Formation of superheavy elements in cold fusion reactions

Robert Smolańczuk*
Nuclear Theory Department, Sołtan Institute for Nuclear Studies, Hoża 69, PL-00-681 Warszawa, Poland
and Nuclear Science Division, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720
(Received 27 April 2000; published 21 March 2001)

We calculate the formation cross sections of transactinides (superheavy elements), as well as heavy actinides (No and Lr), which have been or might be obtained in fusion reactions with the evaporation of only one neutron. We use both more realistic fusion barrier and survival probability of the compound nucleus in comparison with the original phenomenological model [Phys. Rev. C **59**, 2634 (1999)] that prompted the Berkeley experiment on the synthesis of a new superheavy element 118 [Phys. Rev. Lett. **83**, 1104 (1999)]. Calculations are performed for asymmetric and symmetric target-projectile combinations and for reactions with stable and radioactive-ion beams. The formation cross sections measured at GSI-Darmstadt for transactinides and heavy actinides, as well as that for superheavy element 118 reported by the LBNL-Berkeley group, are reproduced within a factor of 2.4, on average. Based on the obtained relatively large cross sections, we predict that optimal reactions with stable beams for the synthesis of so far unobserved superheavy elements 119, 120, and 121 are ^{209}Bi(^{86}Kr, $1n$)^{294}119, ^{208}Pb(^{88}Sr, $1n$)^{295}120, and ^{209}Bi(^{88}Sr, $1n$)^{296}121, respectively. This is because of the magic of both the target and the projectile that leads to larger Q value and, consequently, lower effective fusion barrier with larger transmission probability. The same effect is responsible for relatively large cross sections predicted for the symmetric reactions ^{136}Xe(^{124}Sn, $1n$)^{259}Rf, ^{136}Xe(^{136}Xe, $1n$)^{271}Hs, ^{138}Ba(^{136}Xe, $1n$)^{273}110, and ^{140}Ce(^{136}Xe, $1n$)^{275}112. Although shell effects in the magic nuclei ^{124}Sn, ^{136}Xe, ^{138}Ba, and ^{140}Ce are not as strong as in ^{208}Pb and ^{209}Bi, they act on both the target and the projectile and lead to the prediction of measurable cross sections.

DOI: 10.1103/PhysRevC.63.0346XX PACS number(s): 27.90.+b, 24.60.Dr

I. INTRODUCTION

Low-energy fusion reactions of magic lead or of bismuth nuclei with heavy ions [1,2], which had been named ''cold fusion reactions'' [1,2], turned out to be a very powerful method for producing transactinide (superheavy) elements [3,4]. A reaction of this kind has also been proposed in Refs. [5,6] for the production of a new superheavy element 118 and its α-decay descendants and carried out at Berkeley [7]. A target-projectile-energy combination for the synthesis of element 118 was one of the main results obtained by using our simple reaction model [5]. In the model [5], we assume that the compound nucleus is formed in the subbarrier reaction by quantal tunneling of the Coulomb barrier and then, after thermal equilibration, the compound nucleus undergoes mostly fission and it is less likely that a very heavy nucleus is formed after neutron emission. The ground state of $1n$-evaporation residue is reached by emission of γ quanta. In the entrance channel, one-dimensional static approach with a simplified cutoff Coulomb barrier was used. Since our model is quantal, there was no need for introducing friction as was done in classical models developed in Refs. [8,9]. In the exit channel, the survival probability of the compound nucleus (the neutron-to-total-width ratio) was calculated by using the statistical model formula for the Fermi gas, in which thermal damping of shell effects for a given compound nucleus was introduced through the level density parameters. The objective of the present study is making description of cold fusion reactions more realistic while preserving simultaneously the simplicity of this description. The second objective is indicating additional target-projectile-energy combinations for producing superheavy elements. Since, within the very thin cutoff Coulomb barrier, one may expect underestimated transmission probability through the fusion barrier, our present model is based on a smooth phenomenological fusion barrier with the height and the curvature dependent on the combination of the colliding nuclei. Our present model also takes advantage of a convenient method for calculating the survival probability of the compound nucleus, which, in contrast to the original model [5,10], provides realistic values for this quantity.

In Sec. II, we describe our model. In Sec. III, we compare obtained results with experimental data and with our previous estimates. Furthermore, we indicate the best target-projectile-energy combinations for producing superheavy elements.

II. MODEL

A. Formation cross section

The formation cross section of a very heavy nucleus produced after the evaporation of one neutron reads

$$\sigma_{1n}(E_{HI}) = \pi\lambda\!\!\!^{-2}\sum_{l=0}^{l_{\max}} (2l+1)T_l(E_{HI})P_{1n,l}(E^*), \qquad (1)$$

where E_{HI} is the optimal bombarding energy in the center-of-mass system (the energy corresponding to the maximum of the excitation function), $E^* = E_{HI} - Q$ is the optimal excitation energy, Q is the Q value for a given reaction, $\lambda\!\!\!^{-}$

*Electronic address: smolan@fuw.edu.pl

TABLE I. The comparison of the calculated formation cross section σ_{1n} with the measured one σ_{1n}^{exp} [3,4,7,11,12] for transactinides and heavy actinides produced in cold fusion reactions with the emission of only one neutron. The systematic uncertainty of the measured formation cross sections is equal to a factor of 2 [4]. The measured values for the formation cross section with only statistical errors are given. Model parameter $c=352.93548$ is fixed to reproduce the cross section of ^{208}Pb(^{50}Ti, $1n$)^{257}Rf at the maximum of the measured excitation function [17].

Reaction	σ_{1n}	σ_{1n}^{exp}	Reaction	σ_{1n}	σ_{1n}^{exp}
^{208}Pb(^{48}Ca, $1n$)^{255}No	590 nb	260^{+30}_{-30} nb	^{208}Pb(^{58}Fe, $1n$)^{265}Hs	37 pb	67^{+17}_{-17} pb
^{209}Bi(^{48}Ca, $1n$)^{256}Lr	250 nb	61^{+20}_{-20} nb	^{209}Bi(^{58}Fe, $1n$)^{266}Mt	33 pb	$7.4^{+4.8}_{-3.3}$ pb
^{208}Pb(^{50}Ti, $1n$)^{257}Rf	10.4 nb	$10.4^{+1.3}_{-1.3}$ nb	^{208}Pb(^{62}Ni, $1n$)^{269}110	2.0 pb	$3.5^{+2.7}_{-1.8}$ pb
^{209}Bi(^{50}Ti, $1n$)^{258}Db	6.1 nb	$2.9^{+0.3}_{-0.3}$ nb	^{208}Pb(^{64}Ni, $1n$)^{271}110	17 pb	15^{+9}_{-6} pb
^{208}Pb(^{54}Cr, $1n$)^{261}Sg	580 pb	500^{+140}_{-140} pb	^{209}Bi(^{64}Ni, $1n$)^{272}111	13 pb	$3.5^{+4.6}_{-2.3}$ pb
^{209}Bi(^{54}Cr, $1n$)^{262}Bh	390 pb	240^{+240}_{-132} pb	^{208}Pb(^{70}Zn, $1n$)^{277}112	3.2 pb	$1.0^{+1.3}_{-0.7}$ pb
^{207}Pb(^{58}Fe, $1n$)^{264}Hs	9.7 pb	$8.8^{+16.0}_{-3.5}$ pb	^{208}Pb(^{86}Kr, $1n$)^{293}118	5.9 pb	$2.2^{+2.6}_{-0.8}$ pb

$=\sqrt{\hbar^2/(2\mu E_{HI})}$ is the reduced de Broglie wavelength of the projectile, and μ is the reduced mass. The quantities $T_l(E_{HI})$ and $P_{1n,l}(E^*)$ are the transmission probability through the fusion barrier and the survival probability of the compound nucleus (the neutron-to-total-width ratio) for a given l. We terminate the summation in Eq. (1) at angular momentum l_{max} for which the contribution to σ_{1n} becomes smaller than 1%.

B. Description of the entrance channel

Transmission probability through the fusion barrier $T_l(E_{HI})$ is calculated by means of the WKB approximation,

$$T_l(E_{HI})=\frac{1}{1+\exp[2S_l(E_{HI})]}. \quad (2)$$

Since we deal with low angular momenta ($l_{max}=26$–30) and thin fusion barriers (1.35–1.65 fm), the action integral $S_l(E_{HI})$ between the barrier entrance and exit points for a given angular momentum l may be expressed by the action integral for zero angular momentum,

$$S_l(E_{HI})\approx S_0[E_{HI}-E_{centr}(R)], \quad (3)$$

where $E_{centr}(R)=[\hbar^2 l(l+1)]/[2\mu R^2]$ is the centrifugal energy at a certain distance R connected with the position of the fusion barrier.

For the sake of simplicity, we calculate the action integral assuming the most likely fusion barrier V_{fu} instead of considering the barrier distribution. We approximate the fusion barrier V_{fu} around its top by the inverted parabola,

$$V_{fu}(r)=B_{fu}-q(r-R_{fu})^2, \quad (4)$$

where r is the distance between the centers of the reaction partners. The position of the top of the fusion barrier R_{fu} and the barrier height B_{fu}, as well as the coefficient q describing the curvature of the barrier, are dependent on the combination of the colliding nuclei.

We parametrize the quantity q by the formula

$$q=\frac{B_{fu}-Q}{d^2}, \quad (5)$$

where $d=1$ fm, which corresponds to the barrier thickness of 1.35–1.65 fm. Within this parametrization, we obtain the numerical value of q equal to the height of the fusion barrier relative to the ground state of the compound nucleus, which leads to a good reproduction of the measured formation cross sections [3,4,7,11,12], as shown in Table I.

The height of the fusion barrier

$$B_{fu}=\frac{Z_T Z_P e^2}{R_e} \quad (6)$$

is expressed by the Coulomb energy at an effective distance R_e. The latter is strongly dependent on the atomic number of the target, Z_T, and the projectile, Z_P, which was observed experimentally [13] and theoretically [14]. Here, e is the elementary electric charge. We obtain the expression for the quantitity R_e assuming that the difference between R_e and the distance R_{12} at which the colliding nuclei are at contact is inversely proportional to the height of the fusion barrier. After simple algebra, we obtain the formula

$$R_e=\frac{R_{12}}{1-c/Z_T Z_P}>R_{fu}>R_{12}, \quad (7)$$

where c is the model parameter controlling the height of the fusion barrier. The distance R_{12} at which the colliding nuclei are at contact is the sum of the half-density radii of the target and the projectile and is given by

$$R_{12}=c_T R_T+c_P R_P. \quad (8)$$

Here, R_T and R_P are the nuclear radii of the target and the projectile determined from the root-mean-square charge radii, and c_T and c_P are the coefficients relating R_T and R_P with the half-density radii of the target and the projectile, respectively. The radii R_T and R_P are calculated by using the Nerlo-Pomorska and Pomorski formula [15],

$$R_T = 1.256\left(1 - 0.202\ \frac{N_T - Z_T}{A_T}\right) A_T^{1/3},$$

$$R_P = 1.256\left(1 - 0.202\ \frac{N_P - Z_P}{A_P}\right) A_P^{1/3}. \qquad (9)$$

For the projectiles with $Z_P < 38$,

$$R_P = 1.240\left(1 + \frac{1.646}{A_P} - 0.191\ \frac{N_P - Z_P}{A_P}\right) A_P^{1/3}, \qquad (10)$$

where A_T and N_T are mass and neutron numbers of the target and A_P and N_P are the same quantities for the projectile. The coefficients c_T and c_P relating R_T and R_P with the half-density radii may be deduced from Ref. [16] and are specified in Ref. [5] [Eq. (9) therein]. The model parameter $c = 352.93548$ is fixed to reproduce the cross section of ^{208}Pb(^{50}Ti, $1n$)^{257}Rf at the maximum of the measured excitation function [17]. The choice of this reaction is motivated by the fact that the nice measurement of the excitation function for ^{257}Rf was performed at GSI-Darmstadt [17].

The action integral calculated for the fusion barrier introduced above is given by

$$S_l(E_{HI}) \approx \frac{\pi}{2}\sqrt{\frac{2\mu}{\hbar^2 q}}[B_{fu} + E_{centr}(R) - E_{HI}], \qquad (11)$$

where $R = (R_{12} + R_e)/2$.

C. Description of the exit channel

Using empirical formulas for the Coulomb barrier for proton emission and α-particle emission given in Ref. [18], as well as proton-separation and α-decay energies that may be calculated in the macroscopic-microscopic model [19–21], we are able to determine thresholds for proton and α-particle evaporation. Since for the compound nuclei in question the calculated thresholds for proton and α-particle evaporation are higher than that for neutron emission, one can express the survival probability of the compound nucleus (the neutron-to-total-width ratio) for a given angular momentum l only by the neutron-to-fission-width ratio $(\Gamma_n/\Gamma_f)_l$,

$$P_{1n,l} \approx \frac{(\Gamma_n/\Gamma_f)_l}{1 + (\Gamma_n/\Gamma_f)_l}. \qquad (12)$$

Both widths are dependent on the density of single-particle and collective energy-levels. For low excitations in question ($E^* \lesssim 15$ MeV), shell effects are still present leading to lower level density in the equilibrium configuration $\rho_{eq}(E^*)$ than that in the saddle-point configuration $\rho_{sd}(E^*)$. Lower level density in the equilibrium configuration should lead to slower neutron emission in comparison with fission also for nuclei with comparable thresholds for both processes. The excitation-energy dependence of the neutron-to-total-width ratio measured for heavy actinides at JINR-Dubna [22] may be explained in this way for low excitations. Using the statistical model formula for the Fermi gas, in which thermal damping of shell effects was introduced through the level-density parameters, one obtains values for the neutron-to-fission-width ratio $(\Gamma_n/\Gamma_f)_0$ significantly smaller than 10^{-4} for many very heavy compound nuclei [5,10]. Those values correspond to large times for neutron evaporation allowing deexcitation of the compound nucleus by γ emission. The latter process, however, is not observed for the very heavy compound nuclei. Therefore, in the present study, we use a different method for calculating $(\Gamma_n/\Gamma_f)_l$. Instead of describing $(\Gamma_n/\Gamma_f)_l$ by the formula for the Fermi gas with inserted different level-density parameters for the equilibrium configuration of $1n$-evaporation residue and for the saddle-point configuration of the compound nucleus, we use a constant-temperature formula for $(\Gamma_n/\Gamma_f)_l$ in which we insert different temperatures T_{eq} and $T_{sd} > T_{eq}$ for these configurations, respectively.

Assuming that the rotational energy is not available for neutron evaporation, as well as for fission, an expression for the neutron-to-fission-width ratio reads

$$(\Gamma_n/\Gamma_f)_l = \frac{k_{eq}}{k_{sd}} \cdot kA^{2/3} T_{eq} \exp\left(\frac{B_f + \Delta_{sd}}{T_{sd}} - \left[\frac{\hbar^2 l(l+1)}{2J(eq)T_{eq}} - \frac{\hbar^2 l(l+1)}{2J(sd)T_{sd}}\right] - \frac{S_n + \Delta_{eq}}{T_{eq}}\right). \qquad (13)$$

Here, k_{eq} and k_{sd} are the collective enhancement factors for the equilibrium and the saddle-point configurations, respectively. For the equilibrium configuration of $1n$-evaporation residue with the quadrupole deformation $\beta_2 > 0.15$ and for the saddle-point configuration of all compound nuclei, k_{eq} and k_{sd} equal to 100 [23] are used because of the presence of the rotational bands. This value is consistent with the recent measurements [24]. For spherical and transitional nuclei ($\beta_2 \leqslant 0.15$), $k_{eq} = 1$ is taken. The constant $k = 0.14\ \mathrm{MeV}^{-1}$ is a coefficient obtained in the statistical model and A is mass number of the compound nucleus. The quantities B_f and S_n are the static fission-barrier height and the neutron-separation energy (thresholds for fission and neutron emission), while Δ_{sd} and Δ_{eq} are the energy shifts in the saddle-point of the compound nucleus and the equilibrium configuration of $1n$-evaporation residue, respectively. These energy shifts are used to take into account differences in level densities between even-even, odd, and odd-odd nuclei [25]. Taking as the reference the potential energy surface of an odd nucleus [25], we use $\Delta_{eq} = 12/\sqrt{A}$, 0, and $-12/\sqrt{A}$, for even-even, odd, and odd-odd $1n$-evaporation residues, respectively. The energy shift in the saddle-point configuration of the compound nucleus Δ_{sd} is significantly larger (see for instance Ref. [26]). In our calculation, the value of 1.5 MeV for even-even and 0 for odd compound nuclei is taken. The moments of inertia $J(eq)$ and $J(sd)$ for the equilibrium and the saddle-point deformations are assumed to be equal to those for the rigid body and are calculated taking advantage of the deformation dependence obtained in Ref. [27] and the nuclear radii given by the Nerlo-Pomorska and Pomorski formula [15].

The temperature in the equilibrium configuration of $1n$-evaporation residue $T_{eq} = [(d/dE^*)\ln \rho_{eq}]^{-1}$ must be lower than the temperature in the saddle-point configuration of the compound nucleus T_{sd} because the level density $\rho_{eq}(E^*)$ increases faster with increasing excitation energy E^* in comparison with the level density in the saddle-point configuration of the compound nucleus $\rho_{sd}(E^*)$. The reason for this is thermal damping of the strong ground-state shell-effect. (In the saddle point, there is no shell effect or it is much weaker than in the equilibrium configuration.) The density of the lowest levels in a very heavy nucleus ($E^* \lesssim 3$ MeV) is well described by the constant-temperature formula with the average temperature $T_{eq}^{low} = 0.4$ MeV [28]. Assuming $T_{eq} = (T_{eq}^{low} + T_{sd})/2 = 0.7$ MeV, we obtain $T_{sd} = 1$ MeV. Since the experimentally observed neutron-to-total-width ratio is excitation-energy dependent, the constant temperature $T_{eq} = 0.7$ MeV may be used only for a narrow range of excitation energy around the maximum of the $1n$-channel excitation function for the heaviest nuclei. Within $T_{eq} = 0.7$ MeV and $T_{sd} = 1$ MeV, we obtain realistic values of the order of 10^{-4}–10^{-1} for the $(\Gamma_n/\Gamma_f)_0$ ratio for the nuclei in question. For high excitations for which shell effects are fully damped, experimental data [22] may be well described by the standard statistical model formula for $(\Gamma_n/\Gamma_f)_l$ within equal temperatures.

D. Optimal bombarding energy

Since for almost all nuclei in question, the calculated threshold for fission following neutron emission is lower and only for a few of them comparable with the calculated threshold for two-neutron emission, we determine the optimal excitation energy that corresponds to the maximum of the excitation function as $E^* = S_n + B_f^{ER}$, where B_f^{ER} is the height of the static fission barrier for $1n$-evaporation residue. This excitation energy corresponds to the bombarding energy in the lab system given by

$$E_{lab} = (Q + S_n + B_f^{ER}) \frac{A_T + A_P}{A_T}. \quad (14)$$

This simple expression may be used because of the narrowness of the excitation function for the heaviest atomic nuclei.

Nuclear structure influences the formation cross section through the model input-quantities Q, S_n, B_f, B_f^{ER} and the equilibrium and saddle-point deformations. In order to obtain Q values, we calculate masses of the compound nuclei by means of the macroscopic-microscopic model [19–21] and use measured masses of the targets and the projectiles [29]. All the other input quantities are calculated by using the macroscopic-microscopic model [19–21].

III. RESULTS AND DISCUSSION

A. Comparison with experimental data

We compare the calculated formation cross sections with the measured ones [3,4,7,11,12] in Table I. The experimental cross sections of ^{207}Pb(^{50}Ti, $1n$)^{256}Rf, ^{207}Pb(^{54}Cr,$1n$)^{260}Sg, and ^{207}Pb(^{58}Fe, $1n$)^{264}Hs given in Refs. [30–32] are not included because they were measured at excitation energy significantly higher than the optimal one. Instead, the cross section of ^{207}Pb(^{58}Fe, $1n$)^{264}Hs measured at the excitation energy $E^* = 12.9$ MeV [11] is compared with the calculated value. The systematic uncertainty of the measured formation cross sections is equal to a factor of 2 [4]. In Table I, the measured values for the formation cross section with only statistical errors are given. The obtained results agree with the experimental data within a factor of 2.4, on average. The calculated formation cross section of 5.9 pb for ^{293}118 overestimates a value [7] reported by the LBNL-Berkeley group only by a factor of 2.7 (a value obtained in the original model [5] was by about two orders of magnitude larger).

The calculation of the cross section of the reactions ^{90,92,94,96}Zr(^{124}Sn, $1n$)^{213,215,217,219}Th [33] carried out at GSI-Darmstadt is outside the scope of the present paper because it requires considerable extension of our model. For nuclei like ^{213,215,217,219}Th, fission is not that important (high fission barriers) and, therefore, the excitation functions are broader with their maxima shifted toward higher excitation energies of 20–30 MeV. The extension of the model would have to contain a method of determining the optimal bombarding energy because Eq. (14) is no longer valid for broad excitation functions. For excitations of 20–30 MeV, the low temperature $T_{eq} = 0.7$ MeV cannot be used either. Moreover, evaporation of charged particles would have to be taken into account because of comparable thresholds for neutron evaporation and emission of charged particles. Furthermore, octupole deformation would have to be taken into account in the nuclear-structure-dependent input-quantities and in the moments of inertia. The calculation of the cross sections of lighter nuclei, for example, ^{179}Hg obtained in the cold fusion reaction ^{90}Zr(^{90}Zr, $1n$)^{179}Hg [34], would require also taking into account the γ-emission channel that competes with evaporation of the neutron and the charged particles.

B. Comparison with previous estimates

Our results for the reactions based on ^{207}Pb and ^{208}Pb target nuclei are listed in Table II. The calculated formation cross sections for transitional and spherical nuclei, i.e., for nuclei heavier than ^{282}114, are smaller than the values obtained in Refs. [5,10] mainly because of thicker fusion barrier in comparison with the cutoff Coulomb barrier used in the original model [5,10]. The formation cross section decreases with increasing atomic number due to decrease of both the transmission probability through the fusion barrier and the neutron-to-fission-width ratio. (In the original model, this decrease was mainly due to decreasing Γ_n/Γ_f.) The reversal of this trend in our quantal model and the increase of the cross section of ^{208}Pb(^{86}Kr, $1n$)^{293}118 and ^{207}Pb(^{86}Kr, $1n$)^{292}118 is caused by the magic of ^{86}Kr projectile ($N_P = 50$), which leads to larger Q value and, consequently, to lower effective fusion barrier with larger transmission probability. This effect is not present in the dinuclear-system model exploited by the authors of Ref. [35], who describe the formation of the compound nucleus classically and obtain a very small cross section of 5 fb [35] for the reaction ^{208}Pb(^{86}Kr, $1n$)^{293}118.

TABLE II. The optimal bombarding energy in the laboratory system E_{lab}, the transmission probability through the fusion barrier for zero angular momentum T_0, the neutron-to-fission-width ratio for zero angular momentum $(\Gamma_n/\Gamma_f)_0$, and the formation cross section σ_{1n} calculated for reactions based on ^{207}Pb and ^{208}Pb target nuclei. Reactions with stable projectiles are given on the left-hand side of the table and those with neutron-rich radioactive-ion-beams are placed on its right-hand side. In the calculation of $(\Gamma_n/\Gamma_f)_l$, the minimal fission barriers are used (in the original model [5,10,36], we used the fission-barrier heights B_f and B_f^{ER} for odd nuclei by 0.5 MeV higher because of the assumed specialization energy).

Reaction	E_{lab} (MeV)	T_0	$(\Gamma_n/\Gamma_f)_0$	σ_{1n}	Reaction	E_{lab} (MeV)	T_0	$(\Gamma_n/\Gamma_f)_0$	σ_{1n}
^{207}Pb(^{50}Ti,1n)^{256}Rf	227.8	1.2×10^{-6}	1.7×10^{-1}	3.3 nb	^{208}Pb(^{80}Ge,1n)^{287}114	381.8	3.7×10^{-8}	2.4×10^{-4}	91 fb
^{208}Pb(^{50}Ti,1n)^{257}Rf	228.8	3.0×10^{-6}	2.2×10^{-1}	10.4 nb	^{207}Pb(^{82}Ge,1n)^{288}114	382.4	9.5×10^{-9}	2.3×10^{-4}	23 fb
^{207}Pb(^{54}Cr,1n)^{260}Sg	253.1	1.7×10^{-7}	6.6×10^{-2}	180 pb	^{208}Pb(^{82}Ge,1n)^{289}114	384.1	4.6×10^{-8}	8.7×10^{-4}	400 fb
^{208}Pb(^{54}Cr,1n)^{261}Sg	253.8	3.9×10^{-7}	9.6×10^{-2}	580 pb	^{207}Pb(^{84}Se,1n)^{290}116	415.8	8.1×10^{-8}	2.2×10^{-4}	160 fb
^{207}Pb(^{58}Fe,1n)^{264}Hs	279.4	3.3×10^{-8}	2.0×10^{-2}	9.7 pb	^{208}Pb(^{84}Se,1n)^{291}116	416.9	3.3×10^{-7}	7.8×10^{-4}	2.4 pb
^{208}Pb(^{58}Fe,1n)^{265}Hs	279.7	6.6×10^{-8}	3.8×10^{-2}	37 pb	^{207}Pb(^{86}Se,1n)^{292}116	415.5	2.8×10^{-8}	6.4×10^{-4}	170 fb
^{207}Pb(^{62}Ni,1n)^{268}110	306.4	6.9×10^{-9}	4.0×10^{-3}	380 fb	^{208}Pb(^{86}Se,1n)^{293}116	416.9	1.2×10^{-7}	2.0×10^{-3}	2.3 pb
^{208}Pb(^{62}Ni,1n)^{269}110	306.4	1.2×10^{-8}	1.2×10^{-2}	2.0 pb	^{207}Pb(^{88}Se,1n)^{294}116	413.8	4.6×10^{-9}	1.7×10^{-3}	77 fb
^{207}Pb(^{64}Ni,1n)^{270}110	310.5	1.9×10^{-8}	1.1×10^{-2}	2.6 pb	^{208}Pb(^{88}Se,1n)^{295}116	415.5	2.3×10^{-8}	4.4×10^{-3}	1.0 pb
^{208}Pb(^{64}Ni,1n)^{271}110	310.8	3.9×10^{-8}	3.3×10^{-2}	17 pb	^{207}Pb(^{88}Kr,1n)^{294}118	449.0	2.1×10^{-7}	5.1×10^{-4}	940 fb
^{207}Pb(^{68}Zn,1n)^{274}112	337.2	3.0×10^{-9}	1.0×10^{-2}	360 fb	^{208}Pb(^{88}Kr,1n)^{295}118	450.0	8.5×10^{-7}	1.2×10^{-3}	8.6 pb
^{208}Pb(^{68}Zn,1n)^{275}112	337.7	7.2×10^{-9}	1.5×10^{-2}	1.3 pb	^{207}Pb(^{90}Kr,1n)^{296}118	449.0	6.5×10^{-8}	9.6×10^{-4}	540 fb
^{207}Pb(^{70}Zn,1n)^{276}112	340.9	5.1×10^{-9}	1.6×10^{-2}	940 fb	^{208}Pb(^{90}Kr,1n)^{297}118	450.3	2.9×10^{-7}	2.1×10^{-3}	5.4 pb
^{208}Pb(^{70}Zn,1n)^{277}112	341.9	1.6×10^{-8}	1.8×10^{-2}	3.2 pb	^{207}Pb(^{92}Kr,1n)^{298}118	447.4	9.2×10^{-9}	2.0×10^{-3}	170 fb
^{208}Pb(^{74}Ge,1n)^{281}114	370.8	5.3×10^{-9}	6.0×10^{-3}	330 fb	^{208}Pb(^{92}Kr,1n)^{299}118	449.1	4.9×10^{-8}	3.2×10^{-3}	1.4 pb
^{207}Pb(^{76}Ge,1n)^{282}114	374.5	4.4×10^{-9}	4.5×10^{-3}	200 fb	^{208}Pb(^{92}Sr,1n)^{299}120	483.2	5.2×10^{-7}	8.3×10^{-4}	3.4 pb
^{208}Pb(^{76}Ge,1n)^{283}114	375.4	1.4×10^{-8}	6.6×10^{-5}	9 fb	^{207}Pb(^{94}Sr,1n)^{300}120	483.0	4.8×10^{-8}	8.1×10^{-4}	310 fb
^{208}Pb(^{82}Se,1n)^{289}116	412.3	8.6×10^{-8}	2.3×10^{-4}	190 fb	^{208}Pb(^{94}Sr,1n)^{301}120	484.2	2.2×10^{-7}	1.2×10^{-3}	2.1 pb
^{208}Pb(^{84}Kr,1n)^{291}118	441.0	7.4×10^{-8}	2.0×10^{-4}	140 fb	^{207}Pb(^{96}Sr,1n)^{302}120	481.4	5.3×10^{-9}	1.2×10^{-3}	53 fb
^{207}Pb(^{86}Kr,1n)^{292}118	447.8	3.7×10^{-7}	2.0×10^{-4}	640 fb	^{208}Pb(^{96}Sr,1n)^{303}120	483.1	3.0×10^{-8}	1.4×10^{-3}	340 fb
^{208}Pb(^{86}Kr,1n)^{293}118	448.4	1.2×10^{-6}	5.7×10^{-4}	5.9 pb	^{207}Pb(^{98}Sr,1n)^{304}120	479.4	5.0×10^{-10}	1.7×10^{-3}	7 fb

In the present paper, we predict the fusion-barrier heights by 1.3–3.3 MeV larger than the heights of the cutoff Coulomb barrier calculated in Refs. [5,10]. The parabolic barrier is thicker and, consequently, transmission probabilities of the order of 10^{-9}–10^{-6} obtained in the present paper are much lower than those calculated in Refs. [5,10]. The neutron-to-fission-width ratio for zero angular momentum $(\Gamma_n/\Gamma_f)_0$ calculated by using Eq. (13) is of the order of 10^{-4}–10^{-1} for the compound nuclei in question. The values for the nuclear-structure-dependent quantities Q and S_n for reactions listed in Table II are given in Refs. [5,10]. The values for the quantity B_f for even-even compound nuclei are also listed in Ref. [5]. In the present calculation, the fission-barrier heights for odd compound nuclei smaller by 0.5 MeV than those listed in Ref. [10] are used. This is because, in the calculation of $(\Gamma_n/\Gamma_f)_l$, the minimal fission-barrier heights B_f and B_f^{ER} for odd nuclear systems should be used. In the original model [5,10,36], we used higher B_f and B_f^{ER} for odd nuclei because of the assumed specialization energy of 0.5 MeV. The latter fission-barrier heights are more reliable for the description of the spontaneous fission rather than for the description of disintegration of the compound nucleus.

In the framework of the macroscopic-microscopic model [19–21], we predicted in Ref. [6] α-decay energies and half-lives for the nuclei in the decay chain of ^{293}118. Later on, this decay chain was discussed by the other authors in Ref. [37] by using the Skyrme-Hartree-Fock-Bogoliubov method and in Ref. [38] in the relativistic-mean-field model. Again, in the framework of the macroscopic-microscopic model [19–21], we predicted α-decay chains that might be initiated by the nuclei ^{292}118 and ^{294}119 in Refs. [10,6], respectively. In Ref. [10], we also discussed α-decay chains that might be initiated by ^{270}110 and ^{276}112.

C. Predictions for elements 119, 120, and 121

Table III contains relatively large cross section calculated for the reactions that might lead to the transitional isotopes of so far undiscovered elements 119, 120, and 121. This is again due to the magic of the reaction partners, which leads to larger Q value and, consequently, lower effective fusion barrier with larger transmission probability. In comparison with the previous study [36], we obtain the formation cross sections of ^{294}119 by about two orders of magnitude smaller. For ^{208}Pb(^{87}Rb, 1n)^{294}119, we predict the cross section of 2.7 pb that is by a factor of 1.6 smaller than that calculated for the reaction ^{209}Bi(^{86}Kr, 1n)^{294}119 that might lead to the same isotope of element 119. The reason for this is the larger effective fusion barrier for ^{208}Pb+^{87}Rb in comparison with that for ^{209}Bi+^{86}Kr. The predicted formation cross sections of 4.4 pb, 1.6 pb, and 1.9 pb for ^{209}Bi(^{86}Kr, 1n)^{294}119, ^{208}Pb(^{88}Sr, 1n)^{295}120, and

TABLE III. The optimal bombarding energy E_{lab}, the Q value, the height of the fusion barrier B_{fu}, the transmission probability through the fusion barrier for zero angular momentum T_0, the neutron-separation energy S_n and the static fission-barrier height B_f for the compound nucleus, the neutron-to-fission-width ratio for zero angular momentum $(\Gamma_n/\Gamma_f)_0$, and the cross section σ_{1n} calculated for fusion reactions involving magic reaction partners which might lead to the synthesis of elements 119, 120, and 121. In the calculation of $(\Gamma_n/\Gamma_f)_l$, the minimal (with no specialization energy) fission-barrier heights B_f and B_f^{ER} for odd and odd-odd nuclei are used.

Reaction	E_{lab} (MeV)	Q (MeV)	B_{fu} (MeV)	T_0	S_n (MeV)	B_f (MeV)	$(\Gamma_n/\Gamma_f)_0$	σ_{1n}
^{208}Pb(^{87}Rb,1n)^{294}119	462.9	313.47	339.37	1.1×10^{-6}	8.00	5.45	3.0×10^{-4}	2.7 pb
^{209}Bi(^{86}Kr,1n)^{294}119	453.9	308.67	334.04	1.7×10^{-6}	8.00	5.45	3.0×10^{-4}	4.4 pb
^{208}Pb(^{88}Sr,1n)^{295}120	478.6	323.14	350.06	6.1×10^{-7}	8.25	5.34	3.1×10^{-4}	1.6 pb
^{209}Bi(^{88}Sr,1n)^{296}121	485.2	328.22	354.20	1.4×10^{-6}	8.32	5.34	1.7×10^{-4}	1.9 pb
^{208}Pb(^{89}Y,1n)^{296}121	492.2	331.48	359.19	3.4×10^{-7}	8.32	5.34	1.7×10^{-4}	460 fb

^{209}Bi(^{88}Sr, 1n)^{296}121, respectively, suggest a good chance for the synthesis of superheavy elements 119, 120, and 121 in these reactions.

D. Predictions for symmetric reactions

Our results suggest that it is possible to carry out symmetric reactions involving magic nuclei by using the present-day experimental technique. Although shell effects in the magic nuclei ^{124}Sn, ^{136}Xe, ^{138}Ba, and ^{140}Ce are not as strong as in ^{208}Pb and ^{209}Bi, they act in both the target and the projectile and lead to the prediction of measurable cross sections of the reactions ^{136}Xe(^{124}Sn, 1n)^{259}Rf, ^{136}Xe(^{136}Xe, 1n)^{271}Hs, ^{138}Ba(^{136}Xe, 1n)^{273}110 and ^{140}Ce(^{136}Xe, 1n)^{275}112, which are collected in Table IV. Some of these reactions might be more useful for producing deformed superheavy nuclei than the so-called hot fusion reactions (very asymmetric reactions with the evaporation of several particles). For example, we predict the cross section of 27 pb for the symmetric reaction ^{138}Ba(^{136}Xe, 1n)^{273}110 that might lead to the deformed nucleus ^{273}110. The synthesis of this nucleus at JINR-Dubna in the hot fusion reaction ^{244}Pu(^{34}S, 5n)^{273}110 with the measured cross section of 0.4 pb has been reported in Ref. [39]. This value is almost 70 times smaller than that obtained in the present paper for ^{138}Ba(^{136}Xe, 1n)^{273}110.

We predict that the most promising symmetric reaction for producing transactinide nuclei is ^{136}Xe(^{136}Xe, 1n)^{271}Hs with the calculated cross section of 170 pb.

We obtain the cross section of 1.0 pb for ^{142}Ce(^{136}Xe, 1n)^{277}112 (E_{lab}=656.7 MeV). This means that the use of the heaviest stable isotope of cerium, ^{142}Ce, instead of magic ^{140}Ce, decreases the cross section by a factor of 2.9 (cf. Table IV).

From the present study, we draw the conclusion that the calculated cross section of the reaction ^{208}Pb(^{70}Zn, 1n)^{277}112 is by a factor of 3.2 larger than that calculated for the symmetric reaction ^{142}Ce(^{136}Xe, 1n)^{277}112 that may lead to the same nucleus. This conclusion is in sharp contrast to the suggestion made by the authors of Ref. [40] based on a very recent qualitative concept of ‘‘unshielded fusion’’ that the use of ^{142}Ce(^{136}Xe, 1n)^{277}112 should give orders-of-magnitude better chance for producing ^{277}112 in comparison with the reaction ^{208}Pb(^{70}Zn, 1n)^{277}112 carried out at GSI.

One should keep in mind, however, that heavy-ion fusion is far more complicated than a one-dimensional tunneling model suggests as it is known from subbarrier-fusion excitation-functions. For symmetric and nearly symmetric entrance channels, the c parameter may not necessarily be as

TABLE IV. The same quantities as in Table III calculated for symmetric reactions based on magic reaction partners. In the calculation of $(\Gamma_n/\Gamma_f)_l$, the minimal (with no specialization energy) fission-barrier height B_f^{ER} for odd 1n-evaporation residues is used. We obtain the cross section of 1.0 pb for ^{142}Ce(^{136}Xe, 1n)^{277}112 (E_{lab}=656.7 MeV). This means that the use of the heaviest stable isotope of cerium, ^{142}Ce, instead of magic ^{140}Ce, decreases the cross section by a factor of 2.9. Exchanging targets with projectiles leads to larger optimal bombarding energy. We obtain E_{lab}=603.8 MeV, 656.6 MeV, 684.5 MeV, and 685.7 MeV for ^{124}Sn(^{136}Xe, 1n)^{259}Rf, ^{136}Xe(^{138}Ba, 1n)^{273}110, ^{136}Xe(^{140}Ce, 1n)^{275}112, and ^{136}Xe(^{142}Ce, 1n)^{277}112, respectively. The values of the other quantities remain unchanged.

Reaction	E_{lab} (MeV)	Q (MeV)	B_{fu} (MeV)	T_0	S_n (MeV)	B_f (MeV)	$(\Gamma_n/\Gamma_f)_0$	σ_{1n}
^{136}Xe(^{124}Sn,1n)^{259}Rf	550.5	273.52	307.03	1.2×10^{-8}	7.57	6.62	2.7×10^{-1}	24 pb
^{136}Xe(^{136}Xe,1n)^{271}Hs	628.4	300.63	329.58	9.5×10^{-8}	7.06	5.88	2.7×10^{-1}	170 pb
^{138}Ba(^{136}Xe,1n)^{273}110	647.1	312.80	341.68	5.8×10^{-8}	7.56	5.19	6.8×10^{-2}	27 pb
^{140}Ce(^{136}Xe,1n)^{275}112	664.9	324.71	353.68	2.8×10^{-8}	8.09	4.42	1.5×10^{-2}	2.9 pb

good as for asymmetric channels. Whether one parameter c for all asymmetries in the entrance channel is a sufficiently broad cover for all the physics left out from the model, remains to be seen.

E. Estimated uncertainty of the obtained results

In the present paper, we introduced both more realistic fusion barrier and survival probability of the compound nucleus in comparison with our original model [5,10,36]. We reproduced the measured formation cross sections of transactinides and heavy actinides synthesized in reactions with the emission of only one neutron and indicated the most promising target-projectile-energy combinations for producing transactinide (superheavy) elements. Since our model depends on the input quantities, in particular on Q-value, neutron-separation energy S_n and fission-barrier height B_f for the compound nucleus, the use of different input quantities than those taken from Refs. [19–21] may change the results. An increase (decrease) of Q value by 1 MeV increases (decreases) the formation cross section about 1.7–2.2 times. An increase (decrease) of the height of the fission barrier by 0.5 MeV increases (decreases) usually the formation cross section by a factor of about 1.5–1.6 while the same increase (decrease) of the neutron-separation energy decreases (increases) usually the cross section by a factor of about 1.2–1.3. In the present study, the influence of vibrational excitations on the value of $(\Gamma_n/\Gamma_f)_l$ for spherical and transitional nuclei has not been taken into account. The collective enhancement factor k_{eq} for such nuclei may have a value from the range of 1 to 10 [24]. This in turn may increase the formation cross section for spherical and transitional nuclei listed in Table II (nuclei heavier than $^{282}114$) and Table III by a factor of 1–10.

ACKNOWLEDGMENTS

I am especially grateful to D.C. Hoffman and V. Ninov for many discussions that prompted this study. I am also very thankful to A. Ghiorso, K.E. Gregorich, S. Hofmann, C.M. Lyneis, Yu.Ts. Oganessian, M. Redlich, J. Srebrny, and W.J. Świątecki for valuable discussions and helpful suggestions. This work was supported by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of Nuclear Physics under U.S. Department of Energy Contract No. DE-AC03-76SF00098 and Grant of the Polish Committee for Scientific Research (KBN) No. 2 P03B 099 15.

[1] Yu.Ts. Oganessian, *Classical and Quantum Mechanical Aspects of Heavy Ion Collisions*, Lecture Notes in Physics, Vol. 33 (Springer, Heidelberg, 1975), p. 221.
[2] Yu.Ts. Oganessian, A.S. Iljinov, A.G. Demin, and S.P. Tretyakova, Nucl. Phys. **A239**, 353 (1975).
[3] G. Münzenberg, Rep. Prog. Phys. **51**, 57 (1988), and references therein.
[4] S. Hofmann, Rep. Prog. Phys. **61**, 639 (1998), and references therein.
[5] R. Smolańczuk, Phys. Rev. C **59**, 2634 (1999).
[6] R. Smolańczuk, Phys. Rev. C **60**, 021301(R) (1999).
[7] V. Ninov, K.E. Gregorich, W. Loveland, A. Ghiorso, D.C. Hoffman, D.M. Lee, H. Nitsche, W.J. Świątecki, U.W. Kirbach, C.A. Laue, J.L. Adams, J.B. Patin, D.A. Shaughnessy, D.A. Strellis, and P.A. Wilk, Phys. Rev. Lett. **83**, 1104 (1999).
[8] W.J. Świątecki, Phys. Scr. **24**, 113 (1981).
[9] W.J. Świątecki, Nucl. Phys. **A376**, 275 (1982).
[10] R. Smolańczuk, Phys. Rev. Lett. **83**, 4705 (1999).
[11] V. Ninov (private communication).
[12] H.W. Gäggeler, D.T. Jost, A. Türler, P. Armbruster, W. Brüchle, H. Folger, F.P. Hessberger, S. Hofmann, G. Münzenberg, V. Ninov, W. Reisdorf, M. Schädel, K. Sümmerer, J.V. Kratz, U. Scherer, and M.E. Leino, Nucl. Phys. **A502**, 561c (1989).
[13] H. Gauvin, Y. Le Beyec, M. Lefort, and C. Deprun, Phys. Rev. Lett. **28**, 697 (1972).
[14] C.Y. Wong, Phys. Lett. **42B**, 186 (1972).
[15] B. Nerlo-Pomorska and K. Pomorski, Z. Phys. A **348**, 169 (1994).
[16] R.W. Hasse and W.D. Myers, *Geometrical Relationships of Macroscopic Nuclear Physics* (Springer-Verlag, Berlin, 1988).
[17] S. Hofmann, V. Ninov, F.P. Hessberger, P. Armbruster, H. Folger, G. Münzenberg, H.J. Schött, A.G. Popeko, A.V. Yeremin, A.N. Andreyev, S. Saro, R. Janik, and M. Leino, Z. Phys. A **350**, 277 (1995).
[18] W.E. Parker, M. Kaplan, D.J. Moses, G. La Rana, D. Logan, R. Lacey, J.M. Alexander, D.M. de Castro Rizzo, P. DeYoung, R.J. Welberry, and J.T. Boger, Phys. Rev. C **44**, 774 (1991).
[19] R. Smolańczuk, Phys. Rev. C **56**, 812 (1997).
[20] R. Smolańczuk, Ph.D. thesis, Soltan Institute for Nuclear Studies, Warsaw, 1996.
[21] R. Smolańczuk, J. Skalski, and A. Sobiczewski, Phys. Rev. C **52**, 1871 (1995).
[22] A.N. Andreyev, D.D. Bogdanov, V.I. Chepigin, A.P. Kabachenko, O.N. Malyshev, Yu.Ts. Oganessian, A.G. Popeko, J. Roháč, R.N. Sagaidak, A.V. Taranenko, G.M. Ter-Akopian, and A.V. Yeremin, Report No. JINR E7-94-378, 1994; *Proceedings of the Workshop on Heavy-Ion Fusion: Exploring the Variety of Nuclear Properties*, Padova, Italy, 1994 (World Scientific, Singapore, 1994), p. 260.
[23] S. Bjornholm, A. Bohr, and B.R. Mottelson, *Proceedings of the Symposium on Physics and Chemistry of Fission*, Rochester, 1973 (IAEA, Vienna, 1974), Vol. 1, p. 367.
[24] A.R. Junghans, M. de Jong, H.-G. Clerc, A.V. Ignatyuk, G.A. Kudyaev, and K.-H. Schmidt, Nucl. Phys. **A629**, 635 (1998).
[25] B. Vandenbosch and J.R. Huizenga, *Nuclear Fission* (Academic, New York, 1973).
[26] V.E. Viola, Jr. and B.D. Wilkins, Nucl. Phys. **82**, 65 (1966).
[27] H. Hagelund and A.S. Jensen, Phys. Scr. **15**, 225 (1977).
[28] A. Gilbert and A. Cameron, Can. J. Phys. **43**, 1446 (1965).
[29] G. Audi, O. Bersillon, J. Blachot, and A.H. Wapstra, Nucl. Phys. **A624**, 1 (1997).
[30] F.P. Hessberger, G. Münzenberg, S. Hofmann, W. Reisdorf, K.H. Schmidt, H.J. Schött, P. Armbruster, R. Hingmann, B.

Thuma, and D. Vermeulen, Z. Phys. A **321**, 317 (1985).
[31] G. Münzenberg, S. Hofmann, H. Folger, F.P. Hessberger, J. Keller, K. Poppensieker, B. Quint, W. Reisdorf, K.-H. Schmidt, H.J. Schött, P. Armbruster, M.E. Leino, and R. Hingmann, Z. Phys. A **322**, 227 (1985).
[32] G. Münzenberg, P. Armbruster, G. Berthes, H. Folger, F.P. Hessberger, S. Hofmann, J. Keller, K. Poppensieker, A.B. Quint, W. Reisdorf, K.H. Schmidt, H.J. Schött, K. Sümmerer, I. Zychor, M.E. Leino, R. Hingmann, U. Gollerthan, and E. Hanelt, Z. Phys. A **328**, 49 (1987).
[33] C.C. Sahm, H.-G. Clerc, K.-H. Schmidt, W. Reisdorf, P. Armbruster, F.P. Hessberger, J.G. Keller, G. Münzenberg, and D. Vermeulen, Nucl. Phys. **A441**, 316 (1985).
[34] J.G. Keller, K.-H. Schmidt, F.P. Hessberger, G. Münzenberg, W. Reisdorf, H.G. Clerc, and C.-C. Sahm, Nucl. Phys. **A452**, 173 (1986).
[35] G.G. Adamian, N.V. Antonenko, and W. Scheid, Nucl. Phys. **A678**, 24 (2000).
[36] R. Smolańczuk, Phys. Rev. C **61**, 011601(R) (2000).
[37] S. Ćwiok, W. Nazarewicz, and P.H. Heenen, Phys. Rev. Lett. **83**, 1108 (1999).
[38] M. Bender, Phys. Rev. C **61**, 031302(R) (2000).
[39] Yu.A. Lazarev, Yu.V. Lobanov, Yu.Ts. Oganessian, V.K. Utyonkov, F.Sh. Abdullin, A.N. Polyakov, J. Rigol, I.V. Shirokovsky, Yu.S. Tsyganov, S. Iliev, V.G. Subbotin, A.M. Sukhov, G.V. Buklanov, B.N. Gikal, V.B. Kutner, A.N. Mezentsev, K. Subotic, J.F. Wild, R.W. Lougheed, and K.J. Moody, Phys. Rev. C **54**, 620 (1996).
[40] W.D. Myers and W.J. Świątecki, Phys. Rev. C **62**, 044610 (2000).