Scenario for ultrarelativistic nuclear collisions. II. Geometry of quantum states at the earliest stage

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(Received 3 August 2000; published 7 March 2001)

We suggest that the ultrarelativistic collisions of heavy ions provide the simplest situation for the study of strong interactions which can be understood from first principles and without any model assumptions about the microscopic structure of the colliding nuclei. We argue that the boost-invariant geometry of the collision, and the existence of hard partons in the final states, both supported by the data, make a sufficient basis for the quantum theory of the phenomenon. We conclude that the quantum nature of the entire process is defined by its global geometry, which is enforced by a macroscopic finite size of the colliding objects. In this paper, we study the qualitative aspects of the theory and review its development in two subsequent papers. Our key result is that the effective mass of the quark in the expanding system formed in the collision of the two nuclei is gradually built up reaching its maximum by the time the quark mode becomes sufficiently localized. The chromo-magneto-static interaction of the color currents flowing in the rapidity direction is the main mechanism which is responsible for the generation of the effective mass of the soft quark mode and, therefore, for the physical scale at the earliest stage of the collision.

DOI: 10.1103/PhysRevC.63.044902 PACS number(s): 12.38.Mh, 12.38.Bx, 24.85.+p, 25.75.−q

I. INTRODUCTION

In our previous papers [1–3], we began a systematic theoretical study of the scenario of ultrarelativistic collision of heavy ions. Our main result obtained in Ref. [3] (further quoted as paper [I]) was that the dense system of quark and gluons which is commonly associated with the quark-gluon plasma (QGP) can be formed only *in a single quantum transition*. In this and two subsequent papers we continue to develop this approach in greater detail. We come to a conclusion that ultrarelativistic nuclear collisions is a unique physical phenomenon when the quantum dynamics of the process is enforced by a macroscopic finite size of the colliding objects rather than by a microscopic origin of their constituents.

The entropy (the number of excited degrees of freedom) produced in collisions of heavy ions is a natural measure of the strength of the colored fields interaction. Indeed, before the collision, the quark and gluon fields are assembled into two coherent wave packets (the nuclei) and therefore, the initial entropy equals zero. The coherence is lost, and entropy is created due to the interaction. A search for the QGP in heavy-ion collisions is, in the first place, a search for evidence of entropy production. Though one may wish to rely on the invariant formula $S = \mathrm{Tr}\,\rho \ln \rho$, which expresses the entropy S via the density matrix ρ, at least one basis of states should be found explicitly. It is imperative to design such a basis, and to practically study the collective effects that take place at the earliest (<1 fm) stage of the collision.

In any standard *exclusive* scattering process, no entropy can be produced since the scattering process begins with a pure quantum state of two stable colliding particles and the final state is also given as a pure state of several particles in exactly known quantum states. The only way one can address the quantum problem of entropy production is to consider *inclusive* measurements. Since these measurements are not complete (i.e., are not exclusive), they indeed form an ensemble with finite entropy.

Quantum chromodynamics still cannot provide the theory of nuclear collisions with detailed information about nuclear structure before the collision. We face a formidable task to build a reliable theory of nuclear collision knowing almost nothing about the initial state. We may rely safely only upon the fact that the nuclei are stable bound states of the QCD and therefore, their configuration is dominated by the stationary quark and gluon fields which are genuine constituents of these quantum states. Fortunately, this, at first glance very scarce, information appears to be sufficient for the understanding of many intimate details of the collision process, if the problem is addressed from first principles.

We hope that the final state is defined more accurately and *believe* that a single-particle distribution of quarks and gluons at some early moment after the nuclei have intersected, describe it sufficiently. Thus, we may count upon a reasonably well-defined quantum observable. The measurement of the one-particle distribution is an inclusive measurement. The corresponding operator should count the number of final-state particles defined as the excitations above the perturbative vacuum. As long as we expect that this counting makes sense on the event-by-event basis, the collision is indeed producing the entropy. To develop the theory for this transition process we have to cope with a binding feature that the ''final'' state has to be defined at a finite time. This may look disturbing for readers well versed in scattering theory, because the whole idea of *a scenario as a temporal sequence of different stages* is alien to the standard S-matrix theory. The general framework of an appropriate theory, named quantum field kinetics (QFK), has been developed in our previous papers [1–3]. It is based on a remarkable similarity; the measurement of one-particle distributions is as inclusive as the measurement of the distribution of the final-state electron in deeply inelastic ep-scattering (DIS). This conceptual

0556-2813/2001/63(4)/044902(16)/$20.00

similarity, however, meets difficulties in its practical implementation.

(i) The inclusive DIS directly measures only the electromagnetic fluctuations in the proton. The problem is posed according to the *S*-matrix scattering theory improved by means of the renormalization group. The concept of running coupling emerges precisely in this context. The operator product expansion (OPE) allows one to hide all the unknown information about the proton (commonly associated with large distances) into the local operators of various dimensions. Introduced in this way, structure functions (given explicitly in terms of their momenta) are applicable to DIS process, and only to DIS.

(ii) It is impossible to derive structure functions of pp scattering (not to say about AA) using the OPE method, because in this case the composite QCD operators become essentially nonlocal.[1]

(iii) Historically, the escape was provided by the parton model (the factorization hypothesis), which was successfully applied to various processes that accompany pp scattering (like Drell-Yan pairs production), where the factorization scale can be kept under the data control, since the number of particles in the final state is relatively small. In *AA* collisions, the control over the factorization scale is practically impossible because of the enormously high multiplicity. Furthermore, the phase space of the final state is densely populated and the picture of an independent emission (unitary cut in the Feynman diagram) employed for the derivation of DIS structure functions does not hold any more.

As has been already mentioned, in the *AA* case we need a developing in time scenario which cannot be accessed from the *S*-matrix scattering theory, while the DIS structure functions are constructed within *S*-matrix approach. The QFK method has been developed in order to resolve these problems by addressing not only the nuclear collision as a transient process, but the ep-DIS also. Our major hope was to derive QCD evolution equations and to introduce the structure functions using the framework of an independently initiated theory of nuclear collisions. The first step along this guideline was an immediate success [1,2]. It was demonstrated, that the evolution equations indeed describe a transient process that ends as an electromagnetic fluctuations inclusively probed by the electron.

To give a flavor of how the method works practically, let us start with a qualitative description of the inclusive *e*-*p* DIS measurement (for now, at the tree level without discussion of the effects of interference). In this experiment, the only observable is the number of electrons with a given momentum in the final state. Something *in the past* has to create the electromagnetic field that deflects the electron. *Before* this field is created, the electromagnetic current, which is the source of this field, has to be formed. Since the momentum transfer in the process is very high, the current has to be sufficiently localized. This localization requires, in its turn, that the electric charges which carry this current must be dynamically decoupled from the bulk of the proton *before* the scattering field is created (to prevent a recoil to the other parts of the proton which could spread the emission domain). Such a dynamical decoupling of a quark requires a proper rearrangement of the gluonic component of the proton with the creation of short-wave components of a gluon field. By causality, corresponding gluonic fluctuation must happen *before* the current has decoupled, etc. Thus we arrive at the picture of the sequential-dynamical fluctuations which create an electromagnetic field probed by the electron. The lifetimes of these fluctuations can be very short. Nevertheless, they all *coherently* add up to form a stable proton, unless the interaction of measurement breaks the proper balance of phases. This intervention freezes some instantaneous picture of the fluctuations, but with wrong ''initial velocities'' which results in a new wave function, and collapse of the old one. This qualitative picture has been described many times and with many variations in the literature, starting with the pioneering lecture by Gribov [4], and including a recent textbook [5]; however, the sequential temporal ordering of the fluctuations has never been a key issue. We derive this ordering as a consequence of the Heisenberg equations of motion for the observables. The practical scheme of calculation that emerges in this way appears to be *a special form of quantum mechanics which describes an inclusive measurement as a transient process*. Translated into mathematical language in momentum space, this picture leads to the most general form of the evolution equations, which may then be reduced (under different assumptions) to the known DGLAP, BFKL, or GRV equations [2]. The evolution equations were derived immediately in the closed form of the integral equations avoiding a selective summation of the perturbation series. The standard inclusive *e*-*p* DIS indeed delivers information about quantum fluctuations which may *dynamically* develop in the proton *before* it is destroyed by a hard electromagnetic probe. One of the most amazing features that has been discovered in the framework of QFK is that the QCD evolution equations are an *intrinsic property of the inclusive measurement process*, and they are not limited by the factorization condition.

In paper [I], we studied the problem of loop corrections in the QFK evolution equations. First, we found that they do not corrupt the causal picture of the measurement described above, at the tree level. Second, they indeed provide a *scale* to the entire process. This scale is connected with collective interactions in the final state, which dynamically generate masses for the final states of emission, thus regulating the abundant collinear divergences of the null-plane dynamics. We required that the real parts of all radiative corrections (phase shifts) must vanish along the direction of the initial-state propagation of the colliding objects. Thus, we explicitly accounted for the integrity of the nuclear wave function *before* the collision. This special choice of the renormalization point, is *natural* for ultrarelativistic nuclear collisions, since it allows one to treat nuclei as finite-size quantum objects and incorporate their Lorentz contraction as a classical boundary condition imposed on the space-time evolution of

[1]A similar situation takes place in the ep process if a jet is chosen as the inclusive observable. Then the dynamics of the process is sensitive to the QCD content of the electron (see Sec. IV in paper [I]).

quantum fields after the nuclear coherence is broken.

The net yield of our previous study in paper [I] can be summarized as follows: The interaction between the two ultrarelativistic nuclei switches on almost instantaneously. This interaction explores all possible quantum fluctuations which could have developed by the moment of the collision and freezes (as the final states) only the fluctuations compatible with the measured observable. These snapshots cannot have an arbitrary structure, since the emerging configurations must be consistent with all the interactions which are effective on the time scale of the emission process. In other words, the modes of the radiation field which are excited in the course of the nuclear collision should be the collective excitations of the dense quark-gluon system. This conclusion is the result of an intensive search of the *scale* inherent in the process of a heavy-ion collision. We proved that the scale is determined only by the physical properties of the final state.

Our previous study clearly indicates that a theory that describes both phenomena (i.e., *ep*-DIS and *AA* collisions) from a common point of view can be built on two premises: causality, and the condition of emission. The latter is also known as the principle of cluster decomposition, which must hold in any reasonable field theory. What the ''resolved clusters'' are is a very delicate question. These states should be defined with an explicit reference as to how they are detected. Conventional detectors deal with hadrons and allow one to hypothesize about jets. QGP turns out to be a kind of collective detector for quarks and gluons. In DIS experiment, the new wave function is measured *inclusively* which itself could be the source of the entropy production if the final state had some properties of the collective system. This collective system would then be a detector. It turns out that a dramatic difference in population of the final states is the sole fact that makes DIS and heavy ion collisions so different.

At this moment, the natural line of development of these ideas brought us to the point when any further progress is impossible without explicit knowledge of the *normal modes of the expanding dense quark-gluon system*. There are several conceptual and technical problems of different caliber where this knowledge is crucial.

(1) Most of the entropy is expected to be produced during the initial breakup of the nuclei coherence. Computing the entropy amounts to the digital counting of the exited degrees of freedom. Therefore, the states themselves must be precisely defined. From this point of view, the role of dynamical masses of the normal modes is decisive. They provide an infrared boundary for the space of final states thus making the possible number of the excited states (the entropy) finite.

(2) Only after the infrared boundary for the QCD states is found can we hope to have a self-consistent perturbation theory. This was an original idea which motivated the search of the QGP [6]. A perturbative description at the kinetic stage of the scenario cannot rely on massless QCD, which has no intrinsic scale. It can be effective only if it is based on the interaction of the partons plasmons, i.e., quarks and gluons with the effective masses. Built on these premises, the scenario for the ultrarelativistic nuclear collision promises to be more perturbative than the standard pQCD.

(3) Standard perturbative calculations with massless gauge fields always lead to collinear singularities that require a parameter of resolution for their practical removal. When this singularity is due to the emission into the final state, then this parameter is usually found as a property of the detector. As long as we consider the QGP itself as a detector, no external parameters of this kind can be in the theory. Collinear problems also appear in loop corrections, even in their imaginary parts. Therefore, they are also due to real processes. (In physical gauges, the collinear singularities in the loop corrections can also be connected with the spurious poles of the gluon propagator, which is a consequence of an incomplete gauge fixing and imperfect separation between the longitudinal fields and the fields of radiation.)

As has been discussed in paper [I], the collinear problems in perturbative QCD show up only because the unphysical states are added to the list of the possible final states of the radiation processes. These states can be eliminated from the theory by accounting for the real interactions in the final state which provide effective masses for all radiated fields. In the null-plane dynamics, this appeared to be impossible, since any type of kinetics that may lead to the formation of the effective mass is frozen on the light cone. In order to have meaningful evolution equations for heavy ion collisions we must account for the dynamical masses of the realistic final states in dense expanding matter; the QCD evolution has to provide a kind of self-screening of the collinear singularities. The way the effective mass was *estimated* in paper [I] was crude, and it was our original goal to improve the calculations using the full framework of *wedge dynamics*.

II. OUTLINE OF IDEAS, CALCULATIONS, MAIN RESULTS, AND CONCLUSION

In this section, we review the work presented in this and two subsequent papers [7,8] (hereafter quoted as papers [III] and [IV]). Our approach is strongly motivated by an idea, that collisions of ultrarelativistic heavy ions is the cleanest laboratory where one can study the dynamics of strong interactions. We consider an adequate choice of the interacting quantum states at different stages of the scenario as the issue of first priority. The focus of our previous study was on the QCD evolution equations in the environment of the heavy ion collision. Now, we concentrate on the possible properties of the state that emerges immediately *after* the coherence of the nuclei is broken by the first hard interaction. As in paper [I], we view the dynamics of the early stage as a single quantum process and concentrate on the study of quantum fluctuations subjected to the condition of a simple inclusive measurement (currently, on the inclusive one-particle distributions). We endeavor to take full advantage of approaching the problem from first principles.

A. Heuristic arguments

Collision of ultrarelativistic heavy ions is such a unique physical phenomenon, that it is difficult to find its complete analog throughout everything that has been studied in physics previously. However, we can point to several examples which share some common distinctive patterns with the process under investigation. We begin with these examples in order to help the reader understand the ideas of our new synthesis.

(1) Let an electron-positron pair be created by two photons. If the energy of the collision is large, then the electron and positron are created in the states of freely propagating particles and the cross section of this process accurately agrees with the tree-level perturbative calculation. However, if the energy of the collision is near the threshold of the process, then the relative velocity of the electron and positron is small, and they are likely to form positronium. It would be incredibly difficult to compute this case using scattering theory. Indeed, one has to account for the multiple emission of soft photons which gradually builds up the Coulomb field between the electron and positron and binds them into the positronium. However, the problem is easily solved if we realize that the bound state *is* the final state for the process. We can still use low-order perturbation theory to study the transition between the two photons and the bound state of a pair [9].

(2) Let an excited atom be in a cavity with ideally conducting walls. The system is characterized by three parameters: the size L of the cavity, the wavelength $\lambda \ll L$ of the emission, and the lifetime $\Delta t = 1/\Gamma$ of the excited state. The questions are, in what case will the emitted photon bounce between the cavity walls, and when will the emission field be one of the normal cavity modes? The answer is very simple. If $c\Delta t \ll L$, the photon will behave like a bouncing ball. When the line of emission is very narrow, $c\Delta t \gg L$, the cavity mode will be excited. It is perfectly clear that in the first case, the transition current that emits the photon is localized in the atom. In the second case it is not. By the time of emission, the currents in the conducting walls have to rearrange charges in such a way that the emission field immediately satisfies the proper boundary conditions. We thus have a collective transition in an extended system.

From a practical point of view, these two different problems are united by the method of obtaining their solutions. A part of the interaction (Coulomb interaction in the first case, and the interaction of radiation with the cavity walls in the second case) is attributed to the new ''bare'' Hamiltonian which is diagonalized by the wave functions of the final state modes. The less significant interactions can be accounted for by means of perturbation theory. For us, the most important message is that it is possible to avoid a difficult study of the transient process that physically creates these modes.

(3) Let an experimental device consists of quantum detectors that register photons emitted by a pulse source. Each pulse initiates an ''event.'' Let a sheet of glass is placed somewhere between the source and detectors. If this glass were installed permanently in a fixed position, then the method to account for its presence would be trivial. One must expand the field of the initial light pulse over the system of modes (Fresnel triplets of incident, reflected, and refracted waves) that satisfy the continuity conditions on the glass boundaries. The quantum theory would then treat these triplets as the photons, etc. When the position of the glass sheet is unknown, e.g., it changes in the time periods between the pulses, then such a universal decomposition becomes impossible. Nevertheless, in each particular event there exists an important element of *classical boundary conditions*. Using special tricks (e.g., by measurement of the times of arrival of the precursors), one may determine the glass position and thus to learn how the translational symmetry of free space was actually broken and what are the photons of a particular event. Though the whole set up of this example is artificial, it illustrates the major idea. The quantum theory of an individual event can be fully recovered, even if macroscopic parameters of the theory are not known until the event is completely recorded. Indeed, the prepared at a large distance light pulse can be expanded over any of the systems of the Fresnel triplets (corresponding to different positions of the glass sheet). Only after analysis of the data can it be learned, which of these decompositions is meaningful. One can fill the space between the detectors with gas and account for the interaction between the light and gas (or even for a nonlinear interaction of photons in the gaseous medium) by perturbation theory. Being the noninteracting waves in ''free space,'' the Fresnel triplets will serve as the zeroth-order approximation of a quantum theory. One may also decide not to begin with the triple waves. Then the glass must be treated as an active element. The same triplets will be recovered in the course of a real transient process on the glass surface. The translational symmetry will be broken dynamically.

A very similar picture develops during the heavy ion collision. The normal modes of the final state are formed in the course of real interactions. The mechanism responsible for effective mass of the plasmons is illustrated by the first two examples. The third example points us to an optimal choice of the zeroth-order approximation. Exactly in the same way as the reflected and refracted waves of the Fresnel triplet cannot physically appear before the light front reaches the glass surface, nothing can happen with the nuclei before they overlap geometrically. Only at this instance the interaction determines the collision coordinates in (tz) plane. The symmetry gets uniquely broken, and the normal modes of the propagating colored fields after the interaction exist only inside the future region of the interaction domain. If the coupling is small then we may disregard later interactions. However, the system of the final-state free fields will have a broken translational symmetry, which will be remembered by the normal modes that obey certain macroscopic boundary conditions.

Referring to the above examples, one should keep in mind the source of the major difference between the QED and QCD phenomena. The local gauge symmetry of QED can be extended to a global gauge symmetry which generates the conserved global quantum number (electrical charge) which can be sensed at a distance. The proper field of an electric charge is the main obstacle for the definition of its size. On the other hand, the radiation field of QED appears as a result of the changes in the extended proper fields of accelerated charges, and one can physically create such an object as a front of electromagnetic wave. In QCD, the local gauge invariance of the color group does not correspond to any conserved charge. Hence, we can easily determine the size of the colorless nucleus, but we cannot create a front of color radiation in the gauge-invariant vacuum. These two properties of QCD both work for us. They allow one to use the Lorenz contraction to localize the initial moment of the collision and thus, to impose the classical boundary conditions on the propagating color fields at the later times. The existence of the collective propagating quark and gluon modes at these times is the conjecture that has to be verified by the study of heavy ion collisions.

The finite size of the colliding nuclei, and a strong localization of the initial interaction as its consequence, is a sufficient input for the theory that describes the earliest stage of the collision. The formalism of quantum field theory appears to be a powerful tool that allows one to derive many properties of the quark-gluon system after the collision.

B. Phenomenological input

(i) We consider the rapidity plateau seen event-by-event in nuclear collisions at very large energy as a confirmed by the data indication that the quantum transient process has no scale corresponding to the finite resolution in the t and z directions. By a common wisdom, the absence of this scale must cause the boost-invariant expansion.[2]

(ii) All existing data indicate that, regardless of the nature of the colliding objects, a certain number of particles with large transverse momentum are found in the final state. At high p_t, the cross section reasonably well follows the Rutherford formula. We rely on the universality of Rutherford scattering as an indication that there is no scale parameter of resolution in the transverse (xy) plane that characterize this process. We assume that in nuclear collisions, these hard states created at the very early instance of the collision can be described by the one-particle distribution measured on event-by-event basis.

C. Ideas

(i) The finite size of colliding nuclei plays a crucial role in our approach since it allows for a realistic measurement of the Lorentz contraction thus precisely fixing the time and the coordinate of the collision point. In the laboratory frame, both nuclei are Lorentz contracted to a longitudinal size $R_0/\gamma \sim 0.1$ fm, while the scale relevant for the hadron structure is ~ 0.3 fm. Therefore, in the center-of-mass frame, both nuclei are passing through a ''pin-hole,'' and the detailed information about the microscopic nuclear structure is not essential. A precise measurement of the *velocity*, i.e., the coordinates at two close time moments, is impossible [10]. Hence, a celebrated rapidity plateau in every single collision of two ultrarelativistic ions is a direct consequence of this type of measurement. We accept the fact of the rapidity plateau as a classical boundary condition for the quantum sector of the theory.

(ii) There is no doubt that the entire collision process must develop inside the future light cone of the collision domain. Only there can the dynamics of the propagating color fields become a physical reality. In other words, the resolution of colored degrees of freedom is a consequence of the precise measurement of the coordinate by means of strong interactions.

(iii) The true scale of the entire quantum process coincides with its infrared boundary, which is build dynamically in the course of this process. Namely, the hard partons, which are produced as localized and countable particles at the earliest time of the process, define masses for the soft field states formed at the later times thus bringing the transient process to its saturation.

D. Strategy and theoretical foundations

Addressing the problem of normal modes of the expanding quark-gluon system, we proceed in two major steps. First, we study the classical and quantum properties of the normal modes subjected to the boundary condition of a localized interaction that follows from the relativistic causality (being the free fields in all other respects). Then, we use these modes as a basis for the perturbation theory and compute the effective mass of the quark propagating through the background distribution of hard partons.

(i) We begin in Sec. III A with the qualitative study of free fields, fully incorporating the properties of the geometric background of the expanding matter. Taking the simplest plane-wave of the scalar field as an example, and studying the probability to detect this wave on the space-like hypersurface of constant proper time τ, we conclude that it is capable of passing through the center $t=z=0$. The only price paid for this feature is the full delocalization of the state along the hyperplanes $\tau^2=t^2-z^2=0$. The state is completely delocalized at $\tau p_t \ll 1$, and it is sharply localized in the rapidity direction at $\tau p_t \gg 1$. In this way, we approach an idea of *wedge dynamics*, which employs the proper time τ as the natural direction of the evolution. In Sec. III B, we consider a wave packet and demonstrate that the process of localization at finite time τ is physical; it is accompanied by the gradual redistribution of the charge density and the current of this charge. From this observation, we may anticipate a special role of the magneto-static interactions at the earliest times, when the process of the charge density rearrangement is extremely rapid. Further calculations of paper [IV] give even more evidence that the quantum process of delocalization predicted by wedge dynamics is a material process.

(ii) As a first step towards practical calculations, the fields are described classically and quantized in the scope of wedge dynamics. In Sec. IV of this paper, we accomplish this procedure for the fermion fields. In Sec. V, we derive the expressions for various quantum correlators, which are used for the perturbative calculation of the fermion self-energy in paper [IV]. An important observation made at this point is that the material parts of the field correlators immediately have the form of Wigner distributions. This is a unique property of wedge dynamics which relies on the highly localized states as its one-particle basis.

[2]This plateau in the distribution of the final-state hadrons is clearly seen even in the inclusive jet distribution in ep-DIS data, but only statistically.

(iii) The third one, technically the most complicated paper [III] of this cycle, is dedicated to the vector gauge field in wedge dynamics. Several conceptual and technical problems are addressed there. First of all, the states of the free radiation field are studied classically. Also at the classical level, we compute the retarded Green function of the vector gauge field and explicitly separate the longitudinal (i.e., governed by Gauss law) field and the field of radiation. It is found that if the physical charge density $\rho=\tau j_\tau$ vanishes at the starting point $\tau=0$, then Gauss law of the wedge dynamics, being in fact a constraint, becomes an immediate consequence of the equations of motion. Therefore, Gauss law can be explicitly used to eliminate the unphysical degrees of freedom of the gauge field, and the gauge $A^\tau=0$ can be fixed completely. Using this result, we were able to quantize the gluon field according to the standard procedure of canonical quantization.

The requirement $\rho(\tau=0)=0$ would not be physical in QED, where the long-range proper fields of electric charges would limit the possible localization of the first interaction, and the applicability of the wedge dynamics. On the other hand, in the wedge dynamics of colorless objects built from the colored fields, which are ''stretched'' at $\tau\to 0$ along a very wide rapidity interval, this can be a true initial condition. The later creation of the localized color charges can indeed be initiated by the color currents in the color-neutral (at $\tau=0$) system.

(iv) A distinctive property of the longitudinal gauge fields in wedge dynamics is that they do not look like usual static fields. The Hamiltonian time τ does not coincide with a usual time of some particular inertial Lorentz frame. This is a proper time for all observers that move with all possible rapidities starting from the point $t=z=0$. The system, which is static with respect to this time experiences a permanent expansion, and its Gauss fields have magnetic components. As a consequence, the longitudinal part of the gauge field propagator acquires a contact term,

$$D_{\eta\eta}^{[contact]}=-\frac{\tau_1^2-\tau_2^2}{2}\,\delta(\eta)\,\delta(\vec{r}_t).$$

The component $D_{\eta\eta}$ establishes a connection between the A_η component of the potential and the j_η component of the current. In its turn, A_η is responsible for the η component $E_\eta=\partial_\tau A_\eta$ of the electric field and the x and y components, $B_x=\partial_y A_\eta$, $B_y=-\partial_x A_\eta$ of the magnetic field. The electrical field in the longitudinal η-direction is not capable of producing the scattering with transverse momentum transfer. However, this transfer can be provided by the magnetic forces; the two currents j_η can interact via the magnetic field $\vec{B}_t=(B_x,B_y)$. The origin of these currents is intrinsically connected with the geometry of states in the wedge form of dynamics. The existence of these currents indicates that the delocalization of the nuclear wave packet is more than a formal decomposition in terms of fancy modes. This is a physical phenomenon which plays an important role in the formation of the IR scale of the entire process.

E. Calculation of the effective mass

The first calculation that incorporates both the ideas and technical part of the wedge dynamics is attempted in paper [IV]. We compute the effective ''transverse mass'' $\mu(\tau,p_t)$ of the soft (i.e., $\tau p_t<1$) quark mode propagating through the expanding background of hard (i.e., $\tau k_t>1$) partons.

(i) In order to find the normal modes of the quark field in the expanding quark-gluon system, we solve the Dirac equation with radiative corrections, which can be derived as a projection of the Schwinger-Dyson equation for the retarded quark propagator onto the one-particle initial state. This equation can be converted into a dispersion equation that includes the retarded self-energy and connects the effective transverse mass $\mu(\tau,p_t)$ of the soft mode with its transverse momentum p_t. This equation depends on the current proper time τ as a parameter,

$$\mu(\tau,p_t)=p_t+\int_0^\tau d\tau_2\sqrt{\tau\tau_2}e^{i\mu(\tau,p_t)(\tau-\tau_2)}\Sigma_{ret}(\tau,\tau_2;p_t),$$

and we assumed that $d\ln\mu/d\ln\tau\ll 1$, in deriving it. The solution with this property is indeed found.

(ii) The material part of the self-energy can be divided into several parts corresponding to different processes of the forward quark scattering on the hard partons of the expanding surroundings. First, the quark may scatter on a real (transverse) gluon. The second process is quark-quark scattering, which can be conveniently divided into two subprocesses. In one of them, the interaction is mediated by the radiation part of the gluon field, in the other, the mediator is the longitudinal part. The latter can be split further into the contact and nonlocal parts. Our strategy was to find the leading terms of the self-energy which are singular at $\tau-\tau_2=0$, and thus can significantly contribute to the effective quark mass within a short time. Indeed, since we are looking for the time-dependent $\mu(\tau,p_t)$, this mass has to be formed during a sufficiently short time interval. Accordingly, we have chosen the dimensionless parameter $\xi=(\tau-\tau_2)/\sqrt{\tau\tau_2}$ as a small parameter.

(iii) The distribution of hard quarks and gluons that may provide an effective mass to a soft quark mode with transverse momentum p_t at the time $\tau\leq 1/p_t$ are taken in agreement with the qualitative arguments of Secs. II B and II C,

$$n_f(q_t,\theta)\approx\frac{\mathcal{N}_f}{\pi R_\perp^2}\,\frac{\theta(q_t-p_*)}{q_t^2},$$

$$n_g(k_t,\alpha)\approx\frac{\mathcal{N}_g}{\pi R_\perp^2}\,\frac{\theta(k_t-p_*)}{k_t^2}.$$

They are not related to any dynamical scale and the normalization factors $\mathcal{N}_g$ and $\mathcal{N}_f$ are the only (apart from the coupling α_s) parameters of the theory. The impact cross section

$\pi R_\perp^2$ and the full width $2Y$ of the rapidity plateau are defined by the geometry of a particular collision and the c.m.s. energy, respectively. These are irrelevant for the local screening parameters we are interested in.

(iv) Analysis of the terms that include radiation fields clearly reveals two trends. On the one hand, the integration over the transverse momenta of hard quarks and gluons is capable of creating a singularity when the rapidities corresponding to the two lines in the loop coincide. On the other hand, the interval of rapidities where the collinear geometry is possible is extremely narrow due to the light-cone boundaries (causality) of the forward scattering process. The second factor always wins, and the contribution of the collinear domain is always small. We also found that the observed intermediate collinear enhancement of the forward scattering amplitude is, as a matter of fact, fictitious. It is entirely formed by the integration over the infinitely large transverse momenta which are physically absent in the distribution of the hard partons. (Formally, the infinite transverse momentum is needed to provide a precise tuning of two states with given rapidities to each other.) These collinear singularities are integrable, and they do not lead to a disaster of collinear divergence.

Our way to pick out the leading contributions from the space-time domains, where the phases of the interacting fields are stationary, is a generalization of the known method of isolating the leading terms using the pinch-poles in the plane of complex energy. The wedge dynamics does not allow for a standard momentum representation, since its geometric background is not homogeneous in t and z directions. Nevertheless, the patches of phase space, where the phases of certain field fragments are stationary and effectively overlap, do now the same job as the pinch-poles, and yield the same answers when the homogeneity required for the momentum representation is restored. This way to tackle the problem is genuinely more general, because it addresses the space-time picture of the interacting fields. The role of pinch-poles is taken over by the geometrical overlap of the field patterns with the same rapidity. This observation can serve as a footing for the future development of an effective technique for perturbative calculations in wedge dynamics.

(v) The effect of the nonlocal components of the longitudinal part of the gluon propagator that mediates the quark-quark scattering, was shown to be small also. This interaction cannot lead to the collinear enhancement. However, its yield could be not very small, because the interaction has long range. It occurs, that the nonlocal electro- and magneto-static interactions of charges just almost compensate each other.

(vi) The only term in the quark self-energy which is singular at small time differences is due to the above mentioned contact term in the $D^{\eta\eta}$ component of the gluon propagator. This is the leading contribution to the dispersion equation provided by the magneto-static interaction of the longitudinal currents. Studied in the first approximation, the solution of the dispersion equation indicates that in compliance with the original idea, the effective mass $\mu(\tau,p_t)$ gradually increases with time reaching its maximum when $\tau p_t \approx 1$. This is the major practical result of this study. Evolution of the fields at the later times must be approached with another set of normal modes that, from the very beginning, account for the screening effects developed at the previous stage.

F. Conclusion and perspectives

In a series of papers reviewed in this section, we demonstrated that the field theory is indeed able to describe a scenario. By scenario we mean a continuous smoothly developing temporal sequence of one stage into another. These stages are different only in the respect that each of them is characterized by its individual *optimal set* of normal modes. In contrast with paper [I], we do not focus on the stage of QCD evolution, since we have no clear image of the objects that initiate destruction of nuclear coherence. Instead, we try to understand, what can be the immediate products of this destruction. This is an example of the continuity that stands behind the idea of the scenario. The next stage will be the kinetics of the partons-plasmons, and we anticipate that it will impose new restrictions, which will improve our current results. [Quantum mechanics works remarkably in both directions: any information about the properties of the final state imposes limitations on the possible line of the evolution (including the initial data) at the earlier times exactly in the same way as the known initial data imposes restrictions on the possible final states.] By the same token, we must look for a connection between the objects resolved in the first interaction of two nuclei and the known properties of hadrons and the QCD vacuum. Unfortunately, this appealing opportunity is still distant.

First principles appeared to be a powerful tool for achieving our goals. With minimal theoretical input and with the reference to the simplest data, they allow one to build a self-consistent picture of the initial stage of the collision. Colliding the nuclei, we probably create the theoretically simplest situation for understanding the nature of the process. In the course of this study, we relied only on the fact of boost invariance of the process and an assumption that the field states with large transverse momentum, even at very early times, may be associated with the localized particles and thus can be described by the distribution with respect to their rapidity and transverse momentum. Our strategy of looking for the leading contributions and all our approximations in calculating the material part of the quark self-energy are based on this assumption. If it appears incorrect, then it is most likely that the quark-gluon matter created in the collision of two nuclei never, and in no approximation, can be considered as a system of nearly free and weakly interacting field states.

Our decision to begin the exploration of potentialities of the wedge dynamics with the computation of quark self-energy is motivated only by technical reasons. The gluon propagator of wedge dynamics is a very complicated function, and we preferred to start with the computation of the fermion loop which has only one gluon correlator in it. We hope that the discovery of, in the course of our study, an enormous simplifications (with respect to what we had to start with) will allow us to address the more important problem of the gluon self-energy in a reasonably economic way.

III. FIELD STATES IN THE PROPER-TIME DYNAMICS

The dynamical masses of normal modes at finite density are found from the dispersion equation that includes the corresponding self-energy, i.e., the amplitude of the forward scattering of the mode on the particles that populate the phase space. In paper [I], we found that it is impossible to adequately describe this basic process of forward scattering in the null-plane dynamics. The problem arises due to the singular behavior of the field pattern which is defined as the static field with respect to the Hamiltonian time x^+. This singular behavior alone shows that the choice of the dynamics and the proper definition of the field states is a highly nontrivial and important issue. Besides, if we tried to describe quantum fluctuations in the second nucleus in the same fashion, then it would require a second Hamiltonian time x^-, which is not acceptable. Thus, if we wish to view the collision of two nuclei as a unique quantum process, then it is imperative to find a way to describe quarks and gluons of both nuclei, as well as the products of their interaction, using *the same Hamiltonian dynamics*. An appropriate choice for the gluons is always difficult because the gauge is a global object (as are the Hamiltonian dynamics) and both nuclei should be described using the same gauge condition.

Quantum field theory has a strict definition of *dynamics*. This notion was introduced by Dirac [11] at the end of the 1940s in connection with his attempt to build a quantum theory of the gravitational field. Every (Hamiltonian) dynamics includes its specific definition of the quantum mechanical observables on the (arbitrary) spacelike surfaces, as well as the means to describe the evolution of the observables from the ‘‘earlier’’ spacelike surface to the ‘‘later’’ one.

The primary choice of the degrees of freedom is effective if, even without any interaction, the dynamics of the normal modes adequately reflects the main physical features of the phenomenon. The intuitive physical arguments clearly indicate that the normal modes of the fields participating in the collision of two nuclei should be compatible with their Lorentz contraction. Unlike the incoming plane waves of the standard scattering theory, the nuclei have a well-defined shape and the space-time domain of their intersection is also well defined. Hence, the geometric properties of the expected normal modes follows, in fact, from the uncertainty principle. Indeed, we may view the first touch of the nuclei as the first of the two measurements which are necessary to determine the velocity. Since a precise measurement of the nuclei coordinate at an exactly determined moment appears to be an inelastic process that completely destroys the nuclei, the spectrum of the longitudinal velocities of the final-state components must become extremely wide [10]. These components may also be different by their transverse momenta. With respect to the measurement of the longitudinal velocity, the latter plays a role of an ‘‘adjoint mass.’’ The velocity of a heavier object can be measured with a larger accuracy. Therefore, the separation of the ‘‘heavy’’ final state fragments by their longitudinal velocities requires less time and can be verified earlier than for the ‘‘light’’ ones.[3]

The same conclusion can be reached formally: Of the ten symmetries of the Poincaré group, only rotation around the collision z axis, boost along it, and the translations in the transverse x and y directions survive. The idea of the collision of two plane sheets immediately leads us to the *wedge form*; the states of quark and gluon fields before and after the collision must be confined within the past and future light cones (wedges) with the xy-collision plane as the edge. Therefore, it is profitable to choose, in advance, the set of normal modes which have the symmetry of the localized initial interaction and carry quantum numbers adequate to this symmetry. These quantum numbers are the transverse components of momentum and the rapidity of the particle (which replaces the component p^z of its momentum). In this *ad hoc* approach, all the spectral components of the nuclear wave functions ought to collapse in the two-dimensional plane of the interaction, even if all the confining interactions of the quarks and gluons in the hadrons and the coherence of the hadronic wave functions are neglected.

In the wedge form of dynamics, the states of free quark and gluon fields are defined (normalized) on the spacelike hypersurfaces of the constant proper time τ, $\tau^2=t^2-z^2$. The main idea of this approach is to study the dynamical evolution of the interacting fields along the Hamiltonian time τ. The gauge of the gluon field is fixed by the condition $A^\tau=0$. This simple idea solves several problems. On the one hand, it becomes possible to treat the two different light-front dynamics which describe each nucleus of the initial state separately, as two limits of this single dynamics. On the other hand, after the collision, this gauge simulates a local (in rapidity) temporal-axial gauge. This feature provides a smooth transition to the boost-invariant regime of the created matter expansion (as a first approximation). Particularly, addressing the problem of screening, we will be able to compute the plasmon mass in a uniform fashion, considering each rapidity interval separately.

As it was explained in the first two sections, the feature of the states to collapse at the interaction vertex is crucial for understanding the dynamics of a high-energy nuclear collision. A simple optical prototype of the wedge dynamics is the *camera obscura* (a dark chamber with the pin-hole in the wall). Amongst the many possible *a priori* ways to decompose the incoming light, the camera selects only one. Only the spherical harmonics centered at the pin-hole can penetrate inside the camera. The spherical waves reveal their angular dependence at some distance from the center and

[3]The boost invariance with the fixed center means the absence of a corresponding scale and vice versa. Any relativistic equations, regardless of their physical content, will yield a self-similar solution. For example, the relativistic hydrodynamic equations lead to a known Bjorken solution with the rapidity plateau. In its turn, the Bjorken solution can be obtained as a limit of the Landau solution with an infinite Lorentz contraction of the colliding objects. We favor the arguments that are closer to quantum mechanics and allow for the further connection with the properties of the quantum states.

build up the image on the opposite wall. Here, we suggest to view the collision of two nuclei as a kind of diffraction of the initial wave functions through the ''pin-hole'' $t=0$, $z=0$ in tz plane.

Using the proper time τ as the natural direction of the evolution of the nuclear matter after the collisions has far reaching consequences. The surfaces of constant τ are curved, and the oriented objects like spinors and vectors have to be defined with due respect to this curvature. We have to incorporate the tetrad formalism in order to differentiate them covariantly. The properties of local invariance are modified also, since the different directions in the tangent plane become not equivalent. The physical content of the theory also undergoes an important change. The system of observers that are used to *normalize* the quantum states of wedge dynamics is different from the observers of any particular inertial Lorentz frame.

A. One-particle wave functions in wedge dynamics

In order to study the main kinematic properties of the states of the wedge dynamics, it is enough to consider the one-particle wave functions of the scalar field. Let us take the wave function $\psi_{\theta,p_\perp}(x)$ of the simplest form,

$$\psi_{\theta,p_\perp}(x)=\frac{1}{4\pi^{3/2}}e^{-ip^0t+ip^zz+i\vec{p}_\perp\vec{r}_\perp}$$
$$\equiv\begin{cases}4^{-1}\pi^{-3/2}e^{-im_\perp\tau\cosh(\eta-\theta)}e^{i\vec{p}_\perp\vec{r}_\perp}, & \tau^2>0,\\ 4^{-1}\pi^{-3/2}e^{-im_\perp\tau\sinh(\eta-\theta)}e^{i\vec{p}_\perp\vec{r}_\perp}, & \tau^2<0,\end{cases} \tag{3.1}$$

where $p^0=m_\perp\cosh\theta$, $p^z=m_\perp\sinh\theta$ (θ being the rapidity of the particle), and, as usual, $m_\perp^2=p_\perp^2+m^2$. The above form implies that τ is positive in the future of the wedge vertex and negative in its past. Even though this wave function is obviously a plane wave which occupies the whole space, it carries the quantum number θ (rapidity of the particle) instead of the momentum p_z. A peculiar property of this wave function is that it may be normalized in two different ways, either on the hypersurface where $t=$const,

$$\int_{t=\mathrm{const}}\psi^*_{\theta',p'_\perp}(x)\,i\frac{\overleftrightarrow{\partial}}{\partial t}\psi_{\theta,p_\perp}(x)\,dz\,d^2\vec{r}_\perp$$
$$=\delta(\theta-\theta')\,\delta(\vec{p}_\perp-\vec{p}'_\perp), \tag{3.2}$$

or, equivalently, on the hypersurfaces $\tau=$const in the future and the past light wedges of the collision plane, where $\tau^2>0$,

$$\int_{\tau=\mathrm{const}}\psi^*_{\theta',p'_\perp}(x)\,i\frac{\overleftrightarrow{\partial}}{\partial\tau}\psi_{\theta,p_\perp}(x)\,\tau\,d\eta\,d^2\vec{r}_\perp$$
$$=\delta(\theta-\theta')\,\delta(\vec{p}_\perp-\vec{p}'_\perp). \tag{3.3}$$

Being almost identical mathematically, these two equations are very different physically. Equation (3.2) implies that the state is detected by a particular Lorentz observer equipped by a grid of detectors that cover the whole space, while Eq. (3.3) normalizes the measurements performed by an array of the detectors moving with all possible velocities. At any particular time of the Lorentz observer, this array even does not cover the whole space.

The norm of a particle's wave function always corresponds to the conservation of its charge or the probability to find it. Since the norm given by Eq. (3.3) does not depend on τ, the particle with a given rapidity θ (or velocity $v=\tanh\theta=p^z/p^0$), which is ''prepared'' on the surface $\tau=$const in the past light wedge, cannot flow through the light-like wedge boundaries; the particle is predetermined to penetrate in the future light wedge through its vertex. The dynamics of the penetration process can be understood in the following way.

At large $m_\perp|\tau|$, the phase of the wave function $\psi_{\theta,p_\perp}$ is stationary in a very narrow interval around $\eta=\theta$ (outside this interval, the function oscillates with exponentially increasing frequency); the wave function describes a particle with rapidity θ moving along the classical trajectory. However, for $m_\perp|\tau|\ll1$, the phase is almost constant along the surface $\tau=$const. The smaller τ is, the more uniformly the domain of stationary phase is stretched out along the light cone. A single particle with the wave function $\psi_{\theta,p_\perp}$ begins its life as the wave with the given rapidity θ at large negative τ. Later, it becomes spread out over the boundary of the past light wedge as $\tau\to-0$. Still being spread out, it appears on the boundary of the future light wedge. Eventually, it again becomes a wave with rapidity θ at large positive τ. The size and location of the interval where the phase of the wave function is stationary plays a central role in all subsequent discussions, since it is equivalent to the localization of a particle. Indeed, the overlapping of the domains of stationary phases in space and time provides the most effective interaction of the fields.

The size $\Delta\eta$ of the η interval around the particle rapidity θ, where the wave function is stationary, is easily evaluated. Extracting the trivial factor $e^{-im_\perp\tau}$ which defines the evolution of the wave function in the τ direction, we obtain an estimate from the exponential of Eq. (3.1),

$$2m_\perp\tau\sinh^2(\Delta\eta/2)\sim1. \tag{3.4}$$

The two limiting cases are as follows:

$$\Delta\eta\sim\sqrt{\frac{2}{m_\perp}\tau},\quad \text{when}\quad m_\perp\tau\gg1,\quad\text{and}$$
$$\Delta\eta\sim2\ln\frac{2}{m_\perp\tau},\quad\text{when}\quad m_\perp\tau\ll1. \tag{3.5}$$

In the first case, one may boost this interval into the laboratory reference frame and see that the interval of the stationary phase is Lorentz contracted (according to the rapidity θ) in z direction. This estimate confirms what follows from physical intuition; for a heavier quantum object, the velocity can be measured with the higher accuracy. The states of the wedge dynamics appear to be almost ideally suited for the analysis of the processes that are localized at different times τ and intervals of rapidity η, and are characterized by a different transverse momentum transfer. With respect to any particular process, these states are easily divided into slowly varying fields and localized particles. In this way, one may introduce the distribution of particles and study their effect on the dynamics of the fields. As a result, we can calculate the plasmon mass as a local (at some scale) effect which agrees with our understanding of its physical origin.

B. Dynamical properties of states in wedge dynamics

The property of the wave function to concentrate with the time near the classical world line of a particle with the given velocity has important implications. This is a gradual process and it must be accompanied by the redistribution of the charge density and the current of this charge. To see how this happens explicitly, let us consider a particle in a superposition state $|\theta_0\rangle$ of a normalized wave packet,

$$|\theta_0\rangle=\int_{-\infty}^{\infty}d\theta f(\theta-\theta_0)\boldsymbol{a}_\theta^\dagger|0\rangle,$$

$$\langle\theta_0|\theta_0\rangle=\int_{-\infty}^{\infty}d\theta f^*(\theta-\theta_0)f(\theta-\theta_0)=1, \tag{3.6}$$

where $\boldsymbol{a}_\theta^\dagger$ is the Fock creation operator for the one-particle state with the rapidity θ.[4] The explicit form of the weight function in Eq. (3.6) may vary. Solely for convenience, we take the weight function $f(\theta-\theta_0)$ of the form

$$\begin{aligned} f(\theta-\theta_0)&=[K_0(2\xi)]^{-1/2}e^{-\xi\cosh(\theta-\theta_0)}\\ &\approx(4\xi/\pi)^{1/4}e^{\xi}e^{-\xi\cosh(\theta-\theta_0)}, \end{aligned}\tag{3.7}$$

which provides a sharp localization of the wave packet. In the second of these equations, we used an asymptotic approximation of the Kelvin function $K_0(2\xi)$, which is reasonably accurate starting from $\xi\geqslant 1/2$.

The operator of the four-current density for the complex scalar field Ψ is well known to be

$$J_\mu(x)=\Psi^\dagger(x)i\overleftrightarrow{\partial}_\mu\Psi(x), \tag{3.8}$$

and to obey the covariant conservation law,

$$\begin{aligned} \nabla_\mu J^\mu(x)&=(-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}J_\nu(x)]\\ &=\tau^{-1}[\partial_\tau(\tau J_\tau)+\partial_\eta(\tau^{-1}J_\eta)]\\ &=0. \end{aligned}\tag{3.9}$$

(Here, for simplicity, we consider the two-dimensional case and employ the metric $g^{\tau\tau}=1$, $g^{\eta\eta}=-\tau^{-2}$.) The physical components of the current (which are defined in such a way that the integral form of the conservation law is not altered by the curvilinear metric) are $\mathcal{J}_\tau=\tau J_\tau$ and $\mathcal{J}_\eta=\tau^{-1}J_\eta$. Using Eqs. (3.6) and (3.8), we can compute their expectation values of these components in the state $|\theta_0\rangle$

$$\begin{aligned} \langle\theta_0|\mathcal{J}_\tau|\theta_0\rangle=\tau k_t\int_{-\infty}^{\infty}\frac{d\theta_1 d\theta_2}{4\pi}f^*(\theta_1-\theta_0)f(\theta_2-\theta_0)\\ \times[\cosh(\eta-\theta_1)+\cosh(\eta-\theta_2)]\\ \times e^{i\tau k_t[\cosh(\eta-\theta_1)-\cosh(\eta-\theta_2)]}, \end{aligned}\tag{3.10}$$

$$\begin{aligned} \langle\theta_0|\mathcal{J}_\eta|\theta_0\rangle=k_t\int_{-\infty}^{\infty}\frac{d\theta_1 d\theta_2}{4\pi}f^*(\theta_1-\theta_0)f(\theta_2-\theta_0)\\ \times[\sinh(\eta-\theta_1)+\sinh(\eta-\theta_2)]\\ \times e^{i\tau k_t[\cosh(\eta-\theta_1)-\cosh(\eta-\theta_2)]}. \end{aligned}\tag{3.11}$$

The integrals over θ_1 and θ_2 can be estimated by means of the saddle point approximation even for the relatively small values of ξ, e.g., $\xi\sim 1$, because the hyperbolic functions in the exponents vary sufficiently rapidly near the stationary points. These calculations yield the following result:

$$\begin{aligned} \langle\theta_0|\mathcal{J}_\tau|\theta_0\rangle=\frac{2\tau k_t}{(\pi\xi)^{1/2}}\frac{\cosh(\eta-\theta_0)}{1+\dfrac{\tau^2k_t^2}{\xi^2}\cosh[2(\eta-\theta_0)]}\\ \times\frac{e^{-\tau^2k_t^2/\xi\sinh^2(\eta-\theta_0)}}{\sqrt{1+\dfrac{\tau^2k_t^2}{\xi^2}\sinh^2(\eta-\theta_0)}}, \end{aligned}\tag{3.12}$$

$$\begin{aligned} \langle\theta_0|\mathcal{J}_\eta|\theta_0\rangle=\frac{2k_t}{(\pi\xi)^{1/2}}\frac{\sinh(\eta-\theta_0)}{1+\dfrac{\tau^2k_t^2}{\xi^2}\cosh[2(\eta-\theta_0)]}\\ \times\frac{e^{-\tau^2k_t^2/\xi\sinh^2(\eta-\theta_0)}}{\sqrt{1+\dfrac{\tau^2k_t^2}{\xi^2}\sinh^2(\eta-\theta_0)}}. \end{aligned}\tag{3.13}$$

These dependences are plotted in Fig. 1 up to a common scale factor.

From the left figure, it is easy to see that the evolution of the charge density $\mathcal{J}_\tau$ starts from the lowest magnitude and the widest spread at small τ. Then it gradually becomes narrow and builds up a significant amplitude near the classical trajectory with the rapidity θ_0. This process is accompanied by the charge flow $\mathcal{J}_\eta$ (right figure) which has its maximal values at small τ, and then gradually vanishes at later times, when the process of building the classical particle comes to its saturation. The extra factor τ^{-1} in $\langle\theta_0|\mathcal{J}_\eta|\theta_0\rangle$, which tends to boost current at small τ, is of geometric origin. Thus, the behavior of the local observables in the wave packet confirms the simple arguments of Sec. III A based on the analysis of the domain where the wave function is sta-

[4] We do not describe here the procedure of the scalar field quantization in wedge dynamics. It is exactly the same as quantization of the fermion field in the next section.

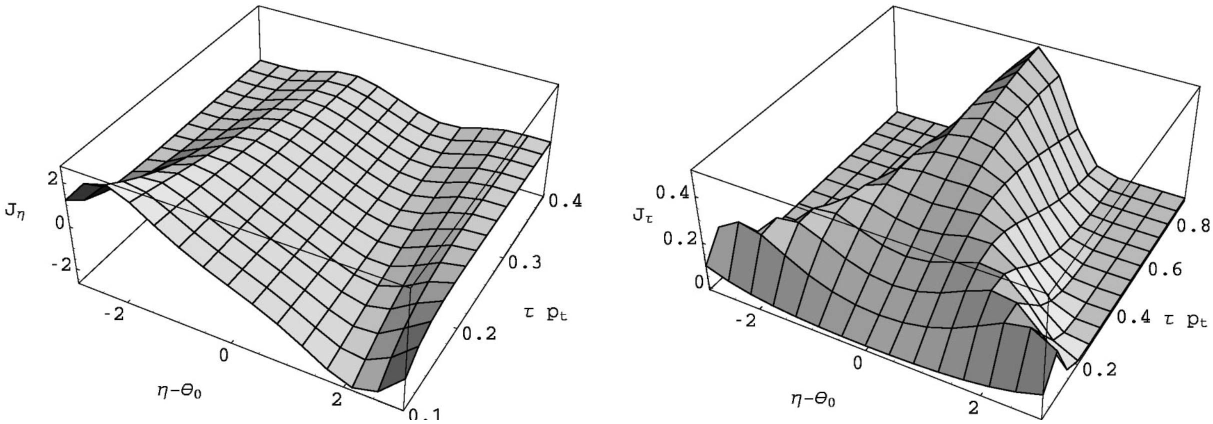

FIG. 1. Charge density in the wave packet (left) and current density (right) evolution.

tionary. One can guess about the possible nature of interactions at the earliest times by making an observation that the η component of the current must produce the x and y components of the magnetic field. These fields are the strongest at the earliest times when $\tau k_t \ll 1$. The magnetic fields of the transition currents provide scattering with the most effective transfer of the transverse momentum. Indeed, at time τ_2 the quark with the transverse momentum $p_t, \tau_2 p_t \ll 1$ interacts with the gluon field and acquires a large transverse momentum $k_t, \tau_2 k_t \gg 1$. This transition is characterized by a drastic narrowing of the charge spread in the rapidity direction, and must be accompanied by a strong η component of the transition current. A similar transition in the opposite direction happens at the time τ_1, when the gluon field interacts with another quark that has large initial transverse momentum k_t, and recovers the soft state with $\tau_2 p_t \ll 1$ in the course of this interaction. This second transition current readily interacts with the magnetic component of the gluon field. These speculations will be justified in paper [IV] of this cycle by the explicit calculation of quark self-energy in the expanding system.

Three remarks are in order. First, the field of a free on-mass-shell particle can be only static, and it is common to think that, in the rest frame of the particle, it is a purely electric field. In the wedge dynamics, the particle is formed during a finite time and this formation process unavoidably generates the magnetic component of the *longitudinal* (i.e., governed by the Gauss law) field. This will be obtained more rigorously in the next paper when the full propagator of the gauge field in wedge dynamics will be found. Furthermore, in wedge dynamics, the source must be called as static if it expands in such a way that $\mathcal{J}_\tau = \tau J_\tau = \mathrm{const}(\tau)$, and its field strength also has a magnetic component. Second, the local color current density may be large even when the system is color-neutral (begins its evolution from the colorless state), as it seems to be the case in heavy-ion collisions. It will be also shown in paper [III], that in order to fix the gauge $A^\tau = 0$ completely, one must require that the physical charge density $\mathcal{J}^\tau = 0$ at $\tau = 0$. Finally, in wedge dynamics we meet a unique structure of phase space, where two variables, the velocity of a particle and its rapidity coordinate, just duplicate each other at sufficiently late proper time τ. The quantum mechanical uncertainty principle does not prohibit one to address them on equal footing, because the one-particle wave packets of the wedge dynamics, evolving in time, become more and more narrow in rapidity direction.

IV. STATES OF FERMIONS

The hypersurfaces of constant Hamiltonian time τ of wedge dynamics are curved. Therefore, all oriented objects like vectors or spinors are essentially defined only in the tangent space and therefore, their covariant derivatives should be calculated in the framework of the so-called tetrad formalism [12,13].[5] The covariant derivative of the tetrad vector includes two connections (gauge fields). One of them, the Levi-Civita connection

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} \mathrm{g}^{\lambda\rho} \left[\frac{\partial \mathrm{g}_{\rho\mu}}{\partial x^\nu} + \frac{\partial \mathrm{g}_{\rho\nu}}{\partial x^\mu} - \frac{\partial \mathrm{g}_{\mu\nu}}{\partial x^\rho} \right],$$

is the gauge field, which provides covariance with respect to the general transformation of coordinates. The second gauge field, the spin connection $\omega^{ab}_\mu(x)$, provides covariance with respect to the local Lorentz rotation. Let $x^\mu = (\tau, x, y, \eta)$ be the contravariant components of the curvilinear coordinates and $x^a = (t, x, y, z) \equiv (x^0, x^1, x^2, x^3)$ are those of the flat Minkowsky space. Then the tetrad vectors e^a_μ can be taken as follows:

$$e^0_\mu = (1,0,0,0), \quad e^1_\mu = (0,1,0,0), \quad e^2_\mu = (0,0,1,0),$$

$$e^3_\mu = (0,0,0,\tau). \tag{4.1}$$

[5] In what follows, we use the Greek indices for four-dimensional vectors and tensors in the curvilinear coordinates, and the Latin indices from a to d for the vectors in flat Minkowsky coordinates. We use Latin indices from r to w for the transverse x and y components $(r, \ldots, w = 1,2)$, and the arrows over the letters to denote the two-dimensional vectors, e.g., $\vec{k} = (k_x, k_y), |\vec{k}| = k_t$. The Latin indices from i to n $(i, \ldots, n = 1,2,3)$ will be used for the three-dimensional internal coordinates $u^i = (x, y, \eta)$ on the hypersurface $\tau = $ const.

They correctly reproduce the curvilinear metric $\mathrm{g}_{\mu\nu}$ and the flat Minkowsky metric g_{ab}, i.e.,

$$\mathrm{g}_{\mu\nu}=g_{ab}e^a_\mu e^b_\nu=\mathrm{diag}[1,-1,-1,-\tau^2],$$
$$g^{ab}=\mathrm{g}^{\mu\nu}e^a_\mu e^b_\nu=\mathrm{diag}[1,-1,-1,-1]. \tag{4.2}$$

The spin connection can be found from the condition that the covariant derivatives of the tetrad vectors are equal to zero,

$$\nabla_\mu e^a_\nu=\partial_\mu e^a_\nu+\omega^a_{\mu b}e^b_\nu-\Gamma^\lambda_{\mu\nu}e^a_\lambda=0. \tag{4.3}$$

The covariant derivative of the spinor field includes only the spin connection,

$$\nabla_\mu\psi(x)=\left[\partial_\mu+\frac{1}{4}\omega^{ab}_\mu(x)\Sigma_{ab}\right]\psi(x), \tag{4.4}$$

where $\Sigma^{ab}=\frac{1}{2}[\gamma^a\gamma^b-\gamma^b\gamma^a]$ is an obvious generator of the Lorentz rotations and γ^a are the Dirac matrices of Minkowsky space. Introducing the Dirac matrices in curvilinear coordinates, $\gamma^\mu(x)=e^\mu_a(x)\gamma^a$, one obtains the Dirac equation in curvilinear coordinates,

$$[\gamma^\mu(x)(i\nabla_\mu+gA_\mu(x))-m]\psi(x)=0, \tag{4.5}$$

where $A^\mu(x)$ is the gauge field associated with the local group of the internal symmetry. The conjugated spinor is defined as $\overline{\psi}=\psi^\dagger\gamma^0$, and obeys the equation

$$(-i\nabla_\mu+gA_\mu(x))\overline{\psi}(x)\gamma^\mu(x)-m\overline{\psi}(x)=0. \tag{4.6}$$

These two Dirac equations correspond to the action

$$\mathcal{A}=\int d^4x\sqrt{-g}\mathcal{L}(x)=\int d^4x\sqrt{-g}\left\{\frac{i}{2}[\overline{\psi}\gamma^\mu(x)\nabla_\mu\psi-(\nabla_\mu\overline{\psi})\gamma^\mu(x)\psi]+g\overline{\psi}\gamma^\mu(x)A_\mu\psi-m\overline{\psi}\psi\right\}, \tag{4.7}$$

from which one easily obtains the locally conserved $U(1)$ current,

$$J^\mu(x)=\overline{\psi}(x)\gamma^\mu(x)\psi(x),$$
$$(-\mathrm{g})^{-1/2}\partial_\mu[(-\mathrm{g})^{1/2}\mathrm{g}^{\mu\nu}(x)J_\nu(x)]=0. \tag{4.8}$$

The Dirac equations (4.5) and (4.6) can be alternatively obtained as the equations of the Hamiltonian dynamics along the proper time τ. The canonical momenta conjugated to the fields ψ and $\overline{\psi}$ are

$$\pi_\psi(x)=\frac{\delta(\sqrt{-\mathrm{g}}\mathcal{L})}{\delta\dot{\psi}(x)}=\frac{i\tau}{2}\overline{\psi}(x)\gamma^0 \quad \text{and}$$
$$\pi_{\overline{\psi}}(x)=\frac{\delta(\sqrt{-\mathrm{g}}\mathcal{L})}{\delta\dot{\overline{\psi}}(x)}=-\frac{i\tau}{2}\gamma^0\psi(x), \tag{4.9}$$

respectively. The Hamiltonian of the Dirac field in the wedge dynamics has the following form:

$$H=\int d\eta d^2\vec{r}\sqrt{-g}\left\{-\frac{i}{2}[\overline{\psi}\gamma^i(x)\nabla_i\psi-(\nabla_i\overline{\psi})\gamma^i(x)\psi]-g\overline{\psi}\gamma^\mu(x)A_\mu\psi+m\overline{\psi}\psi\right\}, \tag{4.10}$$

and the wave equations are just the Hamiltonian equations of motion for the momenta.

The nonvanishing components of the connections are $\Gamma^{\cdot}_{\eta\tau\eta}=\Gamma^{\cdot}_{\eta\eta\tau}=-\Gamma^{\cdot}_{\tau\eta\eta}=-\tau$ and $\omega^{30}_\eta=-\omega^{03}_\eta=1$. Moreover, we have $\gamma^\tau(x)=\gamma^0$ and $\gamma^\eta(x)=\tau^{-1}\gamma^3$. The explicit form of the Dirac equation in our case is as follows:

$$[i\nabla\!\!\!/-m]\psi(x)=\left[i\gamma^0\left(\partial_\tau+\frac{1}{2\tau}\right)+i\gamma^3\frac{1}{\tau}\partial_\eta+i\gamma^r\partial_r-m\right]\times\psi(x)=0. \tag{4.11}$$

The one-particle solutions of this equation must be normalized according to the charge conservation law (4.8). We choose the scalar product of the following form:

$$(\psi_1,\psi_2)=\int\tau d\eta d^2\vec{r}\,\overline{\psi}_1(\tau,\eta,\vec{r})\gamma^\tau\psi_2(\tau,\eta,\vec{r}). \tag{4.12}$$

With this definition of the scalar product, the Dirac equation is self-adjoint. The solutions to this equation will be looked for in the form $\psi(x)=[i\nabla\!\!\!/+m]\chi(x)$, with the function $\chi(x)$ that obeys the ''squared'' Dirac equation,

$$[i\nabla\!\!\!/-m][-i\nabla\!\!\!/+m]\chi(x)=\left[\partial^2_\tau+\frac{1}{\tau}\partial_\tau-\frac{1}{\tau^2}\partial^2_\eta-\partial^2_r+m^2-\gamma^0\gamma^3\frac{1}{\tau^2}\partial_\eta\right]\chi(x)=0. \tag{4.13}$$

The spinor part β_σ of the function $\chi(x)$ can be chosen as an eigenfunction of the operator $\gamma^0\gamma^3$, namely, $\gamma^0\gamma^3\beta_\sigma=\beta_\sigma$, and $\sigma=1,2$. Therefore, the solution of the original Dirac equation (4.5) can be written down as $\psi^\pm_\sigma=w_\sigma\chi^\pm(x)$, with the bispinor operators $w_\sigma=[i\nabla\!\!\!/+m]\beta_\sigma$ that act on the positive- and negative-frequency solutions $\chi^\pm(x)$ of the scalar equation

$$\left[\partial^2_\tau+\frac{1}{\tau}\partial_\tau-\frac{1}{\tau^2}\left(\partial_\eta+\frac{1}{2}\right)^2-\partial^2_r+m^2\right]\chi^\pm(x)=0. \tag{4.14}$$

In what follows, we shall employ only the partial waves with quantum numbers of transverse momentum $\vec{p}_t$ and rapidity θ of massless quarks. In this case, the (already normalized) scalar functions $\chi^\pm(x)$ are

$$\chi^\pm_{\theta,\vec{p}_t}(x)=(2\pi)^{-3/2}(2p_t)^{-1/2}e^{(\theta-\eta)/2}e^{\mp ip_t\tau\cosh(\theta-\eta)}e^{\pm i\vec{p}\vec{r}}. \tag{4.15}$$

Consequently, the one-particle solutions are

$$\Omega^{\pm}_{\sigma,\theta,\vec{p}_t}(x) = (2\pi)^{-3/2}(2p_t)^{-1/2} i\nabla\!\!\!/ \beta_\sigma e^{(\theta-\eta)/2} e^{\mp i p_t \tau \cosh(\theta-\eta)} e^{\pm i\vec{p}\vec{r}}, \tag{4.16}$$

where the spinors β_σ can be chosen in different ways. However, regardless of a particular choice of the spinors β_σ, the polarization sum is always

$$\sum_\sigma \beta_\sigma \otimes \beta_\sigma = \frac{1+\gamma^0\gamma^3}{2}. \tag{4.17}$$

The waves $\Omega^{\pm}_{\sigma,\theta,\vec{p}_t}(x)$ are orthonormalized according to

$$(\Omega^{(\pm)}_{\sigma,\theta,\vec{p}}, \Omega^{(\pm)}_{\sigma',\theta',\vec{p}'}) = \delta_{\sigma\sigma'}\delta(\vec{p}-\vec{p}\,')\delta(\theta-\theta'),$$
$$(\Omega^{(\pm)}_{\sigma,\theta,\vec{p}}, \Omega^{(\mp)}_{\sigma',\theta',\vec{p}'}) = 0. \tag{4.18}$$

These partial waves form a complete set [cf. Eq. (5.16)] and therefore, can be used to decompose the fermion field,

$$\Psi(x) = \sum_\sigma \int d^2\vec{p}_t d\theta [\boldsymbol{a}_{\sigma,\theta,\vec{p}_t}\Omega^{(+)}_{\sigma,\theta,\vec{p}_t}(x) + \boldsymbol{b}^\dagger_{\sigma,\theta,\vec{p}_t}\Omega^{(-)}_{\sigma,\theta,\vec{p}_t}(x)],$$
$$\Psi^\dagger(x) = \sum_\sigma \int d^2\vec{p}_t d\theta [\boldsymbol{a}^\dagger_{\sigma,\theta,\vec{p}_t}\overline{\Omega}^{(+)}_{\sigma,\theta,\vec{p}_t}(x) + \boldsymbol{b}_{\sigma,\theta,\vec{p}_t}\overline{\Omega}^{(-)}_{\sigma,\theta,\vec{p}_t}(x)]. \tag{4.19}$$

The canonical quantization procedure, which identifies the coefficients $\boldsymbol{a}$ and $\boldsymbol{b}$ with the Fock operators, is standard, and it leads to the anticommutation relations,

$$[\boldsymbol{a}_{\sigma,\theta,\vec{p}_t}, \boldsymbol{a}^\dagger_{\sigma',\theta',\vec{p}_t'}]_+ = [\boldsymbol{b}_{\sigma,\theta,\vec{p}_t}, \boldsymbol{b}^\dagger_{\sigma',\theta',\vec{p}_t'}]_+ = \delta_{\sigma\sigma'}\delta(\vec{p}-\vec{p}\,')\delta(\theta-\theta'), \tag{4.20}$$

all other anticommutators being zero. Nontrivial issues of the canonical quantization in wedge dynamics show up only in the gluon sector. They will be discussed in paper [III] of this cycle.

V. FERMION CORRELATORS

The field-theory calculations are based on various field correlators. A full set of these correlators is employed by the Keldysh-Schwinger formalism [14] which will be used below in the form given in Refs. [1–3].[6] This set consists of two Wightman functions, where the field operators are taken in fixed order,

$$G_{[10]}(x_1,x_2) = -i\langle \Psi(x_1)\overline{\Psi}(x_2)\rangle,$$
$$G_{[01]}(x_1,x_2) = i\langle \overline{\Psi}(x_2)\Psi(x_1)\rangle, \tag{5.1}$$

and two differently ordered Green functions

$$G_{[00]}(x_1,x_2) = -i\langle T[\Psi(x_1)\overline{\Psi}(x_2)]\rangle,$$
$$G_{[11]}(x_1,x_2) = -i\langle T^\dagger[\Psi(x_1)\overline{\Psi}(x_2)]\rangle. \tag{5.2}$$

Here, $\langle \cdots \rangle$ denotes the average over an ensemble of the excited modes. The vacuum state (of each particular mode) is a part of this ensemble. In order to find the explicit form of these correlators, we shall employ the modes corresponding to the states with a given transverse momentum $\vec{p}_t$ and an unusual rapidity quantum number θ. Further on, it will be profitable to use the field correlators in the mixed representation when they are Fourier-transformed only by their transverse coordinates $\vec{r}_t$, while the dependence on the proper time τ and the rapidity coordinate η is retained explicitly. Below, we derive the corresponding expressions. The details of the derivation are important, since they help to clarify physical issues related to the the localization of quanta in the wedge dynamics, and are beneficial for the future analysis of collinear singularities in paper [IV]. As for the ''vacuum part'' of the correlators, we obtain the more or less known expressions and put them into the form which is convenient for future calculations.

For the practical calculations, we shall need not the functions $G_{[AB]}$ of Eqs. (5.1) and (5.2), but their linear combinations, the fermion anticommutator $G_{[0]}$ and the density of states $G_{[1]}$,

$$G_{[0]}(x_1,x_2) = G_{[10]}(x_1,x_2) - G_{[01]}(x_1,x_2),$$
$$G_{[1]}(x_1,x_2) = G_{[10]}(x_1,x_2) + G_{[01]}(x_1,x_2), \tag{5.3}$$

and the retarded and advanced Green functions,

$$G_{[ret]}(x_1,x_2) = G_{[00]}(x_1,x_2) - G_{[01]}(x_1,x_2) = \theta(\tau_1-\tau_2)G_{[0]}(x_1,x_2),$$
$$G_{[adv]}(x_1,x_2) = G_{[00]}(x_1,x_2) - G_{[10]}(x_1,x_2) = -\theta(\tau_2-\tau_1)G_{[0]}(x_1,x_2). \tag{5.4}$$

Nevertheless, we have to start with the computation of the simplest correlators, the Wightman functions. Using Eqs. (5.1) and (4.19), we obtain

[6] The indices of the contour ordering, as well as the labels of linear combinations of variously ordered correlators, are placed in square brackets, e.g., $G_{[AB]}$, $G_{[ret]} = G_{[00]} - G_{[01]}$, etc.

$$G_{[10]}(x_1,x_2;p_t)=-i\int\frac{d\theta}{8\pi}[\gamma^+p_te^{-\theta}e^{(\eta_1+\eta_2)/2}+\gamma^-p_te^{+\theta}e^{-(\eta_1+\eta_2)/2}+p_r\gamma^r\gamma^0(\gamma^+e^{-(\eta_1-\eta_2)/2}+\gamma^-e^{+(\eta_1-\eta_2)/2})]$$
$$\times[[1-n^+(\theta,p_t)]e^{-ip_t[\tau_1\cosh(\theta-\eta_1)-\tau_2\cosh(\theta-\eta_2)]}+n^-(\theta,p_t)e^{+ip_t[\tau_1\cosh(\theta-\eta_1)-\tau_2\cosh(\theta-\eta_2)]}]. \quad (5.5)$$

It is useful to keep in mind a simple connection between this expression and the standard one. Since, $\gamma^\pm=\gamma^0\pm\gamma^3$, and $p_te^{\pm\theta}=p^\pm\equiv p^0\pm p^3$, the first line in this formula can be rewritten as $\Lambda(-\eta_1)\not{p}\Lambda(\eta_2)$, where

$$\Lambda(\eta)=\cosh(\eta/2)+\gamma^0\gamma^3\sinh(\eta/2)$$
$$=\mathrm{diag}[e^{\eta/2},e^{-\eta/2},e^{-\eta/2},e^{\eta/2}] \quad (5.6)$$

is the matrix of Lorentz rotation with the boost η. Furthermore, the quantum number θ can be formally changed into p_z. Incorporating the mass-shell delta-function $\delta(p^2)$ and returning to Cartesian coordinates, we obtain

$$G_{[10]}(x_1,x_2)=\Lambda(-\eta_1)\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-x')}[-2\pi i\delta(p^2)\not{p}]$$
$$\times\{\theta(p^0)[1-n^+(p)]$$
$$-\theta(-p^0)n^-(p)\}\Lambda(\eta_2). \quad (5.7)$$

The expression between the two spin-rotating matrices Λ is what is commonly known for this type correlator in flat Minkowsky space, and it explicitly depends on the difference, $x-x'$, of Cartesian coordinates. The matrices $\Lambda(\eta)$ corrupt this invariance, because the curvature of the hypersurface of constant τ causes the effect known as Thomas precession of the fermion spin that can be seen by an observer that changes his rapidity coordinate and thus is subjected to an acceleration in z direction. From the representation (5.7), it is still difficult to see that the correlators (5.1) depend only on the difference $\eta=\eta_1-\eta_2$ [provided the distributions $n^\pm(\theta,p_t)$ are boost invariant]. This fact becomes clear after we change the variable, $\theta=\theta'+(\eta_1+\eta_2)/2$. Then

$$G_{[10]}(x_1,x_2;p_t)$$
$$=-i\int\frac{d\theta'}{8\pi}[\gamma^+p_te^{-\theta'}+\gamma^-p_te^{+\theta'}+p_r\gamma^r\gamma^0(\gamma^+e^{-\eta/2}$$
$$+\gamma^-e^{+\eta/2})]\left[\left[1-n^+\left(\frac{\eta_1+\eta_2}{2}+\theta',p_t\right)\right]\right.$$
$$\times e^{-ip_t[\tau_1\cosh(\theta-\eta/2)-\tau_2\cosh(\theta+\eta/2)]}$$
$$+n^-\left(\frac{\eta_1+\eta_2}{2}+\theta',p_t\right)$$
$$\left.\times e^{+ip_t[\tau_1\cosh(\theta-\eta/2)-\tau_2\cosh(\theta+\eta/2)]}\right]. \quad (5.8)$$

An amazing property of this formula is that the rapidity argument of the distributions $n^\pm(\theta,p_t)$ is shifted by $(\eta_1+\eta_2)/2$ towards the geometrical center of the correlator. Now, the spin rotation in the (tz) plane is virtually eliminated in such a way that both the spin direction and the occupation numbers acquired a reference point exactly in the middle between the endpoints η_1 and η_2. Now, things look exactly as if we had performed the Wigner transform of the correlator. In actual fact, we did not. If the distributions $n^\pm$ are boost-invariant along some finite rapidity interval, then the fermion correlator (5.8) will have the same property.

The Wightman function (5.8) has two different parts. One part is connected with the vacuum density of states. The second ''material'' part is connected with the occupation numbers. The first one is always boost invariant. Furthermore, we may expect that it depends (apart from the spin-rotation effects) only on the invariant interval $\tau_{12}^2=(t_1-t_2)^2-(z_1-z_2)^2$. The invariance of the material part is limited, e.g., by the full width $2Y$ of the rapidity plateau and we have to be careful in the course of further its transformation. In order to extract the dependence on τ_{12}, we must make a second change of variable, $\theta'=\theta''+\psi$, where $\psi(\tau_1,\tau_2,\eta)$ depends on the sign of the interval τ_{12} between the points (τ_1,η_1) and (τ_2,η_2). Let the interval τ_{12} be timelike. Then

$$\tau_{12}^2=\tau_1^2+\tau_2^2-2\tau_1\tau_2\cosh\eta>0,$$

$$\tanh\psi(\eta)=\frac{\tau_1+\tau_2}{\tau_1-\tau_2}\tanh\frac{\eta}{2},$$

$$|\eta|<\eta_0=\ln\frac{\tau_1}{\tau_2},\quad \tanh\psi(\pm\eta_0)=\pm1,\quad \psi(\pm\eta_0)=\pm\infty. \quad (5.9)$$

Then, Eq. (5.8) becomes

$$G_{[10]}(x_1,x_2;p_t)$$
$$=-i\int\frac{d\theta''}{8\pi}[\gamma^+e^{-\psi}p_te^{-\theta''}+\gamma^-e^{\psi}p_te^{+\theta''}$$
$$+p_r\gamma^r\gamma^0(\gamma^+e^{-\eta/2}+\gamma^-e^{+\eta/2})]$$
$$\times\left[\left[1-n^+\left(\frac{\eta_1+\eta_2}{2}+\psi+\theta'',p_t\right)\right]e^{-ip_t\tau_{12}\cosh\theta''}\right.$$
$$\left.+n^-\left(\frac{\eta_1+\eta_2}{2}+\psi+\theta'',p_t\right)e^{+ip_t\tau_{12}\cosh\theta''}\right], \quad (5.10)$$

and we see that the rapidity distributions of particles are shifted by $\psi(\eta)$ towards the direction between the points (τ_1,η_1) and (τ_2,η_2). According to Eq. (5.9), the rapidity ψ may be infinite when this direction is lightlike $(\tau_{12}^2=0)$. Then this shifted argument appears to be beyond the physical rapidity limits $\pm Y$ of the background distribution $n^{\pm}(\theta,p_t)$. This is extremely important, since this lightlike direction is dangerous; it is solely responsible for the collinear singularities in various amplitudes. One may think that the cutoff $\pm Y$ will now appear as a parameter in the final answer. This would be counterintuitive, e.g., for many local quantities related to the central rapidity region, like dynamical masses we are intending to compute. It will be shown later, that the theory is totally protected from collinear singularities even in its vacuum part and no explicit cutoff is necessary.

For the case of a spacelike interval τ_{12}, we introduce

$$\tilde{\tau}_{12}^2=-\tau_{12}^2=-\tau_1^2-\tau_2^2+2\tau_1\tau_2\cosh\eta>0,$$

$$\tanh\tilde{\psi}(\eta)=\frac{\tau_1-\tau_2}{\tau_1+\tau_2}\coth\frac{\eta}{2},\quad |\eta|>\eta_0, \tag{5.11}$$

and rewrite Eq. (5.5) as follows:

$$\begin{aligned}G_{[10]}(x_1,x_2;\vec{p}_t)&=-i\int\frac{d\theta''}{4\pi}\bigg[\frac{1}{2}\gamma^+e^{-\tilde{\psi}}p_te^{-\theta''}+\frac{1}{2}\gamma^-e^{\tilde{\psi}}p_te^{\theta''}\\&\quad+p_r\gamma^r\bigg(\cosh\frac{\eta}{2}-\gamma^0\gamma^3\sinh\frac{\eta}{2}\bigg)\bigg]\\&\quad\times\bigg[\bigg[1-n^+\bigg(\frac{\eta_1+\eta_2}{2}+\tilde{\psi}+\theta'',p_t\bigg)\bigg]e^{-ip_t\tilde{\tau}_{12}\,\mathrm{sign}\,\eta\sinh\theta''}\\&\quad+n^-\bigg(\frac{\eta_1+\eta_2}{2}+\tilde{\psi}+\theta'',p_t\bigg)e^{+ip_t\tilde{\tau}_{12}\,\mathrm{sign}\,\eta\sinh\theta''}\bigg].\end{aligned} \tag{5.12}$$

Now it is easy to see that we are protected from the null-plane singularities in the material sector of the theory on both sides of the lightlike plane. Similar calculations can be done for the second Wightman function $G_{[10]}$ which differs from $G_{[10]}$ by the obvious replacements, $1-n^+\to -n^+$, and $-n^-\to 1-n^-$. The results can be summarized as follows:

$$\begin{aligned}G_{[10]}(\tau_1,\tau_2,\eta;\theta'',p_t)&=[1-n^+(\theta,p_t)]G^{(0)}_{[10]}(\tau_1,\tau_2,\eta;\theta'',p_t)\\&\quad-n^-(\theta,p_t)G^{(0)}_{[01]}(\tau_1,\tau_2,\eta;\theta'',p_t),\\G_{[01]}(\tau_1,\tau_2,\eta;\theta'',p_t)&=-n^+(\theta,p_t)G^{(0)}_{[10]}(\tau_1,\tau_2,\eta;\theta'',p_t)\\&\quad+[1-n^-(\theta,p_t)]G^{(0)}_{[01]}(\tau_1,\tau_2,\eta;\theta'',p_t),\end{aligned} \tag{5.13}$$

where according to Eqs. (5.8), (5.10) and (5.12), $\theta=(\eta_1+\eta_2)/2+\psi+\theta''$. Here, $G^{(0)}_{[\alpha]}$ is the vacuum counterpart of $G_{[\alpha]}$, and $G^{(0)}_{[01]}(x_1,x_2;\theta,\vec{p}_t)=[G^{(0)}_{[10]}(x_1,x_2;\theta,-\vec{p}_t)]^*$. Using Eqs. (5.13), we may easily obtain the field correlators defined by Eqs. (5.3). One of them is the causal anticommutator,

$$\begin{aligned}G_{[0]}(x_1,x_2;\vec{p}_t)&\equiv G_{[10]}(x_1,x_2;\vec{p}_t)-G_{[10]}(x_1,x_2;\vec{p}_t)\\&=G^{(0)}_{[10]}(x_1,x_2;\vec{p}_t)-G^{(0)}_{[10]}(x_1,x_2;\vec{p}_t),\end{aligned} \tag{5.14}$$

which does not include occupation numbers, while the density of states,

$$\begin{aligned}G_{[1]}(x_1,x_2;\vec{p}_t)&\equiv G_{[10]}(x_1,x_2;\vec{p}_t)+G_{[10]}(x_1,x_2;\vec{p}_t)\\&=[1-2n_f(\theta,p_t)]G^{(0)}_{[1]}(x_1,x_2;\vec{p}_t),\end{aligned} \tag{5.15}$$

carries all the information about the phase-space population. In the last equation, we have put $n^-(\theta,p_t)=n^+(\theta,p_t)=n_f(\theta,p_t)$. It is straightforward to check that

$$\begin{aligned}G_{[0]}(\tau,\eta_1,\vec{r}_{t1};\tau,\eta_2,\vec{r}_{t2})&=-i\langle 0|[\Psi(\tau,\eta_1,\vec{r}_{t1}),\overline{\Psi}(\tau,\eta_2,\vec{r}_{t2})]_+|0\rangle\\&=-i\frac{\gamma^0}{\tau}\delta(\eta_1-\eta_2)\delta(\vec{r}_{t1}-\vec{r}_{t2}).\end{aligned} \tag{5.16}$$

This property of the equal-proper-time commutator is the canonical commutation relation which is translated into commutation relations for the Fock operators. It also verifies that the system of wave functions we employ forms a complete set.

In any calculations connected with the local quantities in heavy ion collisions, we would like to rely on the rapidity plateau in all distributions and to avoid its width as a parameter in the final answers. If this is possible (which appears to be the case), then we may consider the occupation numbers as the functions of p_t only, and accomplish the integration over the rapidity θ''. This integration gives the vacuum correlators $G_{[\alpha]}(\tau_1,\tau_2,\eta;\vec{p}_t)$ in closed form.[7] The integrations are straightforward and result in the following representation of the fermion correlators:

$$\begin{aligned}G_{[\alpha]}(\tau_1,\tau_2;\eta,\vec{p}_t)&=\gamma^+p_tg^{L(+)}_{[\alpha]}+\gamma^-p_tg^{L(-)}_{[\alpha]}\\&\quad+p_r\gamma^r\gamma^0\gamma^+g^{T(+)}_{[\alpha]}+p_r\gamma^r\gamma^0\gamma^-g^{T(-)}_{[\alpha]}.\end{aligned} \tag{5.17}$$

The products of three gamma matrices in this expression indicates that the fermion correlators acquire an axial component ($\sim\gamma^r\gamma^5$), which is consistent with the absence of complete Lorentz and rotational symmetry in our problem. In order to obtain the compact expressions for the invariants $g_{[\alpha]}$, one must note that in all domains, we can replace

$$(e^{\mp\psi},\mp\mathrm{sign}\,\eta e^{\mp\tilde{\psi}})\to\frac{\tau_1e^{\mp\eta/2}-\tau_2e^{\pm\eta/2}}{\sqrt{|\tau_{12}^2|}}. \tag{5.18}$$

[7] Some of the integrals over θ'' are defined as distributions by means of analytic continuation.

These transformations lead to the final expressions for the invariants of the fermion correlators that we shall use in our calculations. For the invariants of the causal anticommutator $G_{[0]}$, we have

$$g_{[0]}^{L(\pm)}=i\,\frac{\tau_1 e^{\mp\eta/2}-\tau_2 e^{\pm\eta/2}}{4\sqrt{|\tau_{12}^2|}}\,\theta(\tau_{12}^2)J_1(p_t\tau_{12}),$$

$$g_{[0]}^{T(\pm)}=-\frac{e^{\mp\eta/2}}{4}\,\theta(\tau_{12}^2)J_0(p_t\tau_{12}). \tag{5.19}$$

They are causal in the sense, that they are completely confined to the interior of the future light wedge. Depending on the context, the invariants of the density $G_{[1]}$ will be used in two different representations,

$$\begin{aligned} g_{[1]}^{L(\pm)}&=-\int\frac{d\theta'}{4\pi}\left[1-2n_f\left(\frac{\eta_1+\eta_2}{2}+\theta',p_t\right)\right]\\ &\quad\times e^{\mp\theta'}\sin(p_t[\tau_1\cosh(\theta-\eta/2)-\tau_2\cosh(\theta+\eta/2)])\\ &=\frac{\tau_1 e^{\mp\eta/2}-\tau_2 e^{\pm\eta/2}}{4\sqrt{|\tau_{12}^2|}}\left[\theta(\tau_{12}^2)Y_1(p_t\tau_{12})\right.\\ &\quad\left.+\frac{2}{\pi}\theta(-\tau_{12}^2)K_1(p_t\tilde{\tau}_{12})\right][1-2n_f(p_t)], \end{aligned} \tag{5.20}$$

$$\begin{aligned} g_{[1]}^{T(\pm)}&=-ie^{\mp\eta/2}\int\frac{d\theta'}{4\pi}\left[1-2n_f\left(\frac{\eta_1+\eta_2}{2}+\theta',p_t\right)\right]\\ &\quad\times\cos(p_t[\tau_1\cosh(\theta-\eta/2)-\tau_2\cosh(\theta+\eta/2)])\\ &=i\,\frac{e^{\mp\eta/2}}{4}\left[\theta(\tau_{12}^2)Y_0(p_t\tau_{12})-\frac{2}{\pi}\theta(-\tau_{12}^2)K_0(p_t\tilde{\tau}_{12})\right]\\ &\quad\times[1-2n_f(p_t)]. \end{aligned} \tag{5.21}$$

The first of these representations will be expedient when the quark from the distribution $n_f(p_t)$ in the self-energy loop is interacting with the radiation component of the gluon field which imposes the physical limits on the rapidity $\psi(\eta)$ in the phase, $\Phi=\tau_{12}(\eta)p_t\cosh(\theta'-\psi(\eta))$, in the integrands of Eqs. (5.20) and (5.21). Then, the integration $d\theta'$ will have finite limits defined by the light cone and the localization of states with the large p_t. When the quark interacts with the longitudinal (static) component of the gluon field, no limitations of this kind appear and we are able to use the second analytic representation.

ACKNOWLEDGMENTS

The author is grateful to Berndt Muller, Edward Shuryak, and Eugene Surdutovich for helpful discussions at various stages in the development of this work, and appreciates the help of Scott Payson who critically read the manuscript. This work was partially supported by the U.S. Department of Energy under Contract No. DE-FG02-94ER40831.

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