interaction terms in the mean-field formalism. We find that the ratios K^+/π^+ and K^-/π^- grow to a very large value at larger temperature and chemical potential. In the past, some calculations appear in the literature where it was argued that the average cross section for the processes $\pi\pi \rightarrow K\bar{K}$ increases with temperature or density due to the reduction of

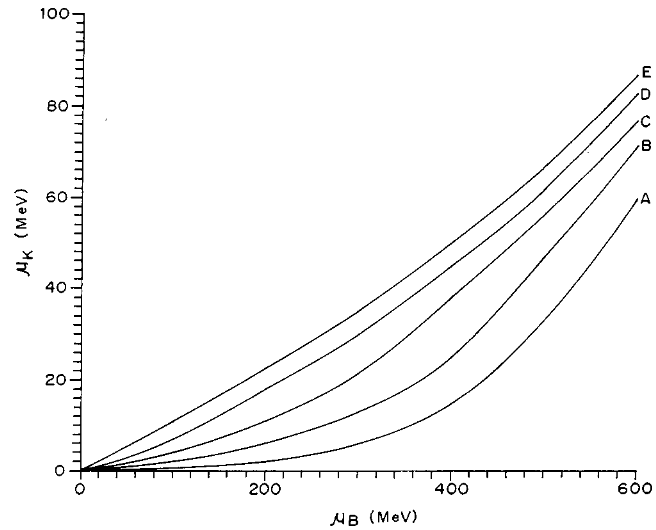

FIG. 4. Variations of the strangeness chemical potential μ_k with respect to the baryon chemical potential μ_B. Curves A, B, C, D, and E correspond to the value of temperature equal to 100, 120, 140, 160, and 180 MeV.

kaon masses [18–20]. However, the density and the temperature dependences of the masses are parametrized in the above models [18]. Here we perform an explicit calculation and hence our results deserve more attention. Relativistic transport models predict a constant K^+/K^- ratio when in-medium mass modification of the K mesons is neglected [21]. These calculations further reveal that the falloff of K^- spectra is steeper than the K^+ spectra possibly due to the decrease of K^- effective mass in the nuclear medium [22]. Therefore, we emphasize that the properties of strange mesons in a hadronic medium at finite temperature and/or density are essential for constructing a realistic EOS for understanding nuclear matter.

Recently Schaffner-Bielich and collaborators [23,24] have explored the properties of strange hadrons in the dense and hot nuclear medium within a coupled channel calculation. They have found that K^- experiences an attractive potential in the medium but the potential changes sign at a sufficiently large temperature. This does not fully support our findings in this paper and hence needs further confirmation. Even if the K^- feels a much shallower attractive potential rather than a repulsive one in heavy-ion collisions, we will not get an enhanced production of K^- due to a reduction in its mass. In order to obtain a maximal in-medium effect for the K^-, they [24] have used an optical potential which scales with density and they have further included the in-medium cross section. Effects from a finite relative momentum and finite temperature are ignored in the calculation. One finds that the production rate of K^- is still not very sensitive to in-medium effects because the in-medium results do not differ from the free case. However, the combined maximum effect of a deeply attractive potential of the K^- and the enhanced cross sections results in a significant deviation of the K^+/K^- ratio from the one of the free case. A recent statistical model calculation [25] based on the canonical ensemble formalism also finds that no in-medium effect is required to get the

K^+/K^- ratio in agreement with the experimental data. In comparison, our results demonstrate little change in the K^+/K^- ratio when we have a very dense medium (i.e., $\mu_B \geqslant 600$ MeV).

One important question which remains unanswered is whether or not the strangeness enhancements presented in this paper signal a QGP formation. We have considered an EOS for a HG and not a plasma of quarks and gluons. So the results obtained here indicate a HG scenario present in an extremely hot and dense situation. Here the masses of the hadrons have changed due to a partial restoration of chiral symmetry. The Alternating Gradient Synchrotron (AGS) value of $K^+/\pi^+ \simeq 0.2$ is compatible with a HG picture presented here at $T \simeq 100$ MeV and $\mu_B = 500$ MeV. However, the value of K^+/π^+ remains almost unaltered when we see the data of the Pb-Pb experiments at CERN SPS that have become available. In comparison, a much higher value of K^+/π^+ at around 0.8 is possible in our model at $T = 160$ MeV and $\mu_B = 500$ MeV. This fact reveals that our thermal HG picture yields a larger enhancement factor for strange particles than what we observe in the present experiment. Therefore, the experimental data may point to a scenario in which a deconfined QGP is formed but the hadronization does not yield a chemically equilibrated HG gas. This conclusion is further supported by the fact that our model fails to account for the anomalously large value reported for $\overline{\Lambda}/\overline{p} \simeq 3-5$ at AGS [2]. All models fail to describe such a large value for $\overline{\Lambda}/\overline{p}$. It was believed that different medium modifications to the masses of nonstrange and strange baryons can explain such a large ratio. However, we do not get such largely different medium modifications for the masses of $\overline{\Lambda}$ and $\overline{p}$, respectively, and this is demonstrated by our results in Fig. 3. Thus our analysis lends further support to a QGP formation as indicated by the experimental data of the unusually large value for $\overline{\Lambda}/\overline{p}$ in the recent experiment [2].

Recently many calculations have appeared which explain the data of heavy-ion collisions in the HG picture alone. However, it is surprising to see that the same data are explained by the models, which employ completely different pictures. For example, thermal HG models using the concepts of local thermal and chemical equilibrium are able to reproduce the data with the help of a set of the thermodynamical parameters T and μ_B of the fireball [26–28]. Similarly expanding the thermal fireball, the multicomponent firestreaks and hadronic transport or the cascade type of the models have also been used to explain the data well [29–33]. In such types of situations, more detailed analysis of the experimental data is needed in order to differentiate between the models. A thermal HG picture employing the idea of the medium modification of the properties of hadrons yields a more suitable EOS for the description of a hot and dense HG. But such a picture contributes a larger enhancement factor for strange particles than is observed in heavy-ion collisions at very large T/μ. In such a case, we conclude that a deconfined QGP picture, in which chemical equilibrium is not achieved after hadronization, appears more suitable [34] for the description of the features of the experimental data for strange particles in the Pb-Pb collisions.

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