Approach for the three-body force effect in a high-energy approximation: Application to hadron-deuteron elastic scattering

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In the framework of the Glauber approximation, using a simple suggested approach for the three-body force effect and taking into account *D*-state and phase variation effects, p-, $\bar{p}$-, and π^{-}-d elastic scattering differential cross sections at different energies are calculated. In general, one or two effects only from the three used effects are not enough to obtain a good fit with the data. Thus, with the *D*-state effect as a principal correction, the three-body force effect plays a role in the scattering process. The reality of the three-body force effect with a small contribution can be accepted to obtain a good fit with the experimental data.

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I. INTRODUCTION

In principle, scattering processes should provide more information than bound states in the search for evidence of three-body force effect [1]. Many authors are concerned with this subject at different energies. At low energy, in Ref. [2] the small discrepancy between the neutron-neutron scattering lengths for the $d(\pi^{-},\gamma)2n$ and $d(n,2n)p$ reactions is interpreted as a result of three-nucleon force effect in the final state of the latter reaction. The improved results for nucleon-deuteron scattering lengths have been obtained by Chen [3] and Osman [4] including the three-body force. Osman in Ref. [4] concluded that the three-body forces which arise from the different meson exchanges are very important, and their inclusion improved all calculations on the three-body system. A new approach to include a three-nucleon force into three-nucleon continuum calculations was presented in Ref. [5] with the results for elastic nucleon-deuteron scattering as well as for the break up process. Neutron-deuteron elastic scattering cross sections were calculated in Ref. [6] at different low energies using modern nucleon-nucleon interactions and the Tucson-Melbourne three-nucleon force. Predictions based on *NN* forces only underestimate nucleon-deuteron data in the minima at energies around 60 MeV. Adding the three-nucleon force fills up those minima and reduces the discrepancies significantly. Three-nucleon forces have been considered in Refs. [7,8] to resolve the discrepancy between experimental and theoretical calculations of the nucleon analyzing power in low-energy nucleon-deuteron scattering. A noticeable improvement in the description of the polarization observables has been obtained in Ref. [8]. On the other hand, in the Faddeev calculations of the $^{2}\mathrm{H}(p,pp)n$ reaction at 14.1 MeV [9] and in the study of *p-d* breakup at 25 MeV [10], the evidence for three-body forces is not entirely clear. In Ref. [11] the inclusion of the Urbana three-nucleon interaction does not significantly modify the calculated analyzing powers in deuteron-proton scattering at low energies. Also, Friar in Ref. [12] concluded that the three-nucleon forces nevertheless play an important role in nuclear physics. At higher energies, authors in Refs. [13,14] from the analysis of spin observables in *p-d* scattering at 800 MeV, show that the inclusion of the three-body force improved the agreement with the data markedly. Also, the three-body forces of the contact interaction's type are required by the high-energy asymptotics of the amplitudes involved Ref. [14]. Primakoff [15] investigated the dynamics of the three-nucleon system in exclusive meson production in *p-d* scattering between 1.5 and 3 GeV. Three-body forces dominate the dynamics of the three-nucleon system at very large momentum transfer ($\geqslant 1$ GeV/c). In Ref. [16] the authors concluded that the effect of the three-body force between nucleons in ^{3}He nucleus on p-^{3}He elastic scattering differential cross section and total cross section at intermediate and high energies is very small and, in general, can be neglected. Its effect is significant only in two small regions, about the first and second minima of the differential cross section.

Thus, different conclusions about the three-body forces role are obtained at different energies by using different approaches. Therefore, more study about the three-body force effect is needed. Also, a general formulation of the three-body force effect in the Glauber theory is absent. Therefore, in this paper, we will be concerned with the three-body force effect, using a simple suggested approach, in the framework of Glauber high-energy approximation. We will apply this approach to the more simple three-body case of hadron-deuteron scattering, where the elastic scattering differential cross section is calculated. In this case, the spin-isospin effects are important at lower energy [17–20]. Also, the ratio of the real part to the imaginary part of hadron-nucleon amplitude in the forward direction, which plays an important role in the minimum region, can be considered as an experimental parameter and at a definite energy must take an experimental value. Therefore, for more accurate calculations and to obtain a realistic estimation of the three-body force role, we will also consider, only, two well-known effects which are important in the study of hadron-deuteron elastic scattering at high energy. The first is the *D*-state effect [21–24] and the second is the effect of phase variation of hadron-nucleon elastic scattering amplitude [25,26]. These two effects play an important role in the minimum region of hadron-deuteron elastic scattering differential cross section.

II. MANY-BODY FORCE EFFECTS IN GLAUBER APPROXIMATION—GENERAL FORMALISM

The potential energy of hadron-nucleus system, taking into account the many-body forces, can be written, in general, in the form [27]

$$V(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=V_t+V_{\mathrm{th}}+V_{\mathrm{fo}}+\cdots, \tag{1}$$

where $\mathbf{r}$ is the position vector of the incident particle, $\mathbf{r}_i$, $i=1,\ldots,A$ are the position vectors of the nucleons in the target nucleus, V_t is the two-body force component, V_{th} is the three-body force component, V_{fo} is the four-body force component, and so on. We believe that this is an approximate formula, where the corrections due to many-body forces are considered as additional terms. Also, the two-body force component V_t is written—in high-energy approximation—as

$$V_t=\sum_{i=1}^{A} V_i(\mathbf{r},\mathbf{r}_i), \tag{2}$$

where $V_i(\mathbf{r},\mathbf{r}_i)$ is the potential energy of the incident particle-ith nucleon target. Here, we note that the overlap effect between the different scattering centers is neglected [28]. In the same way, we can write the three-body force component as follows:

$$V_{\mathrm{th}}=\sum_{\substack{i,j\\ i\neq j}} V_{ij}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j), \tag{3}$$

where $V_{ij}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j)$ is the three-body force correction to $V_i(\mathbf{r},\mathbf{r}_i)$ due to the existence of jth nucleon as a spectator. Also,

$$V_{\mathrm{fo}}=\sum_{\substack{i,j,k\\ i\neq j\neq k}} V_{ijk}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j;\mathbf{r}_k), \tag{4}$$

where $V_{ijk}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j;\mathbf{r}_k)$ is the four-body force correction to V_i which is due to the existence of two spectators jth and kth nucleons, and so on. Therefore, the general form of the potential energy V can be written as

$$V(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=\sum_{i=1}^{A}\Bigg\{V_i(\mathbf{r},\mathbf{r}_i)+\sum_{\substack{j\\ j\neq i}} V_{ij}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j) + \sum_{\substack{j,k\\ i\neq j\neq k}} V_{ijk}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j;\mathbf{r}_k)+\cdots\Bigg\}. \tag{5}$$

This additive form of the potential leads to the additive form of the total phase shift for the hadron-nucleus scattering, which, in high-energy approximation, is defined as

$$\chi(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=-\frac{1}{\hbar v}\int_{-\infty}^{+\infty}V(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)dz, \tag{6}$$

where v is the relative velocity of the incident particle with respect to the target, $\mathbf{s}_i$ is the projection of $\mathbf{r}_i$ on the impact plane, and $\mathbf{r}=\mathbf{b}+z\hat{\mathbf{k}}$ where $\hat{\mathbf{k}}$ is the unit vector in the direction of the z axis, which is, usually, taken in the incident direction of the projectile and $\mathbf{b}$ is the impact vector which is perpendicular to the incident direction. Using Eq. (5) we have

$$\begin{aligned}\chi(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A) &= \sum_{i=1}\Bigg\{-\frac{1}{\hbar v}\int_{-\infty}^{\infty}V_i(\mathbf{r},\mathbf{r}_i)dz\\ &\quad+\sum_{\substack{j\\ j\neq i}}\left(-\frac{1}{\hbar v}\int_{-\infty}^{\infty}V_{ij}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j)dz\right)\\ &\quad+\sum_{\substack{j,k\\ i\neq j\neq k}}\left(-\frac{1}{\hbar v}\int_{-\infty}^{\infty}V_{ijk}(\mathbf{r},\mathbf{r}_i;\mathbf{r}_j;\mathbf{r}_k)dz\right)+\cdots\Bigg\},\end{aligned} \tag{7}$$

which can be written as

$$\chi(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\sum_{i=1}\Bigg\{\chi_i(\mathbf{b},\mathbf{s}_i)+\sum_{\substack{j\\ j\neq i}}\chi_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j) + \sum_{\substack{j,k\\ i\neq j\neq k}}\chi_{ijk}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j;\mathbf{s}_k)+\cdots\Bigg\}, \tag{8}$$

where $\chi_i(\mathbf{b},\mathbf{s}_i)$ is the two-body phase shift of the incident particle-ith target nucleon scattering, $\chi_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)$ is the three-body force correction to χ_i which is due to the existence of the jth nucleon as a spectator, $\chi_{ijk}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j;\mathbf{s}_k)$ is the four-body force correction to χ_i due to existence of jth and kth nucleons as spectators, and so on. Of course, the total hadron-nucleus phase shift in the above equation can be written in the form

$$\chi(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\chi_t(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)+\chi_{\mathrm{th}}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)+\chi_{\mathrm{fo}}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)+\cdots, \tag{9}$$

where χ_t is the total two-body phase shift, χ_{th} is the total three-body force correction, χ_{fo} is the total four-body force correction, and so on.

The total hadron-nucleus profile function in the Glauber formalism is given by [28]

$$\Gamma(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=1-\exp\{i\chi(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\}. \tag{10}$$

Using Eq. (9), we can write this profile function in the form

$$\Gamma(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=1-\exp\{i(\chi_t+\chi_{\mathrm{th}}+\chi_{\mathrm{fo}}+\cdots)\}, \tag{11}$$

where the effects of many-body forces are contained. Also, the two body profile function is defined as

$$\Gamma_t(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=1-\exp\{i\chi_t(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\}. \tag{12}$$

In the same way, we will define the three-body force profile function, four-body force profile function, and other many-body force profile functions as

$$\Gamma_{\rm th}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=1-\exp\{i\chi_{\rm th}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\},\qquad(13)$$

$$\Gamma_{\rm fo}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=1-\exp\{i\chi_{\rm fo}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\},\qquad(14)$$

and so on. These profile functions are related, from their definition, to two-body interaction, three-body force component of interaction, and so on. Thus, from Eqs. (11)–(14), we have

$$\begin{aligned}\Gamma&=1-(1-\Gamma_t)(1-\Gamma_{\rm th})(1-\Gamma_{\rm fo})\cdots\\&=\Gamma_t+\Gamma_{\rm th}+\Gamma_{\rm fo}+\cdots-(\Gamma_t\Gamma_{\rm th}+\Gamma_t\Gamma_{\rm fo}+\cdots+\Gamma_{\rm th}\Gamma_{\rm fo}+\cdots)\\&\quad+(\Gamma_t\Gamma_{\rm th}\Gamma_{\rm fo}+\cdots)-\cdots.\end{aligned}\qquad(15)$$

But, from Eq. (8),

$$\chi_t(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\sum_i \chi_i(\mathbf{b},\mathbf{s}_i),$$

then

$$\begin{aligned}\Gamma_t(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)&=1-\exp\left\{i\sum_i \chi_i(\mathbf{b},\mathbf{s}_i)\right\}\\&=1-\prod_{i=1}^{A}[1-\Gamma_i(\mathbf{b},\mathbf{s}_i)]\\&=\sum_{i=1}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)-\sum_{i>j}\Gamma_i(\mathbf{b},\mathbf{s}_i)\Gamma_j(\mathbf{b},\mathbf{s}_j)\\&\quad+\cdots+(-1)^{(A+1)}\Gamma_1(\mathbf{b},\mathbf{s}_1)\cdots\Gamma_A(\mathbf{b},\mathbf{s}_A),\end{aligned}\qquad(16)$$

where $\Gamma_i(\mathbf{b},\mathbf{s}_i)$ is the incident particle-ith target nucleon profile function. The first term represents the single scattering processes, the second term represents the double scattering processes, and so on. Also

$$\begin{aligned}\Gamma_{\rm th}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)&=1-\exp\left\{i\sum_{i\neq j}\chi_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\right\}\\&=1-\prod_{i\neq j}[1-\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)]\\&=\sum_{i\neq j}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\\&\quad-\sum_{\substack{i<j\\i'>j'}}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\Gamma_{i'j'}(\mathbf{b},\mathbf{s}_{i'};\mathbf{s}_{j'})+\cdots,\end{aligned}\qquad(17)$$

where

$$\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)=1-\exp\{i\chi_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\}.$$

The same thing can be obtained for $\Gamma_{\rm fo}$ and higher order correction terms.

Since the profile function Γ_{ij} is not related to some kind of single scattering as Γ_i and the product $\Gamma_{ij}\Gamma_{i'j'}$ is not related to some kind of double scattering processes and so on, the terms of the last equation (17) cannot take the same physical meaning of terms in Eq. (16). The profile function Γ_{ij} is related to a correction to the hadron-ith target nucleon potential due to the existence of jth nucleon in the nucleus as a spectator through the scattering of the incident hadron on the ith nucleon. This corrected potential represents the hadron-nucleon interaction through the single scattering or double scattering and so on. Therefore, we can consider the first term in Eq. (17) as the first order three-body force correction, the second term as the second order three-body force correction, and so on. Since Γ_{ij} is, in general, small, the product $\Gamma_{ij}\Gamma_{i'j'}$ is of the second order and smaller than it. The same situation for $\Gamma_{\rm fo}$ and other profile functions of many-body force corrections.

Since we shall be concerned with the three-body force only, and the different phase shifts must satisfy the relation

$$|\chi_i|\gg|\chi_{ij}|\gg|\chi_{ijk}|\gg\cdots,\qquad(18)$$

we shall neglect the terms of $\Gamma_{\rm fo}$ and higher orders. The total profile function can be written, from Eq. (15), as

$$\Gamma=\Gamma_t+\Gamma_{\rm th}-\Gamma_t\Gamma_{\rm th}.\qquad(19)$$

Thus, from Eqs. (16) and (17) we have

TABLE I. Form factors parameters for Lacombe *et al.* [31] and Bressel *et al.* [33] deuteron wave functions.

Wave function	i	A_i	c_i $(\mathrm{GeV}/c)^{-2}$	B_i $(\mathrm{GeV}/c)^{-2}$	d_i $(\mathrm{GeV}/c)^{-2}$
Lacombe	1	−0.15469	4.7128	6.58579	50.13
et al. [31]	2	0.49497	20.9128	2.46887	15.58
	3	0.65972	109.59	0.76161	7.575
Bressel	1	−0.13863	4.3	6.138	71.3
et al. [33]	2	0.47869	22.0	3.555	20.99
	3	0.65994	105.56	0.911	7.99

$$\Gamma(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\sum_{i=1}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)-\sum_{i>j}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)\Gamma_j(\mathbf{b},\mathbf{s}_j)+\cdots+(-1)^{(A+1)}\Gamma_1(\mathbf{b},\mathbf{s}_1)\cdots\Gamma_A(\mathbf{b},\mathbf{s}_A)$$
$$+\Bigg\{\sum_{i\neq j}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)-\sum_{\substack{i<j\\ i'>j'}}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\Gamma_{i'j'}(\mathbf{b},\mathbf{s}_{i'};\mathbf{s}_{j'})+\cdots\Bigg\}-\Bigg\{\sum_{i=1}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)-\sum_{i>j}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)\Gamma_j(\mathbf{b},\mathbf{s}_j)+\cdots$$
$$+(-1)^{(A+1)}\Gamma_1(\mathbf{b},\mathbf{s}_1)\cdots\Gamma_A(\mathbf{b},\mathbf{s}_A)\Bigg\}\Bigg\{\sum_{i\neq j}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)-\sum_{\substack{i<j\\ i'>j'}}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\Gamma_{i'j'}(\mathbf{b},\mathbf{s}_{i'};\mathbf{s}_{j'})+\cdots\Bigg\}. \tag{20}$$

The single scattering terms in this equation are

$$\Gamma^s(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\Bigg\{\sum_{i=1}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)\Bigg\}\Bigg\{1-\Bigg[\sum_{i\neq j}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)-\sum_{\substack{i<j\\ i'>j'}}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\Gamma_{i'j'}(\mathbf{b},\mathbf{s}_{i'};\mathbf{s}_{j'})+\cdots\Bigg]\Bigg\}, \tag{21}$$

i.e.,

$$\Gamma^s(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\Gamma_t^s(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\{1-\Gamma_{\text{th}}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\}, \tag{22}$$

where $\Gamma_t^s(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\Sigma_{i=1}^{A}\Gamma_i(\mathbf{b},\mathbf{s}_i)$. The double-scattering terms are

$$\Gamma^d(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\Bigg\{-\sum_{i>j}\Gamma_i(\mathbf{b},\mathbf{s}_i)\Gamma_j(\mathbf{b},\mathbf{s}_j)\Bigg\}\times\Bigg\{1-\Bigg[\sum_{i\neq j}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)-\sum_{\substack{i<j\\ i'>j'}}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)\Gamma_{i'j'}(\mathbf{b},\mathbf{s}_{i'};\mathbf{s}_{j'})+\cdots\Bigg]\Bigg\}, \tag{23}$$

i.e.,

$$\Gamma^d(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\Gamma_t^d(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\{1-\Gamma_{\text{th}}(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\}, \tag{24}$$

where $\Gamma_t^d(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=-\Sigma_{i>j}\Gamma_i(\mathbf{b},\mathbf{s}_i)\Gamma_j(\mathbf{b},\mathbf{s}_j)$. In general, the multiple-scattering term is

$$\Gamma^m(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)=\Gamma_t^m(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)\{1-\Gamma_{\text{th}}(\mathbf{b},\mathbf{s}_1,\ldots\mathbf{s}_A)\}, \tag{25}$$

where $\Gamma_t^m(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_A)$ is multiple-scattering term of m order without three-body force correction.

The second term of Eq. (19) Γ_{th} can be considered as a general correction term. But, $\Gamma_{\text{th}}=1-\exp(i\chi_{\text{th}})$ and with very small values of χ_{th} we get $\exp(i\chi_{\text{th}})\sim 1$. Then, $\Gamma_{\text{th}}\approx 0$ and can be neglected. Therefore, we can say that the factor $(1-\Gamma_{\text{th}})$ is the three-body force factor in the profile function. Thus, we can write, as an approximation, the total profile function in the form

$$\Gamma=\Gamma_t(1-\Gamma_{\text{th}}). \tag{26}$$

III. HADRON-DEUTERON SCATTERING

We will be concerned, in this work, with the simple case of hadron-deuteron scattering. The hadron-deuteron total profile function is given, from Eq. (19), by

$$\Gamma(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)=\Gamma_t(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)+\Gamma_{\text{th}}(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)\{1-\Gamma_t(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)\}, \tag{27}$$

where 1 and 2 denote the proton and neutron in the deuteron, respectively, and

$$\Gamma_t(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)=\Gamma_1(\mathbf{b},\mathbf{s}_1)+\Gamma_2(\mathbf{b},\mathbf{s}_2)-\Gamma_1(\mathbf{b},\mathbf{s}_1)\Gamma_2(\mathbf{b},\mathbf{s}_2),$$
$$\Gamma_{\text{th}}(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)=\Gamma_{12}(\mathbf{b},\mathbf{s}_1;\mathbf{s}_2)+\Gamma_{21}(\mathbf{b},\mathbf{s}_2;\mathbf{s}_1)-\Gamma_{12}(\mathbf{b},\mathbf{s}_1;\mathbf{s}_2)\Gamma_{21}(\mathbf{b},\mathbf{s}_2;\mathbf{s}_1).$$

Through the hadron-ith target nucleon scattering process, it can be imagined that the incident hadron interacts with the target nucleon as well as another nucleon in the target nucleus interacts with the same ith nucleon, i.e., we have two-pion exchange processes. This situation seems to be two body interaction, but the exchange of two field particles, at the same time, leads—by the uncertainty relation of energy and time—to some effects on the hadron-ith target nucleon interaction, which can be considered as some kind of three-body force effect. This effect reduces the radius of hadron-ith nucleon interaction. Therefore, we try to use the profile function Γ_{ij} in the form

$$\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)=(A_{ij}+iB_{ij})\exp\left\{-\frac{1}{\gamma}(|\mathbf{b}-\mathbf{s}_i|^2+|\mathbf{s}_j-\mathbf{s}_i|^2)\right\}, \tag{28}$$

where A_{ij}, B_{ij}, and γ are constants. The parameter γ is related to three-body force radius such that γ

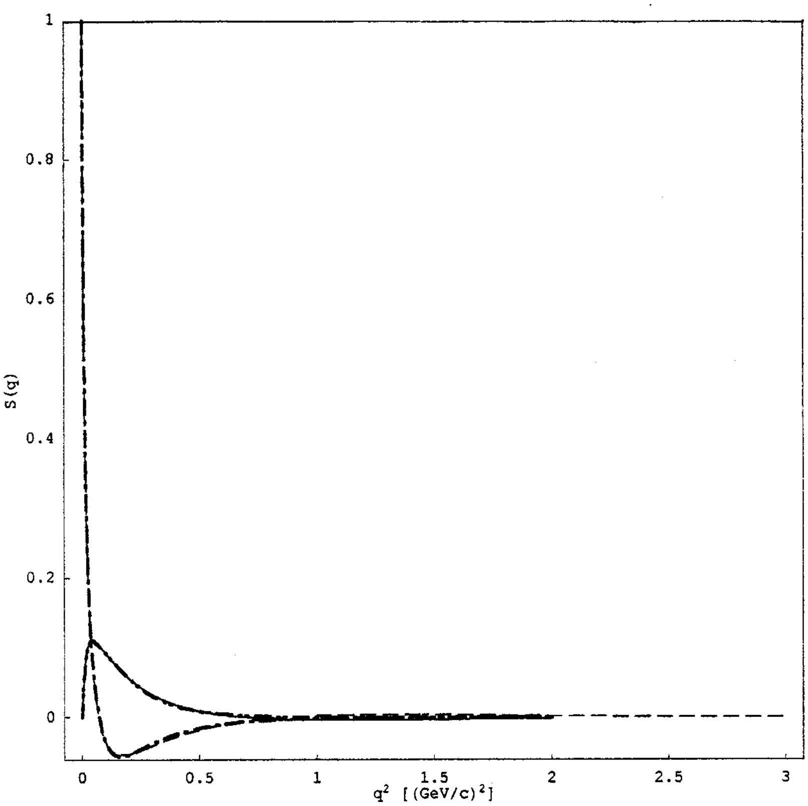

FIG. 1. Deuteron form factors $S_0(q)$ and $S_2(q)$. Dashed and solid curves correspond to the results of Eqs. (43) and (44), respectively, with the deuteron wave functions of Lacombe *et al.* [31]. Dot-dashed and dot-dot-dashed curves correspond to the results of Eqs. (45) and (46), respectively, with the first group of the parameters of Table I which are determined using solid and dashed curves as standard curves.

$\sim$(radius of the three-body force)^2. This phenomenological form is not obtained using a certain type of three-nucleon interaction, it is only a trial function. This profile function is dependent, simultaneously, on the coordinates of the three particles, $\mathbf{b}$, $\mathbf{s}_i$, and $\mathbf{s}_j$. At the same time, it is tends to zero if the distance between the target nucleon i and any one of both other two particles increases to the limit of three-body force radius.

The particle-particle scattering amplitude $f_i(\mathbf{q})$ for the two-body force component is related to $\Gamma_i(\mathbf{b}_i)$ by the relation

$$f_i(\mathbf{q}) = \frac{ik}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}}\Gamma_i(\mathbf{b}_i)d\mathbf{b}_i, \tag{29}$$

where k is the momentum of incident particle, and $\mathbf{q}$ is the two-dimensional momentum transfer vector. We will suggest the momentum representation of the function $\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)$ $[\equiv\Gamma_{ij}(\mathbf{b}_i,\mathbf{s}_{ij})$, where $\mathbf{b}_i=\mathbf{b}\text{-}\mathbf{s}_i$ and $\mathbf{s}_{ij}=\mathbf{s}_i-\mathbf{s}_j]$ in the form

$$F_{ij}(\mathbf{q},\mathbf{q}') = \frac{ik}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}_i + i\mathbf{q}'\cdot\mathbf{s}_{ij}}\Gamma_{ij}(\mathbf{b},\mathbf{s}_i;\mathbf{s}_j)d\mathbf{b}_i d\mathbf{s}_{ij}. \tag{30}$$

From Eqs. (28) and (30) we get

$$F_{ij}(\mathbf{q},\mathbf{q}') = \frac{ik\pi\gamma^2}{2}(A_{ij}+iB_{ij})e^{-(\gamma/4)(q^2+q'^2)}. \tag{31}$$

For the two-body profile function Γ_i, we shall use the form

$$\Gamma_i(\mathbf{b},\mathbf{s}_i) = \frac{(1-\alpha_i)\sigma_i}{4\pi(\beta_i^2+i\delta)}\exp\left\{-\frac{b_i^2}{2(\beta_i^2+i\delta)}\right\}. \tag{32}$$

This form is obtained, by the Fourier transformation, from the usual form with the phase variation effect as suggested by Franco [25], of two-body hadron-nucleon elastic scattering amplitude

TABLE II. Hadron-nucleon parameters.

	k, GeV/c	σ_p, mb	σ_n, mb	α_p	α_n	β_p^2 (GeV/c)$^{-2}$	β_n^2 (GeV/c)$^{-2}$	Ref.
p	1.75	47.5	40.4	−0.35	−0.800	5.8807	4.9819	[36]
	12.8	39.61	39	−0.27	−0.383	8.16	8.16	[36]
$\bar{p}$	2	89	94	0.128	0.128	13.84	13.84	[37]
π^-	9	26.9	25.3	−0.12	−0.23	8.5	8.5	[23]

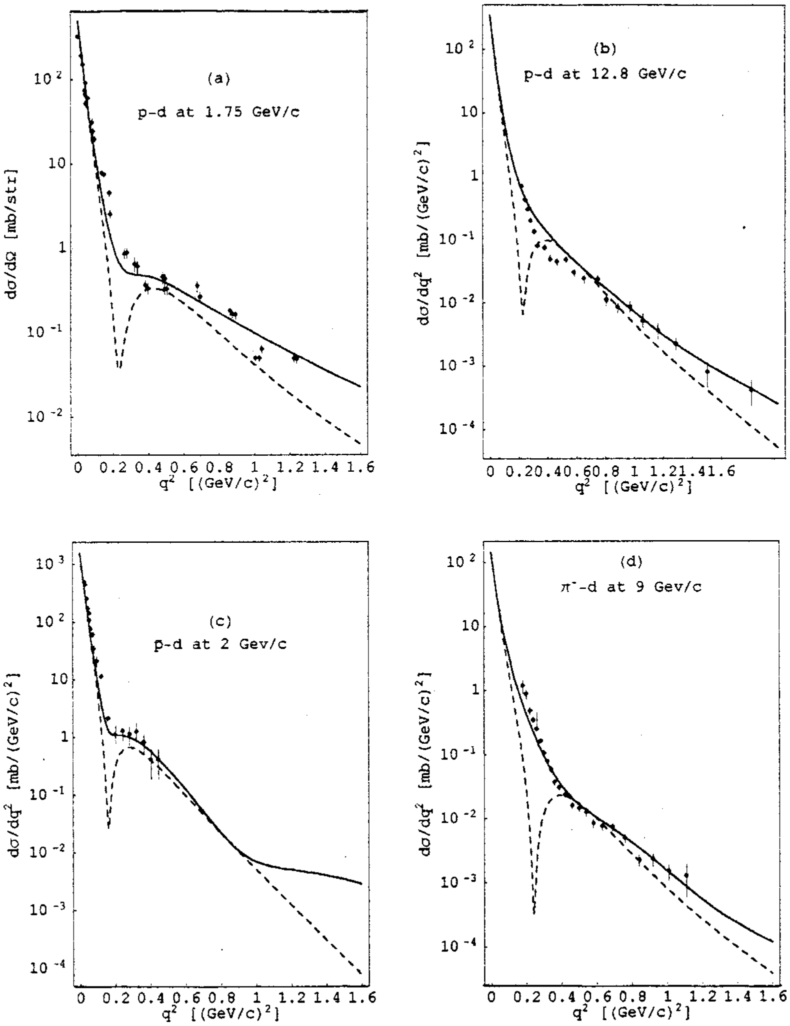

FIG. 2. p-, $\overline{p}$-, and π^--d elastic scattering differential cross sections. The solid and dashed curves correspond to the results with and without (three-body force+phase variation+D state) effects, respectively. The first group of the form factor parameters of Table I is used. The three-body force parameters and phase variation parameter are given in Tables III. The experimental data are taken from Refs. [38–40] and [23] for p-d scattering at 1.75 and 12.8 GeV/c, respectively, and from Refs. [41] and [23] for $\overline{p}$- and π^--d scattering, respectively.

$$f_i(\mathbf{q}) = \frac{k\sigma_i(i+\alpha_i)}{4\pi}\exp\{-(\beta_i^2 + i\,\delta)q^2/2\}, \tag{33}$$

where σ_i is the hadron-nucleon total cross section, α_i is the ratio of real part to imaginary part of the forward scattering amplitude, β_i^2 is the slope parameter, and δ is the phase variation parameter. This nucleon-nucleon amplitude can be considered, approximately, as an experimental form where all its parameters—except δ—are determined from hadron-nucleon experiment.

The hadron-deuteron elastic scattering amplitude, in Glauber high-energy approximation, is given by [29]

$$F(\mathbf{q}) = \frac{ik}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}}\langle i|\Gamma(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)\,\delta\!\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)|i\rangle d\mathbf{b}, \tag{34}$$

where $|i\rangle$ is the deuteron ground state wave vector. Using the coordinates

$$\mathbf{R} = \frac{\mathbf{r}_1+\mathbf{r}_2}{2}, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \tag{35}$$

we have

$$F(\mathbf{q}) = \frac{ik}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}}\langle i|\Gamma(\mathbf{b},\mathbf{s})\,\delta(\mathbf{R})|i\rangle d\mathbf{b}, \tag{36}$$

where $\mathbf{s}$ is the projection of $\mathbf{r}$ on the impact plane. We must note that the total profile function $\Gamma(\mathbf{b},\mathbf{s})$ is dependent only on $\mathbf{b}$ and $\mathbf{s}$ in terms of which the coordinates of the incident, target, and spectator particles are given in the deuteron case.

Thus, using the corrected profile function $\Gamma_t + \Gamma_{\mathrm{th}} - \Gamma_t\Gamma_{\mathrm{th}}$, we can obtain the following form for hadron-deuteron elastic scattering amplitude where the three-body forces are taken into account:

$$F(\mathbf{q}) = T_1(\mathbf{q}) + T_2(\mathbf{q}) + T_3(\mathbf{q}), \tag{37a}$$

where

TABLE III. The values of A, B, δ, and χ^2 for different collisions at used energies.

	K, GeV/c	A	B	δ	χ^2
p-d	1.75	1.35	1.65	1.5	1.06
	12.8	1.4	0.4	10	0.69
$\pi^- - d$	9	1.1	0.25	10	0.47
$\overline{p} - d$	2	1.6	−1.5	12	0.64

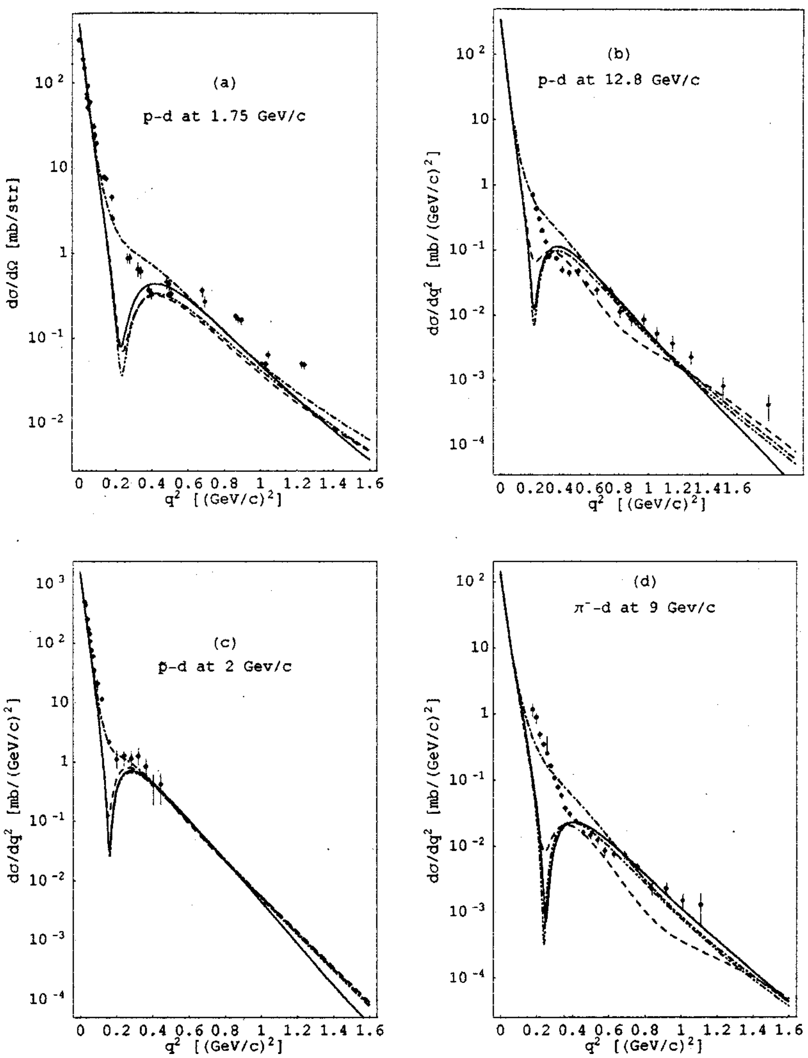

FIG. 3. Same as Fig. 2 except that solid, dashed, dot-dashed, and dot-dot-dashed curves correspond to the results with three-body force, phase variation, D-state effects, and without any effects, respectively.

$$T_1(\mathbf{q})=f_1(\mathbf{q})S\left(\frac{\mathbf{q}}{2}\right)+\frac{1}{(2\pi)^2}\int S(\mathbf{q}_1)F_{12}\left(\mathbf{q},\frac{\mathbf{q}}{2}-\mathbf{q}_1\right)d\mathbf{q}_1+\frac{i}{(2\pi)^3k}\int S(\mathbf{q}_1)f_1(\mathbf{q}_2)\left[F_{12}\left(\mathbf{q}-\mathbf{q}_2,\frac{\mathbf{q}}{2}-\mathbf{q}_1\right)\right.$$
$$\left.+F_{21}\left(\mathbf{q}-\mathbf{q}_2,\frac{\mathbf{q}}{2}+\mathbf{q}_1-\mathbf{q}_2\right)\right]d\mathbf{q}_1d\mathbf{q}_2-\frac{1}{(2\pi)^6k^2}\int S(\mathbf{q}_1)f_1(\mathbf{q}_3)F_{12}\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4,\mathbf{q}_2\right)$$
$$\times F_{21}\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1-\mathbf{q}_2+\mathbf{q}_4,\mathbf{q}_4\right)d\mathbf{q}_1d\mathbf{q}_2d\mathbf{q}_3d\mathbf{q}_4, \tag{37b}$$

$$T_2(\mathbf{q})=f_2(\mathbf{q})S\left(-\frac{\mathbf{q}}{2}\right)+\frac{1}{(2\pi)^2}\int S(\mathbf{q}_1)F_{21}\left(\mathbf{q},\frac{\mathbf{q}}{2}+\mathbf{q}_1\right)d\mathbf{q}_1+\frac{i}{(2\pi)^3k}\int S(\mathbf{q}_1)\left[f_2\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1-\mathbf{q}_2\right)F_{12}\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1+\mathbf{q}_2,\mathbf{q}_2\right)\right.$$
$$\left.+F_{21}\left(\mathbf{q}-\mathbf{q}_2,\frac{\mathbf{q}}{2}+\mathbf{q}_1\right)f_2(\mathbf{q}_2)\right]d\mathbf{q}_1d\mathbf{q}_2-\frac{1}{(2\pi)^6k^2}\int S(\mathbf{q}_1)f_2\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3+\mathbf{q}_4\right)F_{21}(\mathbf{q}_3,\mathbf{q}_2)$$
$$\times F_{12}\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_4,\mathbf{q}_4\right)d\mathbf{q}_1d\mathbf{q}_2d\mathbf{q}_3d\mathbf{q}_4, \tag{37c}$$

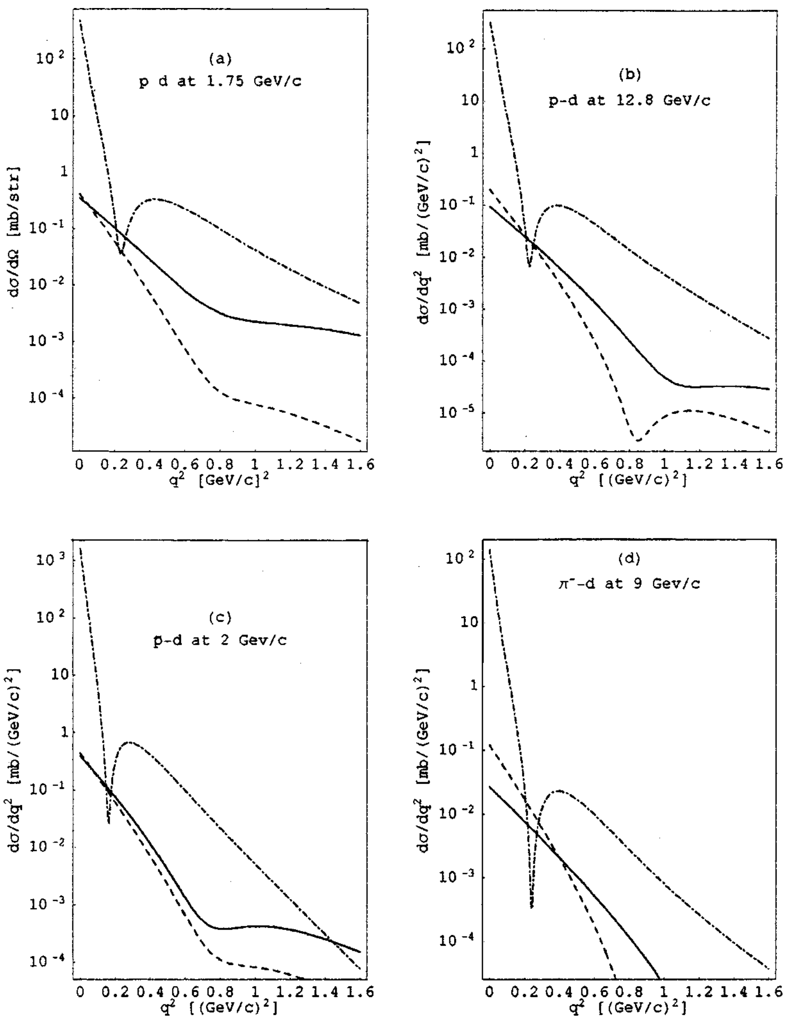

FIG. 4. The contributions of two-body force term (dot-dashed curve), three-body force term (dashed curve), and interference term (solid curve), in p-, $\overline{p}$-, and π^{-}-d elastic scattering differential cross section.

$$T_3(\mathbf{q})=\frac{i}{2\pi k}\int f_1\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1\right)f_2\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1\right)S(\mathbf{q}_1)d\mathbf{q}_1+\frac{i}{(2\pi)^5k}\int S(\mathbf{q}_1)F_{12}\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3,\mathbf{q}_2\right)$$
$$\times F_{21}\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1-\mathbf{q}_2+\mathbf{q}_3,\mathbf{q}_3\right)d\mathbf{q}_1d\mathbf{q}_2d\mathbf{q}_3-\frac{1}{(2\pi)^4k^2}\int S(\mathbf{q}_1)F_{12}\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3,\mathbf{q}_2\right)f_1(\mathbf{q}_3)$$
$$\times f_2\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1-\mathbf{q}_2\right)d\mathbf{q}_1d\mathbf{q}_2d\mathbf{q}_3-\frac{1}{(2\pi)^4k^2}\int S(\mathbf{q}_1)f_1(\mathbf{q}_3)f_2(\mathbf{q}_2)$$
$$\times F_{21}\left(\mathbf{q}-\mathbf{q}_2-\mathbf{q}_3,\frac{\mathbf{q}}{2}+\mathbf{q}_1-\mathbf{q}_3\right)d\mathbf{q}_1d\mathbf{q}_2d\mathbf{q}_3-\frac{i}{(2\pi)^7k^3}\int S(\mathbf{q}_1)f_1(\mathbf{q}_3)$$
$$\times f_2\left(\frac{\mathbf{q}}{2}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_5+\mathbf{q}_4\right)F_{12}\left(\frac{\mathbf{q}}{2}+\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4,\mathbf{q}_2\right)F_{21}(\mathbf{q}_5,\mathbf{q}_4)d\mathbf{q}_1d\mathbf{q}_2d\mathbf{q}_3d\mathbf{q}_4d\mathbf{q}_5. \tag{37d}$$

The quantity $T_1(\mathbf{q})+T_2(\mathbf{q})$ represents the single-scattering term and $T_3(\mathbf{q})$ represents the double-scattering term. We must note that the first order correction terms which are coming from Γ_{12} and Γ_{21} are considered in $T_1(\mathbf{q})$ and $T_2(\mathbf{q})$, respectively. Also, the second order correction term which is coming from $\Gamma_{12}\Gamma_{21}$ is considered in the double-scattering term. The first term of each of $T_1(\mathbf{q})$, $T_2(\mathbf{q})$, and $T_3(\mathbf{q})$ represent the usual Glauber terms and the other terms are related to three-body force correction.

Taking the deuteron D state into account, where the three-body force is also considered, the hadron-deuteron elastic scattering differential cross section—as in Ref. [24]—can be

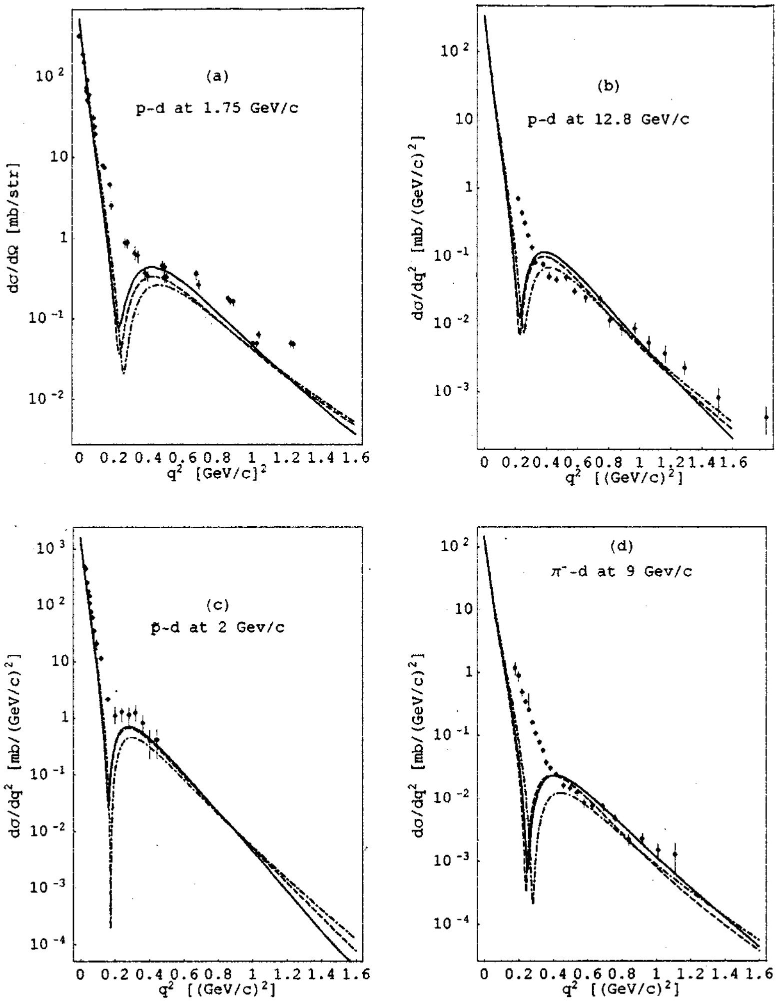

FIG. 5. Same as Fig. 4 except that dashed, dot-dashed, and solid curves correspond to the results of F_t, F_t+F_{th}, and $F(t)$, respectively.

written as

$$\frac{d\sigma}{d\Omega}=|F_0(\mathbf{q})|^2+|F_2(\mathbf{q})|^2 \tag{38}$$

with

$$|F_0(\mathbf{q})|^2=|T_{01}(\mathbf{q})+T_{02}(\mathbf{q})+T_{03}(\mathbf{q})|^2 \tag{39}$$

and

$$|F_2(\mathbf{q})|^2=\frac{3}{4}|T_{21}(\mathbf{q})+T_{22}(\mathbf{q})|^2+\frac{1}{4}|T_{21}(\mathbf{q})+T_{22}(\mathbf{q})+T_{23}(\mathbf{q})|^2, \tag{40}$$

where $T_{01}(\mathbf{q}),T_{02}(\mathbf{q}),T_{03}(\mathbf{q})$, and $T_{21}(\mathbf{q}),T_{22}(\mathbf{q}),T_{23}(\mathbf{q})$ are defined as $T_1(\mathbf{q}),T_2(\mathbf{q}),T_3(\mathbf{q})$, respectively, with

$$S(\pm\mathbf{q})\rightarrow S_0(q) \tag{41}$$

for the first three T's and with

$$S(\pm\mathbf{q})\rightarrow S_2(\mathbf{q}) \tag{42}$$

for the second three T's. $|F_0(\mathbf{q})|^2$ represents the differential cross section in the absence of the D state, $|F_2(\mathbf{q})|^2$ is the D-state correction, $S_0(q)$ and $S_2(q)$ are the spherical and quadruple form factors of deuteron. These form factors are given by

$$S_0(q)=\int_0^\infty[u^2(r)+w^2(r)]j_0(qr)dr, \tag{43}$$

and

$$S_2(q)=\int_0^\infty 2w(r)\left(u(r)-\frac{1}{2\sqrt{2}}w(r)\right)j_2(qr)dr, \tag{44}$$

where $u(r)$ and $w(r)$ are S and D states, respectively, radial wave functions of deuteron and $j_0(qr)$ and $j_2(qr)$ are the Bessel kind functions of zero and second orders, respectively. These form factors can be represented as the sum of three Gauss forms as follows [30]:

$$S_0(q)=\sum_{i=1}^{3}A_ie^{-c_iq^2}, \tag{45}$$

$$S_2(q)=q^2\sum_{i=1}^{3}B_ie^{-d_iq^2}, \tag{46}$$

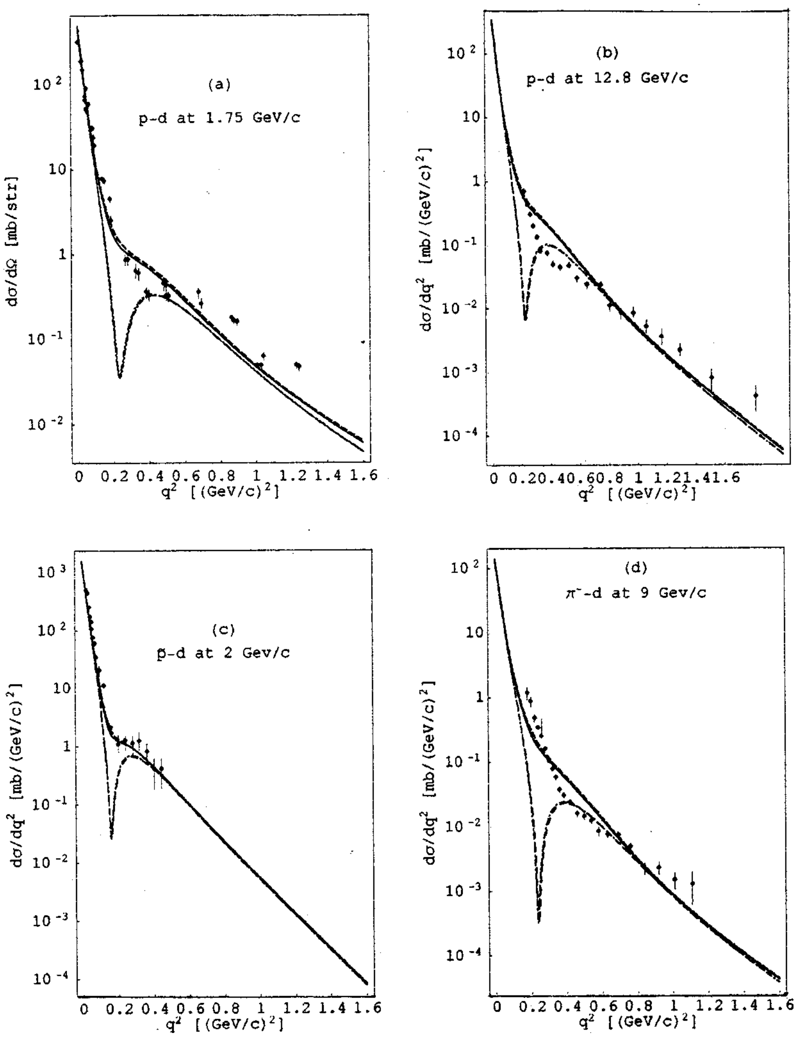

FIG. 6. Same as Fig. 2 except that solid and dashed curves correspond to the results with D-state effect for Lacombe *et al.* wave function [31] and Bressel *et al.* wave function [33], respectively. Dot-dashed curve represents the result without any effects.

where A_i, c_i, B_i, and d_i are fitting parameters, their values are calculated—by using χ^2 method—for the deuteron wave functions $u(r)$ and $w(r)$ as given by Lacombe *et al.* in Ref. [31] where the Paris potential was used in calculations. These functions take the forms

$$u(r)=\sum_{j=1}^{n} C_j e^{-m_j r}, \tag{47}$$

$$w(r)=\sum_{j=1}^{n} D_j\left(1+\frac{3}{m_j r}+\frac{3}{m_j^2 r^2}\right)e^{-m_j r}, \tag{48}$$

where the values of the parameter C_j, D_j, and m_j can be found in Ref. [31]. The values of form factor parameters A_i, c_i, B_i, and d_i are presented in Table I, where the corresponding results of $S_0(q)$ and $S_2(q)$ on the basis of Eqs. (45) and (46), respectively, are given in Fig. 1. Also presented in the same figure are the results of Eqs. (43) and (44) for $S_0(q)$ and $S_2(q)$, respectively, using Lacombe *et al.* wave functions [31]. The χ^2 value of the fitting between the two curves of $S_0(q)$—dot-dashed and dashed curves—is 64×10^{-5} and for $S_2(q)$—dot-dot-dashed and solid curves—is 34×10^{-5}, where the number of points of the data is 38. χ^2 is defined as

$$\chi^2=\sum_i [f(x_i)-f_i]^2, \tag{49}$$

where $f(x_i)$ is the value of the fitting function at the point x_i and f_i is the value of the data at the same point.

IV. RESULTS AND DISCUSSION

Using Eqs. (37)–(40), with hadron-nucleon amplitude in the form (33), three-body force amplitude (31), and deuteron form factors (45) and (46), we can calculate the hadron-deuteron elastic scattering differential cross section for any combination of the considered effects. In fact, the general analytical formulas for the T_{0i} and T_{2i}, $i=1,2,3$, where the D-state, phase variation, and three-body force effects are taken into account, are easily obtained. In the following calculations the form factor parameters for Lacombe *et al.* wave functions in Table I are used and the used hadron-nucleon parameters are given in Table II. The calculations for three-body force case are given with assumptions $A_{12}=A_{21}=A$, $B_{12}=B_{21}=B$. The order of the parameter γ, is determined from the fact that, in the two-pion exchange interaction, where the three-body force takes place, the radius of interac-

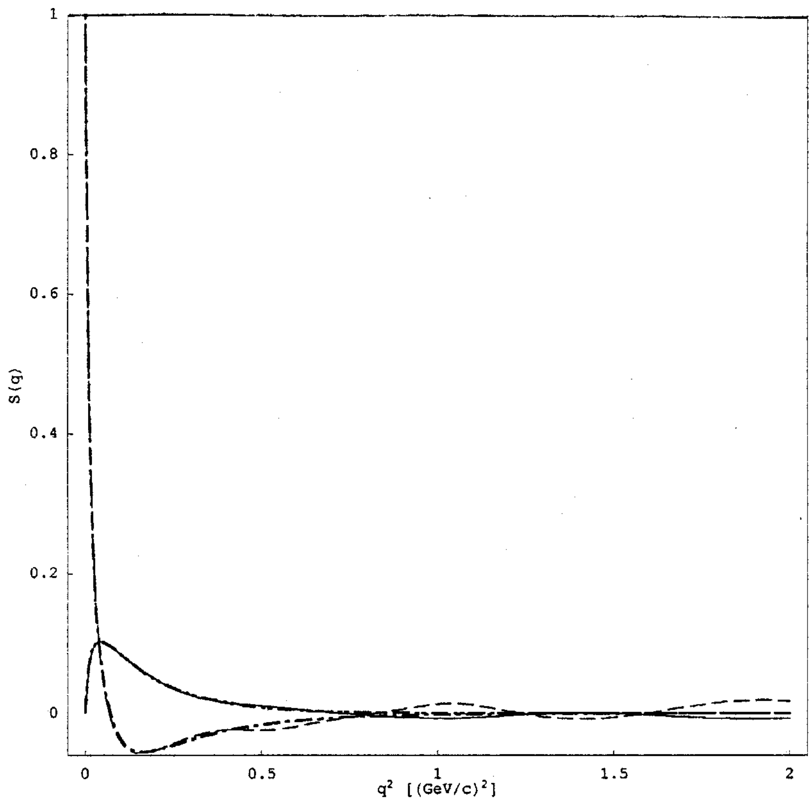

FIG. 7. Deuteron form factors $S_0(q)$ and $S_2(q)$. Dashed and solid curves correspond to the results of Eqs. (43) and (44), respectively, with the deuteron wave functions of Bressel *et al.* [33]. Dot-dashed and dot-dot-dashed curves correspond to the results of Eqs. (45) and (46), respectively, with the second group of the parameters of Table I which are determined using solid and dashed curves as standard curves.

tion is reduced to $r_0/2$, where r_0 is the radius of nuclear force in the two-body (or one-pion exchange) interaction [32]. Therefore, γ is of order $r_0^2/4$. If we take $r_0 = 1.4\,\mathrm{fm}$, we get $\gamma \approx 0.5\,\mathrm{fm}^2 \approx 13\,(\mathrm{GeV}/c)^{-2}$. The parameters A and B are considered as free parameters, which are determined from the fitting with the experimental data using the χ^2 method.

The results of the p-, $\overline{p}$-, and π^--d elastic scattering differential cross sections at different energies with D-state,

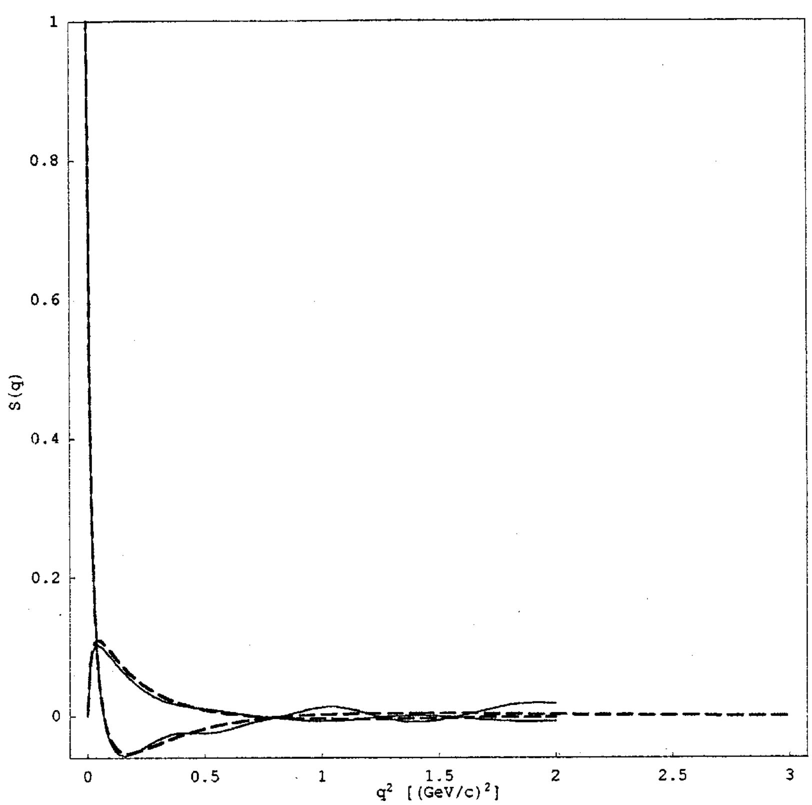

FIG. 8. Deuteron form factors $S_0(q)$ and $S_2(q)$. Dashed curves correspond to the results of Eqs. (43) and (44), respectively, for Lacombe *et al.* [31] wave functions and solid curves correspond to the results of the same equations, respectively, for Bressel *et al.* [33] wave functions.

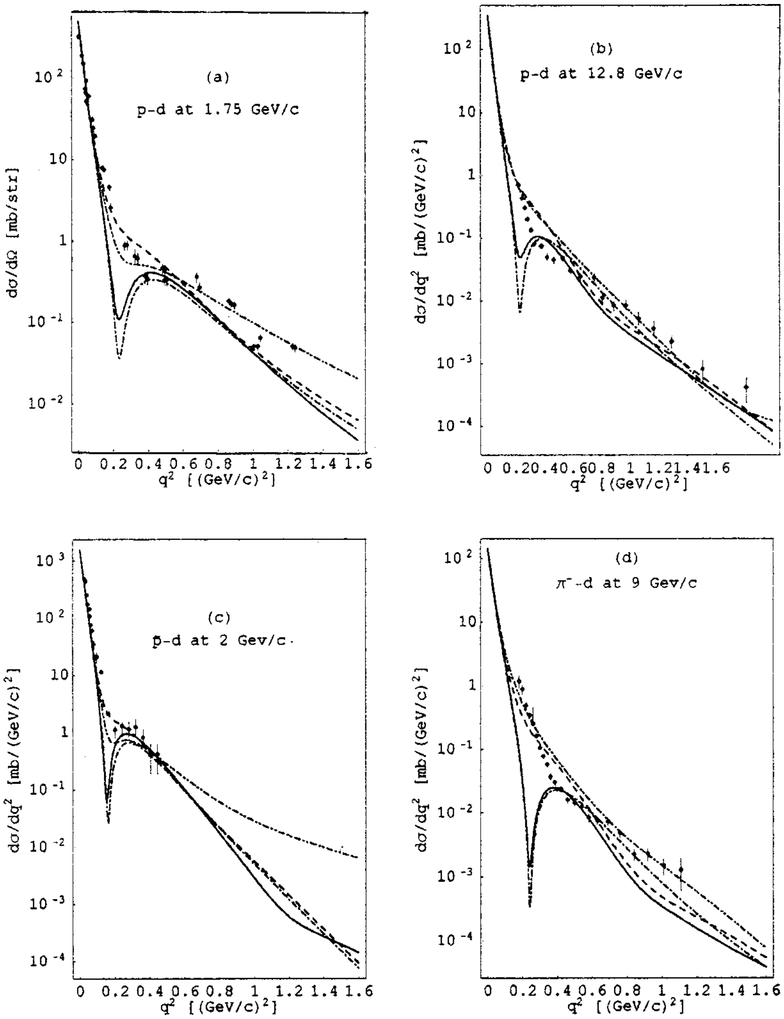

FIG. 9. Same as Fig. 2 except that dashed, dot-dot-dashed, and solid curves correspond to the results with (D state+phase variation), (D state+three-body force), and (three-body force +phase variation) effects, respectively. The dot-dashed curve corresponds to the result without any effects.

three-body force, and phase variation effects are presented in Fig. 2. The values of three-body force parameters A and B, phase variation parameter δ, and the minimum values of χ^2 for the obtained results are given in Table III. A good fit with the experimental data is obtained.

To see the weight of each effect alone, the results of calculations for three-body force, phase variation and D-state effects for different cases are given in Fig. 3. In general, the two-pion exchange three-body force is small and effective in the range $q^2 \geqslant 0.2\,(\mathrm{GeV}/c)^2$ of momentum transfer squared. This range is corresponding to the distance range $r < 0.6$ fm. This is in contrast to what is obtained by Primakoff [15], where the three-body forces dominate the dynamics of the three-nucleon system at very large momentum transfer where $q^2 \geqslant 1\,(\mathrm{GeV}/c)^2$. But, our results are in agreement with the radius of two-pion exchange three-body force ($\sim$0.7 fm), where the radius of the force is not very short as in the case of heavy particle-exchange as η, ρ, and ω particles. The results of Ref. [15] may be in agreement with the radius of the three-body force of two-particle exchange of these heavy particles, where their radii are 0.175, 0.125, and 0.124 fm, respectively.

The total profile function of hadron-deuteron scattering taking into account the three-body force effect is [Eq. (19)]

$$\Gamma = \Gamma_t + \Gamma_{\mathrm{th}} - \Gamma_t \Gamma_{\mathrm{th}}.$$

Therefore, the hadron-deuteron elastic scattering amplitude can be written as

$$F(\mathbf{q}) = F_t(\mathbf{q}) + F_{\mathrm{th}}(\mathbf{q}) + \text{interference term}.$$

The contributions of each term of this amplitude in the elastic scattering differential cross section are presented in Fig. 4. Also, presented in Fig. 5 is the cross section for $F_t(\mathbf{q})$, $F_t(\mathbf{q}) + F_{\mathrm{th}}(\mathbf{q})$, and $F(\mathbf{q})$. We can see that the contributions of three-body force component are, in general, small. The pure three-body force term is, in general, smaller than the interference term. The three-body force effect decreases with increasing energy. The relative roles of pure and interference terms are different at different energies. The different signs of the pure and interference terms in the scattering amplitude $F(t)$ reduce the final three-body force effect on the elastic scattering differential cross section.

However, the phase variation effect as suggested by Franco [25], in general, is not enough to obtain a good fit with the experimental data, Fig. 3. Also, in the case of the D-state effect, the same figure, the free parameters do not exist and the obtained results are corresponding to the deuteron wave functions as given in Ref. [31]. The results, in general, do not describe well the experimental data of hadron-deuteron elastic scattering differential cross section, even at the minimum region, except in the case of $\bar{p}$-d elastic scattering where a good fit is obtained. Therefore, we believe that the three-body force effect is small, but it is important to obtain a good fit with the experimental data.

A similar result, Fig. 6, for the differential cross section of different particles at different energies with *D*-state effect only, can be obtained using the deuteron wave functions which are given by Bressel *et al.* in Ref. [33], where nucleon-nucleon potential with square finite core of radius equal to 0.7 fm was used. The used parameters of the form factors of Eqs. (45) and (46) in the case of Bressel *et al.* wave functions [33] are given in Table I, where χ^2 values of the fitting with the form factors of Eqs. (43) and (44) are 37×10^{-4} for S_0 and 49×10^{-5} for S_2, see Fig. 7. The number of points of Ref. [33] data is 38. Also, the form factors of Eqs. (43) and (44) for Bressel *et al.* [33] and for Lacombe *et al.* [31] wave functions are given in Fig. 8, where a significant difference between the two cases does not exist. This interpretation shows the similarity of the results of both functions.

The wave function of Lacombe *et al.* [31] reproduces quite well the low-energy properties as the contribution percent and satisfies rigorous bounds imposed by the experimental deuteron data for internucleon distances $r\geqslant0.6$ fm. Also, the Bressel *et al.* wave function [33] was calculated using a nucleon-nucleon potential with square finite core of radius 0.7 fm. At the same time, the radius of the two pion-exchange three-body force is 0.7 fm. Thus, the *D*-state and the three-body force have two different regions of action from which most contributions are coming. For the *D* state this region $r\geqslant0.7$ fm and for the three-body force $r\leqslant0.7$ fm. For the other well known deuteron wave functions the same region of action of the *D* state can be considered, see, for example, Refs. [34,35]. Therefore, taking into account the short distance character of high-energy scattering processes, we can interpret the obtained results for both *D* state and three-body force effects. The three-body force with the small radius of order 0.7 fm is more compatible with this short distance character of high-energy scattering, while most contributions of the *D* state are coming from the region of relatively large r ($r\geqslant0.7$ fm).

Now, we try to combine two of the considered effects together. The results, with the wave function of Ref. [31], for different combinations for different particles are given in Fig. 9. The three-body force with the *D*-state effect gives a better agreement with the experimental data for p-d scattering at 1.75 GeV/c and for $\bar{p}$-d at 2 GeV/c. In general, one or two effects only from the three used effects are not enough to obtain a good fit with the data. Thus, with *D*-state effect as a principal correction, the three-body force effect plays a role in the scattering process. The reality of the three-body force effect with small contribution can be accepted to obtain a good fit with the experimental data.

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