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Description of identical superdeformed bands with $\Delta I=4$ bifurcation

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With the supersymmetry scheme including many-body interactions, a parametrization of the excitation energy of superdeformed (SD) states is proposed. The identical SD bands with $\Delta I=4$ bifurcation, ^{149}Gd(1) $-$ ^{148}Gd(6) $-$ ^{148}Eu(1), are investigated. Quantitatively good results are obtained. The result shows that the parametrization can describe the identical SD bands and the $\Delta I=4$ bifurcation simultaneously. It suggests that the $\Delta I=4$ bifurcation in the SD bands may have a bearing on a perturbation exhibiting the SO(5) [or SU(5)] symmetry in the mean field and the identical bands may be interrelated with the supersymmetry.

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It has been observed that, for superdeformed (SD) bands, there exists fascinating phenomena such as the identical bands (IBs) [1], $\Delta I=4$ bifurcation (or $\Delta I=2$ staggering) [2], and even IBs with $\Delta I=4$ bifurcation [3]. Many attempts have been made to describe the properties and explore the underlying physics. In both non-relativistic theory (see, for example, Refs. [4–7]) and relativistic mean field theory [8], the phenomena have been studied. In the pseudo-SU(3) model [9], the pseudospin symmetry model [10], the $C_{4\nu}$-symmetry model [11–13], and other approaches [14–17], the SD bands have also been discussed. The investigations suggest that the phenomenon of IBs may result from a cancellation of contributions to the moment of inertia occurring in mean field methods [4,5]. However, whether there exists a ''heroic'' explanation based on some symmetry of the mean field is not clear [5]. With regard to the $\Delta I=4$ bifurcation, whether the C_4 symmetry is sufficient to induce the bifurcation is still under debate [6,7,18], and other mechanisms have been postulated [15–17]. Meanwhile, a sophisticated scheme to describe the IBs with $\Delta I=4$ bifurcation [3] has not yet been established [18]. After the interacting boson model (IBM) [19] had been exploited to describe SD bands [14,17,20–22], an algebraic model based on the IBM was proposed, in which the SD bands of even-even nuclei in $A\sim 150$ and 190 regions are described well [23,24]. With the extension to supersymmetry [25], quite good results have also been obtained in depicting the SD bands of odd-A nuclei and some of the identical bands [26]. In this paper, by extending the above algebraic approach, we propose a model to describe the identical bands with $\Delta I=4$ bifurcation.

Experimental data show that superdeformed bands exhibit quite good rotational characteristics. The dynamical symmetry group chains to label the states should be the ones ending with SO(3). To describe the SD states in even-even, odd-A, and odd-odd nuclei in a unified way, we propose that the dynamical symmetry is supersymmetry. The states can thus be classified with supersymmetric group chain

$$\begin{array}{ccccccccccc} \mathrm{U}(m,n) & \supset & \mathrm{U}_B(m)\otimes \mathrm{U}_F(n) & \supset\cdots\supset & \mathrm{SO}_{B+F}(3) & \otimes & \mathrm{SU}_F(n') & \supset & \mathrm{Spin}(3), \\ [N] & & [N_B]_m \quad [N_F]_n & & L & & S & & I \end{array}$$

where m is determined by the constituent of the bosons. n is decided by the single particle configuration of the fermion(s), and n' by the total pseudospin. $N=N_B+N_F$ is the total number of particles with N_B, N_F the boson, fermion numbers, respectively. In the framework of supersymmetry [25], even-even nucleus and its odd-A and odd-odd neighbors are the multiplets of the irreducible representation (irrep) $[N]$ of the supergroup $\mathrm{U}(m,n)$.

Since the n and N_F can be decided with the assignment of the single particle configuration, what we should determine is the m and N_B. Since the bosons to describe positive parity SD states should be s, d, and g bosons [17,20,22], and p, f bosons are essential to depict negative parity states [27], the constituent of the bosons should be s, d, g, p, and f bosons, i.e., $m=25$. Symmetry analyses [28] showed that the subset of s, d, g bosons holds $\mathrm{SU}_{\mathrm{sdg}}(5)$, $\mathrm{SU}_{\mathrm{sdg}}(3)$, and other symmetry limits. Examining the geometric shape, hexadecupole deformation parameter β_4 and energy spectrum indicates that the $\mathrm{SU}_{sdg}(5)$ symmetry can describe well-deformed nuclear states as well as the $\mathrm{SU}_{sdg}(3)$ symmetry [29,30]. Other studies manifest that the $\mathrm{SU}_{sdg}(5)$ symmetry can generate a geometric shape with C_4 symmetry [17] and energy spectra exhibiting $\Delta I=2$ staggering [30]. Meanwhile, the potential energy surface of the $SU_{sdg}(5)$ symmetry has two minima displayed with different energies [29]. Then, the $SU_{sdg}(5)$ symmetry can be taken to describe positive parity SD states. On the other hand, it has been known that the subset of p, f bosons has also SU(5) symmetry [denoted as $SU_{pf}(5)$] [27]. We therefore have the group chain for the boson part

$$\begin{array}{ccccccccccccc} U_{sdgpf}(25) & \supset & U_{sdg}(15)\otimes U_{pf}(10) & \supset & SU_{sdg}(5)\otimes SU_{pf}(5) & \supset & SU(5) & \supset & SO(5) & \supset & SO(3), \\ [N_B] & & [N_{sdg}] \quad [N_{pf}] & & IR(5_{PP}) \quad IR(5_{NP}) & & [n_1,n_2,n_3,n_4] & & (\tau_1,\tau_2) & & L_B \end{array}$$

in which SD states can be described. Here the $IR(5_{PP}),IR(5_{NP})$ refer to the irrep $[n_1,n_2,n_3,n_4]_{sdg}$, $[n_1,n_2,n_3,n_4]_{pf}$ of the group $SU_{sdg}(5)$, $SU_{pf}(5)$, respectively. For a nucleus with definite N_B and N_F, all the possible irreps., $[N_{sdg}]$, $[N_{pf}]$, $[n_1,n_2,n_3,n_4]_{sdg}$, $[n_1,n_2,n_3,n_4]_{pf}$, $[n_1,n_2,n_3,n_4]$, (τ_1,τ_2), and L_B can be fixed by the branching rules of the irrep reductions of $U_{sdg}(15)\supset SU_{sdg}(5)$ [28], $U_{pf}(10)\supset SU_{pf}(10)\supset SU_{pf}(5)$ [27], $SU(5)\otimes SU(5)\supset SU(5)$ (cf. Ref. [31]), $SU(5)\supset SO(5)$, $SO(5)\supset SO(3)$ [28] and the trivial one $N_{sdg}=N_B-N_{pf}$ with $N_{pf}\in[0,N_B]$. In practice, it is usual to take $N_{pf}=0$ for positive parity states and $N_{pf}=1$ for negative parity states because only ‘‘octupole vibration’’ has been observed in SD states (see, for example, Ref. [32]). The contribution of the p and f bosons can thus be included on the level of angular momentum coupling. Taking advantage of the spectrum generating principle, we know that the irreps of the SU(5) groups contribute a constant to the energy of all the states labeled by each of them. Consequently, they contribute nothing to the relative excitation energies of the states in a band. The contribution of the bosons to the γ-ray energies in a SD band is thus in fact the one with the SO(5) symmetry. We then get

$$E=E_0(N_B,N_F)+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + C_L L(L+1)+C_S S(S+1)+C_I I(I+1), \qquad (1)$$

where $\vec{L}=\vec{L}_B+\vec{L}_F$ is the total angular momentum of the effective core ($\vec{L}_F$ is the pseudo-orbital angular momentum of the fermion). $\vec{I}$ is the total spin of the nucleus ($\vec{I}=\vec{L}+\vec{S}$ with $\vec{S}$ being the total pseudospin). For the fully stretched pseudospin configuration (i.e., $I=L+S$), with an effective aligned angular momentum i being introduced as $i=C_L S/(C_L+C_I)$, Eq. (1) can be written as

$$E=E_0(N_B,N_F)+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+CI'(I'+1), \qquad (2)$$

where $I'=I-i$, $C=C_L+C_I$, and $E_0(N_B,N_F)$ is a little different from that in Eq. (1) since a constant is involved. Obviously, by adjusting the ratio C_L/C_I, one can get any value of the alignment i. In particular, taking $C_L=0$, one has $i=0$, i.e., the strong coupling limit. If $C_I=0$, one gets $i=S$, i.e., the pseudospin decoupling limit.

It is evident that the variance of the dynamical moment of inertia ($\mathcal{J}^{(2)}$) vs rotational frequency ($\hbar\omega$) cannot be reproduced by Eq. (2) if the parameter C is taken as a constant. In light of the variable moment of inertia model [33,23,24], we can write the C as a function of the angular momentum I'. We thus get

$$E=E_0(N_B,N_F)+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + \frac{C_0}{1+f_1 I'(I'+1)+f_2 I'^2(I'+1)^2} I'(I'+1), \qquad (3)$$

where C_0, f_1, and f_2 are parameters. Including the terms with f_1 and f_2 in the denominator is a way to take into account many-body interactions which induce antipairing driving and pairing damping effects on the $\mathcal{J}^{(2)}$ [34]. It is apparent that the anti-pairing (or pairing) effect can be enhanced if $f_1>0$, $f_2>0$ (or $f_1<0$, $f_2<0$). As $f_1>0$, $f_2<0$ (or $f_1<0$, $f_2>0$), both the antipairing and pairing effects are taken into account. Even though the irreps can be fixed with the branching rules of the irrep reduction, it is difficult to determine the (τ_1,τ_2) of the SO(5) due to the complexity of the microscopic configurations of SD states. Given that SD bands are generated by the nontotally symmetric irrep. $[2N_B-2,2,0,0]$ of the SU(5) group, the (τ_1,τ_2) in practical calculation can be simply given as [26]

$$(\tau_1,\tau_2) = \begin{cases} \left(\left[\frac{L}{2}\right],0\right), & \text{if } L=4k,\ 4k+1 (k=0,1,\cdots), \\ \left(\left[\frac{L}{2}\right]-1,2\right), & \text{if } L=4k+2,\ 4k+3 (k=0,1,\cdots), \end{cases} \qquad (4)$$

where $L=[I']$, and $[a]$ denotes to take the value of the integer part of a.

Since the SD bands ^{148}Gd(6) and ^{148}Eu(1) are believed to be the one fermion excitation states with respect to ^{149}Gd(1) [3,35,36], we take ^{149}Gd(1) as the reference band

TABLE I. Obtained γ-ray energies of the identical bands ^{149}Gd(1), ^{148}Gd(6), and ^{148}Eu(1) in the present supersymmetry scheme with single particle energy being considered (labeled with $C_{\rm Full}$) or not (labeled with $C_{\rm SUSY}$) and the comparison with experimental data (taken from Refs. [2,3,36]).

^{149}Gd(1)			^{148}Gd(6)				^{148}Eu(1)			
Spin	Exp.	Cal.	Spin	Exp.	$C_{\rm SUSY}$	$C_{\rm Full}$	Spin	Exp.	$C_{\rm SUSY}$	$C_{\rm Full}$
27.5	617.8(1)	615.4					28			
29.5	664.2(1)	662.9					30		674.96	651.45
31.5	711.8(1)	711.1					32		722.98	700.09
33.5	759.7(1)	759.4	33		747.29	742.49	34	747.7(1)	771.77	748.62
35.5	808.1(1)	808.6	35		796.73	794.96	36	797.9(2)	820.34	798.25
37.5	857.1(1)	857.9	37	849.44(22)	845.46	843.74	38	848.3(1)	870.57	848.27
39.5	906.7(1)	908.2	39	897.40(16)	895.60	893.83	40	899.5(2)	920.57	898.63
41.5	957.1(1)	958.6	41	945.86(15)	945.86	944.15	42	951.4(2)	971.54	950.26
43.5	1008.7(1)	1010.0	43	996.08(19)	997.15	995.44	44	1003.8(2)	1022.70	1002.22
45.5	1060.7(1)	1061.6	45	1046.83(14)	1048.58	1046.92	46	1057.1(2)	1074.90	1055.25
47.5	1113.8(1)	1114.3	47	1099.39(16)	1101.14	1099.44	48	1110.7(2)	1127.29	1107.91
49.5	1167.2(2)	1167.2	49	1152.20(15)	1153.83	1152.21	50	1165.3(2)	1180.82	1162.01
51.5	1221.8(1)	1221.3	51	1206.26(24)	1207.72	1206.13	52	1220.1(2)	1234.53	1216.07
53.5	1276.5(1)	1275.5	53	1261.00(16)	1261.73	1260.17	54	1275.7(2)	1289.47	1271.27
55.5	1332.0(1)	1331.0	55	1316.57(14)	1317.07	1315.55	56	1330.9(2)	1344.57	1327.50
57.5	1387.6(1)	1386.6	57	1372.10(22)	1372.52	1371.06	58	1387.5(2)	1400.98	1384.76
59.5	1444.2(1)	1443.6	59	1428.55(24)	1429.32	1427.87	60	1443.3(2)	1457.54	1441.54
61.5	1500.5(2)	1500.7	61	1485.16(26)	1486.24	1484.79	62	1498.9(3)	1515.48	1499.77
63.5	1557.8(2)	1559.2	63	1542.40(42)	1544.58	1543.19	64	1555.1(4)	1573.54	1559.30
65.5	1615.7(3)	1617.8	65		1603.01	1601.68	66		1633.04	1619.37
67.5	1672.1(4)	1677.8								
69.5	1729.9(8)	1738.0								

to investigate them. With Eqs. (3) and (4) under the assumption that $E_0(N_B, N_F)$ is a constant for a nucleus [1,14,21], we calculate the γ-ray energies of the SD band ^{149}Gd(1). After a nonlinear least square fitting to the experimental γ-ray energies within the strong coupling scheme (i.e., $i=0$, $I'=I$), we get the E_γ's with spin assignment $I_0=27.5$. The obtained results are listed in Table I. It is obvious that the calculated E_γ's agree with experimental data excellently. With $\mathcal{J}^{(2)}=4\hbar^2/[E_\gamma(I+2)-E_\gamma(I)]$, we get the dynamical moment of inertia of the band. The results are shown in Fig. 1. The figure shows that the $\mathcal{J}^{(2)}$'s of the band ^{149}Gd(1) have been reproduced well.

By evaluating the energy differences ΔE_γ between two consecutive γ-ray transitions of the obtained E_γ's, we discuss the $\Delta I=4$ bifurcation in ^{149}Gd(1). The obtained result (in Cederwall's notation [37]) is illustrated in Fig. 2. The figure indicates that the $\Delta I=4$ bifurcation in the band ^{149}Gd(1) has been described well except for the phase shift at about $\hbar\omega=0.75$ MeV. To trace the source of why the $\Delta I=4$ bifurcation is reproduced well in the present approach, we show the result with $B\equiv 0$, too. The figure displays evidently that $\Delta I=4$ bifurcation cannot be generated if $B\equiv 0$. It manifests that the interaction with SO(5) [or SU(5)] symmetry plays a crucial role in generating the $\Delta I=4$ bifurcation. In the theoretical point of view, recalling the spectrum generating process, one may know that the contribution of the term with SO(5) symmetry to the

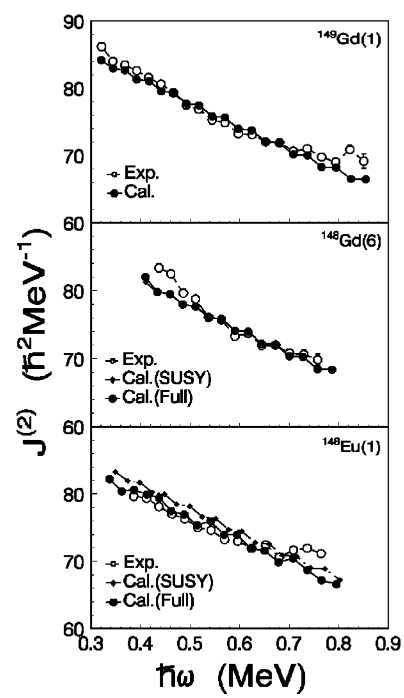

FIG. 1. Calculated results of the dynamical moment of inertia of the SD bands ^{149}Gd(1), ^{148}Gd(6), and ^{148}Eu(1) and the comparison with experimental data (taken from Refs. [2,3,36]).

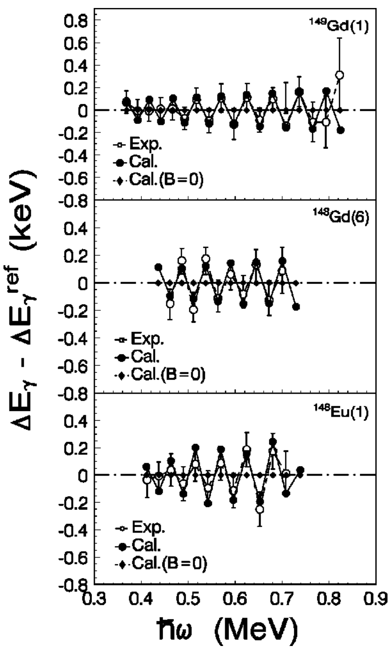

FIG. 2. Calculated results of the energy differences $\Delta E_\gamma - \Delta E_\gamma^{\mathrm{ref}}$ of the SD bands ^{149}Gd(1), ^{148}Gd(6), and ^{148}Eu(1) in the case with $B\neq 0$ and that with $B\equiv 0$ and the comparison with experimental data (taken from Refs. [2,3,36]).

E_γ is $E_\gamma^{\mathrm{SO}(5)}(L=4k,4k+1)=4(\tau_1-1)B=4(2k-1)B$ and $E_\gamma^{\mathrm{SO}(5)}(L=4k+2,4k+3)=6B$ with L referring to the value of the integer part of the I. It gives then $\Delta I=2$ staggering in energy definitely if $B\neq 0$. The emergence of the band with $\Delta I=4$ bifurcation is then an inherent property of the approach. The obtained result in practical calculation shows that the $|B|$ is much smaller than the C_0 (the values are listed below). It means that the interaction holding the SO(5) [or SU(5)] symmetry is, in fact, a perturbation on the rotation. Then the present approach is equivalent to the perturbed SU(3) symmetry scheme with a perturbation holding the SO(5) [or SU(5)] symmetry. The $\Delta I=4$ bifurcation of SD bands may thus result from a perturbation possessing SO(5) [or SU(5)] symmetry on the rotation. Furthermore, since the evaluation is carried out with least square fitting, the B can vanish automatically for the bands that do not exhibit any $\Delta I=2$ staggering. Then the present approach is also available to describe the bands without $\Delta I=4$ bifurcation.

To investigate the identity among the bands ^{148}Gd(6), ^{148}Eu(1), and ^{149}Gd(1), we evaluate the ''supersymmetry part'' of the E_γ's of ^{148}Gd(6) and ^{148}Eu(1) with the fitted parameters for ^{149}Gd(1) ($B=-0.002805$ keV, $C_0=5.582$ keV, $f_1=-1.432\times 10^{-5}$, $f_2=2.867\times 10^{-10}$) and the spin assignment $I_0=27$, 28, respectively. The obtained results are listed in Table I, too. It is obvious that the calculated results do not agree with experimental data well, which can be attributed to the neglect of the effect arisen from the single particle configuration against the reference. The contribution of single particle Routhian must then be taken into account. Therefore, to describe IBs, Eq. (3) should be rewritten as

$$E=E_0'(N_B,N_F)+\varepsilon_F N_F+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + \frac{C_0}{1+f_1 I'(I'+1)+f_2 I'^2(I'+1)^2} I'(I'+1), \qquad (5)$$

where $E_0'(N_B,N_F)$ is a constant. Experiments indicate that, with respect to ^{149}Gd(1), the ^{148}Gd(6) is built upon the $[411]\frac{1}{2}+$ Nillson orbital of neutron hole and the ^{148}Eu(1) on the $[301]\frac{1}{2}-$ orbital of proton hole. Even though many calculations have been completed [6,35,36,38], the single particle rothians have not yet been fixed uniquely. Taking the average value of the results in Refs. [6,35,36,38], we have $\varepsilon_F(\hbar\omega)=0.035(\hbar\omega)^2-0.1\hbar\omega-10.84$ (MeV) for the $[411]\frac{1}{2}+$ orbital and $\varepsilon_F(\hbar\omega)=0.6(\hbar\omega)^2-1.4\hbar\omega-5.31$ (MeV) for the $[301]\frac{1}{2}-$ orbital. Adding the single particle energy to the ''supersymmetry part,'' we finally get the E_γ's of SD bands ^{148}Gd(6) and ^{148}Eu(1). The results are listed in Table I, too. The induced dynamical moment of inertia and the $\Delta E_\gamma - \Delta E_\gamma^{\mathrm{ref}}$ are illustrated in Figs. 1 and 2, respectively.

The table and figures manifest that not only the E_γ's, and $\mathcal{J}^{(2)}$'s, but also the ΔE_γ's of the identical SD bands ^{149}Gd(1), ^{148}Gd(6), and ^{148}Eu(1) have been simultaneously reproduced well. Meanwhile, the spin assignment retains that the alignment of the bands ^{148}Gd(6), ^{148}Eu(1) against the ^{149}Gd(1) agrees with experimental results well. In addition, since the dynamical moment of inertia of a SD band is mainly determined by the parameters B, C_0, f_1, and f_2, the calculated $\mathcal{J}^{(2)}$ of SD bands ^{148}Gd(6) and ^{148}Eu(1) in pure supersymmetry scheme should be exactly the same as that of ^{149}Gd(1). At present, a simultaneously good reproduction of both generic rotational properties and individual characteristics of SD bands ^{148}Gd(6) and ^{148}Eu(1) indicates that, besides that supersymmetry plays a dominant role, single particle rothian contributes a great deal to the detail of the identical SD bands.

In summary, with an algebraic model in terms of supersymmetry with many-body interactions, a four parameter energy level formula for SD bands is proposed in this paper. The identical superdeformed bands with $\Delta I=4$ bifurcation, ^{149}Gd(1), ^{148}Gd(6), and ^{148}Eu(1), are investigated systematically. The calculated results show that, with single particle routhian being taken into account simultaneously, supersymmetry approach reproduces not only the E_γ's and $\mathcal{J}^{(2)}$'s , but also the ΔE_γ's of the IBs ^{149}Gd(1), ^{148}Gd(6), and ^{148}Eu(1) quantitatively well. It suggests that the mean field governing SD states may possess some kind symmetry, e.g., supersymmetry with many-body interactions. Meanwhile, the $\Delta I=4$ bifurcation may have direct bearing on the perturbation with SO(5) [or SU(5)] symmetry on the rotation. Combining the facts that the SO(5) [or SU(5)] symmetry is the symmetry of a five-dimensional space, and the superdeformed states in the $A\sim 190$ mass region can be described quite well with a quantal Hamiltonian expressed in terms of five collective quadrupole coordinates [39], we suggest that the mean field generating SD states may not be the usual three-dimensional space field, but the five-dimensional super-space field spanned by the five effective collective quadrupole coordinates. Such a super-space may hold the orthogonal rotational symmetry SO(5), even the unitary symmetry SU(5).

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