Removal of spurious center of mass effects in nuclear many-body systems

D. R. Bes[1,3] and O. Civitarese[2]
[1]*Departamento de Física, CNEA, Av. Libertador 8250, (1429) Buenos Aires, Argentina*
[2]*Departamento de Física, UNLP, C.C. 67, (1900) La Plata, Argentina*
[3]*Departamento de Física, Universidad Favaloro, Belgrano 1723, (1093) Buenos Aires, Argentina*
(Received 27 November 2000; published 22 March 2001)

The problem of the motion of the center of mass in nuclear many-body systems is revisited. In the first part of the work, the counterterms needed to fulfill the translational and Galilean invariances are solved through the exact albeit perturbative expansion underlying the random-phase approximation. Collective variables are introduced in the second part of the work. The inherent problems of overcounting and infrared divergencies are solved by means of the BRST invariance. Consistency between the two procedures is achieved by use of the same perturbative expansion. The formalism is applied to the calculation of matrix elements of some of the electroweak operators which are active in the (μ^-,e^-) conversion process, to show the influence of center of mass effects upon transitions induced by vector terms of the weak current.

DOI: 10.1103/PhysRevC.63.044323 PACS number(s): 21.60.−n, 24.30.Cz, 24.80.+y

I. INTRODUCTION

The current predictions of effects associated with electroweak interactions in nuclei rely both upon the standard model (SM) and the description of the nuclear structure [1]. Setting up limits on the parameters of nontrivial extensions of the SM is a challenging issue [2], which is heavily dependent upon nuclear model approximations. The interested reader can find a recent review of the subject in Ref. [3].

Among the nuclear structure aspects which can influence the outcome of a SM analysis of nuclear electroweak decays, we have chosen the problem of symmetry violations and its consequences upon the calculation of nuclear matrix elements of electroweak operators. Particularly, we have focused our attention on the restoration of the translational and Galilean invariance of the nuclear Hamiltonians used in the calculation of $I^\pi=1^-$ states. These invariances are required because the nuclear response to multipole excitations is sensitive to the spurious center of mass (c.m.) motion. This is the case, for instance, of lepton-flavor violation processes [4], like the (μ^-,e^-) conversion, where a sizeable part of the calculated transition strength is predicted to proceed by the excitation of $I^\pi=1^-$ states.

The case of the spurious c.m. motion is perhaps one of the most studied examples of broken symmetries in nuclear Hamiltonians. Hereafter we briefly review some of the classic papers existing in the literature.

The removal of spurious 1^- states, i.e., states in which the c.m. is not at rest, has been attempted in shell model treatements of two-body interactions in the harmonic oscillator basis [5]. In Ref. [6] Gloeckner and Lawson have diagonalized a modified Hamiltonian which includes a harmonic term in the variables corresponding to the c.m. The scale at which the decoupling of intrinsic and c.m. excitations becomes operative is defined by an externally introduced parameter associated with the included term. The use of this method is limited naturally by the number of configurations reached by the extra term and the results are strongly dependent upon the truncation of the model space, as emphasized by Mac Grory and Wildenthal [7]. These authors have also suggested that effective interactions, constructed with simple operators, can compensate for the spurious c.m. effects. However, the structure of these operators has not been presented explicitly in Ref. [7].

Bohr and Mottelson have derived the strength of effective, separable, two-body interactions from geometrical arguments involving the self-consistency of either surface and volume fields [8]. Following the same procedure as for these finite-frequency modes, they have also obtained the strength of a schematic residual force from the field generated by a small displacement of the nuclear central potential. Thus the procedure is not directly applicable if one starts from empirical single-particle energies and/or also includes residual interactions which may restore (at least partly) the symmetry. Pyatov and Salamov [9] have constructed a symmetry restoring interaction to be added to the symmetry violating total Hamiltonian. However, as remarked in Ref. [10], this procedure may introduce purely intrinsic terms affecting the intrinsic excitations. Another method was proposed in the work of Meyer ter Vehn [10]. It is based on the identification and subsequent elimination of the coupling between c.m. and intrinsic degrees of freedom. This formal decoupling leads to random-phase approximation (RPA) equations which include also a finite frequency mode for the c.m. motion. The method resembles the Gaussian separation of variables known in field and gauge theories. In all three cases, Refs. [8–10], the treatments are extended to restore also the Galilean invariance of the interaction. Their validity is always restricted to the RPA or equivalent linearized approximations. A related approach was used in Ref. [17] to approximately decouple spurious and physical 1^- states. We would like to quote this work as an example of the effects of the center of mass motion upon physical observables, like the multipole operators of the weak current which are proportional to the coordinate.

A study of the invariance of single-particle and residual interactions in deformed nuclei was published by Birbrair and Sadovnikova [11]. Translational and rotational invariance conditions were taken into account by adding derivative terms to deformed trial central potentials. These derivative

terms are coupled and the decomposition in odd and even multipolarities contains, for an effective potential, infinite terms. This condition casts some doubts on the feasibility of the method. Within the framework of the Landau prescription, Sakamoto and Kishimoto [12] derived the expression for a translational (and rotational) invariant effective interaction from the mean field of deformed nuclei. Their results appear to be the first terms of a series that was summed up by Marshalek [13], and that yields a peculiar, however justified, interaction. The influence of c.m. motion upon proton and neutron contra-rotations was also analyzed by Marshalek [14]. He worked with a Hamiltonian in which the c.m. term is represented by a three-dimensional harmonic oscillator centered at the origin of coordinates. The results of Ref. [14] show that the transformation of angular momentum variables to the c.m. frame is required to describe correctly the relative motion of neutrons against protons.

The present treatment is developed in two steps, dealing with the reconstruction of the translational invariance of the Hamiltonian and with the specific application of the collective treatment to translational motion, respectively. In the first part we proceed along the conceptual lines underlying Refs. [8–10], i.e., we introduce counterterms in the Hamiltonian. As a difference from these previous contributions, we obtain an exact (albeit perturbative) expression for the counterterms (i.e., we go beyond the RPA). Thus the new interactions may influence not only the strength of transitions to $I^{\pi}=1^{-}$ states but also other nuclear properties as well.[1] The calculation of the counterterms is made in Sec. II A and in Appendix A. The solution is applied to the most commonly used single-particle terms in the nuclear Hamiltonian in Sec. II B.

Second, we apply a consistent procedure [15] to eliminate the infrared problems that appear as a consequence of the zero-frequency RPA mode (Sec. III). The collective treatment is simplified by the Abelian nature of the linear displacements. Consequently, this application is only outlined in Sec. III and in Appendix B, since the procedure has been thoroughly discussed in the literature, although, to our knowledge, it has not been previously applied to the case of the translational motion.

As an example about the use of the formalism we present and discuss, in the last part of the paper, some results on the calculation of matrix elements of the vector operator which induces transitions to 1^{-} states in nuclear (μ^{-},e^{-}) conversion processes. To get an insight about the scope of center of mass effects in the (μ^{-},e^{-}) decay, the reader is kindly referred to Ref. [17], where the applied approximate removal of spurious center of mass effects was not able to completely eliminate admixtures to the dominant 1^{-} multipole matrix element.

[1]For instance, the calculation of rotational parameters may be affected by the velocity dependence of the interactions that insure Galilean invariance.

II. HAMILTONIAN

A. Construction of the counterterms

Since the space is homogeneous, the Hamiltonian H and and the momentum component p_{μ} should commute. Similarly, Galilean invariance requires the commutation of the interaction $H-p^2/2m$ with the coordinate r_{μ}. A Hamiltonian derived self-consistently should satisfy these two conditions. However, this is not usually the case: while single-particle energies are empirically obtained from (predominantly) single-particle nuclear states, the residual interaction is obtained either from schematic forces or, through several approximations, from phase shifts in nucleon-nucleon scattering. Therefore

$$\pi_{\mu}\equiv[H,p_{\mu}];\quad \rho_{\mu}\equiv[H,r_{\mu}]-\frac{i}{m}p_{\mu}, \tag{1}$$

are usually nonvanishing operators.

As a consequence, we are forced to include counterterms in the Hamiltonian which we assume to be of the form

$$H_p=\vec{P}\cdot\vec{r};\quad H_r=\vec{R}\cdot\vec{p}, \tag{2}$$

in order to uncouple the three degrees of freedom $(\vec{r},\vec{p})$ from the remaining ones in the problem. Here the (spherical) components P_{μ} and R_{μ} are determined from the conditions

$$0=\pi_{\mu}+iAP_{\mu}+(-1)^{\nu}[P_{-\nu},p_{\mu}]r_{\nu}+(-1)^{\nu}[R_{-\nu},p_{\mu}]p_{\nu}, \tag{3}$$

$$0=\rho_{\mu}-iAR_{\mu}+(-1)^{\nu}[P_{-\nu},r_{\mu}]r_{\nu}+(-1)^{\nu}[R_{-\nu},r_{\mu}]p_{\nu}. \tag{4}$$

The n-body contributions to P,R yield $(n+1)$ terms in the effective interactions (2). The one- and two-body terms of the solutions P_{μ},R_{μ} to these equations are obtained in Appendix A in terms of an expansion similar to the one implicit in the RPA. In particular, the one-body terms are given by

$$P_{\mu(1v)}=\frac{i}{A}\pi_{\mu(1v)}+\frac{\sqrt{3}}{2A^2}[\pi,p]^0_{(00)}r_{\mu(1v)},$$

$$R_{\mu(1v)}=-\frac{i}{A}\rho_{\mu(1v)}+\frac{\sqrt{3}}{2A^2}[\rho,r]^0_{(00)}p_{\mu(1v)}, \tag{5}$$

where the notation (nm) in the subindices labels a n-body operator with m bodies crossing the Fermi sea. The orders of magnitude of the operators P_{μ},R_{μ} are $O(\epsilon/A^{1/2})$ and $O(\epsilon/A)$ for $v=1,0$ respectively. The solution for the particle-hole equations $(v=1)$ coincide with the one found by Meyer ter Vehn for the case in which the boson associated with the degree of freedom corresponding to the center of mass has zero energy [10]. The two-body terms read [cf. Eqs. (A13) and (A16)]

$$P_{\mu(2v)}=\frac{i}{A}\tau_{\mu(2v)} + \frac{1}{2A^2}\sum_{\alpha}\sqrt{\frac{2\alpha+1}{3}}([[\tau_{(2v)},p_{(11)}]^{\alpha}_{(1(v-1))}r_{(11)}] - [[\eta_{(2v)},p_{(11)}]^{\alpha}_{(1(v-1))}p_{(11)}])^{1}_{\mu(2v)},$$

$$R_{\mu(2v)}=-\frac{i}{A}\eta_{\mu(2v)} - \frac{1}{2A^2}\sum_{\alpha}\sqrt{\frac{2\alpha+1}{3}}([[\tau_{(2v)},r_{(11)}]^{\alpha}_{(1(v-1))}r_{(11)}] - [[\eta_{(2v)},r_{(11)}]^{\alpha}_{(1(v-1))}p_{(11)}])^{1}_{\mu(2v)}, \quad (6)$$

where the two-body operators $\tau_{\mu(2v)},\eta_{\mu(2v)}$ are derived from the known operators π_μ,ρ_μ according to Eq. (A12). These expressions are of $O(\epsilon/A)$ for $v=2$, $O(\epsilon/A^{3/2})$ for $v=1$ and $O(\epsilon/A^2)$ for $v=0$.

B. Application to single-particle Hamiltonians

The systematic application of the previous procedure associates residual interactions (2) to any single-particle Hamiltonian. In the present section we study some applications to the most frequent single-particle contributions, namely the harmonic oscillator potential, the spin-orbit and the l^2 terms, and the empirical single-particle Hamiltonian. Details of the calculation are given in Appendix B.

1. Harmonic oscillator case

We assume a single-particle Hamiltonian of the form

$$H^{ho}_{sp}=\frac{1}{2m}p^2+\frac{m\omega^2}{2}r^2. \quad (7)$$

Not surprisingly, the previously described procedure yields the interaction

$$H^{ho}_{p}=-\chi_{ho}\vec{r}\cdot\vec{r},$$
$$\chi_{ho}=\frac{m\omega^2}{2A}, \quad (8)$$

with the strength χ_{ho} equal to the self-consistent value derived in Ref. [8]. The linear solutions (5) are exact for this case. The addition of the single-particle (7) and residual interaction (8) yields the well known two-body Hamiltonian

$$H^{ho}=\frac{1}{2m}p^2+\frac{\chi_{ho}}{2}\sum_{ab}|\vec{r}_a-\vec{r}_b|^2. \quad (9)$$

2. Spin-orbit and the l^2 terms

We start from a single-particle term

$$H^{so}_{sp}=-\chi_{so}(\vec{l}\cdot\vec{s})_{(1v)}, \quad (10)$$

which yields the two-body interactions

$$H^{so}_{p}=\frac{\chi_{so}}{A}\sum_{w}(\vec{p}\times\vec{s})_{[1(v-w)]}\cdot\vec{r}_{(1w)},$$
$$H^{so}_{r}=-\frac{\chi_{so}}{A}\sum_{w}(\vec{r}\times\vec{s})_{[1(v-w)]}\cdot\vec{p}_{(1w)}. \quad (11)$$

Therefore the three spin-orbit contributions may be written as

$$H^{so}=H^{so}_{sp}+H^{so}_{p}+H^{so}_{r} =-\frac{\chi_{so}}{2A}\sum_{ab}(\vec{r}_a-\vec{r}_b) \times(\vec{p}_a-\vec{p}_b)\cdot(\vec{s}_a+\vec{s}_b)+\frac{\chi_{so}}{2A}(\vec{l}\cdot\vec{s})_{(2v)}, \quad (12)$$

i.e., as a two-body spin orbit interaction which is obviously translational and Galilean invariant plus a term that, within the RPA, is only operative for spin-spin correlations ($I^\pi=1^+$ resonances). The imposition of the form (2) to the counterterms results in the existence of higher order terms which should compensate for the $(\vec{l}\cdot\vec{s})_{(2v)}$ interaction. Indeed, Eqs. (3) and (4) may be exactly solved also for the spin-orbit case. They yield [cf. Eq. (6)]

$$P_\mu=\frac{\chi_{so}}{A}\left((\vec{p}\times\vec{s})_{\mu(1v)}-\frac{1}{2A}\sum_{w}(\vec{p}_{(1(v-w))}\times\vec{s}_{(1w)})_{\mu(2v)}\right),$$
$$R_\mu=-\frac{\chi_{so}}{A}\left((\vec{r}\times\vec{s})_{\mu(1v)}-\frac{1}{2A}\sum_{w}(\vec{r}_{(1(v-w))}\times\vec{s}_{(1w)})_{\mu(2v)}\right). \quad (13)$$

Let us consider now the term

$$H^{ll}_{sp}=-\chi_{ll}(\vec{l}\cdot\vec{l}). \quad (14)$$

The resultant interactions are

$$H^{ll}_{p}=\frac{\chi_{ll}}{A}(\vec{p}\times\vec{l})\cdot\vec{r}-\frac{\chi_{ll}}{A}(\vec{l}\times\vec{p})\cdot\vec{r}-\frac{2\chi_{ll}}{3A^2}\langle p^2\rangle\vec{r}\cdot\vec{r},$$
$$H^{ll}_{r}=\frac{\chi_{ll}}{A}(\vec{l}\times\vec{r})\cdot\vec{p}-\frac{\chi_{ll}}{A}(\vec{r}\times\vec{l})\cdot\vec{p}-\frac{2\chi_{ll}}{3A^2}\langle r^2\rangle\vec{p}\cdot\vec{p}. \quad (15)$$

The sum of the three contributions may be written as

$$H^{ll}_{sp}+H^{ll}_{p}+H^{ll}_{r}=-\frac{\chi_{ll}}{2A}\sum_{ab}|(\vec{r}_a-\vec{r}_b)\times(\vec{p}_a-\vec{p}_b)|^2 - \frac{2\chi_{ll}}{3A}\langle p^2\rangle\left(\frac{1}{A}\vec{r}\cdot\vec{r}-(\vec{r}\cdot\vec{r})_{(11)+(10)}\right) - \frac{2\chi_{ll}}{3A}\langle r^2\rangle\left(\frac{1}{A}\vec{p}\cdot\vec{p}-(\vec{p}\cdot\vec{p})_{(11)+(10)}\right)$$

$$+\frac{\chi_{ll}}{A}\vec{l}\cdot\vec{l}+\frac{\chi_{ll}}{A}\sum_{ab}\vec{r}_b\times\vec{p}_a\cdot\vec{r}_a\times\vec{p}_b$$

$$+\frac{\chi_{ll}}{A}(\vec{r}\cdot\vec{r})_{(11)+(10)}(\vec{p}\cdot\vec{p})_{(11)+(10)}$$

$$-\frac{\chi_{ll}}{A}\sum_{\mu,a,b}(-1)^{\mu}r_{a,\mu}p_{b,\mu}(r_{a,\mu+1}p_{b,\mu+1}$$

$$+r_{a,\mu-1}p_{b,\mu-1}). \tag{16}$$

Note that the right-hand side is irrelevant for RPA calculations of $I^{\pi}=1^{-}$ resonances, but for the first line.

Since the form of the counterterms has been chosen in order to eliminate the coupling between the center of mass and the other degrees of freedom, we should not expect a significant mixture of unperturbed excitations in the final normal modes. The only exception could be the mixture between the pair of degenerate excitations appearing with the (physical) choice $\chi_{ll}=1/4\chi_{so}$.

3. Empirical single-particle energies

We have performed three RPA calculations using the empirical single-particle energies as in ^{208}Pb and introducing as interaction: (i) a $\vec{r}\cdot\vec{r}$ term with the self-consistent strength (8); (ii) the same interaction as in (i) with a strength such that there is an eigenvalue as close to zero as allowed by the computational facilities; (iii) the counterterms with the values of $\vec{P}_{(11)},\vec{R}_{(11)}$ as obtained from Eq. (B4).

The results are given in Fig. 1, where the matrix elements of the operator $\vec{r}$, corresponding to transitions between the ground state and excited states, are represented as a function of the excitation energy. The two sides of the figure differ by the scale on the vertical axis. It is such that only the large peak is shown on the left side of the figure. Although the calculation (i) with the self-consistent strength displays a prominent peak at low energies, the peak is finite and located at an energy significantly larger than zero. On the contrary, the results (ii) and (iii) show the peak as close to zero energy as allowed by the computational procedure. This similarity apparently supports the (frequent) use of the procedure (ii) appearing in the literature. However, let us consider now the right side displaying the matrix elements to finite frequency modes, which are the ones that interest us from the physical point of view. In this case the calculations (i) and (ii) are the ones that yield very similar results, while the scale is smaller by two orders of magnitude for (iii), although the excitation pattern is quite similar. We conclude that the admixture of the spurious with the finite frequency modes is not changed significantly by varying the strength of the $\vec{r}\cdot\vec{r}$ interaction and thus the use of this interaction does not insure that we obtain correct matrix elements to excited states. On the contrary, the uncoupling of the spurious mode is accomplished through the counterterms (2).

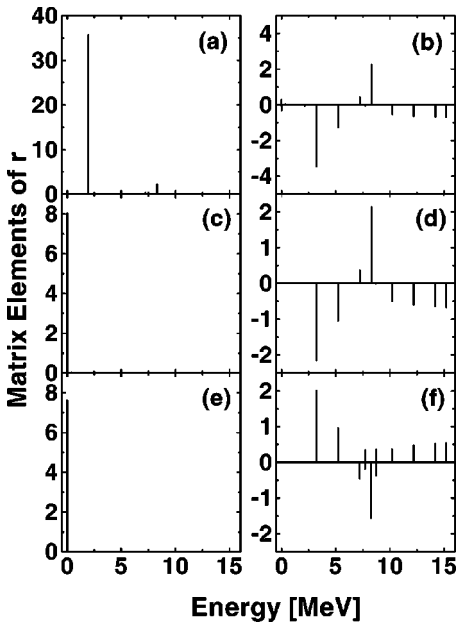

FIG. 1. The matrix elements of the coordinate operator, $\langle 1_n^-||r||g\cdot s\rangle$, to the nth one-phonon state, in units of fm. The results are scaled by a factor 10^{-2} in cases (c) and (e) and by a factor 10^2 in case (f). Cases (a) and (b) show the results of the RPA calculation performed with the $r\cdot r$ interaction of Eq. (8) and using the harmonic oscillator coupling χ_{ho}, cases (c) and (d) correspond to the same interaction with a renormalized coupling which yields a solution at zero energy, cases (e) and (f) show the results obtained with the counterterms of Eq. (A18). The single-particle basis used in the calculations is an empirical one which includes the $N_{osc}=5$, 6, and 7 active shells, and $A=126$ particles.

III. THE COLLECTIVE FORMALISM

The solution (5) guarantees that there is a zero-frequency RPA boson for each direction of space.[2] This consequence of the homogeneity of space gives rise to infrared divergencies, which should be taken care off. One way to solve the problem is through the introduction of collective coordinates,[3] which in the present case represent the coordinates $\mathcal{R}_\mu$ determining the position of a moving frame of reference relative to the laboratory frame. Within this description there is no way to distinguish between the motion of the body in one direction and the displacement of the frame of reference in the opposite one. This gauge-type invariance is expressed by the constraint

$$p_\mu-\mathcal{P}_\mu=0, \tag{17}$$

where $\mathcal{P}_\mu$ is the generator of displacements of the moving frame, hereon the collective momentum ($[\mathcal{R}_{-\mu},\mathcal{P}_\nu]=i(-1)^\mu\delta_{\mu\nu}$). Physical states $|\text{phys}\rangle$ are annihilated by the

[2]Conversely, the existence of a vanishing frequency does not imply that the Hamiltonian commutes with the momentum $\vec{p}$ even at the RPA level.

[3]We follow here the treatment presented in Ref. [15].

left-hand side of Eq. (17) and physical operators $\mathcal{O}_{\rm phys}$ commute with it. As is well known, the constraints may be taken into account by adding to the Hamiltonian terms proportional to Lagrange multipliers $\vec{\Omega}$

$$H \to H - \vec{\Omega}\cdot(\vec{p}-\vec{\mathcal{P}}), \tag{18}$$

and requiring the vanishing of $\vec{B}$, the momentum conjugate to $\vec{\Omega}([\Omega_{-\mu}, B_\nu] = i(-1)^\mu \delta_{\mu\nu})$.

A. The BRST invariance

A gauge theory has an underlying invariance under transformations generated by the charge $\mathcal{Q}$. This is a Hermitian and nilpotent operator that is linear in the constraints and includes fermion ghost operators $\vec{\eta}, \vec{\bar{\pi}}$ with conjugate momenta $\vec{\pi}, \vec{\bar{\eta}}$, $([\eta_{-\mu}, \pi_\nu]_+ = [\bar{\eta}_{-\mu}, \bar{\pi}_\nu]_+ = (-1)^\mu \delta_{\mu\nu})$,

$$\mathcal{Q} = (\vec{p}-\vec{\mathcal{P}})\cdot\vec{\eta} + \vec{B}\cdot\vec{\bar{\pi}}. \tag{19}$$

The subspace of states which are annihilated by the charge $\mathcal{Q}$ consists of physical states plus states $|\chi\rangle$ having zero norm. Both physical operators and operators[4] $\mathcal{O}_\chi$ mapping physical states into zero-norm states, commute with the charge $\mathcal{Q}$. Consequently, there are families of equivalent states and equivalent operators, namely

$$|\text{phys}\rangle \to |\text{phys}\rangle + |\chi\rangle; \quad \mathcal{O}_{\rm phys} \to \mathcal{O}_{\rm phys} + \mathcal{O}_\chi, \tag{20}$$

which yield the same matrix elements as physical operators between physical states. The constraints are automatically taken into account by operating within the subspace carrying zero charge. Following Eq. (20), we may add to the original Hamiltonian H a nil operator to obtain the (equivalent) BRST Hamiltonian

$$H_{\rm BRST} = H - \vec{\Omega}\cdot(\vec{p}-\vec{\mathcal{P}}) + i\vec{\pi}\cdot\vec{\bar{\pi}} + \omega^2\left(\frac{\vec{r}\cdot\vec{B}}{A} - \frac{\vec{B}\cdot\vec{B}}{2mA} - i\vec{\bar{\eta}}\cdot\vec{\eta}\right). \tag{21}$$

In $H_{\rm BRST}$ the original symmetry is restored at the collective level, since it commutes with the momentum $\vec{\mathcal{P}}$. On the contrary, it does not commute with the intrinsic momentum $\vec{p}$, and thus the infrared divergencies become eliminated. Because of the Abelian nature of the gauge generators (17), the ghosts are uncoupled from the remaining degrees of freedom and may be ignored. Since the parameter ω is arbitrary, it should not appear in physical results.

B. Transformation to a moving system

In $H_{\rm BRST}$ [Eq. (21)], the term $\vec{\Omega}\cdot\vec{\mathcal{P}}$ represents the coupling between the collective and the intrinsic motion. This coupling may be eliminated via a transformation setting in motion the intrinsic system, namely

$$T = \exp\left[\frac{i}{A}\vec{\mathcal{P}}\cdot\left(\frac{\vec{B}}{m} - \vec{r}\right)\right], \tag{22}$$

$$T H_{\rm BRST} T^+ = H'_{\rm BRST} + \frac{1}{2mA}\mathcal{P}^2, \tag{23}$$

which explicitly displays the collective kinetic energy. Here $H'_{\rm BRST}$ is the BRST Hamiltonian without the coupling term $\vec{\Omega}\cdot\vec{\mathcal{P}}$. Therefore the translational collective sector has become totally uncoupled from the remaining degrees of freedom of the system.

All (physical) operators must be transformed using Eq. (22). We illustrate the consequences of such transformation for the case of an operator which is represented by a function of $\vec{r}_i$, the coordinate of particle i. The vector $\vec{r}_i$ is not a physical operator, since it does not commute with the constraints (17). This expresses the fact that $\vec{r}_i$ measures the position in the moving frame, which is an artifact. Therefore the dependence of any physical operator on the particle coordinate should be of the form $F(\vec{r}_i + \vec{\mathcal{R}})$, because $\vec{r}_i + \vec{\mathcal{R}}$ is the position in the laboratory frame. Consistently, the constraints (17) are satisfied. We transform now the operator $F(\vec{r}_i + \vec{\mathcal{R}})$ according to Eq. (22), namely

$$T F(\vec{r}_i + \vec{\mathcal{R}}) T^{-1} = F\left[\vec{r}_i + \frac{1}{A}\left(\frac{\vec{B}}{m} - \vec{r}\right) + \vec{\mathcal{R}}\right] \to F\left(\vec{r}_i - \frac{\vec{r}}{A} + \vec{\mathcal{R}}\right), \tag{24}$$

since $\vec{B}$ is a nil operator [c.f. Eq. (C1)]. Therefore the transformation (22) replaces the collective coordinate $\vec{\mathcal{R}}$ by[5] $\vec{\mathcal{R}} - \vec{r}/A$.

In the first place we may apply the formalism to the calculation of the coordinate operator $\vec{r}$ itself. This operator appears, for instance, in the excitation of $I^\pi = 1^-$ states. We recall that, within the RPA, this operator creates a phonon with frequency $\omega_g = 0$, with an amplitude proportional to $1/\sqrt{\omega_g}$ (a clear example of infrared problems). From the practical point of view, this behavior leads to unphysical predictions whenever even small amplitudes of the spurious state are present in a finite frequency RPA mode.

The corresponding physical operator is $\vec{r} + A\vec{\mathcal{R}}$, and it reduces to $A\vec{\mathcal{R}}$ after being boosted: the vector $\vec{r}$ has disappeared from the calculation (and the associated infrared divergencies as well). The problem is reduced to the calculation of the well behaved operator $\vec{\mathcal{R}}$ within the collective sector of the Hilbert space.

We give now a simple example of how the formalism may be applied in nuclear reactions. Let us consider a heavy

[4] Henceforth ''nil'' operators.

[5] Since the collective and intrinsic degrees of freedom are independent of each other, the operator $\vec{\mathcal{R}} - \vec{r}/A$ is different from zero.

nucleus in which the particles are labeled by the subindex i and a point particle labeled by 0. Both systems interact via a two-body force depending on the particle distances, namely

$$\sum_i V(|\vec{r}_0-\vec{r}_i-\vec{\mathcal{R}}|)\rightarrow\sum_i V\left(\left|\vec{r}_0-\vec{r}_i+\frac{1}{A}\vec{r}-\vec{\mathcal{R}}\right|\right), \quad (25)$$

where $\vec{\mathcal{R}}$ is the position of the c.m. of the heavy system. The transformed interaction expresses the fact that the position of particle i should be measured with respect to the center of mass in the intrinsic frame of the heavy system.

C. Hilbert space

In the moving frame of reference the collective variables $\vec{\mathcal{R}}$ are considered to be real variables and thus, as a tradeoff, some original degrees of freedom must join the spurious sector. At the level of elementary modes of excitation these are given by the RPA zero-frequency modes. The total (quadratic) spurious sector reads

$$H^{(2)}_{\rm spur}=\frac{1}{2m}\vec{p}_{(11)}\cdot\vec{p}_{(11)}-\vec{\Omega}\cdot\vec{p}_{(11)}+\omega^2\left(\frac{1}{A}\vec{B}\cdot\vec{r}_{(11)}-\frac{1}{2mA}\vec{B}\cdot\vec{B}\right)$$
$$=\omega(\vec{\Gamma}^+_1\cdot\vec{\Gamma}_1-\vec{\Gamma}^+_0\cdot\vec{\Gamma}_0), \quad (26)$$

where the transformation to normal modes is given in Appendix C. The following commutation relations are satisfied:

$$[\Gamma_{1\mu},\Gamma^+_{1\nu}]=-[\Gamma_{0\mu},\Gamma^+_{0\nu}]=\delta_{\mu\nu}. \quad (27)$$

The unperturbed vacuum state is annihilated by the operators $\vec{\Gamma}_1,\vec{\Gamma}_0$. The two spurious bosons (labeled by 0,1) have the same excitation spectrum as a result of the anomalous commutation relation in Eq. (27) and the form of the quadratic Hamiltonian (26).

In addition to the spurious sector $|n_{0\mu},n_{1\mu}\rangle$, the intrinsic sector displays elementary modes of excitation, which are represented by the finite-frequency RPA modes $|n_\nu\rangle$, $(\omega_\nu>0)$. Therefore the total intrinsic Hamiltonian $H'_{\rm BRST}$ may be written

$$H'_{\rm BRST}=H^{(2)}_{intr}+H_{res},$$
$$H^{(2)}_{intr}=H^{(2)}_{spur}+\sum_\nu \omega_\nu\left(n_\nu+\frac{1}{2}\right),$$
$$H_{res}=H_{(21)}+H_{(20)}-\vec{\Omega}\cdot\vec{p}_{(10)}+\frac{\omega^2}{A}\vec{B}\cdot\vec{r}_{(10)}. \quad (28)$$

IV. TRANSITION OPERATORS TO BE USED IN REALISTIC CALCULATIONS

The conversion of muons into electrons may proceed according to the lepton-flavor violating processes discussed in Refs. [4,16,17]. The main interest of such process lies on the necessary mixing of muon and electron neutrinos. So far there are experimental upper limits for this process [4]. An example of the current calculations is given in Refs. [16,17].

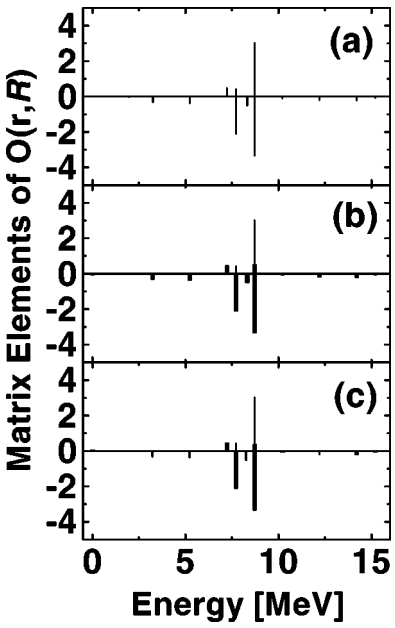

FIG. 2. The matrix elements of the vector operator of Eq. (29). Cases (a), (b), and (c) correspond to cases (b), (d), and (f) of Fig. 1. All values are scaled up by a factor 10.

Considering the dominance of the contributions due to the excitation of 1^- states, see Refs. [16,17], in the context of the RPA diagonalization, it is obvious that, from the nuclear structure point of view, one has to produce an estimation as accurate as possible of the nuclear matrix elements involved in the transitions.

The vector operator exciting $I^\pi=1^-$ states may be written (cf. Appendix D)

$$j_1(qr)Y_{1\mu}|_{(11)}=\sum_{n_\nu}\langle n_\nu|j_1Y_1|\rangle[\gamma^+_{n_\nu,\mu}-(-1)^\mu\gamma_{n_\nu,(-\mu)}]$$
$$-\frac{i}{A}(-1)^\mu\langle[j_1Y_{1\mu},p_{-\mu}]\rangle r_{\mu(11)}, \quad (29)$$

where n_ν denotes a finite-frequency RPA mode and a similar expression should be used for the dipole axial-vector term of the weak current. Its contribution is not affected by the treatment of the spurious sector. The amplitude in the second line may be regularized as in Sec. III B. The results corresponding to the transition matrix elements of the shifted operator (29), obtained in the RPA diagonalization and for transitions involving the states depicted in Fig. 1, are shown in Fig. 2. The similarity between the results obtained with the three different Hamiltonians supports the above claim about the validity of the procedure. The same effects are expected to materialize in the case of realistic calculations and work is in progress to include the counterterms obtained in the previous sections, starting from realistic two-body forces.

V. CONCLUSIONS

We have attacked the problem of the center of mass motion in nuclear spectroscopy calculations in two successive

steps, namely (i) the reconstruction of the translational invariance of the Hamiltonian and (ii) the inclusion of collective variables in order to eliminate infrared singularities. The solutions in both steps are exact, albeit perturbative. They share in common the same perturbation parameter, $A^{-1/2}$, which is a convenient feature if a given order of perturbation is envisioned.

Quite generally, the first step requires the introduction of counterterms, which has been performed following specially Ref. [10]. We have tried the procedure for pure independent-particle Hamiltonians: in the case of harmonic oscillator potential, the counterterms reproduce the dipole force with the self-consistent strength (see Ref. [8]); for spin-orbit and l^2 terms in the central potential the procedure leads to the introduction of two-body terms of similar character; finally, the application of the procedure to an empirical single-particle spectrum insures the elimination of all the matrix elements of the coordinate operator to excited $I^\pi = 1^-$ RPA states, which is not the case for the dipole interaction, no matter how close to zero the lowest RPA energy is made by adjusting the strength of the interaction.

The introduction of the collective formalism is made along the same lines as in Ref. [15], benefitting from the Abelian character of the transformations associated with translational motion. In this case the Hamiltonian displays the collective energy term and this degree of freedom becomes uncoupled from the remaining part of the Hamiltonian. The procedure requires however a nontrivial transformation for all physical operators, which leads, for instance, to the substitution of the intrinsic center of mass operator $(1/A)\vec{r}$ by the collective operator $\vec{\mathcal{R}}$ representing the same magnitude, and thus to the elimination of the infrared singularities associated with the former.

To conclude, we thus suggest the use of the counterterms (2) and of the operators (29) in cases where, like in (μ^-, e^-) conversion, the dominance of the $I^\pi = 1^-$ channels is apparent.

ACKNOWLEDGMENTS

Discussions with E. Maqueda and N. Scoccola are gratefully acknowledged. This work was supported in part by Fundación Antorchas, Consejo Nacional de Investigaciones Cientí ficas y Técnicas through the Carrera del Investigador Cientí fico, Agencia Nacional para la Promoción Cientí fica y Tecnológica Argentina (Contract No. PICT03-00000-00133), and Universidad Favaloro (Proyecto 010/99). O.C. thanks the Institute for Nuclear Theory at the University of Washington for its hospitality.

APPENDIX A: SOLUTION OF THE EQUATIONS THAT DETERMINE THE COUNTERTERMS

We attempt a solution to the inhomogenous equations (3) and (4) in terms of an expansion similar to the one implicit in the RPA. Since this expansion is not always well defined from the point of view of an expansion parameter, we start by discussing the meaning of it.

Let $X_{(nm)}$ be a n-body operator such that there are m bodies crossing the Fermi surface and $(n-m)$ bodies that do not cross[6] $(0 \leqslant n,\ 0 \leqslant m \leqslant n)$. As is well known, within the RPA, the expectation value $X_{(00)}$ is privileged over the particle-hole term $X_{(11)}$, and this last component relative to the terms $X_{(10)}$ representing the scattering of a particle above or below the Fermi surface,

$$O(X_{(00)}) > O(X_{(11)}) > O(X_{(10)}). \tag{A1}$$

Also, within the RPA, the (large) parameter measures the collectivity and, for the case of motion of the center of mass, it should be represented by some power of the number of particles A. A different power of the (small) parameter $A^{-1/2}$ may be assigned to each term in Eq. (A1). Consequently,

$$O(X_{(00)}) = \mathcal{X};\quad O(X_{(11)}) = \mathcal{X}/A^{1/2};\quad O(X_{(10)}) = \mathcal{X}/A, \tag{A2}$$

where $\mathcal{X}$ is another parameter depending on the chosen operator. For instance, in the case of the momentum component p_μ, $p_{\mu(00)} = 0$, and $\mathcal{X} = A$, while

$$O(p_{\mu(11)}) = A^{1/2};\quad O(p_{\mu(10)}) = 1. \tag{A3}$$

We generalize the estimation (A2) for the case of n-body operators, namely

$$O(X_{nm}) = \mathcal{X} A^{-n+m/2}. \tag{A4}$$

As an illustration, we ascribe the following orders to the different terms in the Hamiltonian:

$$O(H_{(00)}) = \epsilon A;\quad O[H_{(11)}(=0)] = \epsilon A^{1/2};$$

$$O(H_{(10)}) = \epsilon O(H_{(22)}) = \epsilon;$$

$$O(H_{(21)}) = \epsilon/A^{1/2};\quad O(H_{(20)}) = \epsilon/A. \tag{A5}$$

Here we have taken the energy parameter ϵ as the order of the single-particle energies. Note that $O(H_{(10)}) = O(H_{(22)})$ as befits the RPA.

This expansion is also valid for the commutation relations. The (n,m) terms in the the commutation of an operator $X = \Sigma_{(n'm')} X_{(n'm')}$ with a one-body operator reads

$$\begin{aligned}[X, (Y_{(11)} + Y_{(10)})]_{(nm)} = {} & [X_{[(n+1)(m+1)]}, Y_{(11)}]_{(nm)} \\ & + [X_{[n(m\pm 1)]}, Y_{(11)}]_{(nm)} \\ & + [X_{(nm)}, Y_{(10)}]_{(nm)}.\end{aligned} \tag{A6}$$

A particular example is

$$[r_{-\mu}, p_\mu]_{(nm)} = \delta_{n0} [r_{-\mu(11)}, p_{\mu(11)}]_{(00)} = iA(-1)^\mu. \tag{A7}$$

[6] We disregard exchange in the associated integrals.

Let us now proceed to solve Eqs. (3) and (4). We may split them into equations corresponding to a definite pair of quantum numbers (nm), and thus to a particular order of magnitude. According to Eq. (A6), the equations involving only the one-body component of the operators $P_{\mu(1v)}$ and $R_{\mu(1v)}$ $(v=1,0)$ may be written as

$$0=\pi_{\mu(1v)}+iAP_{\mu(1v)}+(-1)^{\nu}[P_{-\nu},p_{\mu}]_{(00)}r_{\nu(1v)}+(-1)^{\nu}[R_{-\nu},p_{\mu}]_{(00)}p_{\nu(1v)},$$

$$0=\rho_{\mu(1v)}-iAR_{\mu(1v)}+(-1)^{\nu}[P_{-\nu},r_{\mu}]_{(00)}r_{\nu(1v)}+(-1)^{\nu}[R_{-\nu},r_{\mu}]_{(00)}p_{\nu(1v)}.$$

These equations are of $\mathcal{O}(\epsilon A^{1/2})$ for the case $v=1$ and of $\mathcal{O}(\epsilon)$ for $v=0$. In both cases they uncouple, since the time reversal symmetry of the Hamiltonian requires that

$$[R_{\nu},p_{\mu}]_{(00)}=[P_{\nu},r_{\mu}]_{(00)}=0. \tag{A8}$$

In order to solve the two equations thus simplified, we commute the first and the second one with the operators p_{μ} and r_{μ}, respectively. The expectation value of such commutators yields the relations

$$(-1)^{\nu}[P_{-\nu},p_{\mu}]_{(00)}=\delta_{\nu\mu}\frac{i(-1)^{\mu}}{2A}[\pi_{-\mu(11)},p_{\mu(11)}]_{(00)},$$

$$(-1)^{\nu}[R_{-\nu},r_{\mu}]_{(00)}=-\delta_{\nu\mu}\frac{i(-1)^{\mu}}{2A}[\rho_{-\mu(11)},r_{\mu(11)}]_{(00)}, \tag{A9}$$

where, according to Eq. (A6),

$$\pi_{\mu(11)}=[H_{(10)},p_{\mu(11)}]_{(11)}+[H_{(22)},p_{\mu(11)}]_{(11)},$$

$$\rho_{\mu(11)}=[H_{(10)},r_{\mu(11)}]_{(11)}+[H_{(22)},r_{\mu(11)}]_{(11)}. \tag{A10}$$

Substitution of Eq. (A9) into the first two equations yields the one-body components of the operators P_{μ},R_{μ} [Eq. (5)].

The equations for the two-body components $(2v)(v=0,1,2)$ of the operators P_{μ},R_{μ} read

$$\begin{aligned}0&=\pi_{\mu(2v)}+iAP_{\mu(2v)}+(-1)^{\nu}[P_{-\nu},p_{\mu}]_{[1(v-1)]}r_{\nu(11)}\\&\quad+(-1)^{\nu}[R_{-\nu},p_{\mu}]_{[1(v-1)]}p_{\nu(11)}\\&\quad+(-1)^{\nu}[P_{\nu},p_{\mu}]_{(1v)}r_{\nu(10)}+(-1)^{\nu}[R_{\nu},p_{\mu}]_{(1v)}p_{\nu(10)}\\&=\tau_{\mu(2v)}+iAP_{\mu(2v)}\\&\quad-\sum_{\alpha}\sqrt{\frac{2\alpha+1}{3}}([[P_{(2v)},p_{(11)}]^{\alpha}_{(1(v-1))}r_{(11)}]^{1}_{\mu(2v)}\\&\quad+[[R_{(2v)},p_{(11)}]^{\alpha}_{(1(v-1))}p_{(11)}]^{1}_{\mu(2v)}),\end{aligned}$$

$$\begin{aligned}0&=\rho_{\mu(2v)}-iAR_{\mu(2v)}+(-1)^{\nu}[P_{-\nu},r_{\mu}]_{[1(v-1)]}r_{\nu(11)}\\&\quad+(-1)^{\nu}[R_{-\nu},r_{\mu}]_{[1(v-1)]}p_{\nu(11)}\\&\quad+(-1)^{\nu}[P_{\nu},r_{\mu}]_{(1v)}r_{\nu(10)}+(-1)^{\nu}[R_{\nu},r_{\mu}]_{(1v)}p_{\nu(10)}\\&=\eta_{\mu(2v)}-iAR_{\mu(2v)}\\&\quad-\sum_{\alpha}\sqrt{\frac{2\alpha+1}{3}}([[P_{(2v)},r_{(11)}]^{\alpha}_{[1(v-1)]}r_{(11)}]^{1}_{\mu(2v)}\\&\quad+[[R_{(2v)},r_{(11)}]^{\alpha}_{[1(v-1)]}p_{(11)}]^{1}_{\mu(2v)}),\end{aligned} \tag{A11}$$

where

$$\begin{aligned}\tau_{\mu(2v)}&=\pi_{\mu(2v)}-\sum_{\sigma}\sqrt{\frac{2\sigma+1}{3}}\\&\quad\times\Bigg[\sum_{w}([[P_{(1)},p_{(1)}]^{\sigma}_{[1(v-w)]}r_{(1w)}]^{1}_{\mu(2v)}\\&\quad+[[R_{(1)},p_{(1)}]^{\sigma}_{[1(v-w)]}p_{(1w)}]^{1}_{\mu(2v)})\\&\quad+([[P_{[2(v+1)]},p_{(11)}]^{\sigma}_{(1v)}r_{(10)}]^{1}_{\mu(2v)}\\&\quad+[[R_{[2(v+1)]},p_{(11)}]^{\sigma}_{(1v)}p_{(10)}]^{1}_{\mu(2v)})\Bigg],\end{aligned}$$

$$\begin{aligned}\eta_{\mu(2v)}&=\rho_{\mu(2v)}-\sum_{\sigma}\sqrt{\frac{2\sigma+1}{3}}\\&\quad\times\Bigg[\sum_{w}([[P_{(1)},r_{(1)}]^{\sigma}_{[1(v-w)]}r_{(1w)}]^{1}_{\mu(2v)}\\&\quad+[[R_{(1)},r_{(1)}]^{\sigma}_{[1(v-w)]}p_{(1w)}]^{1}_{\mu(2v)})\\&\quad-([[P_{[2(v+1)]},r_{(11)}]^{\sigma}_{(1v)}r_{(10)}]^{1}_{\mu(2v)}\\&\quad+[[R_{[2(v+1)]},r_{(11)}]^{\sigma}_{(1v)}p_{(10)}]^{1}_{\mu(2v)})\Bigg].\end{aligned} \tag{A12}$$

We try

$$\begin{aligned}P_{\mu(2v)}&=\frac{i}{A}\tau_{\mu(2v)}+\frac{1}{2A^{2}}\sum_{\alpha}\sqrt{\frac{2\alpha+1}{3}}\\&\quad\times(g^{(P)}_{\alpha}[[\tau_{(2v)},p_{(11)}]^{\alpha}_{[1(v-1)]}r_{(11)}]^{1}_{\mu(2v)}\\&\quad+j^{(P)}_{\alpha}[[\eta_{(2v)},p_{(11)}]^{\alpha}_{[1(v-1)]}p_{(11)}]^{1}_{\mu(2v)}),\\R_{\mu(2v)}&=-\frac{i}{A}\eta_{\mu(2v)}+\frac{1}{2A^{2}}\sum_{\alpha}\sqrt{\frac{2\alpha+1}{3}}\\&\quad\times(g^{(R)}_{\alpha}[[\tau_{(2v)},r_{(11)}]^{\alpha}_{[1(v-1)]}r_{(11)}]^{1}_{\mu(2v)}\\&\quad+j^{(R)}_{\alpha}[[\eta_{(2v)},r_{(11)}]^{\alpha}_{[1(v-1)]}p_{(11)}]^{1}_{\mu(2v)}).\end{aligned} \tag{A13}$$

Introducing Eq. (A13) in Eqs. (A11) and using the identity

$$[\tau_{(2v)},r_{(11)}]^{\sigma}=[\eta_{(2v)},p_{(11)}]^{\sigma}(-1)^{\sigma}, \tag{A14}$$

we obtain

$$
\begin{aligned}
0=&-\frac{i}{A}\sum_{\sigma}\sqrt{\frac{2\sigma+1}{3}}\\
&\times\left(1-\frac{g_{\sigma}^{(P)}}{2}[1+(-1)^{\sigma}]\right)\\
&\times[[\tau_{(2v)},p_{(11)}]^{\sigma}_{[1(v-1)]}r_{(11)}]^{1}_{\mu(2v)}+\frac{i}{A}\\
&\times\sum_{\sigma}\sqrt{\frac{2\sigma+1}{3}}\left(1+\frac{j_{\sigma}^{(P)}}{2}+\frac{g_{\sigma}^{(R)}}{2}\right)\\
&\times[[\eta_{(2v)},p_{(11)}]^{\sigma}_{[1(v-1)]}p_{(11)}]^{1}_{\mu(2v)}\\
&+\frac{1}{2\sqrt{3}A^{2}}(g_{1}^{(R)}+j_{1}^{(P)})[[\tau_{(22)},r_{(11)}]^{1}_{(11)},p]^{0}_{0(00)}[pr]^{1}_{\mu(2v)},
\end{aligned}
$$

$$
\begin{aligned}
0=&-\frac{i}{A}\sum_{\sigma}\sqrt{\frac{2\sigma+1}{3}}\left(1+\frac{g_{\sigma}^{(R)}}{2}+\frac{j_{\sigma}^{(P)}}{2}\right)\\
&\times[[\tau_{(2v)},r_{(11)}]^{\sigma}_{[1(v-1)]}r_{(11)}]^{1}_{\mu(2v)}+\frac{i}{A}\\
&\times\sum_{\sigma}\sqrt{\frac{2\sigma+1}{3}}\left(1-\frac{j_{\sigma}^{(R)}}{2}[1+(-1)^{\sigma}]\right)\\
&\times[[\eta_{(2v)},r_{(11)}]^{\sigma}_{[1(v-1)]}p_{(11)}]^{1}_{\mu(2v)}+\frac{1}{2\sqrt{3}A^{2}}\\
&\times(g_{1}^{(R)}+j_{1}^{(P)})[[\tau_{(22)},r_{(11)}]^{1}_{(11)},r_{(11)}]^{0}_{0(00)}[pr]^{1}_{\mu(2v)}.
\end{aligned}
\tag{A15}
$$

The solution appears to be

$$
\begin{aligned}
&g_{\sigma}^{(P)}=j_{\sigma}^{(R)}=1;\quad \sigma=0,2,\\
&g_{\sigma}^{(R)}=j_{\sigma}^{(P)}=-1;\quad \sigma=0,1,2.
\end{aligned}
\tag{A16}
$$

The coefficients $g_{1}^{(P)}$ and $j_{1}^{(R)}$ are not determined. The following terms are left in the right-hand side of Eq. (A15):

$$
\begin{aligned}
0=&-\frac{i}{A}[[\tau_{(2v)},p_{(11)}]^{1}_{[1(v-1)]}r_{(11)}]^{1}_{\mu(2v)}\\
&-\frac{1}{\sqrt{3}A^{2}}[[\tau_{(22)},r_{(11)}]^{1}_{(11)},p]^{0}_{0(00)}[pr]^{1}_{\mu(2v)},\\
0=&+\frac{i}{A}[[\eta_{(2v)},r_{(11)}]^{1}_{[1(v-1)]}p_{(11)}]^{1}_{\mu(2v)}\\
&-\frac{1}{\sqrt{3}A^{2}}[[\tau_{(22)},r_{(11)}]^{1}_{(11)},r_{(11)}]^{0}_{0(00)}[pr]^{1}_{\mu(2v)}.
\end{aligned}
\tag{A17}
$$

Equations (A17) should be satisfied in general only to leading order. We may commute the first (second) one by $r_{(11)}$ ($p_{(11)}$) and consider only the leading order terms in the RPA expansion, which in this case correspond to the one-body terms, namely

$$
\begin{aligned}
0=&\frac{i}{\sqrt{3}A}([[\tau_{(22)},p_{(11)}]^{1}_{(11)},r_{(11)}]^{0}_{0(00)}\\
&-[[\tau_{(22)},r_{(11)}]^{1},p_{(11)}]^{0}_{0(00)})r_{\mu(11)},\\
0=&\frac{i}{\sqrt{3}A}([[\eta_{(22)},r_{(11)}]^{1}_{(11)},p_{(11)}]^{0}_{0(00)}\\
&-[[\eta_{(22)},p_{(11)}]^{1},r_{(11)}]^{0}_{0(00)})p_{\mu(11)}.
\end{aligned}
\tag{A18}
$$

The differences enclosed by the parentheses vanish according to Eq. (A14).

The general solution of Eqs. (3) and (4) may be written as a superposition of the solution (6) of the inhomogenous equations plus solutions of the homogenous equations with $\pi_{\mu}=\rho_{\mu}=0$. However, we must drop the last contributions since the residual interactions should vanish if the Hamiltonian is translational and Galilean invariant.

APPENDIX B: INTERACTIONS ORIGINATED FROM SINGLE-PARTICLE HAMILTONIANS

The calculation of the one-body terms of the operators P_{μ},R_{μ} requires the knowledge of the operators π_{μ},ρ_{μ} and of the expectation values $\langle[\pi_{-}\mu,p_{\mu}]\rangle$, $\langle[\rho_{-}\mu,r_{\mu}]\rangle$ [cf. Eq. (5)]. The notation used as superscripts is obvious,

$$
\begin{aligned}
&\pi_{\mu}^{ho}=im\omega^{2}r_{\mu};\quad (-1)^{\mu}\langle[\pi_{\mu}^{ho},p_{-\mu}]\rangle=-m\omega^{2}A,\\
&\rho^{ho}=0,
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
&\pi_{\mu}^{so}=-i\chi_{so}(\vec{p}\times\vec{s})_{\mu};\quad \langle[\pi_{\mu}^{so},p_{-\mu}]\rangle=0,\\
&\rho_{\mu}^{so}=-i\chi_{so}(\vec{r}\times\vec{s})_{\mu};\quad \langle[\rho_{\mu}^{so},r_{-\mu}]\rangle=0,
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
&\pi_{\mu}^{ll}=i\chi_{ll}(\vec{l}\times\vec{p})_{\mu}-i\chi_{ll}(\vec{p}\times\vec{l})_{\mu};\\
&(-1)^{\mu}\langle[\pi_{\mu}^{ll},p_{-\mu}]\rangle=\frac{4\chi_{ll}}{3}\langle p^{2}\rangle,\\
&\rho_{\mu}^{ll}=i\chi_{ll}(\vec{l}\times\vec{r})_{\mu}-i\chi_{ll}(\vec{r}\times\vec{l})_{\mu};\\
&(-1)^{\mu}\langle[\rho_{\mu}^{ll},r_{-\mu}]\rangle=\frac{4\chi_{ll}}{3}\langle r^{2}\rangle,
\end{aligned}
\tag{B3}
$$

$$
\begin{aligned}
&\langle k||\pi^{sp}||i\rangle=\epsilon_{ki}\langle k||p||i\rangle,\\
&\langle[\pi_{\mu}^{sp},p_{-\mu}]\rangle=-\frac{2}{3}\sum_{ki}\langle k||\pi^{sp}||i\rangle\langle k||p||i\rangle,\\
&\langle k||\rho^{sp}||i\rangle=\epsilon_{ki}\langle k||r||i\rangle+\frac{i}{m}\langle k||p||i\rangle,
\end{aligned}
$$

$$(-1)^{\mu}\langle[\rho^{sp}_{\mu},r_{-\mu}]\rangle=-\frac{2}{3}\sum_{ki}\ \langle k||\rho^{sp}||i\rangle\langle k||r||i\rangle, \tag{B4}$$

where $k(i)$ denotes single-particle states above (below) the Fermi sea and ϵ_{ki} represents the excitation energy of the corresponding particle-hole state.

APPENDIX C: THE TRANSFORMATION TO NORMAL MODES IN THE BRST HAMILTONIAN

The transformation to normal modes in the BRST Hamiltonian (cf. Sec. III C) is the following:

$$\Gamma^{+}_{1\mu}=\frac{1}{\sqrt{2mA\omega}}p_{\mu(11)}-\sqrt{\frac{mA}{2\omega}}\Omega_{\mu}+\frac{i}{A}\sqrt{\frac{mA\omega}{2}}r_{\mu(11)},$$

$$\Gamma^{+}_{0\mu}=\sqrt{\frac{\omega}{2mA}}B_{\mu}+\sqrt{\frac{mA}{2\omega}}\Omega_{\mu}+\frac{i}{A}\sqrt{\frac{mA\omega}{2}}r_{\mu(11)}. \tag{C1}$$

APPENDIX D: EXCITATION OF THE SPURIOUS MODES

The RPA transformation from particle-hole modes $\gamma^{+}_{ki,\mu}$ to normal modes $\gamma^{+}_{n,\mu}$ reads

$$\gamma^{+}_{n,\mu}=\sum_{ki}\ [\lambda_{n,ki}\gamma^{+}_{ki,\mu}-(-1)^{\mu}\mu_{n,ki}\gamma_{ki,-\mu}], \tag{D1}$$

where $k(i)$ denotes states above (below) the Fermi level.[7] Any single-particle operator transforming as the coordinate r_{μ} under time reversal, such as Eq. (29), may be written

$$j_1(qr)Y_{1\mu}(\theta,\phi)|_{(11)}$$
$$=\frac{1}{\sqrt{3}}\sum_{ki}\ \langle k||j_1Y_1||i\rangle[\gamma^{+}_{ki,\mu}-(-1)^{\mu}\gamma_{ki,\mu}]$$
$$=\sum_{n}\ \langle n|j_1Y_1|\rangle[\gamma^{+}_{n,\mu}-(-1)^{\mu}\gamma_{n,-\mu}]. \tag{D2}$$

Let us denote by $n=g$ the zero-frequency mode, and by $n=n_{\nu}$ a finite-frequency one. We can make the following choice:

$$r_{\mu(11)}=-\frac{iA}{\sqrt{2\omega_g}}[\gamma^{+}_{g,\mu}-(-1)^{\mu}\gamma_{g,-\mu}],$$

$$p_{\mu(11)}=\sqrt{\frac{\omega_g}{2}}[\gamma^{+}_{g,\mu}+(-1)^{\mu}\gamma_{g,-\mu}]. \tag{D3}$$

As a consequence, the amplitudes in the RPA transformation read

$$\lambda_{g,ki}=\frac{1}{\sqrt{6\omega_g}}\langle k||p||i\rangle+\frac{i}{A}\sqrt{\frac{\omega_g}{6}}\langle k||r||i\rangle,$$

$$\mu_{g,ki}=-\frac{1}{\sqrt{6\omega_g}}\langle k||p||i\rangle+\frac{i}{A}\sqrt{\frac{\omega_g}{6}}\langle k||r||i\rangle, \tag{D4}$$

and the amplitude to the spurious boson reads

$$\langle g|j_1Y_1|\rangle=\frac{1}{3\sqrt{\omega_g}}\sum_{ki}\ \langle k||j_1Y_1||i\rangle\langle k||p||i\rangle$$
$$=-(-1)^{\mu}\frac{1}{2}\langle[j_1Y_{1\mu},p_{-\mu}]\rangle, \tag{D5}$$

as in Eq. (29).

[7] Coupling to $I^{\pi}=1^{-}$ is assumed.

[1] J. D. Vergados, Phys. Rep. **133**, 1 (1986).
[2] F. Boehm and P. Vogel, *Physics of Massive Neutrinos*, 2nd ed. (Cambridge University Press, Cambridge, 1992).
[3] M. K. Moe and P. Vogel, Annu. Rev. Nucl. Part. Sci. **44**, 247 (1994).
[4] T. S. Kosmas, G. K. Leontaris, and J. D. Vergados, Prog. Part. Nucl. Phys. **33**, 397 (1994).
[5] J. P. Elliot and T. H. R. Skyrme, Proc. R. Soc. London, Ser. A **232**, 56 (1955).
[6] D. H. Gloeckner and R. D. Lawson, Phys. Lett. **53B**, 313 (1974).
[7] J. B. McGrory and B. H. Wildenthal, Phys. Lett. **60B**, 5 (1975).
[8] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, London, 1975), Vol. II, p. 444.
[9] N. I. Pyatov and D. I. Salamov, Nukleonika **22**, 127 (1977).
[10] J. Meyer ter Vehn, Z. Phys. A **289**, 319 (1979).
[11] B. L. Birbrair and V. A. Sadovnikova, Yad. Fiz. **20**, 645 (1974) [Sov. J. Nucl. Phys. **20**, 347 (1975)].
[12] H. Sakamoto and T. Kishimota, Nucl. Phys. **A501**, 205 (1989); **A501**, 242 (1989).
[13] E. R. Marshalek, Phys. Lett. B **244**, 1 (1990).
[14] E. R. Marshalek, Phys. Rev. C **50**, R5 (1994).
[15] D. R. Bes and J. Kurchan, *The Treatment of Collective Coordinates in Many-Body Systems* (World Scientific, Singapore, 1990).
[16] T. S. Kosmas, J. D. Vergados, O. Civitarese, and A. Faessler, Nucl. Phys. **A570**, 637 (1994).
[17] J. Schwieger, A. Faessler, and T. S. Kosmas, Phys. Rev. C **56**, 2830 (1997).