Microscopic study of the ^{168}Er multiphonon band structure

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The dynamic deformation theory with the pairing plus quadrupole interaction is applied to study the multiphonon band structure in ^{168}Er. Three $K^\pi=0^+$, two $K^\pi=2^+$, and $K^\pi=4^+$ bands have been analyzed. The lower four bands are also analyzed in the interacting boson model. The nature of the $K^\pi=0_2$ and 0_3 and $K^\pi=4^+$ bands is discussed.

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I. INTRODUCTION

Recently there has been renewed interest in the multiphonon collective β- and γ-vibrational states in deformed nuclei. Two $K^\pi=2^+\gamma$ phonons can be combined to form $K^\pi=0^+$ and $4^+\gamma\gamma$ bands. Also one can have $K^\pi=0^+2\beta$, 3β and $K^\pi=2^+$ $\beta\gamma$ vibrational bands. The character of the $K^\pi=0_2^+$ band in rare-earth nuclei has been under discussion [1–4], whether it is β-vibrational or 2γ phonon band. Also, the nature of the $K^\pi=4^+$ state in ^{168}Er has been reexamined experimentally in multiple Coulomb excitation [5] and from decay lifetime data [6] on its preferential decay to the one γ-phonon band.

This is also linked to the predictions of the interacting boson model-1 (IBM-1) in the SU(3) limit wherein the β and γ bands in the $(2N-4,2)$ representation are degenerate [7]. With slightly broken SU(3) symmetry, the pairing term in the IBM-1 Hamiltonian raises or lowers the β band above (below) the γ band. Also, the γ-g $E2$ transition is predicted to be stronger than the β-g transition by a factor of 10. There has been considerable emphasis [8] on the strong γ-β $E2$ transition prediction of IBM-1, which was assumed to be weak in the second-order band-mixing approach of the Bohr-Mottelson model [9].

The deformed nucleus ^{168}Er was studied extensively in the (n,γ) reaction by Davidson *et al.* [10] and its band structure was analyzed by Warner *et al.* [11] using the slightly broken SU(3) symmetry. McGowan *et al.* measured the $B(E2)$ excitation strengths to various 2^+ states [12]. Later, Davidson and Dixon [13] repeated the experimental analysis of ^{168}Er in which many more positive and negative parity bands were deduced, presenting a great challenge for nuclear theory.

On the basis of the stronger decay in ^{164,166,168}Er, Kumar from the dynamic deformation theory [14] suggested that such $K^\pi=0_2$ bands may be called $\gamma\gamma$ bands. More recently, Casten and von Brentano [2] cited ^{168}Er as one of the cases where the $K^\pi=0_2$ band decays preferentially to the γ band, thus supporting its 2γ phonon character. However, the band-mixing analysis of interband $E2$ transitions in [1] excludes the 2γ phonon interpretation.

Thus there is a need for a detailed reinvestigation of this deformed nucleus in a microscopic theory to seek the above anomalies. Earlier we successfully used the dynamic deformation theory in the pairing plus quadrupole approximation [15] for analyzing the soft nuclei, ^{152}Gd, ^{154}Gd, and more deformed ^{156}Gd in a multiband analysis [16–18] and $^{154-156}$Dy [19], ^{158,160}Dy [20]. Thus it would be useful to analyze the band structure of ^{168}Er in the DPPQ model. For comparison we also carried out IBM-1 calculations, as in Ref. [11], which enabled us to obtain values not cited in [11]. After a brief introduction of the DPPQ and IBM-1 models in Secs. II and III, we discuss the role of the parametrization in obtaining the final results in Sec. IV. In Sec. V we present the detailed results from our calculations and discuss our conclusions in Sec. VI.

II. THE PAIRING PLUS QUADRUPOLE MODEL

Briefly, in this model one starts from the single particle Nilsson energies in the spherical limit for the two major shells, one each for neutron and protons ($Z=40$, $N=70$ inert core) and adds a quadrupole term to obtain deformed single-particle orbitals (Nilsson orbitals) and wave functions. Then the pairing term is added to obtain the deformed quasiparticle energies and wave functions. This constitutes the Hartree-Fock-Bogoliubov (HFB) method. This is done for each point of a β-γ mesh and by using the cranking model expressions, the parameters of the collective Hamiltonian

$$H_{coll}=\left(\frac{1}{2}\right)C\beta^2+\frac{1}{2}B(\dot{\beta}^2+\beta^2\dot{\gamma}^2)+\frac{1}{2}\sum_{k=1}^{3}\theta_k\omega^2 \quad (1)$$

or its more general form

$$H_{coll}=V(\beta,\gamma)+\frac{1}{2}B_{\beta\beta}(\beta,\gamma)(\dot{\beta}^2)+2B_{\beta,\gamma}(\beta,\gamma)\beta\dot{\beta}\dot{\gamma}$$
$$+B_{\gamma\gamma}(\beta,\gamma)\beta^2\dot{\gamma}^2+\frac{1}{2}\sum_{k=1}^{3}\theta_k(\beta,\gamma)\omega_k^2 \quad (2)$$

are determined. The $E0$, $E2$, and $M1$ operators are also derived microscopically. In a second step the Bohr collective Hamiltonian is constructed and solved for each spin. The summation over the $\beta\gamma$ mesh allows the nucleus to find its own dynamic shape. This constitutes the time-dependent dynamic deformation method. For further details see Refs. [14,15].

In the DPPQ model [15], in order to correct for the inert core the effective quadrupole force parameter X_0 and the kinetic coefficients renormalization factor F_B are varied slightly to fit approximately to the $E(2_1)$ and $E(4_1)$ energies. In this respect it differs from the fully microscopic version called DDM [14] where no varying parameters are used. The values of the quadrupole force constant $=68.5A^{-1.4}$ and inertial renormalization constant $F_B=1.8$ were adopted for constructing the H_{PPQ} for ^{168}Er. However, see further comments on the problem of parametrization below.

III. THE INTERACTING BOSON MODEL-1

The interacting boson model-1 accounts for the collective deformation in terms of the valence nucleon or hole-pair bosons (with no distinction between proton and neutron bosons). In sd-IBM only the $L=0,2$ bosons are considered, and in the multversion the H_{IBM} is given by

$$H_{IBM}=\epsilon n_d+kQQ+k'LL+k''PP \tag{3}$$

neglecting the hexa and octa terms. The four parameters of the Hamiltonian are determined empirically through a least square fit (to level energies) program. In the computer code PHINT, these four parameters are named as EPS, QQ, ELL, and PAIR, respectively. The quadrupole operator Q used for Eq. (3) and also for the $E2$ transition operation $T(E2)$ is given by

$$Q=e_o[(s^+d+d^+s)+\chi(d^+d)^{(2)}]. \tag{4}$$

The coefficients of the terms in Eq. (4) are related to the FBEM code variables E2SD and E2DD, their ratio E2DD/E2SD being equal to χ in Eq. (4). We set $\epsilon=0$ in Eq. (3) and the values of QQ, ELL, and PAIR used are (-16.5, 19.5, and 16.3) keV, respectively, corresponding to $k=8.25$, $k'=19.5$, and $k''=32.6$ keV. For FBEM E2SD $=0.132$ and E2DD $=-0.10$ yield $\chi=-0.76$. For the Hamiltonian we used the SU(3) χ value. The code space limitation allowed the detailed analysis of only four bands, viz., g, γ, β, and β' ($K=0_3$) bands.

IV. PARAMETRIZATION OF THE HAMILTONIAN AND OPERATORS

Since any given nuclear model is only an idealization of some aspects of the nucleus, some parametrization is usually done. In IBM-1 all the energy data are input to freely adjust the three (or four) parameters of the H_{IBM} and the two coefficients of the Q operator are determined in Eq. (4) through two $B(E2)$ values. In a more versatile version, called the consistent Q method [8], several $B(E2)$ values and $B(E2)$ ratios also are input for a simultaneous fit to H and Q. More specifically, the coefficients ϵ and k fix the band spacings, k' fixes the level structure, and k'' affects the band order of the β and γ bands. The coefficient χ in Eq. (4) affects the γ-β coupling.

Seiwert *et al.* [21] used the eight-parameter Gneuss-Greiner model (GGM) and the 12-parameter generalized collective model (GCM) for a fit to the ^{168}Er levels and $B(E2)$ data and demonstrated that basically the geometric and algebraic models yield the same features of the collective aspects of the low-energy part of the nuclear structure. They also discussed the question of weighting of data and the number of parameters. Too many parameters lead to a problem as in GCM [21]. This is also evident from the study of ^{168}Er in IBM-2 [22].

In the DPPQ model, also because of the limitations on the choice of single-particle levels that are input (two different sets were used for the beginning and the end of the $Z=50$–82 major shells [14,15]), the use of an inert core ($Z=40$, $N=70$), the use of a crude quadrupole interaction, and the use of BCS theory for the pairing interaction, etc., one allows a small variation of a few percent in the quadrupole strength constant X_0 (around $X_0=70$). The inertial factor F_B multiplies the kinetic coefficient B's and the moments of inertia θ_k linearly. The charge parameter e_n ($e_p=1+e_n$) for neutron is almost constant at 0.6–0.7.

Some of the structural observables like energy scale, quadrupole deformation β_0, the quadrupole moments, and the intraband $B(E2)$ values are affected only marginally by the variation in these two parameters. However, several other interband transition rates, especially if the absolute values are small, can vary by a factor of 10 or more, and the $B(E2)$ ratios may vary by a factor of 100 or more for slight variation of the input parameters. This is much more true for the higher-lying bands where the levels from several bands overlap. This is also because of the complexity of the underlying Nilsson orbitals and the quantum rules leading to β and γ vibrations. Thus we studied the effect of variation of X_0 from 70 to 63 and $F_B=1.8$ to 2.4. At large X_0, the 2^+ level and the ratio $(E_4^+/E_2^+)=R_4$ are low. At small X_0 the prolate-minimum shifts toward a lower deformation value β_0. We chose a value of $X_0=68.5$ and $F_B=1.8$ corresponding to the correct β_0. Large departures from experiment or between two models must be seen in this context when looking at numerous data. Only an overall predicted trend is useful.

A similar ambiguity about the input parameter set and the resulting predictions exists in the phenomenological IBM-1, in spite of the facility of a least square fit to input data for determining the parameters. The amount of data or the starting values of the parameters can affect the final values of the parameters. The optional use of the boson energy ϵ has its effect on the predicted values. For example, reduction to $\epsilon=0$ also reduced the β-g interaction strength by a factor of 10, while the γ-g strength was unaffected. This affects the $(\beta\text{-}g/\gamma)$ or $(\beta\text{-}g)/(\gamma\text{-}g)$ $B(E2)$ ratios. The γ-β strength was affected only 20%. The reduction of χ from the SU(3) value in H_{IBM} also has similar effects.

V. RESULTS

The DPPQ model calculation reproduces the energy ordering of the K-band structure for the five bands studied, viz., g, γ, $K^\pi=0_2^+=0'$, also denoted β here; $K^\pi=0_3^+=0''$, denoted β'; and $K^\pi=2_2^+\beta\gamma$ bands. The $K^\pi=4^+$ band is predicted to be lower below the $\beta\gamma$ band. At a lower X-parameter value, the band order gets affected for the close-

TABLE I. The level energies and K components in ^{168}Er.

Band K	I	Level energy (keV) Expt.[a]	DPPQ	$K^\pi=0^+$	2^+	4^+
0_1	2	80	79	99.94	0.06	
	4	264	187	99.86	0.10	0.04
	6	549	370	99.77	0.18	0.02
2_1	2	821	1547	0.03	99.97	
	3	896	1580		100	
	4	995	1709	0.47	99.44	0.09
	5	1118	1763		99.88	0.12
	6	1264	1930	1.14	98.65	0.17
0_2	0	1217	1779	100		
	2	1276	1876	99.87	0.13	
	4	1411	2024	99.43	0.56	0.01
	6	1617	2277	98.75	1.24	0.01
0_3	0	1422	2883	100		
	2	1493	2966	99.68	0.32	
	4	1656	3058	96.45	0.62	2.93
	6	1903	3220	94.02	4.41	1.48
2_2	2	1848	3069	0.41	99.59	
	3	1915	3103	100		
	4	2002	3249	2.12	96.06	1.79
	5	2109	3356		98.03	1.97
4_1	4	2056	3038	2.60	1.58	95.82
	5	2170	3143		1.84	98.16
	6	2307	3300	2.35	2.22	95.33

[a]References [13,23].

lying bands. Our notation of $0_2=\beta$, $0_3=\beta'$ agrees with the use of other authors cited here except [2,14]. The latter called them $\gamma\gamma$ and β, respectively. As previously, in general, our calculation yields somewhat compressed g-band energies and raised excited-band energies by up to a factor of 1.5–2.0.

TABLE II. Absolute moments in ^{168}Er.

Quantity	Expt.[a]	DPPQ	DDM[b]	IBM-1[c]
$Q(2_1)$ (eb)	−2.18(2)	−2.18	−2.17	−2.19
$Q(4_g)$	−2.2(10)	−2.75	−2.75	−2.77
$Q(2_\gamma)$	2.25(23)	2.16	2.12	2.05
$Q(2_\beta)$	2.14(14)[d]	−2.14	−2.21	−1.94
$Q(2_{0''})$		−2.14		−1.90
$Q(2_\beta\gamma)$		1.97		
Zero point energy		2.64		
β_0 at V_{min}		0.33		
μ (2_g) (n.m.)	0.642(12)	0.80	0.63	
μ (2_γ)	0.72 (14)	0.97		

[a]Reference [23].
[b]Reference [14].
[c]Present work.
[d]Reference [27].

TABLE III. Absolute moments (in e^2b^2) in ^{168}Er. The states 2_3,2_4, are indicated by 2_β and $2_{0''}$, respectively, and $2_5=2_{\beta\gamma}$ here.

Quantity	Expt.[a]	DPPQ	DDM[b]	IBM-1[c]
$B(E2;0_g\text{-}2_g)$	5.90 (6)	5.80	5.73	5.90
$B(E2;0_g\text{-}2_\gamma)$	0.132 (4)	0.159	0.141	0.130
$B(E2;0_g\text{-}2_\beta)$	<0.02[d]	0.043	0.032[e]	0.0035
$B(E2;0_g\text{-}2_{0''})$	0.005	<0.0025[f]	2×10^{-5} [e]	2×10^{-4}
$B(E2;0_g\text{-}2_{\beta\gamma})$		1.2×10^{-5}		
$B(E2;2\text{-}0)$	1.18	1.16	1.15	1.18
$B(E2;4\text{-}2)$	1.60[g]	1.66	1.66	1.67
$B(E2;6\text{-}4)$		1.84	1.83	1.81
$B(E2;0_\beta\text{-}2_g)$	0.002[f]	0.073		0.004
$B(E2;0_\beta\text{-}2_\gamma)$	0.017[d,f]	0.032[h]	0.29	0.095
$B(E2;2_\gamma\text{-}2_\beta)$	0.033(15)[i]	0.003	0.053	0.025
$B(E2;2_\gamma\text{-}2_g)$	0.05(1)[i]	0.053		0.040
$B(E2;2_\beta\text{-}2_g)$	0.003[i]	0.016		0.001
$B(E2;2_\beta\text{-}4_g)$	0.012[i]	0.020		0.0023
$B(E2;2_\beta\text{-}0_\beta)$		1.14		0.93

[a]Reference [23].
[b]Reference [14].
[c]Present work.
[d]Reference [1].
[e]In [14] these refer to $2_\beta=2_{0''}$ and $2_\gamma\gamma=2_{0'}$, respectively.
[f]Reference [27].
[g]Reference [24].
[h]The values calculated in SCCM, sdg-IBM, and (MPM) are 0.032, 0.047, and 0.003, respectively (see [27]).
[i]Reference [28].

But the orders of the spins are correctly given in Table I. The anharmonicity required for higher-lying multiphonon bands is not achieved with our prolate minimum and the dynamics involved. The calculated K components given in percent allow band grouping of these collective levels. Since the ^{168}Er nucleus is well deformed ($\beta_0=0.33$), K mixing is rather small in all six bands here, although K mixing increases toward the higher-lying bands. At $X_0=65$, the K mixing in the β' and $\beta\gamma$ bands increases up to 30% compared to only a few percent at the higher X_0 adopted here. A further confirmation of the levels of the same band is available in the large intraband transitions. This criterion can also be used for band identification in IBM besides the energy considerations. In our IBM-1 calculation, a close fit to level energies is obtained here but the $K^\pi=4^+$ band lies low as also in [11]. In fact, our IBM parameters are close to those of Ref. [11].

The calculated $\beta_{\rm rms}$, $\gamma_{\rm rms}$ values (not shown) are in close agreement with the DDM values [14]. But DDM predicts greater K admixtures in some states, more specifically for $I\geqslant4$ states. This trend is obtained in the DPPQ model for smaller values of the quadrupole strength parameter X_0, as stated above. This in fact may be the situation in DDM [14], where no parameter adjustment was done, and which was considered the weak point of DDM in [2]. Bohr and Mottel-

TABLE IV. Comparison of experimental and theoretical $B(E2)$ branching ratios from the states of $K^\pi = 2_1^+$ γ band in ^{168}Er.

Transition		Relative $B(E2;I_i \rightarrow I_f)$				
I_i	I_f/I_f	Expt.[a]	DPPQ	IBM[a]	IBM[b]	IBM[c]
2^+	$0_g/2_g$	0.54(10)	0.60	0.66	0.54	0.66
	$4_g/2_g$	0.068(10)	0.063	0.060	0.076	0.058
	$0_\beta/0_g$	0.12[d]	0.20			0.73
3^+	$2_g/4_g$	1.53(20)	1.82	2.08	1.44	2.12
	$2_g/2_\gamma$	0.026(10)	0.028	0.027	0.026	0.022
	$4_g/2_\gamma$	0.017(5)	0.016	0.013	0.018	0.010
4^+	$2_g/4_g$	0.20(4)	0.19	0.30	0.18	0.30
	$6_g/4_g$	0.14(3)	0.08	0.12	0.14	0.12
	$2_g/2_\gamma$	0.016(3)	0.019	0.025	0.015	0.023
5^+	$4_g/6_g$	0.80(16)	1.1	1.39	0.80	1.37
	$4_g/4_\gamma$	0.024(5)	0.04	0.043	0.037	0.3
6^+	$4_g/6_g$	0.12(6)	0.13	0.23	0.09	0.25
	$4_g/4_\gamma$	0.004(2)	0.006	0.010	0.0044	0.009

[a]Reference [11].
[b]Using consistent Q parametrization [8,32].
[c]Present work.
[d]References [1,23], derived from $B(E2;0_\beta\text{-}2_\gamma)/B(E2;0_g\text{-}2_\gamma)$.

son [25] in a macroscopic analysis demonstrated through Mikhailov plots the small band mixing with $M_2 = (0.021 \text{ and } 0.013)M_1$ indicating small K admixtures in γ and β bands.

Next we report the static electric quadrupole and magnetic dipole moments in Table II and compare with IBM-1 and DDM values. The negative quadrupole moments of the 2^+ state in g, β, and $K^\pi = 0_3 = \beta'$ bands and the positive ones for the $K^\pi = 2^+$ bands are given correctly in sign and magnitude. The static moments (electric quadrupole and magnetic dipole) of various states obtained in the DPPQ model compare well with DDM values. Similar values are also noted in IBM-1, where the charge parameter e_0 and χ of the quadrupole operator are freely adjusted. As stated earlier, the input parameters in the DPPQ model have a minor effect here. The same is true for IBM. The small variation of $Q(I = K)$ is consistent with rotational structure analysis of these bands in [25] and near-constancy of coefficient ''A'' in the first term of the BM expansion for level energies.

Some important absolute $B(E2)$ values are given in Table III. The $E2$ excitation probabilities from the ground states 0_1^+ to the various excited 2^+ states decrease progressively from 2_1 to 2_5. Our values compare well with experiment and DDM [14]. The larger 2_γ-0_g $E2$ transition strength compared to the weaker (by a factor of 5) from $2_{0'}$ to ground state is well given in the DPPQ model and DDM. The latter value is, however, somewhat larger than experiment. In IBM-1, the $2_{0'}$–0_g value is ten times weaker (for the $\epsilon = 0$ set). This is also χ-parameter-value-dependent in IBM-1 as pointed out in Sec. IV. Casten *et al.* [26] noted that the

TABLE V. $B(E2)$ branching ratios for transitions from $K^\pi = 0_2$ β band in ^{168}Er.

I_i	I_f/I_f	Expt.[a]	DPPQ	IBM[a]	BM[b]	IBM[c]
0_β	$2_\gamma/2_g$	<5.1	0.44	18		24
1217 keV		56(9)[d]		125[e]		
2_β	$0_g/2_g$	0.72(13)	0.54			0.64
1276 keV	$0_g/4_g$	0.25(2)	0.42	0.31	0.2	0.30
	$2_\gamma/4_g$	2.9(7)	0.12	7.5	2.7	10.8
	$2_\gamma/3_\gamma$	0.83(32)	0.13	0.53	2.25	2.65
	$0_g/0_\beta$	5×10^{-5}	0.007	0.001	0.0002	0.001
4_β	$2_g/2_\gamma$	0.67(19)	33	2.2	0.2	1.56
1411 keV	$6_g/2_\beta$	0.0011(2)	0.011	0.0023	0.0011	0.002
	$6_g/2_\gamma$	3.6(10)	36	4.6	1.1	4.03
	$2_\gamma/3_\gamma$	0.081(27)	0.61	0.063	0.15	0.25
	$2_\gamma/4_\gamma$	0.053(17)	1.3	0.018	0.048	0.019
	$2_\gamma/5_\gamma$	0.13(6)	0.03		0.16	0.29
	$3_\gamma/5_\gamma$	1.6(8)	0.05			1.15
6_β	$6_g/4_g$	6.8(11)	0.43			1.2
1517 keV	$4_\gamma/4_g$	5.6(14)	0.01	1.2	12.7	1.6
	$4_\gamma/4_\beta$	0.0011(3)	0.0001	0.0009	0.005	0.0009
	$4_g/4_\beta$	0.0002(1)	0.007	0.0007	0.0004	0.0006
	$4_\gamma/5_\gamma$	0.34(8)	0.003		0.36	1.4
	$4_\gamma/6_\gamma$	0.12(3)	0.1	0.045	2.7	0.045
	$5_\gamma/6_\gamma$	0.34(8)	3	0.36	7.5	0.03

[a]References [11,23].
[b]Referece [11]: relative values.
[c]Present work.
[d]Reference [2], in Ref. [27] remeasured value is 6.9(14).
[e]Reference [2] (read from their graph).

IBM-1 values of 0.005 to 0.02 for the inverse ratio $B(E2;0_g\text{-}2_\beta/2_\gamma)$ are in fact smaller than the empirical values in this region. In DPPQ model the weaker β-g strength comes from the β-γ space structure of the main $K=0$ and two vectors wherein the $K=0_2$ vector has a node near the middle of the $\beta = 0-0.5$ space (illustrated for ^{156}Gd in [17(b)]).

The intraband $B(E2)$ values in the ground band increase with spin that are reproduced in the three models. The $B(E2;0_\beta\text{-}2_g)$ DPPQ value is almost equal to $B(E2;0_g\text{-}2_\beta)$ and larger than the IBM-1 value. The recently measured value in Coulomb excitation work by the COULEX technique [27] is only 0.002, while the value from the lifetime measured by the GRID technique is 0.011 (range 0.002–0.030) e^2b^2 [28]. The absolute $B(E2;0_\beta\text{-}2_\gamma)$ was derived by Gunther *et al.* [1] that agrees with the recent value in [27]. The DPPQ value agrees with this within a factor of 2. The DDM value of 0.29 for $B(E2,0_{\gamma\gamma}\text{-}2_\gamma)$ does not match with this but comes closer to our $B(E2;0_3\text{-}2_\gamma)$ value of 0.15. The IBM-1 value is 4–5 times larger. Note that in both IBM-1 and DPPQ models, the β-γ strength is affected only slightly by parametrization. The value calculated in the self-

consistent collective coordinate method (SCCM) [29], and sdg-IBM [30] are consistent with our value. But the MPM (multi-phonon method) [31] value is rather low by a factor of 5 (Table III). This value also affects the $B(E2;2_\gamma\text{-}0_\beta/2_g)$ ratio (see below and Table IV).

The $B(E2;2_\gamma\text{-}2_g)/B(E2;2_\beta\text{-}2_g)$ ratio of 36 in sd-IBM is rather large compared to 3.3 in DPPQ and 7.3 in GGM [21]. The absolute $B(E2;2_\beta\text{-}2_g)$ value from lifetime data [28] lies in between the DPPQ and IBM values. The $B(E2;2_\beta\text{-}4_g)$ agrees better with DPPQ, the IBM value is smaller by a factor of 5.

To get a more elegant view of the behavior of absolute values, following [25], we did the Mikhailov plot analysis for γ-g, β-g, and β-γ transitions in which one plots $B(E2)^{1/2}$/CGC versus $\Delta = I_f(I_f+1)\text{-}I_i(I_i+1)$. For γ-g, the intrinsic matrix element M_1 and band-mixing coefficient M_2 from DPPQ $B(E2)$ values agree with experiment [25], but for β-g the M_1 value is larger by a factor of 4–5. In the phenomenological sdg-IBM, Yoshinaga *et al.* [30] varied four coefficients in the $E2$ transition operator (besides the six in H_{IBM}) to fit to M_1 values in the three plots. Their M_2 values for γ-g and β-g are larger by a factor of 2 and 3, respectively. The sd-IBM M_2 values [11] are smaller. Note that we make no fits to M_1 values in the $E2$ operator. Also, β-g values are sensitive to parametrization. This explains the deviations in the three models in the table for the absolute values.

In Tables IV–VIII we compare the intraband and interband $B(E2)$ branching ratios from the five excited bands in ^{168}Er. In Table IV for the γ band, the DPPQ model yields several values that give improved agreement over the IBM-1 values [11]. In consistent Q formalism, one uses the same value of χ of the quadrupole operator Q [see Eq. (4)] both in the Hamiltonian H_{IBM} and the $E2$ transition operator $T(E2)$ and thus fits to the level energies and $B(E2)$ values simultaneously. Warner and Casten [32] obtained improved γ-g $B(E2)$ ratios that are closer to experiment and to the DPPQ values. The $B(E2;2_\gamma\text{-}0_\beta/0_g)$ ratio derived from $B(E2;2_\gamma\text{-}0_\beta)$ in [1] is only 0.12 which is better reproduced in DPPQ calculation. The IBM value is 5–6 times larger, independent of parametrization, and lies within the range of IBM-1 values quoted in [26] for this region.

For the $K^\pi = 0_2$ β band our calculation gives weaker $E2$ transitions to the γ band than to the g band for $X_0 > 68$ (Table V). This is different from the DDM and IBM-1 results. The experimental value for $B(E2;0_\beta\text{-}2_\gamma/2_g)$ is also uncertain. The value used in [11] is about 5, but the value derived from $E2$ transitions from higher spin members of the β band in [2] using Alaga rules is rather large. Gunther *et al.* [1] have pointed out the inadequacy of this process. In recent work [27], this ratio is 6.9 (14). Also in the latter work of [2], the IBM-1 value plotted graphically (see Fig. 2 of Ref. [2]) is much larger. Our IBM value is 24, but for $\epsilon \neq 0$ fit it reduces to 2.5. Using the Gneuss-Greiner model with a fit to input $B(E2)$ data, Seiwert *et al.* [21] obtained this ratio equal to three. Since DPPQ value of $B(E2;0_\beta\text{-}2_\gamma)$ of $0.032\,e^2\text{b}^2$ is consistent with the data (Table III), our $B(E2;0_\beta\text{-}2_g)$ is rather large. For lower X_0 value, say, 65.0, the $B(E2;0_\beta\text{-}2_\gamma)$ value exceeds the $B(E2;0_\beta\text{-}2_g)$ value by a factor of 3. Thus both in IBM and DPPQ, this predicted ratio is somewhat parameter dependent.

TABLE VI. The relative $B(E2)$ values for transitions from $K^\pi = 4^+$ band in ^{168}Er. The absolute $B(E2)$ values are in $e^2\text{b}^2$.

I_i	I_f/I_f	Expt.[a]	DPPQ	Others
4^+	$3_\gamma/2_\gamma$	0.64(18)	0.66	0.56[b]
2056 keV	$2_g/2_\gamma$	<0.005	0.001	
	$2_\gamma/4_\gamma$	2.04(72)	1.86	
	$6_\gamma/4_\gamma$	1.2(7)	0.015	
	$4_g/4_\gamma$	<0.01	0.001	
	$4_\gamma/4_0''$		0.2	
5^+	$3_\gamma/4_\gamma$	1.3(9)	0.63	
2169 keV	$3_\gamma/5_\gamma$	2.8(50)	1.0	
	$4_\gamma/6_\gamma$	5.0(10)	8.3	
4^+	2_γ	0.028(14)[c]	0.08	
		0.039(9)[d]		
5^+	3_γ	0.053(37)[c]	0.047	
$(4^+\text{-}2_\gamma)/(2_\gamma\text{-}0_g)$		1.1(5)[c]	2.5	1.3[e,f]
		1.47(36)[d]		

[a]Using E_γ, I_γ in Refs. [13,23].
[b]From DDM, MPM, sdg-IBM, SCCM, and harmonic limit (see [5]).
[c]Value from lifetime data [6].
[d]Value from Coulomb excitation [5]. Recent absolute $B(E2)$ value is 0.060(11) and relative $B(E2)$ value is 1.9(4) [27].
[e]Reference [14].
[f]QPNM, MPM, sdg-IBM, and SCCM values are 0.73, 0.53, 1.4, and 1.9, respectively.

For other intraband or interband $B(E2)$ ratios, the DPPQ model values are consistent with those of IBM-1, though these are not in as good agreement with experiment as for the γ band. In some transitions, the slight change in the input X_0 or F_B value changes the $B(E2)$ value by a large factor. This may be related to strong band-mixing effects here. Some of our IBM-1 values (last column) differ from [11].

The $I^\pi = 4^+$ state at 2.030 MeV previously assigned to a $K^\pi = 4^+$ band [11] is now listed as a member of a $K^\pi = 0^+$ band in [23] and in [13], hence is ignored here. For the $K^\pi = 4^+$ band at 2.056 MeV, the dominant decay is to the $K^\pi = 2^+$ band. The interband relative $B(E2)$ ratios were deduced by using γ-ray data [23]. Five of the six $B(E2;I_{\gamma\gamma}\text{-}1'_\gamma/I''_\gamma)$ ratios agree with experiment within a factor of 2. Hartlein *et al.* [27] deduced $B(E2;4_{\gamma\gamma}\text{-}3_\gamma/4_\gamma = 0.55(10)$, which is reproduced within the error limits by our calculation as also in other calculations. For the hindered direct decay to the g band, the small $B(E2;I_{\gamma\gamma}\text{-}2_g/2_\gamma) < 0.007$ [27] is supported in our calculation, which is indicative of the K purity of the 4^+ band [27] as also predicted in DPPQ model (Table I). Our absolute $B(E2;4_{\gamma\gamma}\text{-}2_\gamma)$ of 0.08 is close to the recently determined value of 0.060 (11) (=2.2 s.p.u.) in the Coulomb excitation work.

TABLE VII. Relative $B(E2)$ values for transitions from $K^\pi = 2^+$ $\beta\gamma$ band in ^{168}Er.

I_i	I_f/I_f	Expt.[a]	DPPQ
2^+	$0_g/2_g$	0.88	0.3
1848 keV	$2_g/2_\gamma$	0.36	1.0×10^{-4}
	$3_\gamma/2_\gamma$	2.77	0.9
	$2_\beta/2_\gamma$	5.0	4.6
	$2''_0/2_\gamma$	23.7	0.6
3^+	$4_g/2_g$	<0.20	0.25
1915 keV	$2_g/2_\gamma$	0.36	0.003
	$2_\gamma/4_\gamma$	0.35	0.9
4^+	$2_g/4_g$	0.43	1.2
2002 keV	$3_\gamma/4_\gamma$	0.58	3.6
	$2_\gamma/2_{\beta\gamma}$		0.02
	$2_\beta/2_{\beta\gamma}$	<0.002[b]	0.02
	$4_\beta/4_{0''}$	0.25	3.0
	$4_\gamma/4_\beta$	0.26	0.01
	$4_\gamma/4_{0''}$	0.064	0.03
	$4_\beta/2_{\beta\gamma}$	0.014	0.15

[a]Reference [13].
[b]Assuming part placement of 726.6-keV γ ray at $4_{\beta\gamma}$-2_β.

So is the $B(E2;5_{\gamma\gamma}$-$3_\gamma)$ value. Also, the deduced value of 1.9(4) in [27] for the much studied ratio $R = B(E2;4_{\gamma\gamma}$-$2_\gamma)/B(E2;2_\gamma$-$0_g)$ that supports the $\gamma\gamma$ character of the band is consistent with our calculated value. This is also close to the DDM [14] and sdg-IBM [30] values. From the theoretical work dealing with the anharmonicity of γ vibration explicitly, e.g., in SCCM [29], the value of the ratio R also agrees with the above, but the MPM value [31] is low. This is in spite of the fact that their $B(E2;2_\gamma$-$0_g)$ value is also lower than the experiment (with effective charge $e_n = 0.2$), so that their absolute value for $B(E2;4_{\gamma\gamma}$-$2_\gamma)$ is much lower. In the quasiparticle-phonon model (QPNM) of Soloviev *et al.* [33], where the 2γ strength is strongly fragmented, the recalculated value of the ratio is only 0.73.

In Table VII the relative $B(E2)$ values for transitions from the $K_\pi = 2_2^+$ $\beta\gamma$ band are compared with more recent experiment [13]. The calculated $B(E2;I_{\beta\gamma}$-$I_g)$ values of the order of $10^{-6}e^2\mathrm{b}^2$ are rather low and sensitive to the two input parameters, but the $B(E2;I_{\beta\gamma}$-$I'_g/I''_g)$ ratios agree with experiment within a factor of 3. The same is true for $B(E2;I_{\beta\gamma}$-$I'_\gamma/I''_\gamma)$ ratios. The calculated $B(E2;2_{\beta\gamma}$-$2_\beta)$ and $B(E2;4_{\beta\gamma}$-$4_\beta)$ are about 3 s.p.u. and the largest $B(E2;\beta\gamma$-$\gamma)$ is about 1 s.p.u. thus predicting some coupling of the $\beta\gamma$ band with β and γ bands. The calculated $B(E2;2_{\beta\gamma}$-$2_\beta/2_\gamma)$ ratio agrees with experiment. But some $B(E2)$ ratios involving two different bands for final states deviate by large factors.

In Table VIII following [11] we express the relative $B(E2)$ values for the states of the $K=0_3$ or $0''$ band relative to the intraband transition. For the 0_3 state itself the relative values are expressed with respect to the transition to the γ band. For the $0''$ state, both models yield the same decay strength to the $0'$ band but much smaller than experiment. Decay to the g band is stronger in DPPQ model, yet less than experiment. For other states, both models predict weaker decays to the β band, relatively stronger decay to the γ band, and much weaker decay to the g band, as in experiment but not to the same order. While theory predicts greater strength to the γ band, experiment indicates more strength to the β band. This discrepancy does not allow a clear $\beta\beta$ or a $\gamma\gamma$ characterization.

TABLE VIII. Relative $B(E2)$ values for states of the $K=0_3$ band in ^{168}Er.

I_i	KI_f	Expt.[a]	DPPQ	IBM[a]	IBM[b]
0^+	02	53.8	4.4	0.43	0.45
1422 keV	22	100	100	100	100
	$0'2$	7375	0.48	0.46	0.9
2^+	00	0.002	0.044	0.006	0.005
1493 keV	2		0.114		0.008
	4	0.025	0.054	0.02	0.02
	22	0.035	3.36	2.3	1.9
	3	0.05	6.47	4.3	2.4
	4	0.05	2.70	2.0	1.7
	$0'0$	0.175	0.003	0.004	0.009
	2	0.175	0.014	0.01	0.02
	4	75	0.83	0.03	0.04
	$0''0$	0	100	100	100
4^+	02	0.01	0.17	0.005	
1656 keV	6	0.34	0.01	0.01	
	22	0.08	0.03	0.04	
	3	0.08	1.52	0.58	
	4		2.70		
	5	0.48	4.46	2.4	
	6	0.4	1.40	1.1	
	$0'2$	4.0	0.13	0.002	
	4	3.0	0.15	0.01	
	$0''2$	100	100	100	

[a]Reference [11].
[b]Present work.

VI. DISCUSSION

In the present semimicroscopic calculation (two parameters varied in a narrow range of a few percent), we obtain some features of ^{168}Er that are the same as in the fully microscopic DDM obtained earlier by Kumar [14]. But there are differences as well, especially regarding the K mixing or band mixing that was much greater in DDM. Considering the fact that much less band mixing is generally expected in well-deformed nuclei [25], our present results may be considered relevant. The reversing of the spin order obtained in a few cases in [14], and cited as a deficiency of DDM in Ref. [2] is corrected here. However, it is to be noted that in the microscopic calculation where one diagonalizes the Hamiltonian built from regional parameters for each individual

spin and in the well-deformed nucleus where the level spacing is rather small, the accuracy of the scale needed is not achieved, as is possible in a phenomenological model calculation where all level energies and even transition rates are used as input data as in [32].

The energy scale of the band heads remains expanded and in the existing DPPQ method there is no anharmonicity parameter to adjust to the band head energy explicitly, as is possible in the phonon-phonon interaction based SCCM [29] and MPM [31] calculations or in quasiparticle-phonon interaction based QPNM [33]. The problem of the energy scale is partly solved in DDM to yield better one-phonon band-head energies, where the Nilsson approach with shell correction is used. We make no attempt to vary the spherical single-particle energies, which may affect the band head energies. However, the dynamic treatment in the $(\beta\text{-}\gamma)$ space allows a more generalized treatment of band mixing in both versions of this model and thus the method is applicable to a wide range of nuclei and for multiband analysis [16–20].

The smaller K-band mixing also explains the major difference in the prediction of the $B(E2;0_2\text{-}2_\gamma)$ value in DDM and DPPQ models here, a result that earlier led Kumar to suggest 0_2 as $K^\pi=0^+$ $\gamma\gamma$ band [14]. The location of the $K^\pi=0^+$ 2γ phonon band (often not 0_2) is linked with the location of $K^\pi=4^+$ 2γ excitation and the suggestion in [2] of 0_2 as a 2γ phonon state led to increased interest. First, the β-soft nuclei in which $I^\pi=0^{+\prime}$ and $2^+_{0'}$ lie below 2_γ have to be treated differently than in well-deformed nuclei, where $K^\pi=0^{+\prime}$ lies above the $K^\pi=2^+$ band. In the former, $K^\pi=0^{+\prime}$ is one β-phonon band as discussed in [4]. In ^{154}Gd using DPPQ model, we analyzed seven multiphonon bands and obtained results that are consistent with experimental data [17].

The motivation in Ref. [2] for calling the $0'$ band as a 2γ phonon band was from the IBM general prediction of relatively stronger $\beta\text{-}\gamma$ coupling than the $\beta\text{-}g$ coupling and almost equal to the $\gamma\text{-}g$ coupling. Here it may be noted (see Sec. IV), that in the consistent-Q formalism, the relative $\beta\text{-}\gamma/\beta\text{-}g$ strength is larger than in sd-IBM-1 when using the SU(3) value of χ in the Q operator mainly because of the change in small $\beta\text{-}g$ strength. Thus the prediction of the ratio is parameter dependent. The larger $B(E2;2_\gamma\text{-}0_g)$ versus $B(E2;2_\beta\text{-}0_g)$ is supported in DPPQ and other models. Thus the larger $(\beta\text{-}\gamma/\beta\text{-}g)$ transition ratio is generally supported by theory.

In well-deformed nuclei, because of increased β rigidity, $K^\pi=0^{+\prime}$ lies higher and the nucleus prefers γ softness so that the $K^\pi=2^+$ band lies low. The $K^\pi=0^+$, 4^+ 2γ bands are expected below the two quasiparticle states and the anharmonicity of γ vibration pushes the 2γ bands to more than the harmonic value of 2, because of the phonon-phonon interaction [25]. This anharmonicity was reproduced in macroscopic calculation [34] and more recently in the microscopic treatment of γ vibration [35]. In MPM [31] where the two-phonon interaction was treated in TDA by using an eight-phonon basis, and in SCCM [29] by using RPA phonons with mode-mode phonon interaction, the anharmonicity of the γ vibration was obtained through variation of interaction strengths. The $K^\pi=0^+$ 2γ phonon band is predicted to lie still higher in general for well-deformed nuclei including ^{168}Er [29–31]. But as illustrated in Table VI, the ratio $R=B(E2;4_{\gamma\gamma}\text{-}2_\gamma)/B(E2;2_\gamma\text{-}0_g)$ is low in MPM and the $B(E2;2_\gamma\text{-}0_g)$ value is much lower. In QPNM where the γ-vibration strength is fragmented over several states, the ground state to 2_γ and to 2_β $E2$ strengths are well reproduced by treating the former as collective and latter as non-collective, but the value of R is low. The $B(E2)$ ratio R is reasonably reproduced in SCCM [29], and DDM [14], as in DPPQ method. Note the upward revision of the R value in the recent Coulomb excitation work [27].

Our analysis of the relative interband transition rates from the $K^\pi=4^+$ band, its K purity and absolute $B(E2,4_{\gamma\gamma}\text{-}2_\gamma)$ supports the 2γ phonon character of the band in agreement with the above-cited experimental and theoretical works. The absolute transition probability of the $2'_0+$ state to the γ band as deduced by Lehmann *et al.* [28] is supported in our work, but this may be related to band mixing effects as pointed out in [1]. For the $0''$ band the decay patterns to g, γ, and β band are not well reproduced in either model as discussed above, hence no conclusion is drawn either on its collectivity or the phonon character. Thus, in spite of some structural deficiencies in the DPPQ method, its wide application has been exhibited here. We have also given more detailed results from IBM-1 and discussed its common points with DPPQ and the differences.

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