Scenario for ultrarelativistic nuclear collisions. IV. Effective quark mass at the early stage

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Using the framework of wedge dynamics, we compute the effective transverse mass of a soft quark mode propagating in the expanding background of hard quarks and gluons created at the earliest time of the collision. We discover that the wedge dynamics does not require any external infrared or collinear cutoff. The effective mass is produced mainly due to the forward quark-quark scattering mediated by the longitudinal (in the sense of Gauss law) magnetic fields. Contribution of the radiation field is parametrically suppressed.

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I. INTRODUCTION

In the first paper of this cycle [1] (further quoted as paper [I]), we formulated a program that might result in a theory of ultrarelativistic nuclear collisions which is free from collinear problems and naturally establishes the infrared boundary for the space of ''final'' states at the very early stage of the collision ($\leqslant 1$ fm). We have demonstrated that even at very early times (much less than is required for any kinetic process to develop), the collective interactions in a dense system provide the final states of the QCD evolution with finite dynamically generated masses that shield mass singularities in the evolution equations.[1] It was shown also that the null-plane dynamics are incapable of describing local screening effects, because any type of kinetics is frozen on the light cone. It was suggested, that a more adequate approach requires the change of the global Hamiltonian dynamics which is used for the field-theory description of nuclear collisions. We proposed the so-called *wedge dynamics* which employs the proper time τ measured from the first touch of the Lorenz-contracted nuclei as the Hamiltonian time. Our initial estimates in paper [I] were very qualitative. In two consecutive papers [4,5] (further quoted as papers [II] and [III]), we have studied, in detail, the space of states of wedge dynamics. In paper [II], we extended the qualitative analysis of scalar fields initiated in [I], and found that for the charged fields, the early-time evolution of the wave function is accompanied by a gradual rearrangement of the charge distribution, starting from its almost uniform spread along the light cone at $\tau\to 0$, and up to a narrow wave packet with a well defined rapidity at later times. We have shown that this redistribution of the charge leads to currents in the rapidity direction and that these currents are the largest at the earliest τ. The magnetic fields generated by these currents can be responsible for the interactions between the currents at the earliest moments of the QCD evolution. In paper [II], we studied the states of fermions in wedge dynamics and found the fermion field correlators that are used below for the calculation of the quark self-energy in the expanding system. In paper [III], we addressed the issue of gauge fields in wedge dynamics. Several important problems were solved there. The natural gauge condition of wedge dynamics, $A^\tau = 0$, was proved to be completely fixed (at the level of perturbation theory). The second (technically nearly most difficult) problem solved in paper [III], was the separation of the longitudinal (i.e., governed by Gauss law) field and the field of radiation. In that paper, we also quantized the gauge field in the scope of wedge dynamics and explicitly found the Wightman functions and retarded propagator of the gluon field which are used in this paper for the practical calculation of the fermion self-energy.

Our decision to begin the exploration of potentialities of the wedge dynamics with the computation of quark self-energy is motivated only by technical reasons. The gluon propagator of wedge dynamics is a very complicated function, and we preferred to start with the computation of the fermion loop which has only one gluon correlator in it. We hope that the possibility of a technical simplification (compared to what we had to start with) discovered in this paper, will allow us to address a more important problem of the gluon self-energy in a reasonably economic way.

In the course of this study, we employ a single heuristic assumption (supported by the analysis of paper [II]) that the field states with large transverse momentum, even at very early times, may be associated with the localized particles and thus can be described by the distribution with respect to the rapidity and transverse momentum. Our strategy of looking for the leading contributions, as well as all our approximations, in the calculation of the material part of the quark self-energy are based on this assumption. If it appears incorrect, then it is most likely that the quark-gluon matter created in the collision of two nuclei never and in no approximation can be considered as a system of nearly free and weakly interacting field states.

II. FERMION RETARDED SELF-ENERGY

In order to find the normal modes of the quark field in the expanding quark-gluon system, we are going to solve the Dirac equation with the radiative corrections, which can be

[1]The idea that screening effects should be taken into account at the early kinetic stage of a collision has been articulated earlier and with different motivations by Shuryak and Xiong [2] and by Eskola, Muller, and Wang [3]

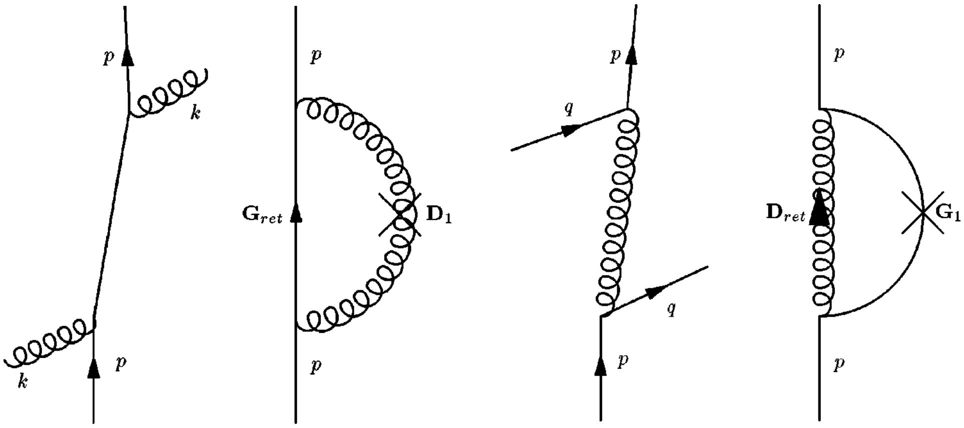

FIG. 1. The retarded forward scattering amplitude is contributed by two subprocesses, $qg \to qg$ and $qq \to qq$.

derived as a projection of the Schwinger-Dyson equation for the retarded quark propagator onto the one-particle initial state. For the quark field without Lagrangian mass, this equation reads

$$i\gamma^{\mu}(x_1)\nabla_{\mu}(x_1)\psi(x_1)=\int d^4x_2\Sigma_{[\mathrm{ret}]}(x_1,x_2)\psi(x_2). \tag{2.1}$$

The covariant derivative $\nabla_{\mu}(x)$ of the spinor field in the curvilinear coordinates of the wedge dynamics includes the spin connection and it was found explicitly in paper [II]. For all calculations below, we employ the mixed representation which is the most profitable in heavy-ion problems. We are looking for the radiative corrections to the wave function with a given transverse momentum $\vec{p}_t$ and rapidity θ with the expectation that within the rapidity plateau nothing will depend on θ. However, we cannot totally eliminate the coordinate η from the theory. We have to keep it explicitly, since the problem of the expanding field system cannot be reduced to (2+1) dimensions. In its expanded form, Eq. (2.1) reads

$$\begin{aligned}\left[i\gamma^0\left(\frac{\partial}{\partial\tau_1}+\frac{1}{2\tau_1}\right)+\frac{i\gamma^3}{\tau_1}\frac{\partial}{\partial\eta_1}-p_r\gamma^r\right]\psi(p_t;\tau_1,\eta_1)\\ =\int_0^{\tau_1}d\tau_2\int_{-\infty}^{\infty}\tau_2 d\eta_2\Sigma_{[\mathrm{ret}]}(p_t;\tau_1,\tau_2;\eta_1-\eta_2)\\ \times\psi(p_t;\tau_2,\eta_2).\end{aligned} \tag{2.2}$$

The retarded self-energy is an object that naturally emerges in the Schwinger-Dyson equation for the retarded propagator in Keldysh-Schwinger formalism [6]. Below, we employ its modified form developed earlier with the view of application to the inclusive and transient processes. We employ the notation used in Refs. [7,8,1].[2] In this notation, the one-loop retarded fermion self-energy in coordinate form is

$$\begin{aligned}\Sigma_{[\mathrm{ret}]}(x_1,x_2)=\frac{ig^2}{2}[t^a\gamma^{\mu}G_{[\mathrm{ret}]}(x_1,x_2)t^b\gamma^{\lambda}D^{ba}_{[1]\lambda\mu}(x_2,x_1)\\ +t^a\gamma^{\mu}G_{[1]}(x_1,x_2)t^b\gamma^{\lambda}D^{ba}_{[\mathrm{adv}]\lambda\mu}(x_2,x_1)].\end{aligned} \tag{2.3}$$

The two subprocesses that contribute this self-energy are depicted in Fig. 1.

The retarded and advanced quark and gluon propagators $G_{[\mathrm{ret}]}$ and $D^{lm}_{[\mathrm{adv}]}$ were found in papers [II] and [III] of this cycle and are connected with the commutators $G_{[0]}$ and $D_{[0]}$,

$$G_{[\mathrm{ret}]}(x_1,x_2)=\theta(\tau_1-\tau_2)G_{[0]}(x_1,x_2),$$

$$D^{lm}_{[\mathrm{adv}]}(x_2,x_1)=-\theta(\tau_1-\tau_2)D^{lm}_{[0]}(x_2,x_1)+D^{lm}_{L}(x_2,x_1), \tag{2.4}$$

where $D^{lm}_L(x_2,x_1)$ is the longitudinal part of the gluon propagator (governed by Gauss law), and it enters in Eq. (2.4) in such a way that the condition $D_{[\mathrm{ret}]}-D_{[\mathrm{adv}]}=D_{[0]}$ is satisfied and the noncausal longitudinal part of the propagator does not violate the causal properties of the commutator $D_{[0]}$. The correlators $G_{[1]}$ and $D_{[1]}$ include densities of vacuum states as well as the information about the occupation numbers (phase-space population). Eventually, we shall prove that an approximation of the boost-invariance (infinite rapidity plateau) is not corrupted by any kind of cutoffs (the vacuum part never is). Therefore, all correlators (G, D, and Σ) will depend on two times τ_1 and τ_2 separately, the difference of rapidities $\eta=\eta_1-\eta_2$, and the difference $\vec{r}=\vec{r}_1-\vec{r}_2$ of distances in xy plane. The latter is Fourier transformed to the transverse momentum dependence. In this mixed representation,

[2]The indices of the field correlators with the Keldysh contour ordering of the field operators (like $G_{[AB]}$) as well as the labels of their linear combinations (like $G_{[\mathrm{ret}]}$) are placed in square brackets.

$$
\begin{aligned}
\Sigma_{[\mathrm{ret}]}(\tau_1,\tau_2;\eta,\vec{p}_t)
&= \frac{ig^2}{2(2\pi)^2}\int d^2\vec{k}_t[t^a\gamma^m(\tau_1)G_{[\mathrm{ret}]}(\tau_1,\tau_2;\eta,\vec{p}_t+\vec{k}_t)\\
&\quad\times t^b\gamma^l(\tau_2)D^{ba}_{[1]lm}(\tau_2,\tau_1;-\eta,\vec{k}_t)\\
&\quad+t^a\gamma^m(\tau_1)G_{[1]}(\tau_1,\tau_2;\eta,\vec{p}_t+\vec{k}_t)\\
&\quad\times t^b\gamma^l(\tau_2)D^{ba}_{[\mathrm{adv}]lm}(\tau_2,\tau_1;-\eta,\vec{k}_t)],
\end{aligned}\tag{2.5}
$$

where $\gamma^\eta(\tau)=\gamma^3/\tau$. As has been shown in [I], all fermion correlators $G_{[\alpha]}$ can be decomposed as

$$
\begin{aligned}
G_{[\alpha]}(\tau_1,\tau_2;\eta,\vec{q}_t)
&=q_t[g^0_{[\alpha]}\gamma^0+g^3_{[\alpha]}\gamma^3]+g^T_{[\alpha]}q_r\gamma^r+ig^A_{[\alpha]}q_r\epsilon^{ru}\gamma^u\gamma^5\\
&=q_t[g^{L(+)}_{[\alpha]}\gamma^+ +g^{L(-)}_{[\alpha]}\gamma^-]+q_r\gamma^r\gamma^0[g^{T(+)}_{[\alpha]}\gamma^+\\
&\quad+g^{T(-)}_{[\alpha]}\gamma^-],
\end{aligned}\tag{2.6}
$$

where, for the sake of brevity, we denote $\vec{q}_t=\vec{p}_t+\vec{k}_t$. A similar decomposition takes place for the self-energy,

$$
\begin{aligned}
\Sigma_{[\mathrm{ret}]}(\tau_1,\tau_2;\eta,\vec{p}_t)&=\Sigma^0\gamma^0+\Sigma^3\gamma^3+\Sigma^Tq_r\gamma^r+i\Sigma^Ap_r\epsilon^{ru}\gamma^u\gamma^5\\
&=\Sigma^{L(+)}\gamma^+ +\Sigma^{L(-)}\gamma^-\\
&\quad+p_r\gamma^r\gamma^0[\Sigma^{T(+)}\gamma^+ +\Sigma^{T(-)}\gamma^-],
\end{aligned}\tag{2.7}
$$

and we obviously have

$$
g^{L(\pm)}_{[\alpha]}=\frac{1}{2}(g^0_{[\alpha]}\pm g^3_{[\alpha]}),\quad g^{T(\pm)}_{[\alpha]}=\frac{1}{2}(g^T_{[\alpha]}\pm g^A_{[\alpha]}),
$$

$$
\Sigma^{L(\pm)}=\frac{1}{2}(\Sigma^0\pm\Sigma^3),\quad \Sigma^{T(\pm)}=\frac{1}{2}(\Sigma^T\pm\Sigma^A).\tag{2.8}
$$

It becomes easier to analyze the various pieces of the quark self-energy if the gluon correlators $D_{[\alpha]lm}$ are taken in the form of the following decomposition,[3]

$$
D_{[\alpha]rs}=\left(\delta_{rs}-\frac{k_rk_s}{k_t^2}\right)\mathcal{D}^{(TE)}_{[\alpha]}+\frac{k_rk_s}{k_t^2}\mathcal{D}^{(2)}_{[\alpha]},
$$

$$
D_{[\alpha]\eta\eta}=\mathcal{D}^{(\eta\eta)}_{[\alpha]},\quad D_{[\alpha]r\eta}=\frac{k_r}{k_t^2}\mathcal{D}^{(r\eta)}_{[\alpha]},\quad D_{[\alpha]\eta s}=\frac{k_s}{k_t^2}\mathcal{D}^{(\eta s)}_{[\alpha]},\tag{2.9}
$$

where the first term in $D_{[\alpha]rs}$ is due to the transverse electric mode, and all invariants of $\mathcal{D}_{[\mathrm{adv}]}$ (except $\mathcal{D}^{(TE)}_{[\mathrm{adv}]}$) have two terms, $\mathcal{D}^{(\cdots)}_{[0]}$ from the transverse magnetic mode of the radiation field, and $\mathcal{D}^{(\cdots)}_{[long]}$ from the longitudinal field. All these components were found in paper [III] and are given in the Appendix in the form which is used in the calculation below. After some algebra, we can present the retarded quark self-energy in the form,

$$
\begin{aligned}
\Sigma_{[\mathrm{ret}]}(\tau_1,\tau_2;\eta,\vec{p}_t)=\frac{i\alpha_sC_F}{2\pi}\int d^2\vec{k}_t[\gamma^+S^{L(+)}+\gamma^-S^{L(-)}\\
+p_r\gamma^r\gamma^0(\gamma^+S^{T(+)}+\gamma^-S^{T(-)})],
\end{aligned}\tag{2.10}
$$

where the scalar invariants of $\Sigma_{[\mathrm{ret}]}$ are the bilinears of the fermion and gluon scalars,

$$
\begin{aligned}
S^{L(\pm)}=\sum_{[\alpha,\beta]}\Bigg\{&q_tg^{L(\pm)}_{[\alpha]}(\mathcal{D}^{(TE)}_{[\beta]}+\mathcal{D}^{(2)}_{[\beta]})+\frac{q_t}{\tau_1\tau_2}g^{L(\mp)}_{[\alpha]}\mathcal{D}^{(\eta\eta)}_{[\beta]}\\
&\mp\frac{(\vec{k}_t\vec{q}_t)}{k_t^2}\left(\frac{g^{T(\pm)}_{[\alpha]}\mathcal{D}^{(r\eta)}_{[\beta]}}{\tau_1}+\frac{g^{T(\mp)}_{[\alpha]}\mathcal{D}^{(\eta r)}_{[\beta]}}{\tau_2}\right)\Bigg\},
\end{aligned}\tag{2.11}
$$

$$
\begin{aligned}
S^{T(\pm)}=\sum_{[\alpha,\beta]}\Bigg\{&\left[\frac{(\vec{p}_t\vec{q}_t)}{p_t^2}-2\frac{(\vec{k}_t\vec{p}_t)(\vec{k}_t\vec{q}_t)}{k_t^2p_t^2}\right]g^{T(\pm)}_{[\alpha]}(\mathcal{D}^{(TE)}_{[\beta]}+\mathcal{D}^{(2)}_{[\beta]})\\
&-\frac{(\vec{q}_t\vec{p}_t)}{p_t^2\tau_1\tau_2}g^{T(\mp)}_{[\alpha]}\mathcal{D}^{(\eta\eta)}_{[\beta]}\\
&\mp\frac{(\vec{k}_t\vec{p}_t)}{k_t^2p_t^2}\left(\frac{g^{L(\pm)}_{[\alpha]}\mathcal{D}^{(r\eta)}_{[\beta]}}{\tau_1}-\frac{g^{L(\mp)}\mathcal{D}^{(\eta r)}_{[\beta]}}{\tau_2}\right)\Bigg\}.
\end{aligned}\tag{2.12}
$$

In these equations, the sum $\Sigma_{[\alpha,\beta]}$ runs over $[\alpha,\beta]=\{[\mathrm{ret},1],[1,\mathrm{adv}]\}$.

III. FERMION MODES IN THE EXPANDING SYSTEM

We shall look for the dispersion law of the fermions in the proper-time dynamics studying the Dirac equation (2.2) with radiative corrections. Since the fermions are massless, it is convenient to use the spinor basis where the Dirac matrices are

$$
\gamma^0=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix},\quad \gamma^l=\begin{pmatrix}0 & -\sigma^l\\ \sigma^l & 0\end{pmatrix},
$$

and the Dirac equation can be split into two separate equations for the left- and right-handed two-component spinors. The latter reads as

[3]In what follows, we use the Greek indices for the four-dimensional vectors and tensors in the curvilinear coordinates (index η is an exception, it always denotes the rapidity direction), and the Latin indices from a to d for the vectors in flat Minkowski coordinates. We use Latin indices from r to w for the transverse x and y components ($r,\ldots,w=1,2$), and the arrows over the letters to denote the two-dimensional vectors, e.g., $\vec{k}=(k_x,k_y)$, $|\vec{k}|=k_t$. The Latin indices from i to n $(i,\ldots,n=1,2,3)$ will be used for the three-dimensional internal coordinates $u^i=(x,y,\eta)$ on the hyper-surface $\tau=\mathrm{const}$.

$$G_R^{-1}(p_t;\tau_1,\eta_1)\psi_R(p_t;\tau_1,\eta_1)$$
$$=\int_0^{\tau_1}d\tau_2\int_{-\infty}^{\infty}\tau_2 d\eta_2\Sigma_R(p_t;\tau_1,\tau_2;\eta_1-\eta_2)$$
$$\times\psi_R(p_t;\tau_2,\eta_2), \tag{3.1}$$

where the matrices of the right-handed differential operator G_R^{-1} and of the right-handed self-energy Σ_R are

$$G_R^{-1}(p_t;\tau,\eta)=\begin{bmatrix} i\left(\partial_\tau+\frac{1}{2\tau}-\frac{1}{\tau}\partial_\eta\right) & p_x-ip_y \\ p_x+ip_y & i\left(\partial_\tau+\frac{1}{2\tau}+\frac{1}{\tau}\partial_\eta\right)\end{bmatrix},$$

$$\Sigma_R(p_t;\tau_1,\tau_2;\eta_1-\eta_2)$$
$$=\begin{bmatrix}\Sigma^{L(-)} & -(p_x-ip_y)\Sigma^{T(+)} \\ -(p_x+ip_y)\Sigma^{T(-)} & \Sigma^{L(+)}\end{bmatrix}. \tag{3.2}$$

The equation for the left-handed spinors differs from Eq. (3.1) only by a change of some signs in matrices (3.2) and leads to the same dispersion law. A solution with positive energy is looked for in the form

$$\psi_R(p_t,\theta;\tau,\eta)=\begin{pmatrix} e^{(\eta-\theta)/2}p_t \\ -e^{-(\eta-\theta)/2}(p_x+ip_y)\end{pmatrix}e^{-i\mu\tau\cosh(\eta-\theta)}, \tag{3.3}$$

where μ is the effective ''transverse mass'' of the mode. For the free on-mass-shell solution we have $\mu=p_t$. To solve Eq. (3.1), we introduce an auxiliary (left-handed) spinor

$$\tilde{\psi}(p_t,\theta';\tau,\eta)=\begin{pmatrix} e^{-(\eta-\theta')/2}p_t \\ -e^{(\eta-\theta')/2}(p_x-ip_y)\end{pmatrix}e^{i\mu\tau\cosh(\eta-\theta')}. \tag{3.4}$$

We insert Eq. (3.3) into Eq. (3.1), multiply it from the left by spinor (3.4) and integrate this along the hypersurface $\tau_1=\text{const}$. Then the left side of the equation becomes

$$\int_{-\infty}^{\infty}\tau_1 d\eta_1\tilde{\psi}(p_t,\theta';\tau_1,\eta_1)G_R^{-1}(p_t;\tau_1,\eta_1)\psi_R(p_t,\theta;\tau_1,\eta_1)$$
$$=4\pi\frac{\mu-p_t}{\mu}p_t^2\delta(\theta-\theta'). \tag{3.5}$$

In deriving this equation, we assumed that μ is independent of τ_1. The weak dependence is admissible, provided $d\mu/d\tau_1\ll\mu/\tau_1$. A solution that has this property does indeed exist. The right-hand side of the equation is, in fact, independent of θ' and is of the following form:

$$p_t^2\int_0^{\tau_1}d\tau_2\int_{-\infty}^{\infty}\tau_1\tau_2 d\eta_2 d\theta d(\eta_1-\eta_2)$$
$$\times e^{i\mu\tau_1\cosh(\eta_1+\theta)}e^{-i\mu\tau_2\cosh\eta_2}[e^{-(\eta_1-\eta_2+\theta)/2}\Sigma^{(L)-}$$
$$+e^{(\eta_1-\eta_2+\theta)/2}\Sigma^{(L)+}+e^{-(\eta_1+\eta_2+\theta)/2}\Sigma^{(T)+}$$
$$+e^{(\eta_1+\eta_2+\theta)/2}\Sigma^{(T)-}], \tag{3.6}$$

where the exponentials are due to the Thomas precession of the spinor field. Next, we integrate both sides with respect to θ. Two rapidity integrals, $d\theta d\eta_2$, on the right absorb the precession factors yielding the product of Hankel functions,

$$\pi^2 H_{1/2}^{(1)}(\mu\tau_1)H_{1/2}^{(2)}(\mu\tau_2)=\frac{2\pi}{\mu\sqrt{\tau_1\tau_2}}e^{i\mu(\tau_1-\tau_2)}. \tag{3.7}$$

Finally, we arrive at the dispersion equation that defines the fermion ''transverse mass'' μ as a function of the transverse momentum and the latest time τ_1,

$$\mu(p_t,\tau_1)-p_t$$
$$=\frac{1}{2}\int_0^{\tau_1}d\tau_2\sqrt{\tau_1\tau_2}e^{i\mu(p_t,\tau_1)(\tau_1-\tau_2)}\int_{-\infty}^{\infty}d\eta$$
$$\times[\Sigma^{L(+)}+\Sigma^{L(-)}+p_t\Sigma^{T(+)}+p_t\Sigma^{T(-)}]. \tag{3.8}$$

As has been discussed in paper [II] for fermions (similar arguments are true for gluons), only the independence of the quark and gluon occupation numbers n_f and n_g on rapidity can provide that the invariants $S^{L(\pm)}$ and $S^{T(\pm)}$ naturally depend only on the difference $\eta=\eta_1-\eta_2$. We shall consider only this case of the local homogeneity; we can do it safely only because no collinear singularities which may require a rapidity cutoff (e.g., $\pm Y$) in the phase space will appear in the theory. Since we are computing an essentially local quantity, such a cutoff would be unphysical. With this reservation, we may rewrite Eq. (3.8) as

$$\mu(p_t)=p_t+\int_0^{\tau_1}d\tau_2\sqrt{\tau_1\tau_2}e^{i\mu(p_t)(\tau_1-\tau_2)}[\Sigma^0(\tau_1,\tau_2)$$
$$+p_t\Sigma^T(\tau_1,\tau_2)], \tag{3.9}$$

where we introduced the notation,

$$\Sigma^0(\tau_1,\tau_2)=\frac{i\alpha_s C_F}{4\pi}\sum_{[\alpha,\beta]}\int d^2\vec{k}_t\int_{-\infty}^{\infty}d\eta q_t g_{[\alpha]}^0$$
$$\times\left[\mathcal{D}_{[\beta]}^{(TE)}+\mathcal{D}_{[\beta]}^{(2)}+\frac{1}{\tau_1\tau_2}\mathcal{D}_{[\beta]}^{(\eta\eta)}\right], \tag{3.10}$$

$$\Sigma^T(\tau_1,\tau_2)=\frac{i\alpha_s C_F}{4\pi}\sum_{[\alpha,\beta]}\int d^2\vec{k}_t\int_{-\infty}^{\infty}d\eta g^T_{[\alpha]}\times\Bigg\{\Bigg[\frac{(\vec{p}_t\vec{q}_t)}{p_t^2}-2\frac{(\vec{k}_t\vec{p}_t)(\vec{k}_t\vec{q}_t)}{k_t^2p_t^2}\Bigg](\mathcal{D}^{(TE)}_{[\beta]}+\mathcal{D}^{(2)}_{[\beta]})+\frac{(\vec{q}_t\vec{p}_t)}{p_t^2\tau_1\tau_2}\mathcal{D}^{(\eta\eta)}_{[\beta]}\Bigg\}. \tag{3.11}$$

Comparing these equations with Eqs. (2.11) and (2.12), we may observe a significant simplification. The terms with the off-diagonal components $\mathcal{D}^{(\eta r)}$ and $\mathcal{D}^{(r\eta)}$ have dropped out. These terms, as it can be seen from Eqs. (A6), (A7), (A10), (A11), and (A18), (A19), are odd with respect to η, while the invariants $g^0=g^{L(+)}+g^{L(-)}$ and $g^T=g^{T(+)}+g^{T(-)}$ are even. Therefore, integration over η eliminates the terms with the off-diagonal components.

IV. PROPAGATORS, DENSITIES OF STATES, AND OCCUPATION NUMBERS IN THE EXPANDING SYSTEM

In this section, we collect condensed information about various correlators of quark and gluon fields derived in papers [II] and [III] which are necessary for the calculation of the quark self-energy. We also discuss our specific choice of occupation numbers $n_g(k_t,\alpha)$ and $n_f(q_t,\theta)$. All field correlators are defined as the expectation values over the distribution of the background particles. The latter are the excitations of the modes allowed by the constraints and the boundary conditions of wedge dynamics. The Fock space of these excitations was constructed in papers [II] and [III]. We have analyzed two sets of quantum numbers that may label the states. Both sets include the transverse momentum $\vec{p}_t$ and polarization index. In one set, the remaining variable was the boost ν (the variable conjugated to the coordinate η); this set proved to be very useful in the practical calculation of the gluon propagators. In the second set, the particles are labeled by their velocity $v_z=\tanh\theta$ in the direction of the collision axis. This representation is used below. The fermion spectral functions are

$$\begin{aligned}G_{[10]}(q_t,\theta;\tau_1,\tau_2)&=[1-n_f^+(q_t,\theta)]G^{(0)}_{[10]}(q_t,\theta;\tau_1,\tau_2)\\&\quad-n_f^-(q_t,\theta)G^{(0)}_{[01]}(q_t,\theta;\tau_1,\tau_2),\\G_{[01]}(q_t,\theta;\tau_1,\tau_2)&=-n_f^+(q_t,\theta)G^{(0)}_{[10]}(q_t,\theta;\tau_1,\tau_2)\\&\quad+[1-n_f^-(q_t,\theta)]G^{(0)}_{[01]}(q_t,\theta;\tau_1,\tau_2).\end{aligned} \tag{4.1}$$

Their gluon counterparts are of a similar form,

$$\begin{aligned}D_{[10]}(k_t,\alpha;\tau_1,\tau_2)&=[1+n_g(k_t,\alpha)]D^{(0)}_{[10]}(k_t,\alpha;\tau_1,\tau_2)\\&\quad+n_g(q_t,\alpha)D^{(0)}_{[01]}(q_t,\alpha;\tau_1,\tau_2),\\D_{[01]}(k_t,\alpha;\tau_1,\tau_2)&=n_g(k_t,\alpha)D^{(0)}_{[10]}(k_t,\alpha;\tau_1,\tau_2)\\&\quad+[1+n_g(q_t,\alpha)]D^{(0)}_{[01]}(q_t,\alpha;\tau_1,\tau_2),\end{aligned} \tag{4.2}$$

where $D^{(0)}_{[\alpha]}$ and $G^{(0)}_{[\alpha]}$ are the vacuum correlators of a given type $[\alpha]$. They are defined as vacuum expectation values of the binary products of field operators,

$$\begin{aligned}G^{(0)}_{[10]}(x_1,x_2)&=-i\langle 0|\Psi(x_1)\overline{\Psi}(x_2)|0\rangle,\\G^{(0)}_{[01]}(x_1,x_2)&=i\langle 0|\overline{\Psi}(x_2)\Psi(x_1)|0\rangle,\\D^{(0)}_{[10]lm}(x_1,x_2)&=-i\langle 0|A_l(x_1)A(x_2)_m|0\rangle,\\D^{(0)}_{[01]lm}(x_1,x_2)&=-i\langle 0|A_m(x_2)A_l(x_1)|0\rangle.\end{aligned} \tag{4.3}$$

In this approximation, the field (anti)commutators,

$$\begin{aligned}G_{[0]}&=G_{[10]}-G_{[01]}=G^{(0)}_{[10]}-G^{(0)}_{[01]}=G^{(0)}_{[0]},\\D_{[0]}&=D_{[10]}-D_{[01]}=D^{(0)}_{[10]}-D^{(0)}_{[01]}=D^{(0)}_{[0]}\end{aligned} \tag{4.4}$$

appear to be insensitive to the presence of the particle distribution, while their counterparts,

$$\begin{aligned}G_{[1]}&=G_{[10]}+G_{[01]}=[1-2n_f]G^{(0)}_{[1]},\\D_{[1]}&=D_{[10]}+D_{[01]}=[1+2n_g]D^{(0)}_{[1]},\end{aligned} \tag{4.5}$$

include the occupation numbers which modify the original vacuum density of states. For the sake of simplicity, we take $n_f^+=n_f^-=n_f$, which corresponds to a neutral system.

The Wightman functions (4.1) and (4.2) (or their various linear combinations $G_{[\beta]}$ and $D_{[\beta]}$) eventually appear under the integrals $d\theta$ and $d\alpha$. One must keep in mind that in order to reduce $G^{(0)}_{[\beta]}$ and $D^{(0)}_{[\beta]}$ to the standard form of the vacuum correlators, at least two shifts of the integration variables is necessary. Only after that will $G^{(0)}_{[\beta]}$ and $D^{(0)}_{[\beta]}$ explicitly depend on the boost-invariant variables η and τ_{12}. The functions $n_g(k_t,\alpha)$ and $n_f(q_t,\theta)$ are not indifferent to this shift. It may well happen that a formal shift in θ or α will drive the stationary points of the wave functions or the singularities of the field correlators outside the physical boundaries of the distributions $n_g(k_t,\alpha)$ and $n_f(q_t,\theta)$. Therefore, different representations of $G_{[1]}$ and $D_{[1]}$ must be used for the study of different subprocesses. One has to account for the reservations stemming from the derivation procedure described in Sec. IV of paper [II]. These different representations of the quark and gluon correlators are quoted in the Appendix.

In our picture, first outlined in paper [I], the fermion vacuum mode with small transverse momentum p_t and zero rapidity is modified by its forward scattering either on gluons with high momentum k_t and rapidity α, $k_t\gg p_t$, or on quarks with high momentum q_t and rapidity θ, $q_t\gg p_t$. These hard modes are created at the earliest moment of the collision and can be treated as well formed particles by the time $\tau\sim 1/p_t$, since at that time $\tau k_t\gg 1$, and $\tau q_t\gg 1$. Therefore, they may be consistently described by the distributions

$$n_f(q_t,\theta)\approx\frac{\mathcal{N}_f}{\pi R_\perp^2}\,\frac{\theta(q_t-p_*)}{q_t^2},$$

$$n_g(k_t,\alpha)\approx\frac{\mathcal{N}_g}{\pi R_\perp^2}\,\frac{\theta(k_t-p_*)}{k_t^2},\tag{4.6}$$

where p_* is the lower bound of the ''hard'' partons distribution. Both distributions (per unit area, per unit rapidity) are chosen on purely dimensional grounds, since we believe that the creation of a parton with large transverse momentum is described by perturbative QCD which has no intrinsic scale.

Currently, the normalization factors $\mathcal{N}_g$ and $\mathcal{N}_f$ are the only (apart from the coupling α_s) parameters of the theory. The cross section $\pi R_\perp^2$ and the full width $2Y$ of the rapidity plateau are defined by the geometry of a particular collision and the c.m.s. energy, respectively. These are irrelevant for the local screening parameters we are interested in. In the first approximation, one may try to extract them from the event-by-event measurement of the high-p_t tail of the collision products and incorporating the standard phenomenology of the fragmentation functions for the analysis.

As was pointed out in paper [I], even in dense systems, the QCD evolution at large Q^2 is not likely to be affected by finite-density effects. Thus, one may also try to employ the known structure functions (without shadowing corrections) and the factorization scheme in order to estimate $\mathcal{N}_g$ and $\mathcal{N}_f$. A most appealing opportunity to find $n_g(k_t,\alpha)$ and $n_f(q_t,\theta)$ from first principles, associating them with the known properties of hadrons and the QCD vacuum, is still very distant.

The distributions (4.6) are used below with the following informal reservations. First, the total energy of any collision is finite and k_t and q_t have (though very high, but finite) upper boundary. Eventually, this leads to the self-energy which is free from collinear singularities in the interaction of charges with the vector gauge field. Second, though the distributions (4.6) are boost-invariant, only the particles which physically affect the forward scattering must be accounted for. There is a strong correlation between the position η where the particle with large transverse momentum q_t is measured (or is interacting) and its rapidity θ. Hence, the limits of integrals $d\alpha$ and $d\theta$ over the rapidities of real particles (which either mediate the scattering or are in the final states) cannot exceed the actual rapidity boundaries of the scattering process. In its turn, this puts an additional requirement on the notion of the distribution itself. It must be normalizeable in the physical volume of the reaction. This volume is defined, in fact, by the light cone (i.e., causality of the forward scattering amplitude). [We remind the reader that the notion of a distribution itself makes sense only after it is prepared (measured) at least in a *gedanken* experiment. Hence, the distributions n_g and n_f must exist, in this sense, both at final time τ_1 and at the initial time τ_2 in the expression for the self-energy. In its turn, this limits the time τ_2 from below.]

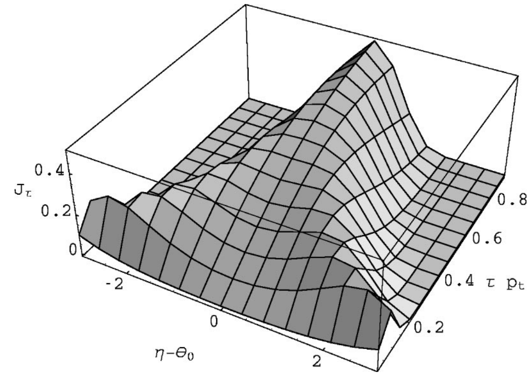

FIG. 2. Evolution of the charge density in the typical state of wedge dynamics.

V. LEADING PART OF THE DISPERSION EQUATION

A. Derivation of the dispersion equation

The most important outcome of this work is that the major contribution to the effective quark mass comes from the $\eta\eta$-component of the propagator of the longitudinal field. This contribution is computed in all details below. All other terms are associated with the propagation of the transverse fields and they appear to be parametrically small in the domain $\tau_1 p_t<1$, $(\tau_1-\tau_2)p_t\ll 1$, $\tau_1-\tau_2\ll\tau_1$, where the dynamical mass of the fermion is effectively formed. The component $D_{\eta\eta}$ of the propagator establishes the connection between the A_η component of the potential and the j_η component of the current. In its turn, A_η is responsible for the η-component $E_\eta=\partial_\tau A_\eta$ of the electric field and the x and y components, $B_x=\partial_y A_\eta$, $B_y=-\partial_x A_\eta$ of the magnetic field. The electrical field in the longitudinal η direction is not capable of producing scattering with transverse momentum transfer. However, this transfer can be provided by the magnetic forces; the two currents j_η can interact via the magnetic field $\vec{B}_t=(B_x,B_y)$. The origin of these currents is intrinsically connected with the geometry of states in the wedge form of dynamics. Any state with a given p_t begins its life being widely spread along the light cone. If the state is charged, then local charge density is small. With time going on, the spread of the wave function diminishes and the charge become localized in a narrower rapidity interval (see Fig. 2). Therefore, any charged state carries a current in the longitudinal (rapidity) direction. The magnetic fields of the *transition currents* provide scattering with the most effective transfer of the transverse momentum. Indeed, at time τ_2 a quark with the transverse momentum $p_t,\tau_2 p_t\ll 1$, interacts with the gluon field and acquires a large transverse momentum $k_t,\tau_2 k_t\gg 1$. This transition is characterized by a drastic narrowing of the charge spread in the rapidity direction, and must be accompanied by a strong η-component of the transition current. A similar transition in the opposite direction happens at time τ_1, when the gluon field interacts with another quark that has large initial transverse momentum k_t, and recovers the soft state with $\tau_2 p_t\ll 1$ in the course of this interaction. This second transition current readily interacts with the magnetic component of the gluon field. Our estimates indicate that the leading contribution comes from the term of $D_{\eta\eta}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)$ which is proportional to $\delta(\eta_1-\eta_2)$ and does not depend on $\vec{k}_t$ [in coordinate representation, this term is just proportional to $\delta(\eta_1-\eta_2)\delta(\vec{r}_1-\vec{r}_2)$]. This is a long-range contact interaction of the two currents, and is not limited by the light-cone boundaries (which suppress the interaction via the strongly localized states of the radiation field). Furthermore, the contact part of the longitudinal propagator is the only one that brings into the integrand of the dispersion equation (3.9) the term which is *singular* at $\tau_1-\tau_2\to 0$. Therefore, it is capable of providing an appreciable contribution into the effective quark mass, which is *defined locally*. This part of the self-energy allows for an exact calculation with a simple analytic answer which is presented below. Estimates of all small terms are explained in Secs. VII and VIII.[4] This contact part of the longitudinal propagator is

$$D_{\eta\eta}^{[contact]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)=\frac{\tau_1^2-\tau_2^2}{2}\delta(\eta). \tag{5.1}$$

Because of the extreme locality of $D_{\eta\eta}^{[contact]}$ provided by $\delta(\eta)$, the invariants $g_{[1]}$ of the fermion density function in Eq. (3.10) lose their kinematic coefficients,

$$g_{[1]}^0=\frac{-2\mathcal{N}_f}{\pi R_t^2}\frac{Y_1(\tau_{12}q_t)}{q_t^2},\quad g_{[1]}^T=\frac{-2\mathcal{N}_f}{\pi R_t^2}\frac{Y_0(\tau_{12}q_t)}{q_t^2}. \tag{5.2}$$

First, we integrate over η, which leads to $\tau_{12}^2=(\tau_1-\tau_2)^2$. Next, we change $d^2\vec{k}_t$ for $d^2\vec{q}_t$ and integrate over the orientation of $\vec{q}_t$ gaining the factor 2π in Σ^0. In Σ^T, $g_{[1]}^T$ is integrated with the weight factor $(\vec{q}_t\cdot\vec{p}_t)/p_t^2$. Therefore, this term identically vanishes after integration over the azimuthal angle. The only remaining integral over the transverse momenta of hard partons is

$$\int_{p_*}^{\infty}Y_1[(\tau_1-\tau_2)q_t]dq_t=\frac{Y_0[(\tau_1-\tau_2)p_*]}{\tau_1-\tau_2}. \tag{5.3}$$

Eventually, we may write the dispersion equation (3.9) as follows:

$$\mu=p_t+\frac{i\alpha_s C_F\mathcal{N}_f}{2\pi R_t^2}\int_0^{\tau_1}d\tau_2\frac{\tau_1+\tau_2}{2\sqrt{\tau_1\tau_2}}\times e^{i\mu(\tau_1-\tau_2)}Y_0[(\tau_1-\tau_2)p_*]. \tag{5.4}$$

B. Study of the dispersion equation

According to the qualitative analysis of paper [II], the dynamics of states is different in the two limiting cases, $\tau p_t<1$ and $\tau p_t>1$. With respect to $p_t\sim 1/\tau$, the states are divided into ''hard'' and ''soft'' states. Therefore, it is natural to take $p_*\sim p_t$ in Eq. (5.4). Further, it is convenient to trade variable τ_2 for $y=(\tau_1-\tau_2)/\tau_1$,

$$\frac{\mu(p_t,\tau_1)}{p_t}=1+\frac{i\alpha_s C_F\mathcal{N}_f\tau_1 p_t}{2\pi R_t^2 p_t^2}\int_0^1 dy\frac{1-y/2}{\sqrt{1-y}}\times e^{i\mu(p_t,\tau_1)\tau_1 y}Y_0(\tau_1 p_t y). \tag{5.5}$$

In this form, the dispersion equation clearly reveals two distinctive regimes. When $\tau p_t<1$, then the function $Y_0(x)$ behaves as a logarithm, and the right-hand side of Eq. (5.5) becomes proportional to $\ln(2/\tau_1 p_t)$, the effective width of the rapidity interval occupied by the state at the early time of the evolution. When $\tau p_t>1$, then $Y_0(x)\sim 1/\sqrt{x}$, and the integral becomes proportional to $1/\sqrt{\tau p_t}$, the effective rapidity width at later times. Thus, the dispersion equation (5.5) clearly reveals two distinctive regimes which were qualitatively analyzed in paper [II]. The solution of Eq. (5.5) is generally complex. Taking $\mu=\mu'+i\mu''$ we can separate real and imaginary parts of this equation,

$$\tau_1\mu'-\tau_1 p_t=-\frac{\alpha_s C_F\mathcal{N}_f}{2\pi(R_t^2/\tau_1^2)}\int_0^1\frac{dy}{2}\left[\frac{1}{\sqrt{1-y}}+\sqrt{1-y}\right]\times e^{-\mu''\tau_1 y}\sin(\mu'\tau_1 y)Y_0(\tau_1 p_t y), \tag{5.6}$$

$$\tau_1\mu''=\frac{\alpha_s C_F\mathcal{N}_f}{2\pi(R_t^2/\tau_1^2)}\int_0^1 dy\left[\frac{1}{\sqrt{1-y}}+\sqrt{1-y}\right]\times e^{-\mu''\tau_1 y}\cos(\mu'\tau_1 y)Y_0(\tau_1 p_t y). \tag{5.7}$$

(The unit upper limit in these integrals corresponds to $\tau_2=0$, and is, as a matter of fact, fictitious. Practically, we are interested only in the domain where $\tau_2 p_t\sim 1$.) We have rearranged the factor in front of the integral in such a way, that at early times, this factor is small. It has been shown in paper [III], that the longitudinal part of the gluon propagator vanishes when the distance r_t exceeds τ_1. Therefore, this factor is proportional to the (small) number of hard partons per transverse area occupied by the soft quark mode. Hence, we can analyze Eqs. (5.6) and (5.7) by successive approximations. It is clear, that in the lowest approximation, we can take $\mu'=p_t$ in the RHS of these equations, and that the imaginary part μ'' can be neglected. Using

$$\sin x\approx x,\quad \cos x\approx 1,\quad Y_0(x)\approx 2\pi^{-1}[\gamma_E+\ln x],$$

as an approximation, and computing the remaining integrals, we arrive at

[4]The authors appreciate discussions with Edward Shuryak, who pointed out that the small effect of the radiation fields is much less surprising than the finite contribution from the longitudinal fields.

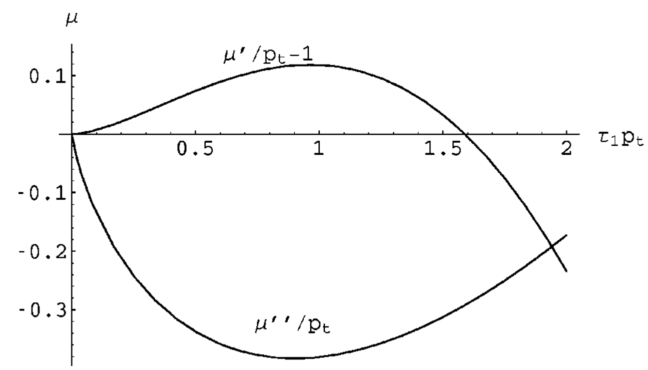

FIG. 3. Self-energy corrections to the real part (μ'/p_t-1, upper curve) and to the imaginary part (μ''/p_t, lower curve) of the effective transverse mass as functions of $\tau_1 p_t$.

$$\frac{\mu'}{p_t}-1=\frac{\alpha_s C_F \mathcal{N}_f}{\pi^2 R_t^2 p_t^2}(\tau_1 p_t)^2 \times\left[\frac{4}{5}\left(-\gamma_E+\ln\frac{2}{\tau_1 p_t}\right)+\frac{104-120\ln 2}{75}\right], \tag{5.8}$$

$$\frac{\mu''}{p_t}=\frac{\alpha_s C_F \mathcal{N}_f}{\pi^2 R_t^2 p_t^2}\tau_1 p_t\left[\frac{4}{3}\left(\gamma_E-\ln\frac{2}{\tau_1 p_t}\right)+\frac{26-24\ln 2}{9}\right]. \tag{5.9}$$

These dependences are plotted n Fig. 3 as functions of the argument $\tau_1 p_t$ up to the prefactor $\alpha_s C_F \mathcal{N}_f/\pi^2 R_t^2 p_t^2$.

Equation (5.5) describes the evolution of the effective transverse mass μ of the state with a given transverse momentum p_t as a function of the proper time τ_1. We see, that the real part μ' gradually grows with time reaching its maximum at $\tau_1 p_t \approx 1$. The mode acquires an ''adjoint mass'' due to the interaction with hard partons, as was anticipated. The curves cannot be trusted above the boundary $\tau_1 \approx p_t^{-1}$, since at later times, the mode becomes ''hard.'' It cannot be viewed as a soft cloud swept with uniformly distributed hard particles. The condition

$$\frac{d\mu}{d\tau_1} \ll \frac{\mu}{\tau_1},$$

which was used in the course of the dispersion equation derivation, is clearly fulfilled near the maximum of the dispersion curve.

One more important dependence is hidden in the prefactor $\alpha_s C_F \mathcal{N}_f/\pi^2 R_t^2 p_t^2$, and is not visible from the figure above. This factor scales as p_t^{-2}, clearly indicating that at large p_t, the effect of screening is small. There is almost no hard particles with $k_t, q_t > p_t$.

VI. CANCELATION OF COLLINEAR TERMS IN THE VACUUM PART OF THE SELF-ENERGY

Usually, the self-energy Σ is studied in the momentum representation, and the first subject of concern is the ultraviolet divergence of this function. It is well known that this divergence can be at most logarithmic. Thus, when we compute the four-dimensional integral over the momentum in the loop, this divergence can show up only in the last of these integrations. We compute the self-energy in the mixed representation. Hence, we cannot see the UV divergence explicitly but we *must* already see (e.g., in ImΣ) the various infrared divergences which emerge due to real processes with massless fields. A corresponding analysis for the case of the null-plane dynamics was done in paper [I]. These divergences must be regularized (or even removed from the theory, as is done by means of dimensional regularization) before the UV renormalization. The primary goal of this section is to demonstrate that in the wedge form of dynamics the quark self-energy is completely protected from collinear problems, and that this is not a surprise. Indeed, in the theory with massless fermions and gauge bosons, the infrared singularities show up in a different way depending on the type of Hamiltonian dynamics (including the gauge condition) which is used to describe the process. In the gauge $A^+=0$ they look like collinear divergences. In the gauge $A^0=0$, they look like an infrared problem of the proper field of the charged particle. In both cases, the problem emerges due to the incomplete gauge fixing, and manifests itself through spurious poles of the gauge field propagators. As has been shown in paper [III], the gauge $A^\tau=0$ is fixed completely, and therefore, the quark self-energy that we compute here is totally free of these problems.

In order to demonstrate this appealing feature we shall compute (the most dangerous in this respect) the vacuum part of the fermion self-energy, concentrating on the terms where the integrand as a function of the rapidity α is not suppressed at $|\alpha|\to\infty$. A self-consistent piece of this type is the contribution of the transverse electric mode of the gluon field. The tensor part of any gluon correlator for this mode is of a very simple form,

$$D^{TE}_{[\alpha]rs}=(\delta_{rs}-k_r k_s/k_t^2)\mathcal{D}^{TE}_{[\alpha]}; \tag{6.1}$$

it has no η components, and the scalar functions $\mathcal{D}^{TE}_{[\alpha]}$ can be computed exactly since they have simple integral representations. We use this piece of the self-energy to explain the principles we base our calculations on. Using Eqs. (3.10) and (3.11), we get

$$[\Sigma^{L(\pm)}_{[\mathrm{ret}]}]^{TE}_{vac}(\tau_1,\tau_2;\eta,\vec{p}_t) =\frac{i\alpha_s C_F}{2\pi}\theta(\tau_{12}^2)\theta(\tau_1-\tau_2)\int d^2\vec{k}_t[q_t g^{L(\pm)}_{[0]}(q_t) \times\mathcal{D}^{TE}_{[1]}(k_t)-q_t g^{L(\pm)}_{[1]}(q_t)\mathcal{D}^{TE}_{[0]}(k_t)], \tag{6.2}$$

where $\vec{q}_t=\vec{k}_t+\vec{p}_t$ and the minus sign in the second term is due to the definition (2.4) of $D_{[\mathrm{adv}]}$. The $\Sigma^{TE}_{[\mathrm{ret}]}$ is fully confined within the light wedge $\tau_{12}^2>0$. Then, the vacuum quark and gluon correlators have the following form:

$$\mathcal{D}^{TE}_{[0]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)=2^{-1}\theta(\tau_{12}^2)J_0(\tau_{12}k_t),$$

$$\mathcal{D}^{TE}_{[1]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)=2^{-1}iY_0(\tau_{12}k_t), \tag{6.3}$$

$$g^{L(\pm)}_{[0]} = i\,\frac{\tau_1 e^{\mp\eta/2} - \tau_2 e^{\pm\eta/2}}{4\sqrt{|\tau_{12}^2|}}\,\theta(\tau_{12}^2) J_1(q_t\sqrt{|\tau_{12}^2|}),$$

$$g^{L(\pm)}_{[1]} = \frac{\tau_1 e^{\mp\eta/2} - \tau_2 e^{\pm\eta/2}}{4\sqrt{|\tau_{12}^2|}}\,Y_1(q_t\sqrt{|\tau_{12}^2|}), \qquad (6.4)$$

where $\tau_{12}^2 = \tau_1^2 + \tau_2^2 - 2\tau_1\tau_2 \cosh\eta$, and the commutators $D^{TE}_{[0]}$ and $G_{[0]}$ include the causal $\theta(\tau_{12}^2)$ by definition, and the terms proportional to $\theta(-\tau_{12}^2)$ in the densities $D^{TE}_{[1]}$ and $G_{[1]}$ are omitted. Integration over the angle φ between the vectors $\vec{p}_t$ and $\vec{k}_t$ involves only the invariants $g_{[\alpha]}$. According to Eqs. (6.2) and (6.4), we have to integrate

$$\int_0^{2\pi} q_t J_1(\tau_{12}q_t)\,d\varphi = 2\pi[k_t J_0(\tau_{12}p_t) J_1(\tau_{12}k_t) + p_t J_1(\tau_{12}p_t) J_0(\tau_{12}k_t)],$$

$$\int_0^{2\pi} q_t Y_1(\tau_{12}q_t)\,d\varphi = 2\pi\{\theta(k_t - p_t)[k_t J_0(\tau_{12}p_t) Y_1(\tau_{12}k_t) + p_t J_1(\tau_{12}p_t) Y_0(\tau_{12}k_t)] + \theta(p_t - k_t)[k_t J_1(\tau_{12}k_t) Y_0(\tau_{12}p_t) + p_t Y_1(\tau_{12}p_t) J_0(\tau_{12}k_t)]\}. \qquad (6.5)$$

This integration is done with the aid of the so called addition theorems [9] for Bessel functions of the argument $q_t = [k_t^2 + p_t^2 + 2k_t p_t \cos\varphi]^{1/2}$. Starting from this point, we can continue in two ways. The most straightforward option is to use the gluon correlators in the integrated form of the Bessel functions, thus sweeping under the rug the singular behavior stemming from $\alpha \to \infty$. This leads to the integral

$$\begin{aligned}
[\Sigma^{L(\pm)}_{[\mathrm{ret}]}]^{TE} = {}& \frac{i\alpha_s C_F}{8}\,\theta(\tau_{12}^2)\,\theta(\tau_1 - \tau_2)\frac{\tau_1 e^{\mp\eta/2} - \tau_2 e^{\pm\eta/2}}{\tau_{12}}\Bigg[-J_0(\tau_{12}p_t)\int_0^\infty k_t^2 [J_1(\tau_{12}k_t) Y_0(\tau_{12}k_t) + J_0(\tau_{12}k_t) Y_1(\tau_{12}k_t)]dk_t \\
& - 2p_t J_1(\tau_{12}p_t)\int_0^\infty k_t J_0(\tau_{12}k_t) Y_0(\tau_{12}k_t)\,dk_t + p_t J_1(\tau_{12}p_t)\int_0^{p_t} k_t J_0(\tau_{12}k_t) Y_0(\tau_{12}k_t)\,dk_t \\
& + J_0(\tau_{12}p_t)\int_0^{p_t} k_t^2 J_0(\tau_{12}k_t) Y_1(\tau_{12}k_t)\,dk_t - p_t Y_1(\tau_{12}p_t)\int_0^{p_t} k_t J_0(\tau_{12}k_t) J_0(\tau_{12}k_t)\,dk_t \\
& - Y_0(\tau_{12}p_t)\int_0^{p_t} k_t^2 J_0(\tau_{12}k_t) J_1(\tau_{12}k_t)\,dk_t\Bigg].
\end{aligned} \qquad (6.6)$$

Here, all the integrals can be computed explicitly as indefinite integrals. The integrals from 0 to p_t yield the regular part of the answer below. Taking the upper limit of improper integrals to be $\Lambda \to \infty$, we get the singular part as the limit,

$$\lim_{\Lambda\to\infty} \frac{\Lambda^2}{\tau}[[J_0(\tau_{12}p_t) + \tau p_t J_1(\tau_{12}p_t)] J_1(\tau\Lambda) Y_1(\tau\Lambda) + \tau p_t J_1(\tau_{12}p_t) J_0(\tau\Lambda) Y_0(\tau\Lambda)]. \qquad (6.7)$$

Using the asymptotic expansion of Bessel functions, we find that the singular part is built from $\delta(\tau_{12})$ and its derivative. The full answer reads as

$$\begin{aligned}
[\Sigma^{L(\pm)}_{[\mathrm{ret}]}]^{TE} = {}& -\frac{i\alpha_s C_F}{16\pi}\,\theta(\tau_{12}^2)\,\theta(\tau_1 - \tau_2)\frac{\tau_1 e^{\mp\eta/2} - \tau_2 e^{\pm\eta/2}}{\tau_{12}} \\
& \times\left\{ p_t^2 \frac{J_2(\tau_{12}p_t)}{\tau_{12}} + \pi\left[\frac{J_0(\tau_{12}p_t)}{\tau_{12}}\left(\delta'(\tau_{12}) - \frac{2}{\tau_{12}}\delta(\tau_{12})\right) - \frac{4p_t J_1(\tau_{12}p_t)}{\tau_{12}}\delta(\tau_{12})\right]\right\}.
\end{aligned} \qquad (6.8)$$

Thus, the answer is singular at the null planes $\tau_{12}^2 = 0$. In order to find the true source of this singularity, we shall proceed in a different manner, keeping the gluon invariants $g^{L(\pm)}$ in the integral form. We shall integrate over k_t first and leave the integral over the gluon rapidity α for the end of calculation. This leads to

$$[\Sigma^{L(\pm)}_{[\mathrm{ret}]}]^{TE}=\frac{i\alpha_s C_F}{8}\,\theta(\tau_{12}^2)\,\theta(\tau_1-\tau_2)\frac{\tau_1 e^{\mp\eta/2}-\tau_2 e^{\pm\eta/2}}{\tau_{12}}$$
$$\times\Bigg\{-\frac{p_t^2}{2\pi}\,\frac{J_2(\tau_{12}p_t)}{\tau_{12}}+\int_{-\infty}^{\infty}\frac{d\alpha}{\pi}\Bigg[J_0(\tau_{12}p_t)\Bigg(\int_0^{\infty}k_t^2J_1(\tau_{12}k_t)\cos(T_{12}k_t)dk_t-\int_0^{\infty}k_t^2Y_1(\tau_{12}k_t)\sin(T_{12}k_t)dk_t\Bigg)$$
$$+p_tJ_1(\tau_{12}p_t)\Bigg(\int_0^{\infty}k_tJ_0(\tau_{12}k_t)\cos(T_{12}k_t)dk_t-\int_0^{\infty}k_tY_0(\tau_{12}k_t)\sin(T_{12}k_t)dk_t\Bigg)\Bigg]\Bigg\},\qquad(6.9)$$

where we remind the reader that $T_{12}=T_1-T_2=\tau_{12}\cosh(\alpha-\psi)$. The integrals in this expression are the well-known Fourier transforms of the Bessel functions,

$$\int_0^{\infty}k_t^2J_1(\tau_{12}k_t)\cos(T_{12}k_t)dk_t=\int_0^{\infty}k_t^2Y_1(\tau_{12}k_t)\sin(T_{12}k_t)dk_t=3T_{12}\tau_{12}[T_{12}^2-\tau_{12}^2]_+^{-5/2},$$
$$\int_0^{\infty}k_tJ_0(\tau_{12}k_t)\cos(T_{12}k_t)dk_t=\int_0^{\infty}k_tY_0(\tau_{12}k_t)\sin(T_{12}k_t)dk_t=-T_{12}[T_{12}^2-\tau_{12}^2]_+^{-3/2},\qquad(6.10)$$

where the distribution x_+^{λ} is defined in a standard way with the due number of subtracted terms of the Taylor expansion in the integral $\int f(x)x_+^{\lambda}dx$ [10]. Three different issues are important here. First, each of the integrals (6.10) is a well defined distribution that includes all necessary regulators which provide the convergence of subsequent integrations. The Bessel functions themselves are *defined* as the Fourier transforms of the (+) distributions and we just recover the original regular form by doing the inverse Fourier transform (6.10) (see Ref. [10]). Second, as will be shown in the next section, the singular behavior of the integrals (6.10) originates from the collinear domain. The (+) prescription that emerges here eliminates them term-by-term. Third, after the result of the term-by-term integration is put back into Eq. (6.9), the singular collinear terms just cancel everywhere, including the null-plane $\tau_{12}^2=0$. This type of cancelation of collinearly singular terms takes place in all other pieces of the vacuum part of the quark self-energy.

All these observations lead us to the conclusion, that even in its vacuum part, the self-energy does not suffer from collinear problems, which seems to be a unique property of the expanding system. We do not continue to study the vacuum part of the self-energy here, since we are currently interested only in its material part which is discussed in the next section. (The full analysis of this part, including the issue of its renormalization, will be published elsewhere.)

VII. RADIATION FIELDS IN THE MATERIAL PART OF THE SELF-ENERGY

We found that the major contribution to the one-loop effective mass of a "soft" quark mode in the background of "hard" quarks and gluons comes from the quark-quark forward scattering mediated by the magnetic component of the longitudinal field. The purpose of this section is to demonstrate that the interactions via transverse fields (including the forward scattering of soft quark on hard gluons) is a secondary effect at least at the very early stage of the nuclear collision. This conclusion may look counterintuitive, since, namely in the interactions of the transverse fields, we expect to encounter the collinear enhancement of the radiation amplitude. As has been shown in Sec. VI, in the vacuum part of the self-energy, the integrals of this type (taken in the limits from 0 to ∞) cancel each other leaving the vacuum sector free from collinear divergences. The statistical weights $\mathcal{N}_g$ and $\mathcal{N}_f$, which are different for the different terms, prevent such a cancelation in the material part. Thus we have to analyze each term of the material part separately.

As in Sec. VI, we consider an isolated piece which corresponds to TE gluons. The gluon correlators of this piece are the most singular and are known not only in the integral representation, but in closed analytic form also. The last circumstance is very helpful for the analysis of the multiple integrals we meet below. (The terms identical to those computed below, also appear in the part of self-energy due to the TM gluons; the remaining terms of TM sector are less singular and, eventually, smaller than considered here.) The corresponding fragment of the quark self-energy (3.10) in the dispersion equation (3.9) is

$$[\Sigma^0(\tau_1,\tau_2)]^{(TE)}_{mat}=\frac{i\alpha_sC_F}{4\pi}\,\theta(\tau_1-\tau_2)\times\int d^2\vec{k}_t\int_{-\infty}^{\infty}d\eta q_t\times[g^0_{[0]}\mathcal{D}^{(TE)}_{[1]}-g^0_{[1]}\mathcal{D}^{(TE)}_{[0]}],\qquad(7.1)$$

$$[\Sigma^T(\tau_1,\tau_2)]^{(TE)}_{mat}=\frac{i\alpha_sC_F}{4\pi}\theta(\tau_1-\tau_2)\int d^2\vec{k}_t\int_{-\infty}^{\infty}d\eta\times\left[\frac{(\vec{p}_t\vec{q}_t)}{p_t^2}-2\frac{(\vec{k}_t\vec{p}_t)(\vec{k}_t\vec{q}_t)}{k_t^2p_t^2}\right]\times[g^T_{[0]}\mathcal{D}^{(TE)}_{[1]}-g^T_{[1]}\mathcal{D}^{(TE)}_{[0]}].\qquad(7.2)$$

The invariants of (anti)commutators are the same as in the vacuum case,

$$g^0_{[0]}(\tau_1,\tau_2;\eta;\vec{q}_t)=i\frac{(\tau_1-\tau_2)\cosh(\eta/2)}{2\tau_{12}}\theta(\tau^2_{12})J_1(q_t\tau_{12}),$$

$$g^T_{[0]}(\tau_1,\tau_2;\eta;\vec{q}_t)=-\frac{\cosh(\eta/2)}{2}\theta(\tau^2_{12})J_0(q_t\tau_{12}), \tag{7.3}$$

and the invariant $\mathcal{D}^{TE}_{[0]}(\tau_2,\tau_1;\eta;\vec{k}_t)$ is given by the first of Eqs. (6.3). They all differ from zero only for the timelike τ_{12}. Hence, the material part of the invariants $g_{[1]}$ and $\mathcal{D}^{(TE)}_{[1]}$ will be needed only in this domain. As has been discussed earlier, the distributions include only "hard" particles which are defined with respect to the soft mode with the transverse momentum p_t by the inequalities, $k_t>p_*$, and $q_t>p_*$, where $p_*\geqslant p_t$. Now, we are interested only in the material part with occupation numbers given by the equations

$$n_f(q_t,\theta)\approx\frac{\mathcal{N}_f}{\pi R^2_\perp}\frac{\theta(q_t-p_*)}{q^2_t},$$

$$n_g(k_t,\alpha)\approx\frac{\mathcal{N}_g}{\pi R^2_\perp}\frac{\theta(k_t-p_*)}{k^2_t}, \tag{7.4}$$

and we must keep in mind the width $2Y$ of the rapidity plateau with the goal to study if this is a significant parameter for the calculation of local quantities. We may also question the validity of these formulas at sufficiently large k_t and q_t, since without a cutoff, the integral $\int dk_t/k_t$ diverges.

The material part of the densities will be employed in two different forms,

$$g^0_{[1]}(\tau_1,\tau_2;\eta;\vec{q}_t)=\int_{-\infty}^{\infty}\frac{d\theta}{\pi}n_f(\theta;q_t)\cosh\theta\sin q_tT_{12}(\theta)$$
$$=-\frac{(\tau_1-\tau_2)\cosh(\eta/2)}{\tau_{12}}n_f(q_t)Y_1(q_t\tau_{12}), \tag{7.5}$$

$$g^T_{[1]}(\tau_1,\tau_2;\eta;\vec{q}_t)=-i\cosh\frac{\eta}{2}\int_{-\infty}^{\infty}\frac{d\theta}{\pi}n_f(\theta;q_t)\cos q_tT_{12}(\theta)$$
$$=-i\cosh(\eta/2)n_f(q_t)Y_0(q_t\tau_{12}), \tag{7.6}$$

$$\mathcal{D}^{(TE)}_{[1]}(\tau_2,\tau_1;\eta;\vec{k}_t)=(\pi i)^{-1}\int_{-\infty}^{\infty}d\alpha n_g(\alpha;k_t)\cos k_tT_{12}(\alpha)$$
$$=iY_0(\tau_{12}k_t)n_g(k_t), \tag{7.7}$$

where the second equation in Eqs. (7.5)–(7.7) is valid only when n_g and n_f are rapidity-independent, and we employ the following notation:

$$T_{12}(\alpha)=\tau_1\cosh(\alpha-\eta/2)-\tau_2\cosh(\alpha+\eta/2)$$
$$=\tau_{12}\cosh(\alpha-\psi),$$
$$\tau^2_{12}=\tau^2_1+\tau^2_2-2\tau_1\tau_2\cosh\eta>0,$$
$$\tanh\psi(\eta)=\frac{\tau_1+\tau_2}{\tau_1-\tau_2}\tanh\frac{\eta}{2},$$
$$|\eta|<\eta_0=\ln\frac{\tau_1}{\tau_2}\approx\frac{\tau_1-\tau_2}{\sqrt{\tau_1\tau_2}}=\xi,\quad \tanh\psi(\pm\eta_0)=\pm1,$$
$$\psi(\pm\eta_0)=\pm\infty, \tag{7.8}$$

where $\xi=(\tau_1-\tau_2)/\sqrt{\tau_1\tau_2}\approx\eta_0$ is the main parameter of our calculations. This parameter is supposed to be small in order that the notion of the current transverse mass $\mu(p_t,\tau_1)$ has the expected meaning of a slowly varying parameter. The geometric mean time $\tau_m=\sqrt{\tau_1\tau_2}$ has a simple interpretation. The two characteristics, one connecting the points $(\tau_2,-\eta_0)$ and (τ_1,η_0), and the second one connecting the points (τ_2,η_0) and $(\tau_1,-\eta_0)$, intersect at the point $(\tau_m,0)$. The proper time τ_m is always inside the domain of the "causal interaction."

Let us start the analysis of the radiation-dominated terms with the invariant $[\Sigma^0]^{(TE)}_{mat}$. (The invariant $[\Sigma^T]^{(TE)}_{mat}$ appears to have an extra small factor ξ.) According to Eqs. (7.4), (7.5), and (7.7) it can be written as a multiple integral,

$$[\Sigma^0(\tau_1,\tau_2)]^{(TE)}_{mat}=\frac{i\alpha_sC_F}{8\pi^2}\int_{-\infty}^{+\infty}d\eta\theta(\tau^2_{12})\Bigg\{\frac{\mathcal{N}_f}{\pi R^2_\perp}\int_{p_*}^{\infty}dq_t\int_0^{2\pi}d\varphi J_0(\tau_{12}k_t)\int_{-\infty}^{\infty}d\theta\cosh\theta\sin T_{12}(\theta)q_t$$
$$+\frac{\mathcal{N}_g}{\pi R^2_\perp}\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}}\int_{p_*}^{\infty}\frac{dk_t}{k_t}\int_0^{2\pi}d\varphi q_tJ_1(\tau_{12}q_t)\int_{-\infty}^{\infty}d\alpha\cos T_{12}(\alpha)k_t\Bigg\}\theta(\tau_1-\tau_2), \tag{7.9}$$

where we choose the integral form of the densities $g_{[1]}$ and $\mathcal{D}^{(TE)}_{[1]}$ in order to find the domain in the multidimensional space where the dominant contribution comes from. Since the two terms in Eq. (7.9) are not expected to interfere (or UV diverge), we are free to change variables in these terms independently. We leave $d^2\vec{k}_t$ in the second term, and change for $d^2\vec{q}_t$ in the first one. The next step is to integrate over the azimuthal angle between $\vec{q}_t$ and $\vec{p}_t$ in the first term of Eq. (6.2), and over the angle between $\vec{k}_t$ and $\vec{p}_t$ in the second term. This integration deals only with the retarded propagators inside the self-energy loop and selects the lowest angular harmonics,

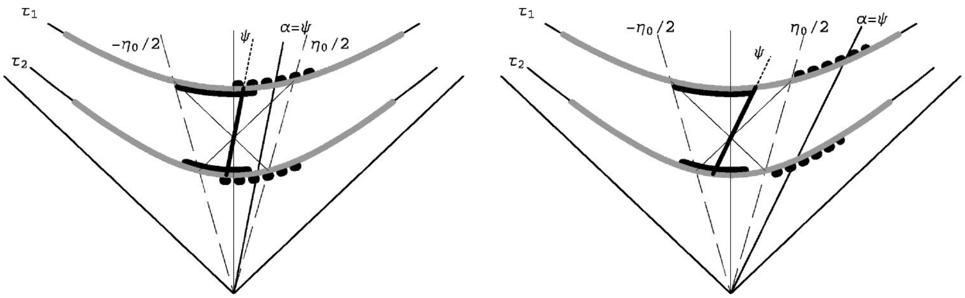

FIG. 4. Geometry of fields in the forward scattering amplitude $qg \to qg$.

$$\int_0^{2\pi} q_t J_1(\tau_{12}q_t)d\varphi = 2\pi[k_t J_0(\tau_{12}p_t)J_1(\tau_{12}k_t) + p_t J_1(\tau_{12}p_t)J_0(\tau_{12}k_t)], \quad k_t > p_t, \tag{7.10}$$

$$\int_0^{2\pi} J_0(\tau_{12}k_t)d\varphi = 2\pi J_0(\tau_{12}p_t)J_0(\tau_{12}q_t), \quad q_t > p_t. \tag{7.11}$$

Only the first of the two terms in Eq. (7.10), corresponding to the collinear geometry in the transverse plane survives in the limit of $k_t \gg p_t$ and has to be retained by our major assumption. The second term describes the deviation from collinearity and is small. However, it is instructive to keep it for a while. After these angular integrations, Eq. (7.9) becomes

$$\begin{aligned}[\Sigma^0(\tau_1,\tau_2)]_{mat}^{(TE)} = {} & \frac{i\alpha_s C_F}{4\pi}\theta(\tau_1-\tau_2)\int_{-\infty}^{+\infty} d\eta\,\theta(\tau_{12}^2)\Bigg\{\frac{\mathcal{N}_f}{\pi R_\perp^2}\int_{p_*}^{\infty} dq_t J_0(\tau_{12}p_t)J_0(\tau_{12}q_t) \\ & \times\int_{-\infty}^{\infty} d\theta\cosh\theta\sin T_{12}(\theta)q_t + \frac{\mathcal{N}_g}{\pi R_\perp^2}\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}}\int_{p_*}^{\infty}\frac{dk_t}{k_t}[k_t J_0(\tau_{12}p_t)J_1(\tau_{12}k_t) \\ & + p_t J_1(\tau_{12}p_t)J_0(\tau_{12}k_t)]\int_{-\infty}^{\infty} d\alpha\cos T_{12}(\alpha)k_t\Bigg\},\end{aligned} \tag{7.12}$$

where the actual limits of integration over η, θ, and α have yet to be put in agreement with the model we employ. Now, we have approached the most subtle point of our analysis. This expression includes triple integrations, any of which (if performed formally) yields singular functions. For the sake of definiteness, let us start with the second term in Eq. (7.12) [which corresponds to the forward scattering of soft quark on a hard gluon from the distribution $n_g(\alpha,k_t)$], rewriting it in its most expanded form,

$$\mathcal{J}_2 = \frac{\mathcal{N}_g}{\pi R_\perp^2}\int d\eta\,\theta(\tau_{12}^2)\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}}J_0[\tau_{12}(\eta)p_t]\int_{-\infty}^{\infty} d\alpha\int_{p_*}^{\infty} J_1[\tau_{12}(\eta)k_t]\cos[k_t\tau_{12}(\eta)\cosh(\alpha-\psi(\eta))]dk_t. \tag{7.13}$$

The first observation is that at large k_t [which is the condition that the distribution $n_g(\alpha,k_t)$ can be measured within a short time] the main contribution to the α integration comes from the domain $\alpha \approx \psi(\eta)$ where the phase of the $\cos T_{12}(\alpha)k_t$ is stationary. This is the domain of collinear interaction when the hard gluon from the distribution $n_g(\alpha,k_t)$ has almost the same rapidity as the virtual quark with transverse momentum $\vec{q}_t = \vec{k}_t + \vec{p}_t$ in the self-energy loop. Obviously, this quark is also hard. Furthermore, its propagator, $G_{[\mathrm{ret}]}(\tau_1,\tau_2;q_t) = \theta(\tau_1-\tau_2)G_{[0]}(\tau_1,\tau_2;q_t)$, is devised only from the free on-mass-shell partial waves which themselves are well localized in the rapidity direction. Hence, we deal with the intuitively very clear case of collinear absorption and emission of the gauge field quantum by a charged particle. All participants of the process are moving with the same velocity. According to the property of localization of states in wedge dynamics studied in paper [II], such a fine tuning of α to ψ is indeed possible. This is illustrated by the left-hand figure in Fig. 4. where the gray segments of the hyperbolas $\tau = \tau_2$ and $\tau = \tau_1$ correspond to the rapidity intervals occupied by the soft quark mode, $\tau p_t < 1$, at the beginning and at the end of the scattering process, respectively. The bold black and the dashed segments show the rapidity intervals where the hard virtual quark and the hard gluon are localized at the same times. All three fields effectively overlap around rapidity $\eta_2 = -\eta/2$ at $\tau = \tau_2$ and around $\eta_1 = +\eta/2$ at $\tau = \tau_1$. The rapidity direction between these points is exactly $\psi(\eta)$. The rapidity α of the external gluon is sufficiently small and is close to the rapidity $\psi(\eta)$.

The maximal rapidity width of the interaction domain is defined by the causality condition $\tau_{12}^2>0$, which immediately establishes the upper boundary $|\eta|<\eta_0$. Since the collinear interaction corresponds to the condition $\alpha\approx\psi(\eta)$, the rapidity of the hard gluon must be within this geometrically defined interval as well. The opposite case is depicted in the right-hand figure of Fig. 4. The rapidity $\psi(\eta)$ is so large, that the external gluon is not localized within the causal boundaries $\pm\eta_0/2$ of the interaction domain. In order to avoid this, we have to impose an even stronger requirement that $|\psi(\eta)|<\eta_0$. According to Eq. (7.8), we have $|\psi(\eta)|>\eta$. Hence, we must further take $|\eta|<\eta_*$, where the boundary η_* is defined by the equation $\psi(\eta_*)=\eta_0$,

$$\frac{\tau_1+\tau_2}{\tau_1-\tau_2}\tanh\frac{\eta_*}{2}=\tanh\eta_0\equiv\frac{\tau_1^2-\tau_2^2}{\tau_1^2+\tau_2^2}, \tag{7.14}$$

which has a solution,

$$\tanh\frac{\eta_*}{2}=\frac{(\tau_1-\tau_2)^2}{\tau_1^2+\tau_2^2},\quad \eta_*\approx\frac{(\tau_1-\tau_2)^2}{\tau_1\tau_2}=\xi^2. \tag{7.15}$$

We remind the reader that $\xi\ll 1$; only this condition allows one to introduce the the time-dependent transverse mass $\mu(p_t,\tau)$. In order to simplify further analysis, it is convenient to present the internal integral over k_t as the difference,

$$-\frac{\tau_{12}}{(T_{12}^2(\alpha)-\tau_{12}^2)_+^{1/2}[T_{12}(\alpha)+(T_{12}^2(\alpha)-\tau_{12}^2)^{1/2}]} - \int_0^{p_*}J_1(\tau_{12}k_t)\cos[T_{12}(\alpha)k_t]dk_t, \tag{7.16}$$

where the first singular term is the integral over k_t, computed from 0 to ∞, and thus, it completely accounts for the domain $k_t\to\infty$. It includes the function

$$f(\eta,\alpha)=[T_{12}^2(\alpha)-\tau_{12}^2]_+^{-1/2}=[\tau_{12}^2(\eta)\sinh^2(\alpha-\psi(\eta))]_+^{-1/2},$$

which is singular at $\alpha=\psi(\eta)$, thus fully accounting for the expected collinear enhancement. This function, however, is a canonical distribution with respect to both its arguments α and η, and it carries the standard regulators for the subsequent integrations. We shall consider the singular and the regular terms separately. Using the above found limits, we may write the singular term as

$$I_2^{sing}=\int_{-\eta_*}^{\eta_*}d\eta\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}(\eta)}\frac{1}{[\tau_{12}(\eta)]_+}\times\int_{-\eta_0}^{\eta_0}d\alpha\frac{e^{-|\alpha-\psi|}}{[\sinh^2|\alpha-\psi|]_+^{1/2}}, \tag{7.17}$$

where, since $\tau_{12}p_t\ll 1$, we put $J_0[\tau_{12}p_t]\approx 1$. After an obvious change of variable, the internal integral of Eq. (7.17) can be split into two,

$$\int_{-\eta_0}^{\eta_0}d\alpha\frac{e^{-|\alpha-\psi|}}{[\sinh^2|\alpha-\psi|]_+^{1/2}}=\left[\int_0^{\eta_0+\psi}+\int_0^{\eta_0-\psi}\right]\frac{\alpha e^{-\alpha}}{\sinh\alpha}\frac{d\alpha}{\alpha_+}.$$

Since by the definition of the $(+)$ distribution,

$$\int_0^{\beta}\frac{\alpha e^{-\alpha}}{\sinh\alpha}\frac{d\alpha}{\alpha_+}=\int_0^{\beta}\frac{e^{-\alpha}d\alpha}{\sinh\alpha}-\int_\epsilon^1\frac{d\alpha}{\alpha}=\ln\frac{1-e^{-2\beta}}{2}, \tag{7.18}$$

we obtain the singular part in the form

$$I_2^{sing}=2\int_0^{\eta_*}\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}(\eta)}\frac{d\eta}{[\tau_{12}(\eta)]_+}\times\ln\left\{\frac{1}{4}[1+e^{-4\eta_0}-2e^{-2\eta_0}\cosh 2\psi(\eta)]\right\}. \tag{7.19}$$

Next, it is convenient to trade η for a new variable y, $\tau_{12}(\eta)=(\tau_1-\tau_2)y$. The helpful relations for this change of variables are

$$\frac{\cosh(\eta/2)d\eta}{\tau_{12}(\eta)}=\frac{-1}{\sqrt{\tau_1\tau_2}}\frac{dy}{\sqrt{1-y^2}},$$

$$\tau_{12}^2(\eta_*)=\frac{(\tau_1-\tau_2)^2}{1+(\tau_1-\tau_2)^2/\tau_1\tau_2},$$

$$y_*\approx 1-\frac{(\tau_1-\tau_2)^2}{2\tau_1\tau_2},\quad \cosh 2\psi=\frac{(\tau_1+\tau_2)^2}{2\tau_1\tau_2}\frac{1}{y^2}-\frac{\tau_1^2+\tau_2^2}{2\tau_1\tau_2}.$$

Taking into account that $\cosh 2\psi\to 1$, when $y\to 1$, we obtain

$$I_2^{sing}=\frac{4}{\sqrt{\tau_1\tau_2}}\int_{y_*}^1\frac{dy}{y\sqrt{1-y^2}}\ln\frac{1-e^{-2\eta_0}}{2}\approx\frac{4}{\sqrt{\tau_1\tau_2}}\frac{\tau_1-\tau_2}{\sqrt{\tau_1\tau_2}}\ln\frac{\tau_1-\tau_2}{\sqrt{\tau_1\tau_2}}=\frac{4}{\sqrt{\tau_1\tau_2}}\xi\ln\xi. \tag{7.20}$$

This formula has two distinctive elements. The first element is the large $\ln\xi$, which is due to the collinear geometry of the interaction. This would lead to a divergence if the interaction domain were unlimited. The second element is the small factor ξ which is due to the small volume occupied by the interaction and it completely suppresses the potential divergence. One may notice that when the mean time $\sqrt{\tau_1\tau_2}$ increases, then $\xi\to 0$, and the corresponding part of the self-energy also tends to zero. This is easy to understand, since with the mean time growing, the system becomes more and more diluted locally.

The regular part of Eq. (7.13) is given by the integral

$$I_2^{reg}=\int_{-\eta_*}^{\eta_*}d\eta\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}(\eta)}\times\int_{-\eta_0}^{\eta_0}d\alpha\int_0^{p_*}J_1(\tau_{12}k_t)\cos[T_{12}(\alpha)k_t]dk_t, \tag{7.21}$$

where, when $-\eta_0<\eta<\eta_0$, then $T_{12}(\alpha)$ varies between its minimal and maximal values,

$$\frac{(\tau_1-\tau_2)^2}{\sqrt{\tau_1\tau_2}}e^{-|\alpha|}<T_{12}(\alpha)<\frac{(\tau_1-\tau_2)^2}{\sqrt{\tau_1\tau_2}}e^{+|\alpha|}.$$

Therefore, when $k_t<p_t$, we have $T_{12}k_t\sim(\tau_1-\tau_2)k_t\ll 1$, and both functions under the integral over k_t can be expanded in Taylor series. All integrations become trivial and yield[5]

$$I_2^{reg}=\frac{\tau_1\tau_2 p_*^2}{\sqrt{\tau_1\tau_2}}\left[\frac{\tau_1-\tau_2}{\sqrt{\tau_1\tau_2}}\right]^4=\frac{\tau_1\tau_2 p_*^2}{\sqrt{\tau_1\tau_2}}\xi^4. \tag{7.22}$$

We have chosen this form of the answer, because our major assumption is valid only as long as $\tau p_t\ll 1$ and because the dispersion equation (3.9) has a kinematic factor $\sqrt{\tau_1\tau_2}$ in it.

The other two terms in Eq. (7.12) can be studied along the same guidelines. The third term is suppressed with respect to $\mathcal{J}_2$ by the factor p_t/k_t, which is small by our major model agreement and it could have been discarded on this ground only. To be on the safe side, let us rewrite it as

$$\mathcal{J}_3=\frac{\mathcal{N}_g}{\pi R_\perp^2}\int_{-\eta_*}^{\eta_*}d\eta\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}}p_tJ_1[\tau_{12}(\eta)p_t]$$
$$\times\int_{-\eta_0}^{\eta_0}d\alpha\left\{-\gamma_E-\ln\left[\frac{\tau_{12}p_t}{2}e^{|\alpha-\psi(\eta)|}\right]\right.$$
$$\left.+\int_0^{p_*}\frac{1-J_0[\tau_{12}(\eta)k_t]\cos(T_{12}(\alpha)k_t)}{k_t}dk_t\right\}, \tag{7.23}$$

where the first term is the integral over k_t from 0 to ∞. As could be expected, the integrand is regular. Since there is an accounted for difference between k_t and q_t, the exactly collinear regime becomes impossible and we do not have the large collinear logarithm in $\mathcal{J}_3$. Overall, this term is also suppressed at least by a factor ξ stemming from $J_1[\tau_{12}(\eta)p_t]$ in the integrand.

The first term in Eq. (7.12) corresponds to the forward quark-quark scattering with high momentum transfer,

$$\mathcal{J}_1=\frac{\mathcal{N}_f}{\pi R_\perp^2}\int_{-\eta_*}^{\eta_*}d\eta J_0[\tau_{12}(\eta)p_t]$$
$$\times\int_{-\eta_0}^{\eta_0}d\theta\left\{\frac{\cosh\theta}{[\sinh^2(\theta-\psi(\eta))]_+^{1/2}}\frac{1}{[\tau_{12}(\eta)]_+}\right.$$
$$\left.-\cosh\theta\int_0^{p_*}J_0[\tau_{12}(\eta)q_t]\sin(T_{12}(\theta)q_t)dq_t\right\}. \tag{7.24}$$

Here, we again recognize the collinear singularity which is, as previously, regulated by the (+) prescription. All further calculations for $\mathcal{J}_1$ are similar to the case of $\mathcal{J}_2$ and the answer reads

$$I_1^{sing}\le\frac{-8\xi+4\xi^2\ln\xi}{\sqrt{\tau_1\tau_2}},\quad I_1^{reg}\approx 2\frac{p_*^2\tau_1\tau_2}{\sqrt{\tau_1\tau_2}}\xi^5. \tag{7.25}$$

These results will serve for us as a reference point for the estimates of the mathematically more complicated part connected with the radiation field of the transverse magnetic TM modes. Before we address this issue, it is expedient to look at the obtained results more attentively and trace the correspondence between the calculations and physical picture in more details.

(1) It has been observed in Sec. VI (for the vacuum part of the quark self-energy) that in the framework of wedge dynamics, the collinear problems do not jeopardize the field theory. In ''material part'' of the self-energy, the collinear interactions were proved to be the most intensive and to lead to a visible enhancement of the interaction between the quark and *radiation* field. However, this enhancement never turns into a disaster of collinear divergence. One of the trivial reasons is that the space-time domain of the interaction is now limited, and large logarithms are multiplied by small phase volumes.

(2) A deeper insight into the wedge dynamics, shows that even intermediate collinear singularities observed in the terms $\mathcal{J}_1$ and $\mathcal{J}_2$ are spurious. In order to reveal this fact, one can notice that the singularity at $\alpha=\psi(\eta)$ is present only in $\mathcal{J}_1$ and $\mathcal{J}_2$. It is absent in $\mathcal{J}_3$, because of the extra negative power of k_t brought by the subleading term of the angular integral $d\varphi$. This extra k_t^{-1} effectively suppresses the distribution $n_g(\alpha,k_t)$ at large k_t. Next, one may ask, what minimal change of $n_g(\alpha,k_t)$ at large k_t is necessary in order that the intermediate collinear singularity does not appear at all. This can be learned by changing the order of integration in Eq. (7.13). One can start from the integral $d\eta$ with an assumption that the integrand only slowly varies within some interval of α around $\alpha=0$. Then, it is easy to see that the singular term I_2^{sing} of Eq. (7.20) *totally* originates from the domain of $k_t\to\infty$. It comes from a logarithmic integral between two infinite limits. This residual piece emerges only because we extend the distribution $n_g\sim k_t^{-2}$ (obtained from a dimensional estimate) to an arbitrarily large k_t. As we have already mentioned, this dependence is unphysical, e.g., be-

[5]Even if we impose no limitations on α the estimate is still as small as

$$\frac{\tau_1\tau_2 p_*^2}{\sqrt{\tau_1\tau_2}}\xi^2\ln\xi.$$

cause the distribution $d^2\vec{k}_t/k_t^2$ is not normalizeable. It has to be modified above some value of k_t and, therefore, the singular term must vanish completely. Thus, in wedge dynamics, the phenomenon of collinear enhancement is intrinsically connected with the basic property of localization inherent in the one-particle states. Only the states with infinitely large k_t can have a precisely given rapidity and be responsible for the singularities like we encounter in Eqs. (7.16) and (7.24).

(3) Our way to pick out the leading contributions (in the mixed representation of wedge dynamics) from the space-time domains, where the phases of the interacting fields are stationary, is a generalization of the known method of isolating the leading terms using the pinch poles in the plane of complex energy. The similarity of two methods can be easily understood since, e.g., the quark density correlator in the self-energy can be presented as a sum of two propagators, $G_{[1]}(q)=G_{[00]}(q)+G_{[11]}(q)$. In the plane of the complex energy $q^0=k^0+p^0$, the (Feynman-type) propagator $G_{[00]}(q)$ has poles in the second and fourth quadrants, while the (anti-Feynman-type) propagator $G_{[11]}(q)$ has poles in the first and third quadrants. The radiation part of the retarded gauge field propagator $D_{[\mathrm{ret}]}(k)$ has poles in the third and fourth quadrants. Therefore, in both terms of $G_{[1]}(q)D_{[\mathrm{ret}]}(k)=G_{[00]}(q)D_{[\mathrm{ret}]}(k)+G_{[11]}(q)D_{[\mathrm{ret}]}(k)$, the integration path along the real axis of the complex k^0 plane is pinched between two poles (one from $D_{[\mathrm{ret}]}$, and the second from $G_{[00]}$ or $G_{[11]}$) giving the leading contribution when p^0 is small, and the three-momenta $\boldsymbol{k}$ and $\boldsymbol{q}=\boldsymbol{k}+\boldsymbol{p}$ are are very close to each other. Similar arguments are valid for the second part, $G_{[\mathrm{ret}]}D_{[1]}$, of the quark self-energy. The term $G_{[1]}(q)D_{[long]}(k)$ is exceptional, because the propagator $D_{[long]}(k)$ of the longitudinal field has no poles corresponding to the propagation.

The wedge dynamics does not allow for a standard momentum representation, since its geometric background is not homogeneous in the t and z directions; accordingly, we do not have familiar pinch-poles in our calculations. Nevertheless, the patches of phase space where the phases of certain field fragments are stationary and effectively overlap, do now the same job as the pinch-poles, and yield the same answers when the homogeneity required for the momentum representation is restored. The way wedge dynamics tackles the problem is genuinely more general, because it addresses the space-time picture of the interacting fields.[6] The momentum space is now split into the subspaces of rapidity and transverse momentum; the correlation between the particle's rapidity and its location is increasing with the increase of its transverse momentum. The role of pinch-poles is taken over by the geometrical overlap of the field patterns with the same rapidity. This observation can serve as a footing for the future development of an effective technique for perturbative calculations in wedge dynamics. The arguments of the localization are not applicable to the longitudinal part of the gluon field. In the term $G_{[1]}D_{[long]}$, no patch in space-time is dynamically selected, since $D_{[long]}$ is not assembled from the propagating waves that could match the virtual quark in the loop by their phase. This is in line with the absence of pinch-poles due to $D_{[long]}(k)$ in the momentum picture.

The arguments presented above allow one to estimate the contribution of the radiation fields of the TM mode in a very economical way. Let us consider the group $\mathcal{D}^{(2)}_{[0]}g^0_{[1]}$, which is very similar to the term $\mathcal{J}_1$ studied above, as an example. Now, the invariant $\mathcal{D}^{(2)}_{[0]}$, as can be inferred from Eq. (A4), is known only in the integral representation, and not in an analytic form. Let us, therefore, employ the analytic form of $g^0_{[1]}$, given by Eq. (A22). It is easy to see, that the integration over α in $\mathcal{D}^{(2)}_{[0]}$ leads to the same causal step-function $\theta(\tau_{12}^2)$ and, as previously shown, we have $|\eta|<\eta_0$. The expanded form of this term is

$$\mathcal{J}_4=\frac{\mathcal{N}_f}{\pi R_\perp^2}\int_{-\eta_0}^{\eta_0} d\eta\,\theta(\tau_{12}^2)\frac{(\tau_1-\tau_2)\cosh\eta/2}{\tau_{12}}\int d\alpha \times\tanh\left(\alpha-\frac{\eta}{2}\right)\tanh\left(\alpha+\frac{\eta}{2}\right)\int_{p_*}^{\infty}Y_1[\tau_{12}(\eta)q_t] \times\sin[T_{12}(\alpha)q_t]dq_t, \tag{7.26}$$

where we have integrated out the azimuthal angle φ assuming that $k_t,q_t\gg p_t$ [the first correction is smaller by the factor $(p_t/k_t)^2\ll 1$]. The internal integral over q_t can be transformed into

$$-\frac{\cosh(\alpha-\psi(\eta))}{[\tau_{12}^2(\eta)\sinh^2(\alpha-\psi(\eta))]_+^{1/2}} -\int_0^{p_*}Y_1(\tau_{12}q_t)\sin[T_{12}(\alpha)k_t]dk_t, \tag{7.27}$$

which brings us very close to Eq. (7.24) for $\mathcal{J}_1$. Once again, we encounter a collinear singularity at $\alpha=\psi(\eta)$, and exactly the same arguments force us to set the same limits in the integrals, as in Eq. (7.24). We do not have to continue the calculations to understand the smallness of $\mathcal{J}_4$, mention only, that due to the narrow limits of two rapidity integrations in Eq. (7.26), the product of the two hyperbolic tangents in the integrand will add extra ξ^2 to the order of smallness of $\mathcal{J}_4$. By the same argument as used previously, we can easily learn that the singular term in Eq. (7.27) is spurious.

In this group, associated with the TM mode of the radiation field, the leading (still parametrically small, and equal to $\mathcal{J}_1$) contribution comes from the $D^{\eta\eta}$ component of the gluon correlators. We may summarize by the observation that only the overlap of the domains of stationary phase in two correlators matters. It can be visualized via the partial-wave expansion of any of the correlators in the self-energy loop.

[6]It is well known that the threshold behavior of the imaginary part of the photon self-energy can be derived from the pinch geometry of the poles of the electron propagators [11]. Since at the threshold, the e^+e^- pair is created with zero relative velocity, the pinch indeed corresponds to the overlap of the stationary phases of the e^+ and e^- wave functions in the maximal possible volume.

VIII. NONLOCAL PART OF THE LONGITUDINAL PROPAGATOR IN THE MATERIAL PART OF THE SELF-ENERGY

The longitudinal part of the gluon propagator contributes, to the invariant Σ^0, the term

$$[\Sigma^0(\tau_1,\tau_2)]^{[long]}_{mat}=\frac{i\alpha_s C_F}{4\pi}\,\theta(\tau_1-\tau_2)\int d^2\vec{k}_t\int_{-\infty}^{\infty}d\eta q_t g^0_{[1]}\left[\mathcal{D}^{(2)}_{[long]}+\frac{1}{\tau_1\tau_2}\mathcal{D}^{(\eta\eta)}_{[long]}\right]. \tag{8.1}$$

Using Eqs. (A16) and (A17) for the gluon invariants, and the first of Eqs. (A22) for the quark invariant $g^0_{[1]}$, we arrive at the following expression which accounts for the nonlocal part of the longitudinal propagator (the contact part was studied in Sec. V)

$$\begin{aligned}[\Sigma^0(\tau_1,\tau_2)]^{[long]}_{mat}&\\ =\frac{i\alpha_s C_F\mathcal{N}_f}{2\pi^2R_\perp^2}\,\theta(\tau_1-\tau_2)\int_{p_*}^{\infty}dq_t\int_{-\infty}^{\infty}d\eta\frac{(\tau_1-\tau_2)\cosh\eta/2}{|\tau_{12}^2|^{1/2}}\,\frac{k_t\cosh\eta}{2}&\\ \times\int_{\tau_2}^{\tau_1}e^{-tq_t\sinh|\eta|}\left(1-\frac{t^2}{\tau_1\tau_2}\right)dt\left[\theta(\tau_{12}^2)Y_1(\tau_{12}q_t)+\frac{2}{\pi}\theta(-\tau_{12}^2)K_1(\tilde{\tau}_{12}q_t)\right],&\end{aligned} \tag{8.2}$$

where we have integrated out the dependence on the azimuthal angle φ in the approximation of $q_t \gg p_t$. The key observation that allows one to judge about the smallness of this term is that the limits of integration over variable t are very close, and the factor $[1-t^2/\tau_1\tau_2]$ is very small. This factor reflects a known competition between the electric and magnetic interaction of moving charges which reduces the net yield almost to zero. Let us replace t by the dimensionless $u=t/\sqrt{\tau_1\tau_2}$. Then this part of the integral becomes

$$\frac{1}{\sqrt{\tau_1\tau_2}}\int_{\tau_2}^{\tau_1}\left[1-\frac{t^2}{\tau_1\tau_2}\right]\cdots dt=\int_{\sqrt{\tau_2/\tau_1}}^{\sqrt{\tau_1/\tau_2}}(1-u^2)\cdots du=\int_{\sqrt{1-\xi^2/4}-\xi/2}^{\sqrt{1-\xi^2/4}+\xi/2}(1-u^2)\cdots du\approx-\frac{\xi^3}{3}f(\xi), \tag{8.3}$$

where the limiting behavior $f(\xi)\sim$ const$/\xi$ when $\xi\to 0$ can be conjectured from the behavior of the functions $Y_1(x)$ and $K_1(x)$ at small x. However, the Laplace transforms of these functions in Eq. (8.2) are singular functions of η and we have to be careful in estimating these terms. In fact, the ξ^2 order of smallness is not altered by the remaining integrations. First, it is useful to notice that the last factor in square brackets in Eq. (8.2) is nothing but the invariant $g^0_{[1]}$ which, according to its integral representation (A22), equals zero at $\xi=0$. Next, it is profitable to change the variables of integration in the following way. We trade η for y according to $\tau_{12}(\eta)=(\tau_1-\tau_2)y$ in the domain $\tau_{12}^2>0$. In the complimentary domain $\tau_{12}^2=-\tilde{\tau}_{12}^2<0$, we change η for y using $\tilde{\tau}_{12}(\eta)=(\tau_1-\tau_2)y$. We also replace ξq_t by a new variable q:

$$\begin{aligned}[\Sigma^0(\tau_1,\tau_2)]^{[long]}_{mat}=&\frac{i\alpha_s C_F\mathcal{N}_f}{2\pi^2R_\perp^2}\,\theta(\tau_1-\tau_2)\frac{1}{\xi}\int_{\sqrt{1-\xi^2/4}-\xi/2}^{\sqrt{1-\xi^2/4}+\xi/2}(1-u^2)du\int_{\xi p_*}^{\infty}qdq\\ &\times\left\{\int_0^1\frac{dy}{\sqrt{1-y^2}}\left[1+\frac{\xi^2}{2}(1-y^2)\right]\frac{\pi}{2}Y_1(\tau_m qy)e^{-\tau_m qu\sqrt{1-y^2}\sqrt{1+\xi^2(1-y^2)/2}}\right.\\ &\left.+\left(\int_0^1+\int_1^{\infty}\right)\frac{dy}{\sqrt{1+y^2}}\left[1+\frac{\xi^2}{2}(1+y^2)\right]K_1(\tau_m qy)e^{-\tau_m qu\sqrt{1+y^2}\sqrt{1+\xi^2(1+y^2)/2}}\right\}.\end{aligned} \tag{8.4}$$

where, we remind the reader that $\tau_m=\sqrt{\tau_1\tau_2}$. When the argument is small, the functions $Y_1(x)$ and $K_1(x)$ are

$$\frac{\pi}{2}Y_1(x)\approx-\frac{1}{x}+\ln\frac{x}{2}\left(\frac{x}{2}+O(x^3)\right),$$

$$K_1(x)\approx\frac{1}{x}+\ln\frac{x}{2}\left(\frac{x}{2}+O(x^3)\right).$$

It is easy to see now that in the sum of the two integrals dy over the interval (0,1), the leading singularities dy/y exactly cancel each other. Furthermore, it is safe to take the limits of $\xi\to 0$, $u\to 1$ in the integrand, and even to set the lowerlimit of the integral dq to be zero. The resulting integral is convergent, ξ independent, and yields a term of the order ξ^2. In the remaining integral, the variable y runs from 1 to ∞, the integrand is not singular at finite y and q, and it is exponentially suppressed at large y and q. The behavior of the integral at $\xi\to 0$ is not singular and the basic upper estimate const$\times\xi^2$ remains unchanged for the entire integral (8.4).

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APPENDIX: WIGHTMAN FUNCTIONS AND PROPAGATORS OF WEDGE DYNAMICS

In this appendix, we put all field correlators into a form which is needed for the practical calculation of the self-energy. The density of states $D_{[1]}$ and the causal part $D_{[0]}$ of the gluon propagator are used in the form of decomposition over the transverse modes,

$$D^{lm}_{[10]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)=-i(2\pi)^2\int d\alpha\sum_\lambda v^{(\lambda)l}_{\alpha,\vec{k}_t}(\tau_2,\eta_2)\,\overset{*}{v}{}^{(\lambda)m}_{\alpha,\vec{k}_t}(\tau_1,\eta_1),$$

$$D^{lm}_{[01]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)=-i(2\pi)^2\int d\alpha\sum_\lambda v^{(\lambda)l}_{\alpha,-\vec{k}_t}(\tau_2,\eta_2)\,\overset{*}{v}{}^{(\lambda)m}_{\alpha,-\vec{k}_t}(\tau_1,\eta_1),\tag{A1}$$

where

$$v^{(TE)}_{\vec{k},\alpha}(x)=\frac{1}{4\pi^{3/2}k_t}\begin{bmatrix}k_y\\-k_x\\0\end{bmatrix}e^{-ik_t\tau\cosh(\alpha-\eta)};$$

$$v^{(TM)}_{\vec{k},\alpha}(x)=\frac{1}{4\pi^{3/2}k_t}\begin{bmatrix}k_xf_1\\k_yf_1\\-f_2\end{bmatrix},\tag{A2}$$

are the transverse electric and transverse magnetic modes of the radiation field found previously in paper [II]. Here, we denoted

$$f_1(\tau,\eta)=i\tanh(\alpha-\eta)(e^{-ik_t\tau\cosh(\alpha-\eta)}-1),$$

$$f_2(\tau,\eta)=\frac{e^{-ik_t\tau\cosh(\alpha-\eta)}-1}{\cosh^2(\alpha-\eta)}+ik_t\tau\frac{e^{-ik_t\tau\cosh(\alpha-\eta)}}{\cosh(\alpha-\eta)}.\tag{A3}$$

Starting from this form, we get the components of the commutator $D_{[0]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)$,[7]

$$D^{rs}_{[0]}=\int\frac{d\alpha}{2\pi}\left\{\left[\delta_{rs}-\frac{k_rk_s}{k_t^2}\right]\sin k_tT_{12}+\frac{k_rk_s}{k_t^2}\tanh\left(\alpha+\frac{\eta}{2}\right)\times\tanh\left(\alpha-\frac{\eta}{2}\right)[\sin k_tT_{12}\underline{-\sin k_tT_1+\sin k_tT_2}]\right\},\tag{A4}$$

$$D^{\eta\eta}_{[0]}=\int\frac{d\alpha}{2\pi}\frac{1}{k_t^2\cosh^2\left(\alpha+\frac{\eta}{2}\right)\cosh^2\left(\alpha-\frac{\eta}{2}\right)}\times[(1+k_t^2T_1T_2)\sin k_tT_{12}-k_tT_{12}\cos k_tT_{12}\underline{+\sin k_tT_2-\sin k_tT_1-k_tT_2\cos k_tT_2+k_tT_1\cos k_tT_1}],\tag{A5}$$

where $T_1=\tau_1\cosh(\alpha-\eta/2)$, $T_2=\tau_2\cosh(\alpha+\eta/2)$, $T_{12}=T_1-T_2$. In the first of these equations, the coefficients of the tensors $(\delta_{rs}-k_rk_s/k_t^2)$ and k_rk_s/k_t^2 are the invariants $\mathcal{D}^{(TE)}_{[0]}$ and $\mathcal{D}^{(2)}_{[0]}$ of Eq. (2.9), respectively. The latter is due to the TM mode of the radiation field. Up to the factor k_t^{-2}, Eq. (A5) defines the invariant $\mathcal{D}^{(\eta\eta)}_{[0]}$. The underlined terms are connected with the boundary conditions imposed on the TM mode at $\tau=0$. They cancel with the underlined terms in the longitudinal part of the gauge field propagator given by Eqs. (A14) and (A15) thus providing the causal behavior of the components $D^{rs}_{[\mathrm{adv}]}$ and $D^{\eta\eta}_{[\mathrm{adv}]}$ of the retarded propagator $D_{[\mathrm{adv}]}(\tau_2,\tau_1)$. In the body of the paper, we call the residues of this cancellation as the diagonal components of $D^{[0]}$. The "off-diagonal" components of the commutator $D^{ij}_{[0]}$ are

[7] In all formulas below, the gluon rapidity α is counted from the reference point $(\eta_1+\eta_2)/2$, the geometric center of the coordinates η_1 and η_2 of the vertices in the self-energy loop. Thus, it corresponds to the rapidity θ' in the integral representation of the quark correlators in paper [II].

$$D^{r\eta}_{[0]}=\frac{-ik_r}{k_t^2}\int\frac{d\alpha}{2\pi}\frac{\tanh\left(\alpha+\frac{\eta}{2}\right)}{\cosh^2\left(\alpha-\frac{\eta}{2}\right)}[\sin k_tT_{12}-k_tT_1\cos k_tT_{12}+\sin k_tT_2-\sin k_tT_1+k_tT_1\cos k_tT_1], \quad (A6)$$

$$D^{\eta r}_{[0]}=\frac{ik_r}{k_t^2}\int\frac{d\alpha}{2\pi}\frac{\tanh\left(\alpha-\frac{\eta}{2}\right)}{\cosh^2\left(\alpha+\frac{\eta}{2}\right)}[\sin k_tT_{12}+k_tT_2\cos k_tT_{12}+\sin k_tT_2-\sin k_tT_1-k_tT_2\cos k_tT_2]. \quad (A7)$$

The commutator is not a symmetric tensor. However, by examination, these components are odd with respect to the rapidity difference $\eta=\eta_1-\eta_2$, and hence they do not contribute to the effective quark mass we are computing in this paper.

The tensor of the gluon density $D^{ij}_{[1]}(\tau_2,\tau_1;\eta_2-\eta_1;\vec{k}_t)$ has the ''diagonal'' components,

$$D^{rs}_{[1]}=-i\int\frac{d\alpha}{2\pi}\left\{\left[\delta_{rs}-\frac{k_rk_s}{k_t^2}\right]\cos k_tT_{12}+\frac{k_rk_s}{k_t^2}\tanh\left(\alpha+\frac{\eta}{2}\right)\tanh\left(\alpha-\frac{\eta}{2}\right)\times(\cos k_tT_{12}-\cos k_tT_1-\cos k_tT_2+1)\right\}, \quad (A8)$$

$$D^{\eta\eta}_{[1]}=-i\int\frac{d\alpha}{2\pi}\frac{1}{k_t^2\cosh^2\left(\alpha+\frac{\eta}{2}\right)\cosh^2\left(\alpha-\frac{\eta}{2}\right)}\times[(1+k_t^2T_1T_2)\cos k_tT_{12}+k_tT_{12}\sin k_tT_{12}-\cos k_tT_2-\cos k_tT_1-k_tT_2\sin k_tT_2-k_tT_1\sin k_tT_1+1], \quad (A9)$$

from which one can infer the invariants $\mathcal{D}^{(TE)}_{[1]}$ and $\mathcal{D}^{(2)}_{[1]}$ of Eq. (2.9) exactly in the same way as it was done for the invariants of the commutator $D_{[0]}$. The off-diagonal components,

$$D^{r\eta}_{[1]}=\frac{-k_r}{k_t^2}\int\frac{d\alpha}{2\pi}\frac{\tanh\left(\alpha+\frac{\eta}{2}\right)}{\cosh^2\left(\alpha-\frac{\eta}{2}\right)}[\cos k_tT_{12}+k_tT_1\sin k_tT_{12}-\cos k_tT_2-\cos k_tT_1-k_tT_1\sin k_tT_1+1], \quad (A10)$$

$$D^{\eta r}_{[1]}=\frac{k_r}{k_t^2}\int\frac{d\alpha}{2\pi}\frac{\tanh\left(\alpha-\frac{\eta}{2}\right)}{\cosh^2\left(\alpha+\frac{\eta}{2}\right)}[\cos k_tT_{12}-k_tT_2\sin k_tT_{12}-\cos k_tT_2-\cos k_tT_1-k_tT_2\sin k_tT_2+1], \quad (A11)$$

are also nonsymmetric and odd with respect to the rapidity difference η. They also do not contribute to the effective quark mass. Equations (A8)–(A11) give the components of the vacuum density of states of the gauge field in the wedge dynamics. In order to incorporate the ''material'' part given by the distribution of real gluons, the integrand of each of Eqs. (A8)–(A11) must be multiplied by the common factor $[1+2n_g(k_t,\alpha)]$.

The full tensor of the longitudinal part of the propagator that defines the field $A(\tau_1)$ via the current $j(\tau_2)$ at all preceding times,

$$A^{[long]}_l(\tau_1)=\int_0^{\tau_1}\tau_2d\tau_2d\eta_2D^{[long]}_{lm}(\tau_2,\tau_1;\eta_1-\eta_2,\vec{k}_t)j^m(\tau_2), \quad (A12)$$

was found in paper [III] in the following form:

$$D^{[long]}_{lm}(\tau_2,\tau_1;\eta_1-\eta_2,\vec{k}_t)=\int\frac{d\nu d^2\vec{k}}{(2\pi)^3k_\perp^2}\begin{bmatrix}k_rk_s[Q_{-1,i\nu}(k_\perp\tau_2)-Q_{-1,i\nu}(k_\perp\tau_1)] & k_r\nu[Q_{1,i\nu}(k_\perp\tau_2)-Q_{-1,i\nu}(k_\perp\tau_1)]\\ \nu k_s[Q_{-1,i\nu}(k_\perp\tau_2)-Q_{1,i\nu}(k_\perp\tau_1)] & \nu^2[Q_{1,i\nu}(k_\perp\tau_2)-Q_{1,i\nu}(k_\perp\tau_1)]\end{bmatrix}_{lm}e^{-i\nu(\eta_1-\eta_2)}. \quad (A13)$$

The diagonal components of this longitudinal part of the gluon propagator are just the differences between the vector potentials of the ''static'' gluon fields at the final time τ_1 and the initial time τ_2,

$$D^{[long]}_{rs}=\frac{k_rk_s}{k_t^2}\left\{\frac{\coth|\eta|}{2}(e^{-\tau_1k_t\sinh|\eta|}-e^{-\tau_2k_t\sinh|\eta|})-\underline{\int\frac{d\alpha}{2\pi}\tanh\left(\alpha+\frac{\eta}{2}\right)\tanh\left(\alpha-\frac{\eta}{2}\right)[\sin k_tT_1-\sin k_tT_2]}\right\}, \quad (A14)$$

$$D^{[long]}_{\eta\eta}=-\frac{\tau_1^2-\tau_2^2}{2}\,\delta(\eta)-\left\{\left[\left(\frac{\cosh\eta}{k_t^2\sinh^3|\eta|}+\frac{\tau_1\cosh\eta}{k_t\sinh^2\eta}+\frac{\tau_1^2\cosh\eta}{2\sinh|\eta|}\right)e^{-\tau_1 k_t\sinh|\eta|}\right]-[\tau_1\to\tau_2]\right\}$$
$$\underline{-\int\frac{d\alpha}{2\pi}\frac{1}{k_t^2\cosh^2\left(\alpha+\frac{\eta}{2}\right)\cosh^2\left(\alpha-\frac{\eta}{2}\right)}[\sin k_tT_1-\sin k_tT_2-k_tT_1\cos k_tT_1+k_tT_2\cos k_tT_2].} \tag{A15}$$

By the derivation, these components include $\theta(\tau_1-\tau_2)$ of the following origin. The source current which acts at the moment τ_2 produces the simultaneous longitudinal electric field $E(\tau_2)$. The gauge field potential is rebuilt from the electric field at the time $\tau_1>\tau_2$ by integrating the electric field $E(\tau)$ over all times from 0 to τ_1. The underlined terms in Eqs. (A14) and (A15) cancel out in the full assembly of the retarded propagator $D_{[\mathrm{adv}]}(\tau_2,\tau_1)$ with the underlined terms in the radiation part, Eqs. (A4) and (A5). In the body of the paper, we call the residue of this cancelation as the diagonal components of $D^{[long]}$, which can be conveniently written as

$$D^{[long]}_{rs}=-\frac{k_rk_s}{k_t^2}\frac{k_t\cosh\eta}{2}\int_{\tau_2}^{\tau_1}e^{-tk_t\sinh|\eta|}dt, \tag{A16}$$

$$D^{[long]}_{\eta\eta}=\frac{\tau_1^2-\tau_2^2}{2}\,\delta(\eta)+\frac{k_t\cosh\eta}{2}\int_{\tau_2}^{\tau_1}e^{-tk_t\sinh|\eta|}t^2dt. \tag{A17}$$

Once again, the ''off-diagonal'' components of the longitudinal part of propagator,

$$D^{[long]}_{r\eta}=\frac{ik_r}{k_t^2}\left\{-\frac{\operatorname{sgn}\eta}{\sinh^2\eta}(e^{-\tau_1k_t\sinh|\eta|}-e^{-\tau_2k_t\sinh|\eta|})+\frac{k_t\tau_2}{\sinh\eta}e^{-\tau_2k_t\sinh|\eta|}-\frac{k_t\tau_1\cosh^2\eta}{\sinh\eta}e^{-\tau_1k_t\sinh|\eta|}\right.$$
$$\left.-\int\frac{d\alpha}{2\pi}\frac{\tanh\left(\alpha-\frac{\eta}{2}\right)}{\cosh^2\left(\alpha+\frac{\eta}{2}\right)}[\sin k_tT_1-\sin k_tT_2+k_tT_2\cos k_tT_2]\right\}, \tag{A18}$$

$$D^{[long]}_{\eta r}=\frac{ik_r}{k_t^2}\left\{-\frac{\operatorname{sgn}\eta}{\sinh^2\eta}(e^{-\tau_1k_t\sinh|\eta|}-e^{-\tau_2k_t\sinh|\eta|})-\frac{k_t\tau_1}{\sinh\eta}e^{-\tau_1k_t\sinh|\eta|}+\frac{k_t\tau_2\cosh^2\eta}{\sinh\eta}e^{-\tau_2k_t\sinh|\eta|}\right.$$
$$\left.-\int\frac{d\alpha}{2\pi}\frac{\tanh\left(\alpha+\frac{\eta}{2}\right)}{\cosh^2\left(\alpha-\frac{\eta}{2}\right)}[-\sin k_tT_1+\sin k_tT_2+k_tT_1\cos k_tT_1]\right\}, \tag{A19}$$

are odd with respect to η and do not contribute the effective quark mass.

The gauge-field correlators have several distinctive features. First, the lengthy expression for each component is such that the gauge field correlators obey the boundary condition $A_\eta(\tau=0,\vec{r}_t)=0$ which provides continuity of the field at $\tau=0$, and allows for a complete fixing of the gauge. Second, in the rs and $\eta\eta$ components of the propagator

$$D^{lm}_{[\mathrm{adv}]}(\tau_2,\tau_1,\eta;k_t)=-\theta(\tau_1-\tau_2)D^{lm}_{[0]}(\tau_2,\tau_1,\eta;k_t)+D^{lm}_L(\tau_2,\tau_1,\eta;k_t), \tag{A20}$$

the boundary terms cancel between the transverse and longitudinal parts. This fact provides causal behavior of the $\Sigma(\tau_1,\tau_2)$ that defines the dispersion law.

The fermion invariants $g_{[\alpha]}$ were derived in paper [II]. For the sake of completeness, we reproduce the final answers here:

$$g_{[0]}^{L(\pm)} = i\,\frac{\tau_1 e^{\mp\eta/2} - \tau_2 e^{\pm\eta/2}}{4\sqrt{|\tau_{12}^2|}}\,\theta(\tau_{12}^2) J_1(q_t\sqrt{|\tau_{12}^2|}), \quad g_{[0]}^{T(\pm)} = -\frac{e^{\mp\eta/2}}{4}\,\theta(\tau_{12}^2) J_0(q_t\sqrt{|\tau_{12}^2|}), \tag{A21}$$

$$g_{[1]}^{L(\pm)} = -\int \frac{d\theta'}{4\pi}\left[1-2n_f\left(\frac{\eta_1+\eta_2}{2}+\theta',p_t\right)\right] e^{\mp\theta'}\,\sin(p_t[\tau_1\cosh(\theta-\eta/2)-\tau_2\cosh(\theta+\eta/2)])$$

$$= \frac{\tau_1 e^{\mp\eta/2} - \tau_2 e^{\pm\eta/2}}{4\sqrt{|\tau_{12}^2|}}\left[\theta(\tau_{12}^2) Y_1(q_t\sqrt{|\tau_{12}^2|}) + \frac{2}{\pi}\theta(-\tau_{12}^2) K_1(q_t\sqrt{|\tau_{12}^2|})\right][1-2n_f(q_t)],$$

$$g_{[1]}^{T(\pm)} = -ie^{\mp\eta/2}\int \frac{d\theta'}{4\pi}\left[1-2n_f\left(\frac{\eta_1+\eta_2}{2}+\theta',p_t\right)\right]\cos(p_t[\tau_1\cosh(\theta-\eta/2)-\tau_2\cosh(\theta+\eta/2)])$$

$$= i\,\frac{e^{\mp\eta/2}}{4}\left[\theta(\tau_{12}^2) Y_0(q_t\sqrt{|\tau_{12}^2|}) - \frac{2}{\pi}\theta(-\tau_{12}^2) K_0(q_t\sqrt{|\tau_{12}^2|})\right][1-2n_f(q_t)]. \tag{A22}$$

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