# Scenario for ultrarelativistic nuclear collisions. III. Gluons in the expanding geometry

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We derive expressions for various correlators of the gauge field and find the propagators in Hamiltonian dynamics which employs a new gauge $A^\tau=0$. This gauge is a part of the wedge form of relativistic dynamics suggested earlier as a tool for the study of quantum dynamics in ultrarelativistic heavy ion collisions. We prove that the gauge is completely fixed. The gauge field is quantized and the field of radiation and the longitudinal fields are unambiguously separated. The new gauge puts the quark and gluon fields of the colliding hadrons in one Hilbert space and thus allows one to avoid factorization.

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## I. INTRODUCTION

In two previous papers of this cycle [1,2] (further quoted as papers [I] and [II]), we explained the physical motivation of the ''wedge form of dynamics'' as a promising tool to explore the processes which take place during the collision of two heavy ions. In compliance with the general definition of dynamics given by Dirac [3], the wedge form includes a specific definition of the quantum mechanical observables on spacelike surfaces, as well as the means to describe the evolution of the observables from an ''earlier'' spacelike surface to a ''later'' one. Unlike the other forms, the wedge form explicitly refers to the two main geometrical features of the phenomenon, i.e., the strong localization of the initial interaction and, as a consequence, the absence of translational invariance in the temporal and longitudinal directions.

In the wedge form of dynamics, the states of the quark and gluon fields are defined on the spacelike hypersurfaces of the constant proper time $\tau$, $\tau^2=t^2-z^2$. The states of fermion fields were discussed in paper [II]. In this paper, we continue the study of the gluons and augment our previous consideration by the gauge condition $A^\tau=0$. This simple idea solves several problems. First, this gauge condition is boost-invariant and thus complies with the symmetry of the collision. Second, it becomes possible to treat two different light-front gauges (which describe gluons from each nucleus of the initial state separately) as the two limits of this single gauge. Therefore, the new approach keeps important connections with the theory of deep inelastic $ep$-scattering (DIS). This fact is vital for the subsequent calculations since $e-p$ DIS is the only existing source of data on nucleon structure in high-energy collisions. The approach based on quantum field kinetics (QFK) allows one to treat both the nuclear collision and $e$-$p$ DIS as the similar transient processes. Third, after the collision, this kind of gauge becomes a local temporal axial gauge, thus providing a smooth transition to the Bjorken regime of the boost-invariant expansion.

Most of this paper is technical, and any relevant physical discussion of the results appears only after their mathematical derivation. These results were summarized in paper [II]. Since the first interaction of two *finite-sized* nuclei is strongly localized, the geometrical symmetry of the final state is manifestly broken and the observables of wedge dynamics are essentially defined on the curved spacelike hypersurface. For the fermion field this have led to an obvious Thomas precession. Similar orientation effects happen in the case of the vector field also. The dynamics of the gauge field is rich, and the procedure of its quantization triggers many puzzles that can be traced back to the classical roots of the gauge field theory.

In Sec. II A, we derive equations of motion for the gauge field in the gauge $A^\tau=0$, find the Hamiltonian variables and the normalization condition. The equations of motion are linearized and the modes of the free radiation field are obtained in Sec. II B. In Sec. II C, the retarded propagator of the perturbation theory is found as the response function of the field on the external current. This part of the calculation turned out to be the most durable, since the gauge condition is inhomogeneous and none of the modern methods is effective. However, the old-fashioned variation of parameters does work. The most important result of this paper, separation the transverse and longitudinal parts of the gluon propagator is obtained here. We essentially base calculation of the quark self-energy in expanding quark-gluon system on this result. These calculations are presented in the next paper. In Sec. II D, we show that the previously obtained propagator solves the initial data problem for the gauge field. Unlike in the homogeneous axial gauges, the propagators of the gauge $A^\tau=0$ do not have any spurious poles.

Section III is devoted to the quantization of the vector field in the gauge $A^\tau=0$. We begin in Sec. III A with the proof of the fact that the gauge $A^\tau=0$ can be completely fixed provided the physical charge density, $\tau j_\tau$ vanishes at $\tau=0$, the moment of the first touch of the nuclei. This is exactly what can be expected from the nuclei colorlessness. Then, the Gauss law can be unquestionably used to eliminate the unphysical degrees of freedom in the equations of motion. We continue in Sec. III B with a computation of the Wightman functions, and study the causal properties of the commutators in Sec. III C. The latter appears to be abnormal; the Riemann function is not symmetric and penetrates the exterior of the light cone. However, the behavior of the observables is fully causal and the procedure of the canonical quantization is accomplished in Sec. III D. Even though it is impossible to introduce transverse and longitudinal currents (as it is customary for the homogeneous gauge conditions) and thus fully separate the dynamics of the corresponding fields, we found it useful to discriminate the various field patterns by the type of their propagation. The propagator of the transverse field is sensitive to the light cone boundaries while the longitudinal and instantaneous parts of the field do not propagate. These coordinate form of these two fragments of the response function is derived in Sec. IV. Ultimately, the longitudinal part of the gluon propagator appeared to be of the greatest importance for the dynamics of the screening effects at the early stage of the collision.

In Appendix D, we study the limiting behavior of the propagator in the central rapidity region and in the vicinity of the null planes and show that propagators of the gauges $A^0=0$ and $A^{\pm}=0$ are recovered. It is important that the spurious poles are recovered only in the unphysical limit of infinite rapidity. This result is of practical importance because it establishes the connection between the new approach and the existing theory of the deeply inelastic processes at high energies.

## II. THE CLASSICAL TREATMENT

The final goal of this paper is to build a quantum theory of the vector gauge field in the expanding geometry of nuclear collision. Development of a quantum theory always begins with its classical counterpart which provides the one-particle wave functions (which later serve as quantum states) and the classical Green functions (which later become the propagators of quantum theory). Furthermore, the quantum propagator of gauge field includes the longitudinal part which can be found only by classical analysis. The classical part of this program is the subject of this section.

### A. Classical equations of motion

We now consider the case of pure glue-dynamics. We denote the gluon field in the fundamental representation of the color group as $A_\mu(x)=t^a A^a_\mu(x)$. Consequently, we have the field tensor

$$F_{\mu\nu}=t^a F^a_{\mu\nu}=\mathcal{D}_\mu A_\nu-\mathcal{D}_\nu A_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu-i\mathrm{g}[A_\mu,A_\nu],$$

where $\mathcal{D}_\mu=\partial_\mu-i\mathrm{g}[A_\mu(x),\dots]$ is the covariant derivative on the local color group. The gauge invariant action of the theory looks as follows:

$$\mathcal{S}=\int \mathcal{L}(x)d^4x$$
$$=\int\left[-\frac{1}{4}\mathrm{g}^{\mu\lambda}(x)\mathrm{g}^{\nu\sigma}(x)F_{\mu\nu}(x)F_{\lambda\sigma}(x)-j^\mu A_\mu\right]\sqrt{-\mathrm{g}}d^4x. \tag{2.1}$$

Its variation with respect to the gluon field yields the Lagrangian equations of motion,

$$\partial_\lambda[(-\mathrm{g})^{1/2}\mathrm{g}^{\mu\lambda}\mathrm{g}^{\nu\sigma}F_{\mu\nu}]-i\mathrm{g}(-\mathrm{g})^{1/2}[A_\lambda,\mathrm{g}^{\mu\lambda}\mathrm{g}^{\nu\sigma}F_{\mu\nu}]$$
$$=(-\mathrm{g})^{1/2}j^\sigma, \tag{2.2}$$

where $j^\mu$ is the color current of the fermion fields and $\mathrm{g}=\det|\mathrm{g}_{\mu\nu}|$. The equations are twice covariant, i.e., with respect to the gauge transformations in color space and the arbitrary transformations of the coordinates. In what follows, we shall employ the special coordinates associated with the constant proper time hypersurfaces inside the light cone of the collision point $t=z=0$. The new coordinates parameterize the Minkowski coordinates $(t,x,y,z)$ as $(\tau\cosh\eta,x,y,\tau\sinh\eta)$. In addition, we impose the gauge condition $A_\tau=0$. The corresponding gauge transformation is well defined. Indeed, let $A_\mu(x)$ be an arbitrary field configuration and $A'_\mu(x)$ its gauge transform with the generator

$$U(\tau,\eta,\vec{r}_\perp)=P_\tau\exp\left\{-\int_0^\tau A_\tau(\tau',\eta,\vec{r}_\perp)d\tau'\right\}, \tag{2.3}$$

then the new field, $A'_\mu=UA_\mu U^{-1}+\partial_\mu UU^{-1}$, obeys the condition $A^\tau=0$. Imposing this gauge condition we arrive at the system of four equations:

$$\mathcal{C}(x)=\frac{1}{\tau}\partial_\eta\partial_\tau A_\eta+\tau\partial_r\partial_\tau A_r$$
$$-i\mathrm{g}\left\{\frac{1}{\tau}[A_\eta,\partial_\tau A_\eta]+\tau[A_r,\partial_\tau A_r]\right\}-\tau j^\tau=0, \tag{2.4}$$

$$-\partial_\tau\tau\partial_\tau A_r+\frac{1}{\tau}\partial_\eta(\partial_\eta A_r-\partial_r A_\eta)+\tau\partial_s(\partial_s A_r-\partial_r A_s)$$
$$-i\mathrm{g}\left\{\frac{1}{\tau}\partial_\eta[A_\eta,A_r]+\tau\partial_s[A_s,A_r]+\frac{1}{\tau}[A_\eta,F_{\eta r}]\right.$$
$$\left.+\tau[A_s,F_{sr}]\right\}-\tau j^r=0, \tag{2.5}$$

$$-\partial_\tau\frac{1}{\tau}\partial_\tau A_\eta+\frac{1}{\tau}\partial_r(\partial_r A_\eta-\partial_\eta A_r)-i\mathrm{g}\left[\frac{1}{\tau}\partial_r[A_r,A_\eta]\right.$$
$$\left.+\frac{1}{\tau}[A_r,F_{r\eta}]\right]-\tau j^\eta=0. \tag{2.6}$$

Here, we use the Latin indices from $r$ to $w$ for the transverse $x$ and $y$ components $(r,\dots,w=1,2)$. We shall also use the arrows over the letters to denote the two-dimensional vectors, e.g., $\vec{k}=(k_x,k_y)$, $|\vec{k}|=k_\perp$. The Latin indices from $i$ to $n$ $(i,\dots,n=1,2,3)$ will be used for the three-dimensional internal coordinates $u^i=(x,y,\eta)$ on the hypersurface $\tau=\mathrm{const}$. The metric tensor has only diagonal components $\mathrm{g}_{\tau\tau}=-\mathrm{g}_{xx}=-\mathrm{g}_{yy}=1,\mathrm{g}_{\eta\eta}=-\tau^2$. The first of these equations (2.4) contains no second order time derivatives and is a constraint rather than a dynamical equation. The constraint weakly equals to zero in classical Hamiltonian dynamics and serves as a condition imposed on physical states in the quantum theory. The canonical momenta of the theory are as follows:

$$p^\tau=0,\quad p^\eta=\frac{1}{\tau}F_{\tau\eta}=\frac{1}{\tau}\dot{A}_\eta,\quad p_r=\tau F_{\tau r}=\tau\dot{A}_r. \tag{2.7}$$

Hereafter, the dot above the letter denotes a derivative with respect to the Hamiltonian time $\tau$. Because of the gauge condition, the canonical momenta do not contain the color commutators. After excluding the velocities, the Hamiltonian can be written down in the canonical variables,

$$H=\int d\eta d\vec{r}_\perp \tau\left\{\frac{1}{2}p^\eta p^\eta+\frac{1}{2\tau^2}p^r p^r+\frac{1}{2\tau^2}F_{\eta r}F_{\eta r}+\frac{1}{4}F_{rs}F_{rs}+j^\eta A_\eta+j^r A_r\right\}. \tag{2.8}$$

Then Eqs. (2.5) and (2.6) are immediately recognized as the Hamiltonian equations of motion. The Poisson bracket of the constraint $\mathcal{C}$ with the Hamiltonian vanishes, thus creating the generator of the residual gauge transformations which are tangent to the hypersurface. Conservation of the constraint is a direct consequence of the Lagrange (or Hamiltonian) classical equations of motion as well.

The normalization condition for the one-particle solutions is obviously derived from the charge conservation law. For the gauge field, this is impossible. Therefore, we shall accept the condition which supports self-adjointness of the homogeneous system after its linearization. This leads to a natural definition for the scalar product of the states of the vector field in the gauge $A^\tau=0$,

$$(V,W)=\int_{-\infty}^{\infty}d\eta\int d^2\vec{r}\,\tau g^{ik}V_i^* i\overleftrightarrow{\partial}_\tau W_k, \tag{2.9}$$

where $g^{ik}$ is the metric tensor of the three-dimensional internal geometry of the hypersurface $\tau=$const. This norm of the one-particle states prevents them from flowing out of the interior of the past and future light wedges of the interaction plane.

### B. Modes of the free radiation field. Field of the static source

As a tool for the future development of the perturbation theory, we need to find the propagators and Wightman functions when the nonlinear self-interaction of the gluon field is switched off. In this case, the system of equations for the nonvanishing components of the vector potential and the constraint look as follows:

$$\left[\partial_\tau\tau\partial_\tau-\frac{1}{\tau}\partial_\eta^2-\tau\partial_s^2\right]A_r+\partial_r\left[\tau\partial_s A_s+\frac{1}{\tau}\partial_\eta A_\eta\right]=-\tau j^r, \tag{2.10}$$

$$\left[\partial_\tau\frac{1}{\tau}\partial_\tau-\frac{1}{\tau}\partial_s^2\right]A_\eta+\frac{1}{\tau}\partial_\eta\partial_s A_s=-\tau j^\eta, \tag{2.11}$$

$$\mathcal{C}(x)=\frac{1}{\tau}\partial_\eta\partial_\tau A_\eta+\tau\partial_\tau\partial_r A_r-\tau j^\tau=0, \tag{2.12}$$

where $j^\mu$ includes all kinds of the color currents. An explicit form of the solution for the homogeneous system is found in Appendix A. In compliance with the gauge condition (which explicitly eliminates one of four field components) we find three modes $V^{(\lambda)}$ of the free vector field. Two transverse modes obey Gauss law without the charge and have the unit norm (see Appendix A) with respect to the scalar product (2.9):

$$V^{(1)}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_\perp}\begin{pmatrix}k_y\\-k_x\\0\end{pmatrix}H^{(2)}_{-i\nu}(k_\perp\tau)e^{i\nu\eta+i\vec{k}\vec{r}};$$

$$V^{(2)}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_\perp}\begin{pmatrix}\nu k_x R^{(2)}_{-1,-i\nu}(k_\perp\tau)\\ \nu k_y R^{(2)}_{-1,-i\nu}(k_\perp\tau)\\ -R^{(2)}_{1,-i\nu}(k_\perp\tau)\end{pmatrix}e^{i\nu\eta+i\vec{k}\vec{r}}. \tag{2.13}$$

The mode $V^{(2)}$ is constructed from the functions $R^{(j)}_{\mu,-i\nu}(k_\perp\tau)=R^{(j)}_{\mu,-i\nu}(k_\perp\tau|s)$ corresponding to the boundary condition of vanishing gauge field at $\tau=0$. This guarantees continuous behavior of the field at $\tau=0$. Indeed, as $\tau\to 0$, the normal and the tangential directions become degenerate. As long as $A^\tau=0$ is the gauge condition, continuity requires that $A^\eta\to 0$ as $\tau\to 0$.

It is instructive to know the physical components of the electric and magnetic fields of these modes, $\mathcal{E}^m=\sqrt{-g}g^{mn}\dot{A}_n$ and $\mathcal{B}^m=-(2\sqrt{-g})^{-1}e^{mln}F_{ln}$:

$$\mathcal{E}^{(1)m}_{\vec{k},\nu}(x)=i\mathcal{B}^{(2)m}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_\perp}\begin{pmatrix}k_y\\-k_x\\0\end{pmatrix}\dot{H}^{(2)}_{-i\nu}(k_\perp\tau)e^{i\nu\eta+i\vec{k}\vec{r}},$$

$$\mathcal{E}^{(2)m}_{\vec{k},\nu}(x)=i\mathcal{B}^{(1)m}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_\perp}\begin{pmatrix}\nu k_x\\ \nu k_y\\ -k_\perp^2\end{pmatrix}H^{(2)}_{-i\nu}(k_\perp\tau)e^{i\nu\eta+i\vec{k}\vec{r}}. \tag{2.14}$$

The mode $V^{(2)}$ can be obtained from the mode $V^{(1)}$ by a simple interchange of its electric and magnetic fields. Using standard wave-guide terminology, one may call mode $V^{(1)}$ as the ‘‘transverse electric mode’’ and the mode $V^{(2)}$ as the ‘‘transverse magnetic mode.’’ Equations (2.14) indicate, that the field strength tensor of the free radiation field obeys the condition, $(F^*)^*=-F$. Therefore, certain linear combinations of the modes $V^{(1)}$ and $V^{(2)}$ may be analytically continued to Euclidean space as self-dual solutions of the field equations.

An equivalent full set of the transverse modes carries (instead of the boost $\nu$) the quantum number $\theta$ (rapidity), i.e., $k_0=k_\perp\cosh\theta$, $k_3=k_\perp\sinh\theta$. These functions can be obtained by means of the Fourier transform,

$$v^{(\lambda)}_{\vec{k},\theta}(x)=\int_{-\infty}^{+\infty}\frac{d\nu}{(2\pi)^{1/2}i}e^{-i\nu\theta}V^{(\lambda)}_{\vec{k},\nu}(x), \tag{2.15}$$

and have the following form:

$$v^{(1)}_{\vec{k},\theta}(x)=\frac{1}{4\pi^{3/2}k_\perp}\begin{pmatrix}k_y\\-k_x\\0\end{pmatrix}e^{-ik_\perp\tau\cosh(\theta-\eta)+i\vec{k}\vec{r}};$$

$$v^{(2)}_{\vec{k},\theta}(x)=\frac{1}{4\pi^{3/2}k_\perp}\begin{pmatrix}k_xf_1\\k_yf_1\\-f_2\end{pmatrix}e^{i\vec{k}\vec{r}},\qquad(2.16)$$

where

$$f_1(\tau,\eta)=k_\perp\sinh(\theta-\eta)\int_0^\tau e^{-ik_\perp\tau'\cosh(\theta-\eta)}d\tau'$$
$$=i\tanh(\theta-\eta)(e^{-ik_\perp\tau\cosh(\theta-\eta)}-1),$$
$$f_2(\tau,\eta)=k_\perp^2\int_0^\tau e^{-ik_\perp\tau'\cosh(\theta-\eta)}\tau'd\tau'$$
$$=\frac{e^{-ik_\perp\tau\cosh(\theta-\eta)}-1}{\cosh^2(\theta-\eta)}+ik_\perp\tau\frac{e^{-ik_\perp\tau\cosh(\theta-\eta)}}{\cosh(\theta-\eta)}.\qquad(2.17)$$

The norm of the Coulomb mode $V^{(3)}$, as defined by Eq. (2.9), equals zero, and it is orthogonal to $V^{(1)}$ and $V^{(2)}$. Though this solution obeys the equations of motion without the current, it does not obey Gauss law without a charge. Therefore, it should be discarded in the decomposition of the radiation field. However, it should be kept if we consider the radiation field in the presence of a static source with the $\tau$-independent density $\rho(\vec{k},\nu)=\tau j^\tau_{k\nu}(\tau)=\mathrm{const}(\tau)$. In this case, its definition can be completed using Gauss' law:

$$V^{(3)}_{\vec{k},\nu}(x)=\frac{\rho(\vec{k},\nu)}{(2\pi)^3ik_\perp^2}\begin{pmatrix}k_rQ_{-1,i\nu}(k_\perp\tau)\\\nu Q_{1,i\nu}(k_\perp\tau)\end{pmatrix}e^{i\nu\eta+i\vec{k}\vec{r}}.\qquad(2.18)$$

The coordinate form of this solution is noteworthy. The physical components, $\mathcal{E}^m=\sqrt{-g}g^{ml}\dot{A}_l$, of the electric field of the "$\tau$-static" source can be written in the integral form,

$$\mathcal{E}^i_{(stat)}(\tau,\vec{r}_1,\eta_1)=\int d\vec{r}_2d\eta_2K_i(\tau;\vec{r}_1-\vec{r}_2,\eta_1-\eta_2)\times\rho(\vec{r}_2,\eta_2),\qquad(2.19)$$

with the kernel

$$K_i(\tau;\vec{r},\eta)=\int\frac{d\nu d^2\vec{k}}{(2\pi)^3}\frac{e^{i\nu\eta+i\vec{k}\vec{r}}}{ik_\perp^2}\begin{pmatrix}k_rs_{1,i\nu}(k_\perp\tau)\\\nu k_\perp^2s_{-1,i\nu}(k_\perp\tau)\end{pmatrix}$$
$$=-\frac{\theta(\tau-r_\perp)}{4\pi}\begin{pmatrix}\tau\cosh\eta(\partial/\partial x^r)\\\partial/\partial(\tau\sinh\eta)\end{pmatrix}\frac{1}{R_{12}},\qquad(2.20)$$

where $R_{12}=(r_\perp^2+\tau^2\sinh^2\eta)^{1/2}=[(\vec{r}_1-\vec{r}_2)^2+\tau^2\sinh^2(\eta_1-\eta_2)]^{1/2}$, is the distance between the points $(\vec{r}_1,\eta_1)$ and $(\vec{r}_2,\eta_2)$ in the internal geometry of the surface $\tau=\mathrm{const}$. The technical details of the derivation of the last expression will be presented in Sec. IV. Equation (2.20) is an analog of Coulomb's law of electrostatics, except that now the source has a density which is static with respect to the Hamiltonian time $\tau$. In fact, *the source is static if it expands in such a way that its physical component* $\mathcal{J}^\tau=\tau j^\tau(\tau,\eta,\vec{r})$ *does not depend on* $\tau$. These expressions will be helpful in recognizing the origin of various terms in the full propagator which is calculated below.

### C. Propagator in the gauge $A^\tau=0$

The calculation of the propagator in the gauge $A^\tau=0$ (associated with the system of the curved surfaces $\tau=\mathrm{const}$) meets several problems. Three methods are commonly used in field theory. One of them strongly appeals to the Fourier analysis in the plane Minkowski space which is not applicable now because the metric itself is coordinate-dependent. The second method uses the path-integral formulation which is also ineffective because of the explicit coordinate dependence of the gauge-fixing term in the Lagrangian. One could also try to study the spectrum of the matrix differential operator, to find its eigenfunctions, and to use the standard expression for the resolvent. However, the extension of the system for the nonzero eigenvalues leads to unwieldy equations. On the other hand, the Green function of the perturbation theory must coincide with the one which solves the problem of the gauge field interaction with the classical "external" current. For this reason, we shall compute the Green function in a most straightforward way; we shall look for the partial solution of the inhomogeneous system using the old-fashioned method of "variation of parameters." This method will immediately separate the radiation and the longitudinal parts of the retarded propagator. All other methods would require an additional analysis for this purpose.

Let us start the derivation of the propagator in the gauge $A^\tau=0$ by obtaining the separate differential equations for the $\eta$ component of the magnetic field, $\Psi=\partial_yA_x-\partial_xA_y$, the transverse divergence of the electric field, $\varphi=\tau(\partial_x\dot{A}_x+\partial_y\dot{A}_y)$, and the $\eta$ component of the electric field, $a=\dot{A}_\eta/\tau$. In terms of the Fourier components with respect to the spatial coordinates, these equations read as

$$\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]\Psi_{\vec{k},\nu}(\tau)=-j^\psi(\vec{k},\nu,\tau),\qquad(2.21)$$

$$\left[\partial_\tau\tau\partial_\tau+\frac{\nu^2}{\tau}\right]\varphi(\vec{k},\nu,\tau)-i\tau\nu k_\perp^2a(\vec{k},\nu,\tau)$$
$$=-\partial_\tau[\tau^2j^\varphi(\vec{k},\nu,\tau)],\qquad(2.22)$$

$$[\partial_\tau\tau\partial_\tau+\tau k_\perp^2]a(\vec{k},\nu,\tau)-\frac{i\nu}{\tau}\varphi(\vec{k},\nu,\tau)=-\partial_\tau[\tau^2j^\eta(\vec{k},\nu,\tau)],\qquad(2.23)$$

where $j^\psi=\partial_yj_x-\partial_xj_y$, $j^\varphi=\partial_xj_x+\partial_yj_y$. Using the constraint conservation, which may be explicitly integrated to

$$\varphi(\vec{k},\nu,\tau)+i\nu a(\vec{k},\nu,\tau)-\tau j^{\tau}(\vec{k},\nu,\tau)=-\rho_0(\vec{k},\nu)=\mathrm{const}(\tau), \tag{2.24}$$

one easily obtains two independent equations for $\varphi(\vec{k},\nu,\tau)$ and $a(\vec{k},\nu,\tau)$:

$$\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]\varphi(\vec{k},\nu,\tau)$$
$$=k_\perp^2[\rho(\vec{k},\nu,\tau)-\rho_0(\vec{k},\nu)]-\frac{1}{\tau}\partial_\tau(\tau^2 j^{\varphi}(\vec{k},\nu,\tau))\equiv f^{\varphi}, \tag{2.25}$$

$$\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]a(\vec{k},\nu,\tau)$$
$$=\frac{-i\nu}{\tau^2}[\rho(\vec{k},\nu,\tau)-\rho_0(\vec{k},\nu)]-\frac{1}{\tau}\partial_\tau(\tau^2 j^{\eta}(\vec{k},\nu,\tau))\equiv f^{\eta}. \tag{2.26}$$

The constant of integration $\rho_0(\vec{k},\nu)$ has the meaning of the arbitrary static charge density and it should be retained until Gauss' law is explicitly imposed on the solution. In what follows, we shall not write it explicitly, keeping in mind that it is included in the true charge density $\rho(\vec{k},\nu,\tau)$. Since wedge dynamics has a selected time moment $\tau=0$, the constant of integration $\rho_0(\vec{k},\nu)$ can be associated with the initial data, namely, with the charge density at $\tau=0$. As we shall see soon, a proper choice of $\rho_0$ will be needed in order to fix the gauge $A^{\tau}=0$ completely.

Equations (2.21), (2.25), and (2.26) can be solved by the method of ‘‘variation of parameters’’:

$$\mathcal{F}(\tau)=\frac{\pi i}{4}\int_0^{\tau}\tau_2 d\tau_2\mathcal{H}(\tau,\tau_2)f(\tau_2), \tag{2.27}$$

where $\mathcal{F}$ stands for any one of the unknown functions in these equations, and $f$ for the corresponding right-hand side. The kernel

$$\mathcal{H}(\tau,\tau_2)=H_{i\nu}^{(1)}(k_\perp\tau)H_{i\nu}^{(2)}(k_\perp\tau_2)-H_{i\nu}^{(2)}(k_\perp\tau)H_{i\nu}^{(1)}(k_\perp\tau_2)$$

is the usual bilinear form which is built from the linearly-independent solutions of the homogeneous equation. (The Wronskian of these solutions is exactly $4/i\pi\tau_2$.) Taking $\mathcal{F}=\Psi$, we obtain the first equation for the components $A_x(\vec{k},\nu,\tau)$ and $A_y(\vec{k},\nu,\tau)$ of the vector potential:

$$\Psi(\vec{k},\nu,\tau_1)\equiv i[-k_yA_x+k_xA_y]$$
$$=\frac{i\pi}{4}\int_0^{\tau_1}\tau_2 d\tau_2\mathcal{H}(\tau_1,\tau_2)i[-k_y j^x(\tau_2)+k_x j^y(\tau_2)]. \tag{2.28}$$

In order to find the second equation for the $x$ and $y$ components and the equation for $A_\eta(\vec{k},\nu,\tau)$, we must integrate twice, i.e.,

$$\Phi(\vec{k},\nu,\tau_1)\equiv i[k_xA_x+k_yA_y]$$
$$=\frac{i\pi}{4}\int_0^{\tau_1}\frac{d\tau'}{\tau'}\int_0^{\tau'}\mathcal{H}(\tau',\tau_2)\tau_2 d\tau_2\left[-k_\perp^2\rho(\vec{k},\nu,\tau_2)+\frac{1}{\tau_2}\partial_{\tau_2}(\tau_2^2 j^{\varphi}(\vec{k},\nu,\tau_2))\right], \tag{2.29}$$

$$A_\eta(\vec{k},\nu,\tau_1)=\frac{i\pi}{4}\int_0^{\tau_1}\tau' d\tau'\int_0^{\tau'}\mathcal{H}(\tau',\tau_2)\tau_2 d\tau_2$$
$$\times\left[\frac{i\nu}{\tau_2^2}\rho(\vec{k},\nu,\tau_2)+\frac{1}{\tau_2}\partial_{\tau_2}(\tau_2^2 j^{\eta}(\vec{k},\nu,\tau_2))\right]. \tag{2.30}$$

The integration over $\tau_2$ recovers the electric fields at the moment $\tau'$, whilst the integration over $\tau'$ gives the vector potential at the moment $\tau_1$. It is convenient to start with the second of these integrations which has the limits $\tau_2<\tau'<\tau_1$. Let us consider the main line of the calculations in detail, using the $\eta$ component as an example. The first integration follows the formula (B1),

$$k_\perp^{\mu+1}\int_{\tau_2}^{\tau_1}(\tau')^{\mu}H_{i\nu}^{(j)}(k_\perp\tau')d\tau'=R_{\mu,i\nu}^{(j)}(k_\perp\tau_1)-R_{\mu,i\nu}^{(j)}(k_\perp\tau_2), \tag{2.31}$$

and the terms emerging from the lower limit $\tau_2$ can be conveniently transformed according to the relation (see Appendix B)

$$R_{\mu,i\nu}^{(1)}(k_\perp\tau_2)H_{i\nu}^{(2)}(k_\perp\tau_2)-R_{\mu,i\nu}^{(2)}(k_\perp\tau_2)H_{i\nu}^{(1)}(k_\perp\tau_2)$$
$$=\frac{4}{i\pi}s_{\mu,i\nu}(k_\perp\tau_2). \tag{2.32}$$

As a result, one obtains, e.g., the following formula for $A_\eta(\vec{k},\nu,\tau_1)$:

$$A_\eta(\vec{k},\nu,\tau_1)=\frac{i\pi}{4k_\perp^2}\int_0^{\tau_1}\tau_2 d\tau_2\left[R_{1,i\nu}^{(1)}(k_\perp\tau_1)H_{i\nu}^{(2)}(k_\perp\tau_2)-R_{1,i\nu}^{(2)}(k_\perp\tau_1)H_{i\nu}^{(1)}(k_\perp\tau_2)-\frac{4}{i\pi}s_{1,i\nu}(k_\perp\tau_2)\right]$$
$$\times\left[\frac{i\nu}{\tau_2^2}\rho(\vec{k},\nu,\tau_2)+\frac{1}{\tau_2}\partial_{\tau_2}(\tau_2^2 j^{\eta}(\vec{k},\nu,\tau_2))\right]. \tag{2.33}$$

In order to eliminate the charge density $\rho$ from the integrand and to separate the transverse and the longitudinal parts of the propagator, all the terms of this formula should be integrated by parts, explicitly accounting for the charge conservation, which reads as

$$i\tau[k_x j^x(\vec{k},\nu,\tau)+k_x j^y(\vec{k},\nu,\tau)+\nu j^{\eta}(\vec{k},\nu,\tau)] + \partial_\tau \rho(\vec{k},\nu,\tau)=0. \tag{2.34}$$

We have in sequence

$$\begin{aligned} &i\nu\int_0^{\tau_1}\frac{d\tau_2}{\tau_2}\rho(\tau_2)H^{(j)}_{i\nu}(k_\perp\tau_2)\\ &=i\nu\int_0^{\tau_1}\frac{dR^{(j)}_{-1,i\nu}(k_\perp\tau_2)}{d\tau_2}\rho(\tau_2)d\tau_2\\ &=i\nu R^{(j)}_{-1,i\nu}(k_\perp\tau_1)\rho(\tau_1)-\nu\int_0^{\tau_1}\tau_2 d\tau_2 R^{(j)}_{-1,i\nu}(k_\perp\tau_2)\\ &\quad\times[k_x j^x(\tau_2)+k_y j^y(\tau_2)+\nu j^{\eta}(\tau_2)], \end{aligned} \tag{2.35}$$

$$\begin{aligned} &i\nu\int_0^{\tau_1}d\tau_2 H^{(j)}_{i\nu}(k_\perp\tau_2)\partial_{\tau_2}(\tau_2^2 j^{\eta}(\tau_2))\\ &=\tau_1^2 j^{\eta}(\tau_1)H^{(j)}_{i\nu}(k_\perp\tau_1)+\int_0^{\tau_1}\tau_2 d\tau_2[R^{(j)}_{1,i\nu}(k_\perp\tau_2)\\ &\quad+\nu^2 R^{(j)}_{-1,i\nu}(k_\perp\tau_2)]j^{\eta}(\tau_2). \end{aligned} \tag{2.36}$$

In a similar way we have

$$\begin{aligned} &i\nu\int_0^{\tau_1}\frac{d\tau_2}{\tau_2}\rho(\tau_2)s_{1,i\nu}(k_\perp\tau_2)\\ &=i\nu\int_0^{\tau_1}\frac{dQ_{-1,i\nu}(k_\perp\tau_2)}{d\tau_2}\rho(\tau_2)d\tau_2\\ &=i\nu Q_{-1,i\nu}(k_\perp\tau_1)\rho(\tau_1)-\nu\int_0^{\tau_1}\tau_2 d\tau_2 Q_{-1,i\nu}(k_\perp\tau_2)\\ &\quad\times[k_x j^x(\tau_2)+k_y j^y(\tau_2)+\nu j^{\eta}(\tau_2)], \end{aligned} \tag{2.37}$$

$$\begin{aligned} &\int_0^{\tau_1}d\tau_2 s_{1,i\nu}(k_\perp\tau_2)\partial_{\tau_2}(\tau_2^2 j^{\eta}(\tau_2))\\ &=\tau_1^2 j^{\eta}(\tau_1)s_{1,i\nu}(k_\perp\tau_1)\\ &\quad+\nu^2\int_0^{\tau_1}\tau_2 d\tau_2[Q_{-1,i\nu}(k_\perp\tau_2)-Q_{1,i\nu}(k_\perp\tau_2)]j^{\eta}(\tau_2). \end{aligned} \tag{2.38}$$

Assembling these pieces together and repeating the same calculations for the function $\Phi$ one obtains three different terms which contribute to the field $A$ produced by the current $j$; $A=A^{(tr)}+A^{(L)}+A^{(inst)}$.

The transverse field $A^{(tr)}$ is defined by the integral terms in the right-hand side (RHS) of Eqs. (2.35) and (2.36). It can be conveniently written down in the following form:

$$A^{(tr)}_l(x_1)=\int d^4x_2\,\theta(\tau_1-\tau_2)\Delta^{(tr)}_{lm}(x_1,x_2)j^m(x_2), \tag{2.39}$$

where

$$\Delta^{(tr)}_{lm}(x,y)=-i\int_{-\infty}^{\infty}d\nu\int d^2\vec{k}\sum_{\lambda=1,2}[V^{(\lambda)}_{\nu\vec{k};l}(x)V^{(\lambda)*}_{\nu\vec{k};m}(y) - V^{(\lambda)*}_{\nu\vec{k};l}(x)V^{(\lambda)}_{\nu\vec{k};m}(y)], \tag{2.40}$$

which can be easily recognized as the Riemann function of the original homogeneous hyperbolic system. The Riemann function solves the boundary value problem for the evolution of the free radiation field. It is obtained immediately in the form of the bilinear expansion over the full set of solutions (2.13) of the homogeneous system. In fact, this is a sole evidence that $\Delta^{(tr)}$ may be associated with the transverse part of the propagator. Then the remaining part is the propagator (response function) for the longitudinal field.

The dynamical longitudinal field $A^{(L)}$ originates from the integral terms in the RHS of Eqs. (2.37) and (2.38):

$$\begin{aligned} A^{(L)}_l(\tau_1,\eta_1,\vec{r}_1)=&\int_0^{\tau_1}\tau_2 d\tau_2\int d\eta_2 d^2\vec{r}_2\\ &\times\Delta^{(L)}_{lm}(\tau_2;\eta_1-\eta_2,\vec{r}_1-\vec{r}_2)j^m(\tau_2,\eta_2,\vec{r}_2). \end{aligned} \tag{2.41}$$

The kernel of this representation,

$$\begin{aligned} &\Delta^{(L)}_{lm}(\tau_2;\eta_1-\eta_2,\vec{r}_1-\vec{r}_2)\\ &=\int\frac{d\nu d^2\vec{k}}{(2\pi)^3k_\perp^2}\begin{bmatrix}k_r\\ \nu\end{bmatrix}_l\begin{bmatrix}k_sQ_{-1,i\nu}(k_\perp\tau_2)\\ \nu Q_{1,i\nu}(k_\perp\tau_2)\end{bmatrix}_m\\ &\quad\times e^{i\nu(\eta_1-\eta_2)+i\vec{k}(\vec{r}_1-\vec{r}_2)}, \end{aligned} \tag{2.42}$$

does not allow for the bilinear expansion with two temporal arguments, and, as we shall see in a while, the retarded character of the integration in Eq. (2.41) is not sensitive to the light cone boundaries. In fact, the electric field $E^{(L)}_l=\dot{A}^{(L)}_l$ is simultaneous with the current $j^m$.

The instantaneous part of the solution comes from the boundary terms in Eqs. (2.35)–(2.38) which were generated via integration by parts. It depends on a single time variable $\tau_1$. Using two functional relations, Eq. (2.32) and

$$\begin{aligned} &R^{(1)}_{1,i\nu}(x)R^{(2)}_{-1,i\nu}(x)-R^{(2)}_{1,i\nu}(x)R^{(1)}_{-1,i\nu}(x)\\ &=-\frac{4}{i\pi}\frac{x}{\nu^2}\frac{ds_{1,i\nu}(x)}{dx}\\ &=-\frac{4}{i\pi}[Q_{1,i\nu}(x)-Q_{-1,i\nu}(x)] \end{aligned} \tag{2.43}$$

(see Appendix B), its Fourier transform can be presented in the form

$$A_l^{(inst)}(\vec{k},\nu;\tau_1)=\frac{\rho(\vec{k},\nu,\tau_1)}{(2\pi)^3 ik_\perp^2}\begin{bmatrix} k_r Q_{-1,i\nu}(k_\perp\tau_1) \\ \nu Q_{1,i\nu}(k_\perp\tau_1)\end{bmatrix}_l, \tag{2.44}$$

which leads to the Poisson-type integral,

$$A_m^{(inst)}(\tau_1,\eta_1,\vec{r}_1)=\int d\vec{r}_2 d\eta_2 \mathcal{K}_m(\tau_1;\vec{r}_1-\vec{r}_2,\eta_1-\eta_2)\times\rho(\tau_1,\vec{r}_2,\eta_2), \tag{2.45}$$

with the *instantaneous* kernel,

$$\mathcal{K}_m(\tau;\vec{r},\eta)=\int\frac{d\nu d\vec{k}}{(2\pi)^3}\frac{e^{i\nu\eta+i\vec{k}\vec{r}}}{ik_\perp^2}\begin{bmatrix} k_r Q_{-1,i\nu}(k_\perp\tau) \\ \nu Q_{1,i\nu}(k_\perp\tau)\end{bmatrix}_m. \tag{2.46}$$

The potential $A^{(inst)}$ given by Eq. (2.44) coincides with the potential $V^{(3)}$ of Eq. (2.18) of the $\tau$-static source. Therefore, this term represents the instantaneous distribution of the potential at the moment $\tau_1$, corresponding to the charge density taken at the same moment. Next, we have to recall that the charge density $\rho(\vec{k},\nu,\tau_1)$ in Eq. (2.44) still includes an arbitrary constant $\rho_0(\vec{k},\nu)$, which may be interpreted as the charge density at $\tau=0$. This constant has appeared because we used only the conservation (2.24) of the constraint (which is the consequence of the equations of motion) and did not used the Gauss law explicitly. Now we can see that imposing the constraint indeed affects only the static potential of the charge distribution and puts it in agreement with Gauss law. If the initial data allow one to put $\rho_0=\rho(\tau=0)=0$, then it immediately solves two problems. First, the conservation of the constraint just duplicates the Gauss law, and the latter can be used to remove the unphysical degrees of freedom without any reservations. Eventually, it allows to fix the gauge $A^\tau=0$ completely (see Sec. III A). Second, it becomes possible to eliminate the charge density $\rho$ completely, and to replace it by the components $j^n$ of the current. Replacement follows an evident prescription,

$$\rho(\tau_1,\eta_2,\vec{r}_2)=\int_0^{\tau_1}d\tau_2\frac{\partial\rho}{\partial\tau_2}=-i\int_0^{\tau_1}\tau_2 d\tau_2[k_s j^s(\tau_2,\eta_2,\vec{r}_2)+\nu j^\eta(\tau_2,\eta_2,\vec{r}_2)],$$

and leads to the standard form of the $A^{(inst)}$ representation (an artificial contribution of any $\rho_0$ would correspond to the recognizable static pattern in the longitudinal part of the propagator and is easily handled),

$$A_l^{(inst)}(\tau_1,\eta_1,\vec{r}_1)=\int_0^{\tau_1}\tau_2 d\tau_2\int d\eta_2 d^2\vec{r}_2\Delta_{lm}^{(inst)}(\tau_1;\eta_1-\eta_2,\vec{r}_1-\vec{r}_2)j^m(\tau_2,\eta_2,\vec{r}_2), \tag{2.47}$$

with the kernel given by the formula,

$$\Delta_{lm}^{(inst)}(\tau_1;\eta_1-\eta_2,\vec{r}_1-\vec{r}_2)=-\int\frac{d\nu d^2\vec{k}}{(2\pi)^3k_\perp^2}\begin{bmatrix} k_r Q_{-1,i\nu}(k_\perp\tau_1) \\ \nu Q_{1,i\nu}(k_\perp\tau_1)\end{bmatrix}_l\begin{bmatrix} k_s \\ \nu\end{bmatrix}_m\times e^{i\nu(\eta_1-\eta_2)+i\vec{k}(\vec{r}_1-\vec{r}_2)}. \tag{2.48}$$

Two parts of the propagator, given by Eqs. (2.42) and (2.48) can be combined in one elegant formula for the propagator of the field $A^{(long)}=A^{(L)}+A^{(inst)}$,

$$\Delta_{lm}^{(long)}(\tau_1;\eta_1-\eta_2,\vec{r}_1-\vec{r}_2)=\int\frac{d\nu d^2\vec{k}}{(2\pi)^3k_\perp^2}\begin{bmatrix} k_r k_s[Q_{-1,i\nu}(k_\perp\tau_2)-Q_{-1,i\nu}(k_\perp\tau_1)] & k_r\nu[Q_{1,i\nu}(k_\perp\tau_2)-Q_{-1,i\nu}(k_\perp\tau_1)] \\ \nu k_s[Q_{-1,i\nu}(k_\perp\tau_2)-Q_{1,i\nu}(k_\perp\tau_1)] & \nu^2[Q_{1,i\nu}(k_\perp\tau_2)-Q_{1,i\nu}(k_\perp\tau_1)]\end{bmatrix}_{lm}\times e^{i\nu(\eta_1-\eta_2)+i\vec{k}(\vec{r}_1-\vec{r}_2)}. \tag{2.49}$$

This expression will be used for practical calculation of quark self-energy in paper [IV], where it will be transformed into the mixed representation. The coordinate form of $A^{(L)}$ that reveals its causal properties, is analyzed in Sec. IV

Equations (2.39)–(2.42) and (2.47), (2.48) present the propagator in a split form. Different constituents of this form are as a preliminary identified as the transverse, the longitudinal and the instantaneous parts of the propagator. It would be useful to learn if the same kind of splitting is possible for the current itself. An affirmative answer (as in the cases of the Coulomb and radiation gauges) would be helpful for the design of the perturbation theory. To answer this question, one should substitute the different pieces of the solution into the left-hand side of the original system of differential equations. This leads to the following expressions for the Fourier components of the three currents:

$$\tau j_{(tr)}^m(\vec{k},\nu;\tau)=\tau j^m(\vec{k},\nu;\tau)+\frac{1}{ik_\perp^2}\begin{bmatrix} k_r s_{1,i\nu}(k_\perp\tau) \\ \nu k_\perp^2 s_{-1,i\nu}(k_\perp\tau)\end{bmatrix}^m\frac{\partial\rho}{\partial\tau}-\frac{\nu}{k_\perp^2}\frac{\partial}{\partial\tau}\left(\dot{s}_{-1,i\nu}(k_\perp\tau)\begin{bmatrix} k_r\tau^3 j^\eta \\ -\tau(k_x j^x+k_y j^y)\end{bmatrix}^m\right), \tag{2.50}$$

$$\tau j_{(L)}^m(\vec{k},\nu;\tau)=\frac{1}{k_\perp^2}\frac{\partial}{\partial\tau}\left(\begin{bmatrix} k_r\tau^2 \\ \nu\end{bmatrix}^m[Q_{-1,i\nu}(k_\perp\tau)(k_x j^x+k_y j^y)+Q_{1,i\nu}(k_\perp\tau)j^\eta]\right), \tag{2.51}$$

$$\tau j^m_{(inst)}(\vec{k},\nu;\tau)=\frac{-1}{ik_\perp^2}\frac{\partial}{\partial\tau}\left(\begin{bmatrix} k_r\tau Q_{-1,i\nu}(k_\perp\tau)\\ \nu\tau^{-1}Q_{1,i\nu}(k_\perp\tau)\end{bmatrix}^m\frac{\partial\rho}{\partial\tau}\right)-\frac{1}{ik_\perp^2}\begin{bmatrix} k_r s_{1,i\nu}(k_\perp\tau)\\ \nu k_\perp^2 s_{-1,i\nu}(k_\perp\tau)\end{bmatrix}^m\frac{\partial\rho}{\partial\tau}. \tag{2.52}$$

Provided that the current is conserved, these three currents, added together, give the full current on the right hand side of the system. Therefore, the solution is correct. However, none of these three currents carries any signature of being longitudinal or transversal in the usual sense. None of them has zero divergence since the operator of the divergence does not commute with the differential operator of the system. No desired simplification is possible in our case.

In fact, the above splitting of the potential has no real physical meaning. To see it explicitly, let us find the divergence of the electric field, $\mathrm{div}\,\mathbf{E}=\partial_m\mathcal{E}^m$ [again, for brevity, in the Fourier representation]:

$$\mathrm{div}\,\mathbf{E}^{(tr)}(\vec{k},\nu;\tau)=i[Q_{-1,i\nu}(k_\perp\tau)-Q_{1,i\nu}(k_\perp\tau)]\times\left(\nu\tau^2 j^\eta-\frac{\nu^2}{k_\perp^2}(k_x j^x+k_y j^y)\right), \tag{2.53}$$

$$\mathrm{div}\,\mathbf{E}^{(L)}(\vec{k},\nu;\tau)=i\left(\tau^2+\frac{\nu^2}{k_\perp^2}\right)[(k_x j^x+k_y j^y)Q_{-1,i\nu}(k_\perp\tau)+\nu j^\eta Q_{1,i\nu}(k_\perp\tau)], \tag{2.54}$$

$$\mathrm{div}\,\mathbf{E}^{(inst)}(\vec{k},\nu;\tau)=\rho(\vec{k},\nu;\tau)-i\left(\tau^2 Q_{-1,i\nu}(k_\perp\tau)-\frac{\nu^2}{k_\perp^2}Q_{1,i\nu}(k_\perp\tau)\right)\times[(k_x j^x+k_y j^y)+\nu j^\eta]. \tag{2.55}$$

Only the divergence of the true retarded component of the field $\mathbf{E}^{(tr)}$ turns out to be zero. The term which prevents the $\mathrm{div}\,\mathbf{E}^{(tr)}$ from being zero is due to the nonsymmetry of the propagator, $\Delta^{\eta r}\neq\Delta^{r\eta}$. It appears when the $\theta$ function in Eq. (2.39) is differentiated with respect to Hamiltonian time $\tau$. This term is vital for obtaining the expression that obeys the Gauss constraint, $\mathrm{div}\,\mathbf{E}(\vec{k},\nu;\tau)=\rho(\vec{k},\nu;\tau)$.

The known examples, when the transverse and the longitudinal fields are separated at the level of the equations of motion, are related to a narrow class of homogeneous gauges. The impossibility of a universal separation of the transverse and longitudinal currents thus appears to be a rule rather than exception. It reflects a general principle; the radiation field created at some time interval has the preceding and the subsequent configurations of the longitudinal field as the boundary condition. The dynamics of the longitudinal field falls out of any scattering problem in its *S*-matrix formulation. However, this dynamics is, in fact, a subject of the QCD evolution in the inelastic high-energy processes.

### D. Initial data problem in the gauge $A^\tau=0$

We obtained the expression for the (retarded) propagator as the response function between the ''external'' current and the potential of the gauge field. We must also verify that the same propagator solves the Cauchy problem for the gauge field. This can be easily done by presenting the initial data at the surface $\tau=\tau_0$ in the form of the source density at the hypersurface $\tau=\tau_0$, i.e.,

$$\sqrt{-\mathrm{g}}J^n(\tau_2)=\sqrt{-\mathrm{g}(\tau_0)}\mathrm{g}^{nm}(\tau_0)[\delta'(\tau_2-\tau_0)\overline{A}_m(\vec{r},\eta)+\delta(\tau_2-\tau_0)\overline{A}'_m(\vec{r},\eta)], \tag{2.56}$$

where $\overline{A}_m(\vec{r},\eta)$ and $\overline{A}'_m(\vec{r},\eta)$ are the initial data for the potential and its normal derivative on the hypersurface $\tau=\tau_0$. Usually, it is assumed that the external current vanishes for $\tau<\tau_0$. Substituting this source into Eqs. (2.39), (2.41), and (2.45), and taking the limit of $\tau_1\to\tau_0$, we may verify that the standard prescription for the solution of the initial data problem,

$$A_l(x_1)=\int_{(\tau_2=\tau_0)}d^2\vec{r}_2 d\eta_2\Delta_{lm}(x_1,x_2)\frac{\overleftrightarrow{\partial}}{\partial\tau_2}A^m(x_2), \tag{2.57}$$

holds with the same propagator $\Delta_{lm}(x_1,x_2)$ that was used to solve the emission problem. For example, in the limit of $\tau\to\tau_0$, the $\eta$ component of the vector potential is a sum of three terms,

$$A^{(tr)}_\eta(\tau_0+0)=\frac{i\pi}{4k_\perp^2}\{[R^{(2)}_{1,i\nu}(k_\perp\tau_0)H^{(1)}_{i\nu}(k_\perp\tau_0)-R^{(1)}_{1,i\nu}(k_\perp\tau_0)H^{(2)}_{i\nu}(k_\perp\tau_0)][\nu\overline{A}_\phi-k_\perp^2\overline{A}_\eta]-\tau_0\nu[R^{(2)}_{1,i\nu}(k_\perp\tau_0)R^{(1)}_{-1,i\nu}(k_\perp\tau_0)-R^{(1)}_{1,i\nu}(k_\perp\tau_0)R^{(2)}_{-1,i\nu}(k_\perp\tau_0)]\overline{A}'_\phi\}, \tag{2.58}$$

$$A^{(L)}_\eta(\tau_0+0)=\frac{-\nu}{k_\perp^2}\left\{-s_{1,i\nu}(k_\perp\tau_0)\overline{A}_\phi+\tau_0 Q_{-1,i\nu}(k_\perp\tau_0)\overline{A}'_\phi-\frac{\nu k_\perp^2}{\tau_0}s_{-1,i\nu}(k_\perp\tau_0)\overline{A}_\eta+\frac{\nu}{\tau_0}Q_{1,i\nu}(k_\perp\tau_0)\overline{A}'_\eta\right\}, \tag{2.59}$$

$$A^{(inst)}_\eta(\tau_0+0)=\frac{\nu}{k_\perp^2}Q_{1,i\nu}(k_\perp\tau_0)\left[\tau_0\overline{A}'_\phi+\frac{\nu}{\tau_0}\overline{A}'_\eta\right], \tag{2.60}$$

where we have denoted $\overline{A}_\phi = k_x\overline{A}_x + k_y\overline{A}_y$. Equation (2.60) follows from Eq. (2.45) and takes care of the consistency between the charge density at the moment $\tau_0$ and the initial data for the gauge field. Using relations (2.32) and (2.44) and adding up Eqs. (2.58)–(2.60) we come to a desired identity, $A_\eta(\tau_0+0) = \overline{A}_\eta$.

When the initial data $\overline{A}_m(\vec{r},\eta)$ and $\overline{A}'_m(\vec{r},\eta)$ correspond to the free radiation field, then only the part of the full propagator, $\Delta^{(tr)}_{lm}(x_1,x_2)$, ''works'' here, and only Eq. (2.58) may be retained. The other two equations acquire the status of being constraints imposed on the initial data. Since the current is absent, we have $A^{(L)}=0$ on the left-hand side of Eqs. (2.59). Then the right hand side confirms that the kernel $\mathcal{K}$ is orthogonal to the free radiation field modes. Since the charge density $\rho$ vanishes, we have $A^{(inst)}=0$, which is equivalent to Gauss law for the free gauge field. The two transverse modes already obey these constraints. This fact provides a reliable footing for the canonical quantization of the free field in the gauge $A^\tau=0$. Indeed, the Riemann function coincides with the commutator of the free gauge field. It can be found via its bilinear decomposition over the physical modes. Thus, one can avoid technical problems of inverting the constraint equations (see Sec. III). The longitudinal part of the propagator will be studied in details in Sec. IV.

### E. Gluon vertices in the gauge $A^\tau=0$

The terms proportional to the first and the second powers of the coupling constant in the classical wave equations may be viewed as the external current and allow one to define the explicit form of the three- and four-gluon vertices. One should start from the solution of the Maxwell equations,

$$A^{a'}_{k'}(z_1) = \int d^4x \Delta^{a'a}_{k'k}(z_1,x)\sqrt{-\mathrm{g}(x)}\mathcal{J}^k_a(x), \quad (2.61)$$

with the color current of the form

$$\begin{aligned}\sqrt{-\mathrm{g}(x)}\mathcal{J}^k_a(x) = & -gf_{abc}\sqrt{-\mathrm{g}(x)}\mathrm{g}^{kn}(x)\mathrm{g}^{ml}(x)\\ & \times[\partial_m(A^b_l(x)A^c_n(x)) + A^b_l(x)\partial_m A^c_n(x)\\ & + A^b_m(x)\partial_n A^c_l(x)] - g^2\sqrt{-\mathrm{g}(x)}f_{abc}f_{cdh}\\ & \times\mathrm{g}^{kn}(x)\mathrm{g}^{ml}(x)A^b_l(x)A^d_m(x)A^h_n(x).\end{aligned} \quad (2.62)$$

In perturbation calculations, every field $A(x)$ in the RHS of this expression is a part of some correlator $\Delta(x,z_N)$. The components of the metric depend only on the time $\tau$ while the derivatives affect only the spatial directions $u^n = (\vec{r},\eta)$. Moreover, in these directions, all the gluon correlators depend only on the differences of the coordinates and can be rewritten in terms of their spatial Fourier components. After symmetrization over the outer arguments $z_N$, one immediately obtains

$$\begin{aligned}V^{kln}_{abc}(p_1,p_2,p_3;\tau) = & -i\tau f_{abc}\delta(p_1+p_2+p_3)[\mathrm{g}^{ln}(p_2-p_3)^k\\ & + \mathrm{g}^{nk}(p_3-p_1)^l + \mathrm{g}^{kl}(p_1-p_2)^n],\end{aligned} \quad (2.63)$$

where $p^n = \mathrm{g}^{nk}p_k$, and the components of the momentum in the curvilinear coordinates are equal to $p_k = (p_x,p_y,\nu)$. The four-gluon vertex has no derivatives and is the same as usual.

## III. QUANTIZATION

The second quantization of the field has several practical goals. We would like to have an expansion of the operator of the free gluon field like

$$\begin{aligned}A_i(x) = \sum_{\lambda=1,2}\int d^2\vec{k}d\nu[&c_\lambda(\nu,\vec{k})V^{(\lambda)}_{\nu\vec{k};i}(x)\\ & + c^\dagger_\lambda(\nu,\vec{k})V^{(\lambda)*}_{\nu\vec{k};i}(x)],\end{aligned} \quad (3.1)$$

with the creation and annihilation operators which obey the commutation relations

$$\begin{aligned}&[c_\lambda(\nu,\vec{k}),c^\dagger_{\lambda'}(\nu',\vec{k}')] = \delta_{\lambda\lambda'}\delta(\nu-\nu')\delta(\vec{k}-\vec{k}'),\\ &[c_\lambda(\nu,\vec{k}),c_{\lambda'}(\nu',\vec{k}')] = [c^\dagger_\lambda(\nu,\vec{k}),c^\dagger_{\lambda'}(\nu',\vec{k}')] = 0.\end{aligned} \quad (3.2)$$

Once obtained, the commutation relations (3.2) allow one to find various correlators of the free gluon field as the averages of the binary operator products over the state of the perturbative vacuum and express them via the solutions $V^{(1)}_{\nu\vec{k};i}(x)$ and $V^{(2)}_{\nu\vec{k};i}(x)$. For example, the Wightman functions,

$$\begin{aligned}i\Delta_{10,ij}(x,y) &= \langle 0|A_i(x)A_j(y)|0\rangle\\ &= \sum_{\lambda=1,2}\int d\nu d^2\vec{k}V^{(\lambda)}_{\nu\vec{k};i}(x)V^{(\lambda)*}_{\nu\vec{k};i}(y)\\ &= i\Delta_{01,ji}(y,x),\end{aligned} \quad (3.3)$$

serve as the projectors onto the space of the on-mass-shell gluons and should be known explicitly in order to have a good definition of the production rate of gluons in the final states. The Fock creation and annihilation operators are also needed in order to define the occupation numbers and to introduce the gluon distributions into various field correlators defined as the averages over an ensemble. With these two Wightman functions at hand, one immediately obtains the expression for the commutator of the free field operators,

$$\begin{aligned}\Delta_{0,ij}(x,y) &= -i\langle 0|[A_i(x),A_j(y)]|0\rangle\\ &= \Delta_{10,ij}(x,y) - \Delta_{01,ij}(x,y),\end{aligned} \quad (3.4)$$

which should coincide with the Riemann function of the homogeneous field equations. The program of second quantization does not reveal any technical problems if we give preference to the holomorphic quantization which is based on commutation relations (3.2) for the Fock operators. However, if we prefer to start with the canonical commutation relations for the field coordinates and momenta, then one should *postulate* them and derive Eq. (3.2) as the consequence.

The way to obtain the canonical commutation relations in cases of the scalar and the spinor fields is quite straightforward. For the vector gauge field, we meet a well-known problem, viz., an excess of the number of the components of the vector field over the number of the physical degrees of freedom. For example, in the so-called radiation gauge, $A^0=0$ and $\mathrm{div}\,\mathbf{A}=0$, we write the canonical commutation relations in the following form [6]:

$$[A_i(\mathbf{x},t),E_j(\mathbf{y},t)]=\delta^{tr}_{ij}(\mathbf{x}-\mathbf{y})=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left(\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right)e^{-ik(x-y)},$$

$$[A_i(\mathbf{x},t),A_j(\mathbf{y},t)]=[E_i(\mathbf{x},t),E_j(\mathbf{y},t)]=0, \tag{3.5}$$

thus eliminating the longitudinally polarized photons from the dynamical degrees of freedom. The function $\delta^{tr}_{ij}$ plays a role as the unit operator in the space of the physical states. Here, $i,j=1,2,3$ and the number of relations postulated by Eqs. (3.5) apparently exceeds the actual number required by the count of the independent degrees of freedom, $\lambda=1,2$, of the free gauge field. The Fourier transform of the function $\delta^{tr}_{ij}$ is easily guessed because the basis of the plane-wave solutions is very simple [6], and it can be obtained rigorously by solving the system of the constraint equations [7,8]. A similar guess or procedure in our case is not so obvious. We have the gauge condition $A^\tau=0$ as the primary constraint and Gauss law as the secondary one. The latter can be resolved in a way which allows one to exclude the $\eta$ components of the potential and the electric field from the set of independent canonical variables. Thus, only $x$ and $y$ components are subject to the canonical commutation relations. To resolve the constraints, one needs the integral operators with the kernels built from the solutions of the Maxwell equations in the gauge $A^\tau=0$. Therefore we shall proceed in two steps. In Sec. III B, we shall sketch the results for the Wightman functions (3.3). These, will be used for the explicit calculation of the free field commutator (3.4) in Sec. III C and for the study of its causal behavior.

### A. Fixing of the gauge $A^\tau=0$

Only the independent components of transverse fields are the subject for quantization. In order to eliminate the extraneous degrees of freedom we have to incorporate the Gauss law. In Sec. II C, the latter was shown to be a consequence of the equations of motion only when the evolution begins with the zero charge density at $\tau=0$. Are these initial data physical or do they mean that the QCD evolution begins from nothing? Addressing the hadron collisions, the question can be asked more specifically: do we really need any resolved *ad hoc* color charges (dipoles, quadrupoles, etc.) to initiate the color interaction? The answer is negative for three reasons. First, as has been demonstrated in paper [II], at $\tau\to 0$ the states of wedge dynamics are widely spread along the null planes with zero density at $\tau=0$. Second, the nucleus which fluctuate into the expanded state of wedge dynamics is colorless even locally. Third, as will be demonstrated in paper [IV] [9] the interactions which are the strongest at very early times, are magneto-static by their nature. The collision of two nuclei is more likely to begin with the magnetic interaction of the color currents in the locally color-neutral system than with the electric interaction of color charges.

In Abelian case considered here the gauge transformation is

$$A'_\mu(x)=A_\mu(x)+\partial_\mu\chi(x). \tag{3.6}$$

Since we have $A_\tau(x)=0$, we must also have $\chi(x)=\chi(\vec{r}_t,\eta)$. The boundary condition, $A_\eta(\tau=0,\eta)=0$, cannot be altered by the gauge transform (3.6). Therefore, we must require that $\chi(x)=\chi(\vec{r}_t)$. Hence, the allowed gauge transform is reduced to $x$ and $y$ components of the vector potential,

$$A'_r(x)=A_r(x)+\partial_r\chi(\vec{r}_t). \tag{3.7}$$

The Gauss constraint (2.12),

$$\partial_\eta\partial_\tau A_\eta(\tau,\eta)=-\tau^2\partial_r\dot{A}_r(\tau,\eta)-\tau^2 j^\tau(\tau,\eta),$$

is a hyperbolic differential equation for the function $A_\eta(\tau,\eta)$ which can be integrated [the Riemann function of this equation, $R(\tau,\eta)=1$]. With the boundary conditions, $A_\eta(0,\eta)=0$ and $A_\eta(\tau,-\infty)=0$, this equation has a unique solution,

$$A_\eta(\tau,\eta)=\int_{-\infty}^{\eta}d\eta\int_0^\tau d\tau[-\tau^2\partial_r\dot{A}_r(\tau,\eta)-\tau^2 j^\tau(\tau,\eta)]. \tag{3.8}$$

The residual gauge transform (3.7) changes only the integrand of Eq. (3.8)

$$\partial_r A'_r(\tau)\to\partial_r A_r(\tau)+\Delta_\perp\chi(\vec{r}_t).$$

As a consequence of the boundary conditions, the transverse divergence of the field must vanish at $\tau=0$, $\partial_r A_r(0,\eta)=0$. Therefore, we must also have

$$\Delta_\perp\chi(\vec{r}_t)=0.$$

$\chi(x,y)$ must be a harmonic function. Demanding, that $\chi(x,y)$ vanishes at $|\vec{r}_t|\to\infty$, we find that $\chi(x,y)=0$. The gauge $A^\tau=0$ is fixed completely. The Gauss law can be unambiguously used to eliminate $A_\eta$ from the list of canonical variables.

### B. Gluon correlators in the gauge $A^\tau=0$

In this section, we shall write down components of the field correlator $\Delta_{10,ij}(x,y)$ in the curvilinear coordinates $u=(\tau,\eta,\vec{r})$. We shall denote their covariant components as $\Delta_{10,ik}(u_1,u_2)$. Later we shall transform them to the standard

Minkowski coordinates and find the correlators of the temporal axial and the null plane gauges as their limits in the central rapidity region and in the vicinity of the null-planes, respectively. The most convenient (for this purpose) basis consists of the transverse modes $v^{(\lambda)}$. The mode $v^{(1)}$ gives the following contribution to the correlator $\Delta_{10,ik}$:

$$i\Delta^{(1)}_{10,rs}(1,2)=\int_{-\infty}^{\infty}\frac{d\theta}{2}\int\frac{d^2\vec{k}}{(2\pi)^3}\left(\delta_{rs}-\frac{k_rk_s}{k_\perp^2}\right)\times e^{i\vec{k}(\vec{r}_1-\vec{r}_2)}e^{-ik_\perp\tau_1\cosh(\theta-\eta_1)+ik_\perp\tau_2\cosh(\theta-\eta_2)}. \tag{3.9}$$

Realizing that $d\theta/2=dk^3/2k^0$, we recognize a standard representation of this part of the correlator in terms of the on-mass-shell plane waves decomposition.

The second part of the correlator is determined by the mode $v^{(2)}$ and has the following components:

$$\Delta^{(2)}_{10,rs}(1,2)=-i\int_{-\infty}^{\infty}\frac{d\theta}{2}\int\frac{d^2\vec{k}}{(2\pi)^3}\frac{k_rk_s}{k_\perp^2}\times e^{i\vec{k}(\vec{r}_1-\vec{r}_2)}f_1(\theta,\tau_1,\eta_1)f_1^*(\theta,\tau_2,\eta_2), \tag{3.10}$$

$$\begin{aligned}\Delta^{(2)}_{10,r\eta}(1,2)&=-i\int_{-\infty}^{\infty}\frac{d\theta}{2}\int\frac{d^2\vec{k}}{(2\pi)^3}e^{i\vec{k}(\vec{r}_1-\vec{r}_2)}\frac{k_r}{k_\perp^2}\times f_1(\theta,\tau_1,\eta_1)f_2^*(\theta,\tau_2,\eta_2)\\&=\Delta^{(2)}_{10,\eta r}(2,1),\end{aligned} \tag{3.11}$$

$$\Delta^{(2)}_{10,\eta\eta}(1,2)=-i\int_{-\infty}^{\infty}\frac{d\theta}{2}\int\frac{d^2\vec{k}}{(2\pi)^3}\frac{e^{i\vec{k}(\vec{r}_1-\vec{r}_2)}}{k_\perp^2}\times f_2(\theta,\tau_1,\eta_1)f_2^*(\theta,\tau_2,\eta_2). \tag{3.12}$$

One may easily see that all components of $\Delta_{10}(1,2)$ vanish when either $\tau_1$ or $\tau_2$ go to zero.

### C. Causal properties of the field commutators in the gauge $A^\tau=0$

Causal properties of the radiation field commutator may be studied starting from the representation (3.4). Using Eqs. (3.9) and (3.10) we may conveniently write the contribution of the two transverse modes in the following form:

$$i\Delta^{(1)}_{0,rs}(1,2)=-i\int\frac{d^2\vec{k}}{(2\pi)^3}\left(\delta_{rs}-\frac{k_rk_s}{k_\perp^2}\right)e^{i\vec{k}\vec{r}}\times\int_{-\infty}^{\infty}d\theta\sin k_\perp\Phi_{12}, \tag{3.13}$$

$$i\Delta^{(2)}_{0,rs}(1,2)=-i\int\frac{d^2\vec{k}}{(2\pi)^3}\frac{k_rk_s}{k_\perp^2}e^{i\vec{k}\vec{r}}\int_{-\infty}^{\infty}d\theta\times\left[1-\frac{\cosh 2\eta}{\sinh^2\theta+\cosh^2\eta}\right](\sin k_\perp\Phi_{12}-\sin k_\perp\Phi_1+\sin k_\perp\Phi_2), \tag{3.14}$$

where we have introduced the following notation: $2\eta=\eta_1-\eta_2$, $\vec{r}=\vec{r}_1-\vec{r}_2$, $\Phi_i=\tau_1\cosh(\theta-\eta_i)$, $\Phi_{12}=\Phi_1-\Phi_2$. The sum of Eqs. (3.13) and (3.14) can be rearranged as follows:

$$\begin{aligned}i\Delta_{0,rs}(1,2)=i\int\frac{d^2\vec{k}d\theta}{(2\pi)^3}e^{i\vec{k}\vec{r}}\Big[&-\delta_{rs}\sin k_\perp\Phi_{12}\\&+\frac{k_rk_s}{k_\perp^2}[\sin k_\perp\Phi_1-\sin k_\perp\Phi_2]\\&+k_rk_s\cosh(\eta_1-\eta_2)\\&\times\int_0^{\tau_1}d\tau'\int_0^{\tau_2}d\tau''\sin[k_\perp\tau'\cosh(\theta-\eta)\\&-k_\perp\tau''\cosh(\theta+\eta)]\Big].\end{aligned} \tag{3.15}$$

Rewriting the integration $d^2\vec{k}d\theta$ into the three dimensional integration $d^3\mathbf{k}/|\mathbf{k}|$ in Cartesian coordinates, the first integral in Eq. (3.15),

$$\begin{aligned}D_0(1,2)&=\int\frac{d^2\vec{k}d\theta}{(2\pi)^3}e^{i\vec{k}\vec{r}}\sin k_\perp\Phi_{12}\\&=\frac{\mathrm{sign}(t_1-t_2)}{2\pi}\delta[(t_1-t_2)^2-(\mathbf{r}_1-\mathbf{r}_2)^2],\end{aligned} \tag{3.16}$$

is easy to calculate and to recognize as the commutator of the massless scalar field. It differs from zero only if the line between the points $x_1$ and $x_2$ has a lightlike direction. We integrate the first and the third terms in the integrand of Eq. (3.15) in this way. To reduce the two integrals in the second term to the same type, we must exclude the factor $1/k_\perp^2$ using the fundamental solution of the two-dimensional Laplace operator,

$$\frac{k_rk_s}{k_\perp^2}e^{i\vec{k}\vec{r}}=\partial_r\partial_s\int\frac{d^2\vec{\xi}}{2\pi}\ln|\vec{\xi}-\vec{r}|e^{i\vec{k}\vec{\xi}}. \tag{3.17}$$

After that, we arrive at the final result,

$$\Delta_{0,rs}(1,2)=-\delta_{rs}D_0(1,2)-\cosh(\eta_1-\eta_2)\times\partial_r\partial_s\int_0^{\tau_1}d\tau_1\int_0^{\tau_2}d\tau_2 D_0(1,2)+\partial_r\partial_s\int\frac{d^2\vec{\xi}}{(2\pi)^2}\ln|\vec{\xi}-\vec{r}|\times[\delta(\tau_1^2-\vec{\xi}^2)-\delta(\tau_2^2-\vec{\xi}^2)]. \tag{3.18}$$

From this form, it immediately follows that the commutator of the potentials vanishes at $\tau_1=\tau_2$. An even stronger result is found for the commutator of the two electric fields,

$$[E_r(1),E_s(2)]=\frac{\partial^2}{\partial\tau_1\partial\tau_2}i\Delta_{0,rs}(1,2)=\left[-\delta_{rs}\frac{\partial^2}{\partial\tau_1\partial\tau_2}-\cosh(\eta_1-\eta_2)\times\frac{\partial^2}{\partial x^r\partial x^s}\right]iD_0(1,2). \tag{3.19}$$

This commutator vanishes everywhere except on the light cone, in full compliance with the microcausality principle for the electric field which is an observable. However, this does not happen for the commutator of the potentials since they are defined nonlocally. It vanishes neither at spacelike nor at timelike separation because the line of integration which recovers the potential at the point $x_2$, in general, intersects (e.g., at some point $x_3$) with the light cone which has its vertex at the point $x_1$, and the commutator of the electric fields at the points $x_1$ and $x_3$ is not zero.

Similar results take place for the commutator of the $\eta$-components of the potential and the electric field. The field commutator,

$$[E_\eta(1),E_\eta(2)]=\frac{\partial^2}{\partial\tau_1\partial\tau_2}i\Delta_{0,\eta\eta}(1,2)=-i\nabla_\perp^2 D_0(1,2), \tag{3.20}$$

is entirely causal, while the commutator of the potentials,

$$[A_\eta(1),A_\eta(2)]=i\Delta_{0,\eta\eta}(1,2)=-i\nabla_\perp^2\int_0^{\tau_1}\tau_1 d\tau_1\int_0^{\tau_2}\tau_2 d\tau_2 D_0(1,2), \tag{3.21}$$

does not vanishes at spacelike distances, except for $\tau_1=\tau_2$. Finally, the formally designed commutator between the $r$ and $\eta$ components of the electric field (the two observables),

$$[E_r(1),E_\eta(2)]=\frac{\partial^2}{\partial\tau_1\partial\tau_2}i\Delta_{0,r\eta}(1,2)=-\frac{\partial^2}{\partial x^r\partial\eta}\left(\frac{\tau_2}{\tau_1}D_0(1,2)\right), \tag{3.22}$$

is entirely confined to the light cone, while the commutator of the potentials (which are not the observables),

$$[A_r(1),A_\eta(2)]=i\Delta_{0,r\eta}(1,2)=-\int_0^{\tau_1}\frac{d\tau_1}{\tau_1}\int_0^{\tau_2}\tau_2 d\tau_2\frac{\partial^2}{\partial x^r\partial\eta}D_0(1,2), \tag{3.23}$$

does not vanish at the spacelike distance, even at $\tau_1=\tau_2$. This result, however, is not a subject for any concern since the potentials are defined nonlocally and commutation relations for the electric and magnetic [cf. Eq. (2.14)] fields are reproduced correctly. Moreover, we have argued above that the $\eta$ components of $A$ and $E$ are not the canonical variables since the constraints express them via the $x$ and $y$ components.

The ‘‘acausal’’ behavior of the Riemann function, $\Delta_0^{\mu\nu}(1,2)$, may cause doubts whether the gauge $A^\tau=0$ allows for meaningful retarded and advanced Green functions which, by causality, should vanish at spacelike distances. Fortunately, this anomalous behavior appears only for the gauge-variant potential; the response functions for observable electric and magnetic fields are causal. This can be easily seen, e.g., from Eqs. (2.21), (2.25), and (2.26), which are the usual inhomogeneous relativistic wave equations for the various physical components of the field strengths $\mathcal{E}$ and $\mathcal{B}$.

### D. Canonical commutation relations in the gauge $A^\tau=0$

A proof of the commutation relations (3.2) for the Fock operators follows the standard guidelines [6]. First, the creation and annihilation operators are *defined* via the relations

$$c_\lambda(\nu,\vec{k})=(V^{(\lambda)}_{\nu\vec{k}},A)=i\mathrm{g}^{ij}\int d^3\mathbf{x}[V^{(\lambda)*}_{\nu\vec{k};j}(x)\dot{A}_i(\mathbf{x},\tau)-\dot{V}^{(\lambda)*}_{\nu\vec{k};j}(x)A_i(\mathbf{x},\tau)],$$

$$c^\dagger_\lambda(\nu,\vec{k})=(A,V^{(\lambda)}_{\nu\vec{k}})=i\mathrm{g}^{ij}\int d^3\mathbf{x}[A_i(\mathbf{x},\tau)\dot{V}^{(\lambda)}_{\nu\vec{k};j}(x)-\dot{A}_i(\mathbf{x},\tau)V^{(\lambda)}_{\nu\vec{k};j}(x)]. \tag{3.24}$$

This results in the following expression for the commutator:

$$[c_\lambda(\nu,\vec{k}),c^\dagger_{\lambda'}(\nu',\vec{k}')]=\int d^3\mathbf{x}d^3\mathbf{y}\mathrm{g}^{ij}(x)\mathrm{g}^{lm}(y)\times\{[A_i(\mathbf{x},\tau),\dot{A}_l(\mathbf{y},\tau)](\dot{V}^{(\lambda)*}_{\nu\vec{k};j}(x)V^{(\lambda')}_{\nu'\vec{k}';n}(y)-\dot{V}^{(\lambda')*}_{\nu'\vec{k}';j}(x)V^{(\lambda)}_{\nu\vec{k};n}(y))+[A_i(\mathbf{x},\tau),A_l(\mathbf{y},\tau)]\dot{V}^{(\lambda)*}_{\nu\vec{k};j}(x)\dot{V}^{(\lambda')}_{\nu'\vec{k}';n}(y)+[\dot{A}_i(\mathbf{x},\tau),\dot{A}_l(\mathbf{y},\tau)]V^{(\lambda')*}_{\nu'\vec{k}';j}(x)V^{(\lambda)}_{\nu\vec{k};n}(y)\}. \tag{3.25}$$

Most of the terms in the second line vanish due to the commutation relations. Next, we rely on the following guess about the form of the commutator:

$$[A_i(x),A_j(y)]=\sum_{\lambda=1,2}\int d\nu d^2\vec{k}(V^{(\lambda)}_{\nu\vec{k};i}(x)V^{(\lambda)*}_{\nu\vec{k};j}(y) - V^{(\lambda)*}_{\nu\vec{k};i}(x)V^{(\lambda)}_{\nu\vec{k};i}(y)), \qquad (3.26)$$

which leads to the proper equal-time commutation relations for the independent canonical variables. Finally, explicitly using the orthogonality relations for the eigenmodes $V^{(\lambda)}$, we immediately obtain the commutation relations (3.2).

## IV. LONGITUDINAL PROPAGATOR AND STATIC FIELDS

In this section, we shall find the explicit expressions for the kernels (2.42) and (2.46) which represent the longitudinal and instantaneous components of the gauge field produced by the ''external'' current $j^\mu$. The calculations are lengthy and their details can be found in Appendix C. Here, we present only the final answers.

The components of the longitudinal propagator $\Delta^{(L)}_{lm}(\tau_2,\vec{r},\eta)$ are already obtained in the form of the three-dimensional integrals (2.42). $\Delta^{(L)}$ depends on the differences of the curvilinear spatial coordinates, $\vec{r}=\vec{r}_1-\vec{r}_2$ and $\eta=\eta_1-\eta_2$, but *not* on the difference of the temporal arguments $\tau_1$ (of the field) and $\tau_2$ (of the source). Introducing the shorthand notation for the distance in the $(xy)$ plane, $r_\perp=|\vec{r}|$, and for the full distance $R_2=R(\tau_2)=[(\vec{r}_1-\vec{r}_2)^2+\tau_2^2\sinh^2(\eta_1-\eta_2)]^{1/2}$ between the two points of the surface $\tau_2=$const, we obtain

$$\Delta^{(L)}_{rs}=-\frac{\theta(\tau_2-r_\perp)}{4\pi}\left[\frac{1}{r_\perp^2}\left(1-\frac{\tau_2\cosh\eta}{R_2}\right)\left(\delta_{rs}-\frac{2x^rx^s}{r_\perp^2}\right) - \frac{2x^rx^s}{r_\perp^2}\frac{\tau_2\cosh\eta}{R_2^3}\right],$$

$$\Delta^{(L)}_{\eta s}=-\frac{\theta(\tau_2-r_\perp)}{4\pi}\frac{x^s}{r_\perp^2}\frac{\tau_2\sinh\eta}{R_2}\frac{\tau_2^2-r_\perp^2}{R_2^2},$$

$$\Delta^{(L)}_{r\eta}=-\frac{\theta(\tau_2-r_\perp)}{4\pi}\frac{x^r}{r_\perp^2}\frac{\tau_2^3\sinh\eta}{R_2^3},$$

$$\Delta^{(L)}_{\eta\eta}=\frac{\tau_2^2}{2}\delta(\vec{r})\delta(\eta)+\frac{\theta(\tau_2-r_\perp)}{4\pi}\left[2\frac{\eta\coth\eta-1}{\sinh^2\eta} + \frac{\tau_2\cosh\eta}{R_2\sinh^2\eta}\left(3-\frac{r_\perp^2}{R_2^2}\right)-\frac{2\cosh\eta}{\sinh^3|\eta|}L_2\right], \qquad (4.1)$$

where $L_2=L(\tau_2)=\ln[(\tau_2\sinh|\eta|+R_2)/r_\perp]$. By examination of Eq. (2.41), one may see that after the replacement of $\tau_2$ by $\tau_1$, the same kernel, $\Delta^{(L)}_{lm}(\tau_1,\vec{r},\eta)$, determines the components $E^{(L)}_m(\tau_1)$ of the longitudinal part of the electric field via the components $j^m(\tau_1)$ of the current at the same time. These propagators do not respect the light cone, but have a remarkable property that the longitudinal fields at the surface of the constant proper time $\tau$ do not exist at the distance $r_\perp$ from their sources that exceed $\tau$. This establishes the upper limit for the possible dynamical correlations between the longitudinal fields in the $(xy)$ plane.

One more representation of the longitudinal propagator is interesting at least in two respects. First, for the practical calculations in paper [IV], we shall need the longitudinal part of the propagator in the mixed representation, $\Delta^{(long)}_{lm}(\tau_1,\tau_2;\eta_1-\eta_2,\vec{k}_t)$, which can be shown to be

$$\Delta^{[long]}_{rs}=\frac{k_rk_s}{k_t^2}\left\{-\frac{k_t\cosh|\eta|}{2}\int_{\tau_2}^{\tau_1}e^{-tk_t\sinh|\eta|}dt-\int\frac{d\alpha}{2\pi}\tanh\left(\alpha+\frac{\eta}{2}\right)\tanh\left(\alpha-\frac{\eta}{2}\right)[\sin k_tT_1-\sin k_tT_2]\right\}, \qquad (4.2)$$

$$\Delta^{[long]}_{\eta\eta}=-\frac{\tau_1^2-\tau_2^2}{2}\delta(\eta)+\frac{k_t\cosh|\eta|}{2}\int_{\tau_2}^{\tau_1}e^{-tk_t\sinh|\eta|}t^2dt-\int\frac{d\alpha}{2\pi}\frac{\sin k_tT_1-\sin k_tT_2-k_tT_1\cos k_tT_1+k_tT_2\cos k_tT_2}{k_t^2\cosh^2\left(\alpha+\frac{\eta}{2}\right)\cosh^2\left(\alpha-\frac{\eta}{2}\right)}, \qquad (4.3)$$

where the last integral terms in Eqs. (4.2) and (4.3) provide that the longitudinal part of the field obey the boundary condition we imposed at $\tau\to0$. These terms cancel with the similar terms in the radiation part of the retarded propagator $\Delta_{[ret]}(\tau_1,\tau_2)$. Eventually, the last fact guarantee that acausal terms (which were the subject for concern in the course of the canonical quantization in Sec. III D) do not contribute to the dispersion equation that we derive in the next paper. Second, the first very simple by its structure contact term in the component $D^{[long]}_{\eta\eta}$ of the propagator

$$\Delta^{[contact]}_{\eta\eta}(\tau_1,\tau_2;\eta_2-\eta_1;\vec{k}_t)=-\frac{\tau_1^2-\tau_2^2}{2}\delta(\eta), \qquad (4.4)$$

appears to be the only part of the enormously complicated full retarded propagator $\Delta_{lm}^{[ret]}$ which significantly contributes the amplitude of the forward quark-quark scattering at the earliest stage of the collision.

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## APPENDIX A: MODES OF THE FREE GAUGE FIELD

Here, we shall obtain the complete set of the one-particle solutions to the homogeneous system of the Maxwell equations with the gauge $A^\tau=0$, that is, Eqs. (2.10) and (2.11). This gauge condition explicitly depends on the coordinates, thus introducing effective nonlocality in the path integral that represents the action. Therefore, it becomes impossible to invert the differential operators using the standard symbolic methods. An explicit form of the one-particle solutions becomes necessary in order to find the Wightman functions of the free vector field, to establish the the explicit form of the field commutators, and to separate the propagators of the transverse and the longitudinal fields. It is natural to look for the solution in the form of the Fourier transform with respect to the spatial coordinates,

$$A_i(x)=\int_{-\infty}^{\infty} d\nu \int d^2\vec{k} e^{i\nu\eta} e^{i\vec{k}\vec{r}} A_i(\vec{k},\nu,\tau). \tag{A1}$$

Then, the system of second order ordinary differential equations for the Fourier transforms takes the following form:

$$\left[\tau\partial_\tau^2+\partial_\tau+\frac{\nu^2}{\tau}+\tau k_y^2\right]A_x(\vec{k},\nu,\tau)-\tau k_x k_y A_y(\vec{k},\nu,\tau) -\frac{\nu k_x}{\tau}A_\eta(\vec{k},\nu,\tau)=0, \tag{A2}$$

$$-\tau k_x k_y A_x(\vec{k},\nu,\tau)+\left[\tau\partial_\tau^2+\partial_\tau+\frac{\nu^2}{\tau}+\tau k_x^2\right]A_y(\vec{k},\nu,\tau) -\frac{\nu k_y}{\tau}A_\eta(\vec{k},\nu,\tau)=0, \tag{A3}$$

$$-\frac{\nu k_x}{\tau}A_x(\vec{k},\nu,\tau)-\frac{\nu k_y}{\tau}A_y(\vec{k},\nu,\tau) +\left[\frac{1}{\tau}\partial_\tau^2-\frac{1}{\tau^2}\partial_\tau+\frac{1}{\tau}k_\perp^2\right]A_\eta(\vec{k},\nu,\tau)=0. \tag{A4}$$

In this form, the system is manifestly symmetric and self-adjoint. An additional equation of the constraint reads

$$\mathcal{C}(\vec{k},\nu,\tau)=\frac{1}{\tau}\nu\partial_\tau A_\eta+\tau\partial_\tau[k_x A_x(\vec{k},\nu,\tau)+k_y A_y(\vec{k},\nu,\tau)]=0. \tag{A5}$$

Let us rewrite the homogeneous system of the Maxwell equations in terms of the variables

$$\Phi=\partial_x A_x+\partial_y A_y, \quad \Psi=\partial_y A_x-\partial_x A_y, \quad \text{and} \quad \mathsf{A}=A_\eta. \tag{A6}$$

One immediately sees that the equation for the Fourier component $\Psi(\vec{k},\nu,\tau)$ of the longitudinal magnetic field $\Psi(x)$ decouples,

$$\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]\Psi(\vec{k},\nu,\tau)=0. \tag{A7}$$

Then, the other two equations of motion take shape, i.e.,

$$[\tau^2\partial_\tau^2+\tau\partial_\tau+\nu^2]\Phi_{\vec{k},\nu}(\tau)-i\nu k_\perp^2\mathsf{A}_{\vec{k},\nu}(\tau)=0, \tag{A8}$$

$$\left[\partial_\tau^2-\frac{1}{\tau}\partial_\tau+k_\perp^2\right]\mathsf{A}_{\vec{k},\nu}(\tau)+i\nu\Phi_{\vec{k},\nu}(\tau)=0. \tag{A9}$$

The additional constraint equation can be conveniently rewritten as

$$\mathcal{C}(\vec{k},\nu,\tau)=\frac{i\nu}{\tau}\partial_\tau\mathsf{A}_{\vec{k},\nu}(\tau)+\tau\partial_\tau\Phi_{\vec{k},\nu}(\tau)=0. \tag{A10}$$

This is an independent equation. However, the conservation of the constraint along the Hamiltonian time $\tau$ is a consequence of the equations of motion, and it *can* be employed to obtain the independent equations for the components of the vector field. This is easily done in terms of the auxiliary functions,

$$\varphi_{\vec{k},\nu}(\tau)=\tau\dot{\Phi}_{\vec{k},\nu}(\tau) \quad \text{and} \quad a_{\vec{k},\nu}(\tau)=\tau^{-1}\dot{\mathsf{A}}_{\vec{k},\nu}(\tau), \tag{A11}$$

which are directly connected to the "physical" components of the electric field, $\mathcal{E}^m=\sqrt{-\mathrm{g}}\mathrm{g}^{ml}\dot{A}_l$:

$$\partial_\tau\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]\varphi_{\vec{k},\nu}(\tau)=0, \tag{A12}$$

$$\partial_\tau[\tau^2\partial_\tau^2+\tau\partial_\tau+\nu^2+k_\perp^2\tau^2]a_{\vec{k},\nu}(\tau)=0. \tag{A13}$$

As a result, we see that the functions $\varphi_{\vec{k},\nu}(\tau)$ and $a_{\vec{k},\nu}(\tau)$ obey inhomogeneous Bessel equations,

$$\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]\varphi_{\vec{k},\nu}(\tau)=\vec{k}^2c_\varphi, \tag{A14}$$

$$\left[\partial_\tau^2+\frac{1}{\tau}\partial_\tau+\frac{\nu^2}{\tau^2}+k_\perp^2\right]a_{\vec{k},\nu}(\tau)=\tau^{-2}c_a, \tag{A15}$$

where $c_\varphi$ and $c_a$ are arbitrary constants. We may now cast the solution of these equations in the form of the sum of the partial solution of the inhomogeneous equation and a general solution of the homogeneous equation,

$$\Psi_{\vec{k},\nu}(\tau)=aH^{(2)}_{-i\nu}(k_\perp\tau)+a^*H^{(1)}_{-i\nu}(k_\perp\tau), \tag{A16}$$

$$\varphi_{\vec{k},\nu}(\tau)=cH^{(2)}_{-i\nu}(k_\perp\tau)+c^*H^{(1)}_{-i\nu}(k_\perp\tau)+c_\varphi s_{1,i\nu}(k_\perp\tau), \tag{A17}$$

$$a_{\vec{k},\nu}(\tau)=\gamma H^{(2)}_{-i\nu}(k_\perp\tau)+\gamma^*H^{(1)}_{-i\nu}(k_\perp\tau)+c_a s_{-1,i\nu}(k_\perp\tau), \tag{A18}$$

where $s_{\mu,\nu}(x)$ is the so-called Lommel function [4,5].

Furthermore, it is useful to notice that the system of the Maxwell equations (2.10) and (2.11) also has an infinite set of the $\tau$-independent solutions of the form

$$W_i(\eta,\vec{r})=\partial_i\chi(\eta,\vec{r}), \tag{A19}$$

where $\chi$ is an arbitrary function of the spatial coordinates $\eta$ and $\vec{r}$. Thus, they are the pure gauge solutions of the Abelian theory, compatible with the gauge condition.

In order to find the coefficients one should integrate Eqs. (A17) and (A18) with respect to the Hamiltonian time $\tau$, thus finding the functions $\Phi$ and $\mathsf{A}$. Next, it is necessary to solve Eqs. (A6) for the Fourier components of the vector potential and to substitute them into the original system of Eqs. (A2)–(A5). Using functional relations from Appendix B, one obtains that $c+\nu\gamma=0$ and $c_a-\nu c_\varphi=0$.

One of the solutions [already normalized according to Eq. (2.9)] is found immediately:

$$V^{(1)}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_\perp}\begin{pmatrix} k_y\\ -k_x\\ 0\end{pmatrix}H^{(2)}_{-i\nu}(k_\perp\tau)e^{i\nu\eta+i\vec{k}\vec{r}}. \tag{A20}$$

Initially, the components of the vector mode $V^{(2)}$ appear in the following form required by the convergence of the integral,

$$\begin{pmatrix} \nu k_r R^{(2)}_{-1,-i\nu}(k_\perp\tau|S)\\ -R^{(2)}_{1,-i\nu}(k_\perp\tau|s)-i\nu[e^{\pi\nu/2}/\sinh(\pi\nu/2)]\end{pmatrix}e^{i\nu\eta+i\vec{k}\vec{r}}.$$

However, it can be gauge transformed to the more compact form,

$$V^{(2)}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_\perp}\begin{pmatrix} k_r\nu R^{(2)}_{-1,-i\nu}(k_\perp\tau|s)\\ -R^{(2)}_{1,-i\nu}(k_\perp\tau|s)\end{pmatrix}e^{i\nu\eta+i\vec{k}\vec{r}}. \tag{A21}$$

The third solution (the last one by the count of the nonvanishing components of the vector potential in the gauge $A^\tau=0$) has the following form:

$$V^{(3)}_{\vec{k},\nu}(x)=\begin{pmatrix} k_r Q_{-1,i\nu}(k_\perp\tau)\\ \nu Q_{1,i\nu}(k_\perp\tau)\end{pmatrix}e^{i\nu\eta+i\vec{k}\vec{r}}. \tag{A22}$$

The modes $V^{(1)}$ and $V^{(2)}$ are the normalized solutions of the Maxwell equations. They are orthogonal and obey the normalization condition,

$$(V^{(1,2)}_{\vec{k},\nu},V^{(1,2)}_{\vec{k}',\nu'})=\delta(\nu-\nu')\delta(\vec{k}-\vec{k}'),\quad (V^{(1)}_{\vec{k},\nu},V^{(2)}_{\vec{k},\nu})=0, \tag{A23}$$

which can be easily verified by means of Eq. (B5). The norm of these solutions is given by Eq. (2.9). A normalization coefficient of the mode $V^{(3)}$ is not defined, as this mode has a zero norm. It is also orthogonal to $V^{(1)}$ and $V^{(2)}$:

$$(V^{(3)}_{\vec{k},\nu},V^{(3)}_{\vec{k}',\nu'})=(V^{(1)}_{\vec{k},\nu},V^{(3)}_{\vec{k},\nu})=(V^{(2)}_{\vec{k},\nu},V^{(3)}_{\vec{k},\nu})=0. \tag{A24}$$

Thus this mode drops out from the decomposition of the free gauge field.

The conservation of the constraint can be obtained as a consequence of Eqs. (A12) and (A13) in the form

$$\tau\partial_\tau[\varphi_{\vec{k},\nu}(\tau)+\nu a_{\vec{k},\nu}(\tau)]\equiv\tau\partial_\tau\mathcal{C}_{\vec{k},\nu}(\tau)=0, \tag{A25}$$

which reassures us of the consistency between the dynamic equations and conservation of the Gauss law constraint.

One can explicitly check that the modes $V^{(1)}$ and $V^{(2)}$ obey the constraint equation (A10), which expresses Gauss law. The mode $V^{(3)}$ does not. This mode corresponds to the longitudinal field which cannot exist without the source.

## APPENDIX B: MATHEMATICAL MISCELLANY

This appendix contains a list of mathematical formulas for the functions which appear in various calculations in the body of the paper and Appendix A. The components of the vector field are expressed via two types of integrals. The first of them was studied in Ref. [6]:

$$\begin{aligned} R^{(j)}_{\mu,\nu}(x|\mathsf{S})&=\int x^\mu H^{(j)}_\nu(x)dx\\ &=x[(\mu+\nu-1)H^{(j)}_\nu(x)\mathsf{S}_{\mu-1,\nu-1}(x)\\ &\quad-H^{(j)}_{\nu-1}(x)\mathsf{S}_{\mu,\nu}(x)],\end{aligned} \tag{B1}$$

where $\mathsf{S}_{\mu,\nu}$ stands for any of the two Lommel functions, $s_{\mu,\nu}$ or $S_{\mu,\nu}$ [5,6]. [Whenever we omit the indicator $|\mathsf{S})$, the function $R^{(j)}_{\mu,\nu}(x|s)$ is assumed.] The second type of integrals,

$$Q_{\mu,\nu}(x)=\int_0^x x^\mu dx s_{-\mu,\nu}(x), \tag{B2}$$

is a new one. The functions $R^{(j)}_{\mu,\nu}(x|\mathsf{S})$ are introduced as indefinite integrals. The preliminary choice of the lower limit and, consequently, the choice of which of the functions, $s_{\mu,\nu}$ or $S_{\mu,\nu}$, is used is motivated by the requirement of convergence and regular behavior. One can easily prove that

$$R^{\binom{2}{1}}_{-1,\mp i\nu}(x|S)-R^{\binom{2}{1}}_{-1,\mp i\nu}(x|s)=\frac{\mp ie^{\pi\nu/2}}{\nu\sinh(\pi\nu/2)},$$

$$R^{\binom{2}{1}}_{1,\mp i\nu}(x|S)-R^{\binom{2}{1}}_{1,\mp i\nu}(x|s)=\frac{\pm i\nu e^{\pi\nu/2}}{\sinh(\pi\nu/2)}. \tag{B3}$$

We often use the following relation between Lommel functions [4,5]:

$$\mathsf{S}_{1,i\nu}(k_\perp\tau)=1-\nu^2\mathsf{S}_{-1,i\nu}(k_\perp\tau). \tag{B4}$$

From the integral representations (B1) and (B2), it is straightforward to derive the functional relations

$$R^{(j)}_{-1,i\nu}(k_\perp\tau)+\frac{1}{\nu^2}R^{(j)}_{1,i\nu}(k_\perp\tau)=-\frac{\tau}{\nu^2}\frac{\partial}{\partial\tau}H^{(j)}_{i\nu}(k_\perp\tau), \tag{B5}$$

$$Q_{-1,i\nu}(k_\perp\tau)-Q_{1,i\nu}(k_\perp\tau)=-\frac{\tau}{\nu^2}\frac{\partial}{\partial\tau}s_{1,i\nu}(k_\perp\tau)$$
$$=\tau\frac{\partial}{\partial\tau}s_{-1,i\nu}(k_\perp\tau). \tag{B6}$$

The Wronskian of the Hankel and Lommel functions,

$$W\{s_{1,i\nu}(x),H^{(j)}_{i\nu}(x)\}=-\frac{1}{x}R^{(j)}_{1,i\nu}(x), \tag{B7}$$

must be obtained in order to prove orthogonality of $V^{(2)}$ and $V^{(3)}$. To prove Eq. (B7), one should use the following representation for the Lommel function:

$$s_{1,i\nu}(x)=\frac{\pi}{4i}[H^{(1)}_{i\nu}(x)R^{(2)}_{1,i\nu}(x)-H^{(2)}_{i\nu}(x)R^{(1)}_{1,i\nu}(x)], \tag{B8}$$

which follows from Eq. (B1) and, consequently,

$$s'_{1,i\nu}(x)=\frac{\pi}{4i}[H^{(1)\prime}_{i\nu}(x)R^{(2)}_{1,i\nu}(x)-H^{(2)\prime}_{i\nu}(x)R^{(1)}_{1,i\nu}(x)]. \tag{B9}$$

In order to prove relation (2.32) one should use the representation (B1) for the functions $R^{(j)}_{\mu,\nu}$ and the Wronskian of the two independent Hankel functions. The proof of relation (2.43) begins with replacing the functions $R^{(j)}_{-1,i\nu}$ by $R^{(j)}_{1,i\nu}$ by means of Eq. (B5). The final result follows from Eqs. (B9) and (B6).

## APPENDIX C: CALCULATION OF THE LONGITUDINAL PART OF THE PROPAGATOR

The kernels (2.44) and (2.46) of the longitudinal and instantaneous parts of the propagator are given in the form of the three-dimensional Fourier integrals $d\nu d^2\vec{k}$. Here, we describe the major steps of the calculations which lead to Eqs. (4.1).

We permanently use the following integral representation for the Hankel functions:

$$e^{-\pi\nu/2}e^{\pm i\nu\eta}H^{\binom{2}{1}}_{\mp i\nu}(k_\perp\tau)=\frac{\pm i}{\pi}\int_{-\infty}^{\infty}e^{\mp ik_\perp\tau\cosh(\theta-\eta)}e^{\pm i\nu\theta}d\theta, \tag{C1}$$

which allows one to calculate many integrals by changing the order of integration. The Lommel function $S_{1,i\nu}$ has a similar representation,

$$S_{1,i\nu}(x)=x\int_0^\infty \cosh u\cos\nu u e^{-x\sinh u}du. \tag{C2}$$

Integrating by parts, and using Eq. (B4), we find the integral representation for $S_{-1,i\nu}$,

$$\nu S_{-1,i\nu}(x)=\int_0^\infty \sin(\nu u)e^{-x\sinh u}du. \tag{C3}$$

We start with the integral representation (B2) of the functions $Q_{\pm1,i\nu}$ and perform an integration over $\nu$. To compute the integrals from the function $s_{1,i\nu}$ it can be conveniently decomposed in the following way:

$$s_{1,i\nu}(x)=S_{1,i\nu}(x)-h_{i\nu}(x),$$
$$h_{i\nu}(x)=\frac{e^{-\pi\nu/2}}{2}\frac{\pi\nu/2}{\sinh(\pi\nu/2)}[H^{(1)}_{i\nu}(x)+H^{(2)}_{-i\nu}(x)], \tag{C4}$$

which allows one to find

$$\int_{-\infty}^{\infty}S_{1,i\nu}(k_\perp\tau)e^{i\nu\eta}d\nu=\pi k_\perp\tau\cosh\eta e^{-k_\perp\tau\sinh|\eta|},$$
$$\int_{-\infty}^{\infty}\nu S_{-1,i\nu}(k_\perp\tau)e^{i\nu\eta}d\nu=i\pi\,\mathrm{sign}\,\eta e^{-k_\perp\tau\sinh|\eta|}. \tag{C5}$$

The similar Fourier integrals from the function $h_{i\nu}$ are calculated using the representation (C1) for the Hankel functions and the integral,

$$\frac{\pi\nu/2}{\sinh(\pi\nu/2)}=\frac{1}{2}\int_{-\infty}^{\infty}d\theta\frac{e^{i\nu\theta}}{\cosh^2\theta}. \tag{C6}$$

This yields, for example,

$$\int_{-\infty}^{\infty}d\nu e^{i\nu\eta}h_{i\nu}(k_\perp\tau)=\int_{-\infty}^{\infty}\frac{d\theta}{\cosh^2\theta}\sin[k_\perp\tau\cosh(\theta-\eta)]. \tag{C7}$$

After integration over $\nu$ we obtain the following integral for $\Delta^{(L)}_{rs}$:

$$\Delta^{(L)}_{rs}=\int\frac{d^2\vec{k}}{(2\pi)^3}\frac{k_rk_s}{k_\perp^2}e^{i\vec{k}\vec{r}}\int_0^{\tau_2}\frac{d\tau}{\tau}\bigg(\pi k_\perp\tau\cosh\eta e^{-k_\perp\tau\sinh|\eta|}$$
$$-\int_{-\infty}^{\infty}\frac{d\theta}{\cosh^2\theta}\sin[k_\perp\tau\cosh(\theta-\eta)]\bigg), \tag{C8}$$

and similar integrals for the other components. The first term in this formula is calculated in the following way. After integration over $\tau$ we continue:

$$\Delta'_{rs} = -\frac{\partial_r\partial_s}{8\pi^2}\coth|\eta|\int\frac{d^2\vec{k}}{k_\perp^2}e^{i\vec{k}\vec{r}}[1-e^{-k_\perp\tau_2\sinh|\eta|}]$$
$$= -\frac{\partial_r\partial_s}{4\pi}\coth|\eta|\int_0^\infty\frac{dk_\perp}{k_\perp}J_0(k_\perp r_\perp)[1-e^{-k_\perp\tau_2\sinh|\eta|}]$$
$$= -\frac{\partial_r\partial_s}{4\pi}\coth|\eta|\ln\left[\frac{\tau_2\sinh|\eta|+\sqrt{r_\perp^2+\tau_2^2\sinh^2\eta}}{r_\perp}\right]. \tag{C9}$$

To work out the second term, one should introduce $k_z = k_\perp \sinh\theta$ and $k_0 = k_\perp\cosh\theta = |\mathbf{k}|$ and change $d^2\vec{k}d\theta$ for the three-dimensional integration $d^3\mathbf{k}$. With $t=\tau\cosh\eta$, $\mathbf{r} = (x,y,\tau\sinh\eta)$, this leads to

$$\Delta''_{rs} = \frac{\partial_r\partial_s}{2\pi^3}\int_0^{\tau_2}\frac{d\tau}{\tau}\int\frac{d^3\mathbf{k}}{k_0^3}e^{i\mathbf{kr}}\sin k_0 t$$
$$= \frac{\partial_r\partial_s}{4\pi}\int_0^{\tau_2}\frac{d\tau}{\tau}\left(\theta(r_\perp^2-\tau^2)\frac{\tau\cosh\eta}{\sqrt{r_\perp^2+\tau^2\sinh^2\eta}}+\theta(\tau^2-r_\perp^2)\right)$$
$$= \frac{\partial_r\partial_s}{4\pi}\left(\theta(r_\perp-\tau_2)\coth|\eta| \times\ln\left[\frac{\tau_2\sinh|\eta|+\sqrt{r_\perp^2+\tau_2^2\sinh^2\eta}}{r_\perp}\right]+\theta(\tau_2-r_\perp)\ln\frac{\tau_2}{r_\perp}\right). \tag{C10}$$

Adding Eqs. (C9) and (C10), we obtain the first of the equations (4.1).

## APPENDIX D: GLUON CORRELATORS IN THE CENTRAL RAPIDITY REGION AND NEAR THE LIGHT WEDGE

In this section, we compare the correlators of the gauge $A^\tau=0$ with the similar correlators in the three other gauges, $A^0=0$, $A^+=0$, and $A^-=0$. We shall start with the simplest on-mass-shell Wightman function $\Delta_{10}^{\mu\nu}$. These type of correlators, $\Delta_{01}^{\mu\nu}$, $\Delta_0^{\mu\nu}$, and $\Delta_1^{\mu\nu}$ share the same polarization sum of the free gauge field. They correspond to the densities of the final states of the radiation field and are important for various calculations. The same polarization sum appears in expressions for the transverse part of the propagators, $\Delta_{ret}^{\mu\nu}$, $\Delta_{adv}^{\mu\nu}$, $\Delta_{00}^{\mu\nu}$, and $\Delta_{11}^{\mu\nu}$. For our immediate purpose we shall include the projector $d^{\mu\nu}$ of the gauge $A^\tau=0$ to the formal Fourier representation,

$$iD_{10}^{\mu\nu}(x_1,x_2) = \int\frac{d^3k}{(2\pi)^3 2k^0}d^{\mu\nu}(k;x_1,x_2)e^{-ik(x_1-x_2)}, \tag{D1}$$

with the ‘‘extraneous’’ dependence of the Fourier transform on the time and spatial coordinates. This dependence disappear in some important limits. Therefore, we discover the domains where the wedge dynamic simplifies and describes the processes which are approximately homogeneous in space and time. These domains are (i) the central rapidity region, $\eta_{1,2}\ll 1$ (or $x_{1,2}^3\sim 0$), where the projector in the integrand of Eq. (D1) is

$$d^{\mu\nu}(k,u) = -g^{\mu\nu}+\frac{k^\mu u^\nu+u^\mu k^\nu}{ku}-\frac{k^\mu k^\nu}{(ku)^2}, \tag{D2}$$

with the gauge-fixing vector, $u^\mu=(1,0,0,0)$, which approximately coincide with the local normal to the hypersurface $\tau = \text{const}$; and (ii) the vicinities of two null-planes, $\eta\to\pm\infty$ (or $x^\mp\to 0$), where

$$d^{\mu\nu}(n_\pm,k) = -g^{\mu\nu}+\frac{k^\mu n_\pm^\nu+k^\nu n_\pm^\mu}{(kn_\pm)}, \tag{D3}$$

with the null-plane vectors $n_\pm^\mu=(1,0,0,\mp 1)$.

Equations (3.9)–(3.12) almost fit our needs. In all three cases ($x^3\to 0$, $k^0x^0\gg 1$, as well as $x^-\to 0$, $k^-x^+\gg 1$, and $x^+\to 0$, $k^+x^-\gg 1$) the functions $f_1$ and $f_2$ can be approximated by the following expressions:

$$f_1 \approx i\tanh(\theta-\eta)e^{-ik_\perp\tau\cosh(\theta-\eta)+i\vec{k}\vec{r}}$$
$$= \frac{k^0x^3-k^3x^0}{k^0x^0-k^3x^3}e^{-ikx}$$
$$= \frac{k^+x^- - k^-x^+}{k^+x^- + k^-x^+}e^{-ikx}, \tag{D4}$$

$$f_2 \approx ik_\perp\tau\frac{e^{-ik_\perp\tau\cosh(\theta-\eta)+i\vec{k}\vec{r}}}{\cosh(\theta-\eta)}$$
$$= ik_\perp^2\tau^2\frac{e^{-ikx}}{k^0x^0-k^3x^3}$$
$$= 2ik_\perp^2\tau^2\frac{e^{-ikx}}{k^+x^-+k^-x^+}. \tag{D5}$$

(We have omitted the time independent terms in $f_1$ and $f_2$ which set the potentials of the mode $v^{(2)}$ to zero at $\tau=0$. These kind of terms would correspond to the residual gauge symmetry and is not kept in the axial and the null-plane gauges as well. Thus, we cannot really claim the correspondence of the longitudinal fields between the wedge dynamics and these three dynamics.)

Transformation of the correlator $\Delta^{lm}(1,2)$ to the Minkowski coordinates is carried out according to the formula

$$D^{\mu\nu}(x_1,x_2) = a_i^\mu(x_1)g^{il}(x_1)\Delta_{lm}(u_1,u_2)g^{mk}(x_2)a_m^\nu(x_2), \tag{D6}$$

where the matrix of the transformation is defined in the standard way,

$$a_i^\mu(x) = \frac{\partial x^\mu}{\partial u^i},\quad a_\eta^0(x)=x^3,\quad a_\eta^3(x)=x^0,\quad a_s^r=\delta_s^r. \tag{D7}$$

These are the only components of the tensor $a_i^\mu(x)$ which participate in the transformation. In this way, we obtain

$$D^{00}(1,2)=x_1^3x_2^3\Delta^{\eta\eta}(1,2);\quad D^{03}(1,2)=x_1^3x_2^0\Delta^{\eta\eta}(1,2),$$
$$D^{30}(1,2)=x_1^0x_2^3\Delta^{\eta\eta}(1,2);\quad D^{33}(1,2)=x_1^0x_2^0\Delta^{\eta\eta}(1,2),$$
$$D^{0r}(1,2)=x_1^3\Delta^{\eta r}(1,2);\quad D^{r0}(1,2)=x_2^3\Delta^{r\eta}(1,2);$$
$$D^{3r}(1,2)=x_1^0\Delta^{\eta r}(1,2);\quad D^{r3}(1,2)=x_2^0\Delta^{r\eta}(1,2),$$
$$D^{rs}(1,2)=\Delta^{rs}(1,2). \tag{D8}$$

Every additional factor $g^{\eta\eta}=\tau^{-2}$ finds a counterpart which prevents singular behavior at $\tau=0$. In the above approximation, the expression for the $\Delta^{\eta\eta}(x_1,x_2)$ component of the correlator has the form

$$\Delta^{\eta\eta}(x_1,x_2)=\int\frac{d^3k}{(2\pi)^3 2k^0}\frac{k_\perp^2 e^{-ik(x_1-x_2)}}{(k^0x_1^0-k^3x_1^3)(k^0x_2^0-k^3x_2^3)}$$
$$=\int\frac{d^3k}{(2\pi)^3 2k^0}\frac{4k_\perp^2 e^{-ik(x_1-x_2)}}{(k^+x_1^-+k^-x_1^+)(k^+x_2^-+k^-x_2^+)}. \tag{D9}$$

Therefore, in the limit of $x_{1,2}^3\to 0$ we obtain that $D^{00},D^{0i}\to 0$, while $d^{33}(k,u)\to k_\perp^2/k_0^2$, thus reproducing the corresponding components of the gauge $A^0=0$. The other components are reproduced one by one as well, and one can expect a smooth transition between the gauge of the the wedge dynamic and the local temporal axial gauge of the reference frame comoving with the dense quark-gluon matter created in the collision.

In the limits of $x_{1,2}^\mp\to 0$ we obtain that

$$D^{00},D^{03},D^{30},D^{33}\to\frac{k_\perp^2}{(k^-)^2}=\frac{k^+}{k^-},\quad \text{if } x^-\to 0, \tag{D10}$$

and

$$D^{00},D^{03},D^{30},D^{33}\to\frac{k_\perp^2}{(k^+)^2}=\frac{k^-}{k^+},\quad \text{if } x^+\to 0. \tag{D11}$$

These limits, after they are found for all components, lead to the well known expressions of the projectors $d^{\mu\nu}(n,k)$ in the null-plane gauge (the gauge $A^+=0$ in the vicinity of $x^+=0$, and $A^-=0$ in the vicinity of $x^-=0$). Therefore we obtained an expected result; in the limit of the light-cone propagation, the gauge $A^\tau=0$ recovers the null-plane gauges $A^+=0$ and $A^-=0$.

Some remarks are in order. First, the concept of the structure functions relies heavily on the null-plane dynamics which essentially uses these gauges. For two hadrons (or two nuclei) we have two different null-plane dynamics which do not share the same Hilbert space of states. Now we have an important opportunity to describe both nuclei and the fields produced in their interaction within the same dynamic and the same Hilbert space. Second, one may trace back the origin of the poles $(ku)^{-1}$ in the polarization sums of axial gauges $(uA)=0$ and see that they appear in the course of the approximation of the less-singular factor $[k_\perp\cosh(\theta-\eta)]^{-1}$ in various limits of the propagator of the gauge $A^\tau=0$.

Further, contrary to the naive expectation that we obtain the gauge $A^+=0$ at $x^-=0$ and the gauge $A^-=0$ at $x^+=0$, we obtained them in the opposite correspondence. First of all, let us notice that the result is mathematically consistent. Indeed, the gauge condition $A^\tau=0$ may be rewritten in the form

$$A^\tau=\frac{1}{2}(A^+e^{-\eta}+A^-e^{\eta})=0. \tag{D12}$$

Thus the limit of $\eta\to\infty$ $(x^-\to 0)$ indeed leads to $A^-=0$ and the limit of $\eta\to-\infty$ $(x^+\to 0)$ leads to $A^+=0$ as the limiting gauge conditions. Recalling that

$$A^\eta=\frac{1}{2}(A^+e^{-\eta}-A^-e^{\eta})=A^+e^{-\eta}=-A^-e^{\eta}, \tag{D13}$$

we immediately realize that in the vicinities of both null planes, the tangent component $A^\eta=0$. This fact has a very simple geometrical explanation; the normal and tangent vectors of the null plane are degenerate. Once $A^\tau=0$, we have $A^\eta=0$ and consequently, $A^+=0$ and $A^-=0$ at $\eta\to\pm\infty$. This result naturally follows from the geometry of the system of the surfaces where we define the field states. These are subject to dynamical evolution in the direction which is normal to the hypersurface.


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