Nucleon-nucleon interaction in the Skyrme model

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We consider the interaction of two Skyrmions in the framework of the sudden approximation. The widely used product *Ansatz* is investigated. Its failure in reproducing an attractive central potential is associated with terms that violate *G* parity. We discuss the construction of alternative *Ansätze* and identify a plausible solution to the problem.

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I. INTRODUCTION

Nucleon-nucleon (NN) interactions are relatively simple at large distances and become rapidly more complex as one moves inward. In the best phenomenological models existing at present, which reproduce low-energy observables accurately, they are described by the consensual one-pion exchange potential (OPEP), supplemented by theoretical two-pion exchange potentials (TPEP), and parametrized at short distances [1,2]. The OPEP is responsible for a strong tensor component, which is mostly important in few-body systems, such as the deuteron. Two-pion exchange, on the other hand, gives rise to the central potential, that survives to all averages and is responsible for most properties of large systems and nuclear matter. Quantum chromodynamics (QCD) is the basic framework for the study of strong processes and should have, in principle, an important role in the description of nuclear forces. However, at present, the non-Abelian character of this theory prevents low energy calculations and one has to resort to effective theories, which must reflect the main features of QCD. Thus, in nuclear physics applications, besides the usual space-time invariances, one requires these theories to have approximate chiral symmetry. The latter is usually restricted to the $SU(2)\times SU(2)$ sector, for most processes involve only the quarks u and d. This symmetry is explicitly broken by the small quark masses and, at the effective level, by the pion mass.

Chiral symmetry has no influence over the OPEP, but is crucial to the TPEP, which depends on an intermediate pion-nucleon (πN) amplitude [1]. In the case of NN interactions, the importance of this symmetry was stressed already in the early 1970s, by Brown and Durso [3] and by Chemtob, Durso, and Riska [4], who used it to constrain the form of the TPEP. In that decade it also became popular to describe nuclear processes by means of the linear σ model [5], containing a fictitious particle called σ that, to some extent, simulates the TPE. The elimination of this unobserved degree of freedom gave rise to nonlinear theories, which underlie modern descriptions of the interaction. The first theoretical framework to incorporate nonlinear chiral dynamics into the NN problem was proposed by Skyrme. This remarkable model for the nucleon, developed in the 1960s [6] and revived in the 1980s [7], describes baryons as topological solitons, objects extended in space that rotate according to the laws of quantum mechanics. The quark condensate appears as an intrinsic feature, corresponding to a nonvanishing classical content of the vacuum, whose intensity is given by the pion decay constant f_π. Skyrmions correspond to distortions of this condensate that carry topological charges. One then works with pion fields which are unusually strong, in the sense that their amplitudes may be comparable to f_π. Thus, in spite of its well-known limitations [8], the Skyrme model remains a unique laboratory for studying chiral symmetry in the nonperturbative regime.

In the early 1990s Weinberg restated the role of perturbative chiral symmetry in nuclear interactions [9] and motivated interest in the TPEP. Initially, several authors explored the pion-nucleon sector of nonlinear Lagrangians [10], but the corresponding potentials could not reproduce even the medium range attraction in the scalar channel. This happened because the TPEP is based on an intermediate πN amplitude, that can only be well described with the help of other degrees of freedom [11]. Accordingly, in a later stage, agreement with empirical πN information was enforced and descriptions could reproduce NN scattering data [12–14].

In the case of perturbative calculations, the delta is by far the most important non-nucleonic degree of freedom and is largely responsible for the intermediate range scalar attraction. As the Skyrme model incorporates the delta from the very beginning, one expects that it should yield a good qualitative NN potential. However, it fails to do so.

Skyrme himself considered NN interactions, already in the 1960s, using the so-called product *Ansatz* (PA) [6]. The basic idea underlying the PA is that solutions corresponding to baryon number $B=1$ can be used as building blocks to construct approximate solutions with an arbitrary value of B. The great advantage of this approach is that the baryon number of the composite system is automatically equal to the number of individual $B=1$ Skyrmions, irrespectively of their relative positions. In the PA, the Skyrmions that constitute a larger system are assumed to retain their shape all the time, what is known as sudden approximation. In this framework, the construction of the NN potential is rather simple and the fact that each nucleon has a profile function which falls off rapidly with the distance allows one to assume that, for medium and large distances, the $B=2$ system is not consider-

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TABLE I. Empirical values for the subthreshold coefficients a_{00}^+, a_{01}^+ and the constant K, which determines the intensity of the central potential.

	Ref. [11]	Ref. [28]	Ref. [29]	Ref. [29]
$a_{00}^+(m_\pi^{-1})$	-1.46 ± 0.10	-1.30 ± 0.02	-1.27 ± 0.03	-1.15 ± 0.03
$a_{01}^+(m_\pi^{-3})$	1.14 ± 0.02	1.35 ± 0.14	1.27 ± 0.03	1.23 ± 0.03
K(MeV)	7.19	9.51	8.84	8.74

ably different from the superposition of two with $B=1$.

In the 1980s, the product *Ansatz* was used by Jackson *et al.* [15] and Vinh Mau *et al.* [16] to calculate the NN potential, who found out a fully repulsive central component, in disagreement with very well established phenomenology. This result was carefully investigated in Ref. [17]. This puzzle motivated several attempts to construct improved versions of those early works. Among them, one notes the symmetrized product *Ansatz* by Nyman and Riska [18], which could produce an intermediate range scalar attraction. However, as pointed out by Sternheim and Kälbermann [19], there is a violation of baryon number conservation in this *Ansatz*. Exact numerical calculations were also used, which allowed one to evaluate the reliability of the sudden approximation at short distances [20]. Lattice calculations, using a method developed by Manton and collaborators, gave rise to a torus-like baryon density, believed to correspond to the true $B=2$ ground state and having almost twice the nucleon mass [21]. The scalar potential associated with this configuration does show some medium to long range attraction [22]. However, it is worth recalling that lattice results depend on the definitions adopted for collective coordinates and, as the full treatment is rather difficult, one usually resorts to approximations [23,24].

In this work we consider the scalar interaction between Skyrmions, in order to explore the possibility of obtaining the central attraction at large distances by relaxing some of the constraints present in usual calculations. We employ the sudden approximation because it gives rise to a constructive interaction, in which nondeformed nucleons are the main building blocks, as in perturbative calculations. Our presentation is divided as follows. In Sec. II we study the asymptotic behavior of the scalar potential in the standard product *Ansatz* approximation, in order to understand why it does not yield attraction. In Sec. III we discuss the construction of alternative solutions, which must be constrained to have the correct baryon number. Finally, in Sec. IV we analyze a possible solution to the problem, and present concluding remarks.

II. CENTRAL POTENTIAL

The structure of the central potential has been studied recently, in the framework of chiral perturbation theory [14,25,26]. In momentum space, the leading contribution has the generic form

$$V_C(t)=-\frac{2}{f_\pi^2 m_\pi^2}[f_\pi^2(a_{00}^+ + a_{01}^+ t)]\sigma_N(m;t), \tag{1}$$

where f_π is the pion decay constant, the a_{0i}^+ are subthreshold coefficients [11], and $\sigma_N(m;t)$ is the intermediate nucleon contribution to the scalar form factor, that depends on both the momentum transfer t and the baryon mass m. In general, the scalar form factor is defined in terms of the symmetry breaking Lagrangian as

$$\langle N(\boldsymbol{p}')|-\mathcal{L}_{sb}|N(\boldsymbol{p})\rangle=\langle N(\boldsymbol{p}')|\hat{m}\overline{q}q|N(\boldsymbol{p})\rangle=\sigma(t)\overline{u}(\boldsymbol{p}')u(\boldsymbol{p}), \tag{2}$$

where $\hat{m}$ is the average of the masses of u and d quarks.

In configuration space, Eq. (1) becomes

$$V_C(d)=-\frac{2}{f_\pi^2 m_\pi^2}[f_\pi^2(a_{00}^+ + a_{01}^+\boldsymbol{\nabla}^2)]\sigma_N(m;d), \tag{3}$$

where d is the internucleon distance and $\sigma(m;d)$ is the Fourier transform of $\sigma(m;t)$. In order to allow this result to be compared with the corresponding one in the Skyrme model, we note that, in the large N_c limit, the nucleon and the delta are degenerate and very heavy. In this case, one has [25,26]

$$\begin{aligned}\sigma(N_c\to\infty;d)&=\sigma_N(N_c\to\infty;d)+\sigma_\Delta(N_c\to\infty;d)\\&=3\sigma_N^{\mathrm{HB}}(d)\\&=\frac{9m_\pi^6}{128\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\left[\frac{d}{dx}\frac{e^{-x}}{x}\right]^2,\end{aligned} \tag{4}$$

where $x=m_\pi d$, σ_Δ is the contribution from the delta intermediate state, whereas σ_N^{HB} is the heavy baryon limit of σ_N. The relationship between nucleon and delta contributions to σ in the large N_c limit was discussed in Ref. [27].

Using Eq. (4) into Eq. (3), the central potential becomes

$$\begin{aligned}V_C^{\mathrm{HB}}(d)=&-\frac{2}{f_\pi^2 m_\pi^2}\left[\frac{3m_\pi^6 g_A^2}{128\pi^2}\right]\left[a_{00}^+\left(1+\frac{2}{x}+\frac{1}{x^2}\right)\right.\\&\left.+4m_\pi^2 a_{01}^+\left(1+\frac{3}{x}+\frac{11}{2x^2}+\frac{6}{x^3}+\frac{3}{x^4}\right)\right]\frac{e^{-2x}}{x^2}\end{aligned} \tag{5}$$

and, at very large distances, it behaves as

$$V_C^{\mathrm{HB}}(d)\to -K\left(\frac{e^{-x}}{x}\right)^2. \tag{6}$$

The sign of the constant K is determined by the values of the subthreshold coefficients in the combination $(a_{00}^+ + 4m_\pi^2 a_{01}^+)$.

In Table I we display empirical values for the a_{0i}^{+} and it is possible to note that the correct sign of V_C comes mainly from a_{01}^{+}, since a_{00}^{+} in isolation would give rise to a repulsive interaction.

In order to study the central potential in the Skyrme model, we recall that the standard soliton Lagrangian density is written as [6,8]

$$\mathcal{L}=\mathcal{L}_{\sigma}+\mathcal{L}_4, \tag{7}$$

where

$$\mathcal{L}_{\sigma}=\frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)+m_\pi^2\frac{f_\pi^2}{4}\mathrm{Tr}(U+U^\dagger-2) \tag{8}$$

corresponds to the nonlinear σ model and

$$\mathcal{L}_4=\frac{1}{32e^2}\mathrm{Tr}[\partial_\mu UU^\dagger,\partial_\nu UU^\dagger]^2 \tag{9}$$

is the stabilizing term. In these expressions, e is a free parameter, called Skyrme constant, whereas the dynamical variable U is a 2×2 unitary matrix, given by

$$U=e^{i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}}F}=\cos F+i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}}\sin F, \tag{10}$$

where $\boldsymbol{\tau}$ are the isospin Pauli matrices and F is the chiral angle, whose boundary conditions determine the baryon number of a particular configuration. The function F and the isospin direction $\hat{\boldsymbol{\pi}}$ are related to the pion field $\boldsymbol{\pi}$ of the nonlinear σ model [5] by $\boldsymbol{\pi}=f_\pi\sin F\hat{\boldsymbol{\pi}}$.

In the $B=1$ case, a static solution is obtained using the condition $\hat{\boldsymbol{\pi}}=\hat{\boldsymbol{r}}$, the so-called hedgehog *Ansatz*, with boundary conditions $F(r=0)=\pi$ and $F(r\to\infty)=0$ [8]. The quantization of this baryon is achieved by rotating the static solution with the help of collective coordinates, as a rigid body. This procedure endows the Skyrmion with spin and isospin and corresponds to multiplying the pion field by the rigid body rotation matrix D,

$$\pi_i\to\pi_\alpha^q=D_{\alpha i}\pi_i\,. \tag{11}$$

The matrices D satisfy the completeness relations $D_{\alpha i}D_{\alpha j}=\delta_{ij}$, $D_{\alpha i}D_{\beta i}=\delta_{\alpha\beta}$ and, in the case of nucleons, the correspondence with the ordinary formalism is achieved by using

$$\langle N|D_{\alpha i}|N\rangle=-\tfrac{1}{3}\langle N|\tau_\alpha\sigma_i|N\rangle, \tag{12}$$

σ_i being the spin Pauli matrices.

The scalar form factor in the Skyrme model can be obtained directly from Eqs. (8) and (10) and reads

$$\sigma^{\mathrm{Sk}}(d)=\langle N|-\mathcal{L}_{sb}(d)|N\rangle=-m_\pi^2f_\pi^2[\cos F(d)-1]. \tag{13}$$

On the other hand, the asymptotic form of the chiral angle is determined by $\mathcal{L}_\sigma$ as [30]

$$F_\infty(d)=-\left(\frac{3g_Am_\pi^2}{8\pi f_\pi^2}\right)\left(\frac{d}{dx}\frac{e^{-x}}{x}\right) \tag{14}$$

and hence, for large distances, the leading term in Eq. (13) yields $\sigma_\infty^{\mathrm{Sk}}(d)=\sigma(N_c\to\infty;d)$. In order to test this relationship further, we write

$$\sigma_\infty^{\mathrm{Sk}}(d)=m_\pi^2f_\pi\langle N|\sqrt{f_\pi^2-\boldsymbol{\pi}^q\cdot\boldsymbol{\pi}^q}-f_\pi|N\rangle\simeq\frac{m_\pi^2}{2}\boldsymbol{\pi}^2$$
$$=\frac{m_\pi^2}{2}[\langle N|\pi_\alpha^q|N\rangle\langle N|\pi_\alpha^q|N\rangle+\langle N|\pi_\alpha^q|\Delta\rangle\langle\Delta|\pi_\alpha^q|N\rangle]. \tag{15}$$

Using Eqs. (11) and (12) in the last expression, one finds that N and Δ intermediate states determine, respectively, 1/3 and 2/3 of the total value of $\sigma_\infty^{\mathrm{Sk}}(d)$, in agreement with Ref. [27]. This relative proportion is identical to that found recently in the framework of perturbation theory [26].

For systems with $B=2$, the standard point of departure for constructing approximate solutions is the product *Ansatz* (PA). It uses two undistorted $B=1$ hedgehog solutions, whose centers are located at two fixed points equidistant from origin along the z axis, so that the hedgehog space coordinates are given by $\boldsymbol{y}=\boldsymbol{r}+d\hat{\boldsymbol{z}}/2$ and $\boldsymbol{w}=\boldsymbol{r}-d\hat{\boldsymbol{z}}/2$. Denoting the composite field by $U(\boldsymbol{y},\boldsymbol{w})$, one writes

$$U(\boldsymbol{y},\boldsymbol{w})=U(\boldsymbol{y})U(\boldsymbol{w}). \tag{16}$$

In this configuration, the $B=2$ condition is automatically fulfilled, for any distance between their centers [31]. As the PA keeps the identities of constituent Skyrmions, it allows the direct incorporation of spin and isospin, through collective rotations of individual hedgehogs.

The potential is a function of the distance d and given by

$$V(d)=-\int d^3r\,\mathcal{L}_{\mathrm{int}}(\boldsymbol{r},d\hat{\boldsymbol{z}}), \tag{17}$$

where $\mathcal{L}_{\mathrm{int}}$ is obtained by using the field $U(\boldsymbol{y},\boldsymbol{w})$ in the Skyrme Lagrangian, Eqs. (7)–(9), and subtracting the self-energies $\mathcal{L}[U(\boldsymbol{y})]$ and $\mathcal{L}[U(\boldsymbol{w})]$. This potential works well in the isospin-dependent channels, since the OPEP is reproduced for distances larger than 2 fm and it is also possible to identify the roles of ρ and $A1$ mesons [32]. On the other hand, problems occur in the scalar-isoscalar channel, where the interaction is repulsive at all distances, in sharp contradiction with phenomenology.

Using the definitions $F_r'=dF(r)/dr$, $s_r=\sin F(r)$, and $c_r=\cos F(r)$, the central potential is given by

TABLE II. Coefficients of the multipole expansion of V_C for the product *Ansatz* and the Argonne potential, as defined in Eq. (22).

	$\mathcal{L}_\sigma$	$\mathcal{L}_4$	$\mathcal{L}_\sigma+\mathcal{L}_4$	Argonne
K(MeV)	7.29	-8.52	-1.23	4.80
α_1	0.93	4.31	24.4	1.0
α_2	1.78	12.9	79.1	6.0

$$V_C^{pa}(d)=\frac{2f_\pi}{e}\frac{4\pi}{3}\int_0^\infty dz\int_0^\infty \rho d\rho\Bigg\{-\frac{3m_\pi^2}{16e^2f_\pi^2}(1-c_y)(1-c_w)+\Bigg[\left(F_y'^2+\frac{s_y^2}{y^2}\right)\left(F_w'^2+\frac{s_w^2}{w^2}\right)+\frac{2s_y^2s_w^2}{y^2w^2}-(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})^2\left(F_y'^2-\frac{s_y^2}{y^2}\right)\left(F_w'^2-\frac{s_w^2}{w^2}\right)\Bigg]\Bigg\}. \tag{18}$$

In order to study its asymptotic structure, we note that the pion fields exist effectively only in the neighborhood of the hedgehog centers. When the distance d is large the Skyrmion located at $(0,0,d/2)$ is in the presence of the asymptotic region of $U(\boldsymbol{y})$, we expand F_y, F_y', and $\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}}$ around the point $\boldsymbol{w}=0$ and write

$$F_y\simeq \alpha e^{-m_\pi w_z}\left(1+\frac{f_1}{x}+\frac{f_2}{x^2}\right)\frac{e^{-x}}{x}, \tag{19}$$

$$F_y'\simeq -\alpha e^{-m_\pi w_z}\left(1+\frac{g_1}{x}+\frac{g_2}{x^2}\right)\frac{e^{-x}}{x}, \tag{20}$$

$$\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}}\simeq \frac{w_z}{\sqrt{\rho^2+w_z^2}}\left(1+\frac{m_\pi\rho^2}{w_z x}-\frac{3m_\pi^2\rho^2}{2x^2}\right), \tag{21}$$

where f_i, g_i are dimensionless polynomials of $w_z\equiv(z-d/2)$ and ρ, which are not displayed here. These expressions were tested order by order, by using them in Eq. (18) and checking that the potential did have the asymptotic structure, as in Eq. (6). We found out that it was necessary to expand $F(\boldsymbol{y})$ up to order d^{-2}, in order to have accurate results.

Replacing Eqs. (19)–(21) into Eq. (18), we obtain an asymptotic contribution of the form

$$V_C^{pa}(d)\to -K\left[1+\frac{\alpha_1}{x}+\frac{\alpha_2}{x^2}\right]\frac{e^{-2x}}{x^2}, \tag{22}$$

for both $\mathcal{L}_\sigma$ and $\mathcal{L}_4$, separately. The values of the parameters K and α_i are displayed in Table II, based on the numerical constants $m_\pi=139$ MeV, $f_\pi=93$ MeV, and $e=4.0$. For the sake of comparison, we also present the values of those parameters in the case of the phenomenological Argonne potential [33].

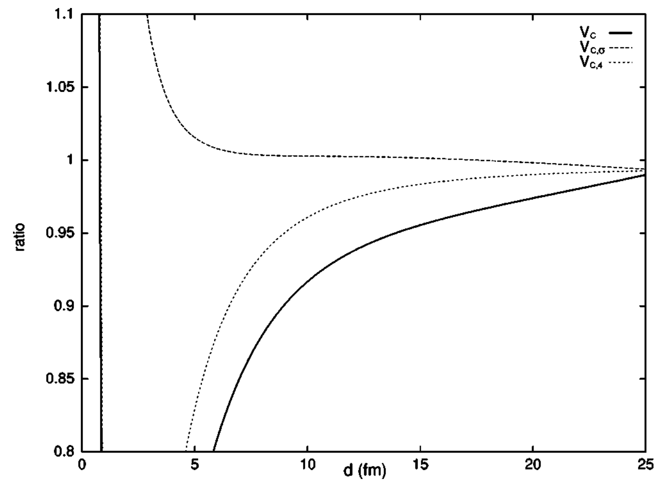

FIG. 1. Ratios between the multipole expansion and the exact numerical result for the scalar potential (solid) and separate contributions of $\mathcal{L}_\sigma$ (dashed) and $\mathcal{L}_4$ (dotted).

Inspecting this table, one notes that the part of the potential due to $\mathcal{L}_\sigma$ is attractive, but is superseded by a repulsive contribution coming from the stabilizing term. The net sign of the potential is, then, the outcome of a large cancellation. On the other hand, the dependence of Eq. (22) on d is similar to those of both the perturbative chiral calculation, Eq. (6), and of the phenomenological Argonne potential. We stress that this correct geometry is a general feature of the model, because it depends only on the form of the $B=1$ solution, and not on the specific *Ansatz* used to obtain the $B=2$ result. In Fig. 1 we display the ratios between the full and asymptotic PA potentials, as given by Eqs. (18) and (22), with the purpose of illustrating their convergence. The interplay between the attractive contribution from $\mathcal{L}_\sigma$ and the repulsive one from $\mathcal{L}_4$ can also be seen in Fig. 2, where we present the function $dV_C^{pa}(d)/dz$, corresponding to the integrand in z of expression (18), for several values of d. One notes that the contributions in the neighborhood of the Skyrmion centers are large and positive but, on the other hand, a negative region develops as the distance d increases.

These features of the central potential allow us to identify clearly the stabilizing term as the responsible for its repulsive character. Therefore, mechanisms which can reduce the importance of $\mathcal{L}_4$ may help in producing an attractive interaction. In the next section we discuss a class of such mechanisms, associated with deformations of the QCD vacuum.

III. CONSTRUCTING $B=2$ SOLUTIONS

We consider here $B=2$ solutions, constructed by using the hedgehog $B=1$ Skyrmions as building blocks, in the framework of the sudden approximation. In general, an *Ansatz* is a prescription of the form

$$U(\boldsymbol{y},\boldsymbol{w})=f[U(\boldsymbol{y}),U(\boldsymbol{w})], \tag{23}$$

where f is a function, chosen according to physical criteria. The construction of such a function should follow some guidelines: (1) the baryon number of the composite configu-

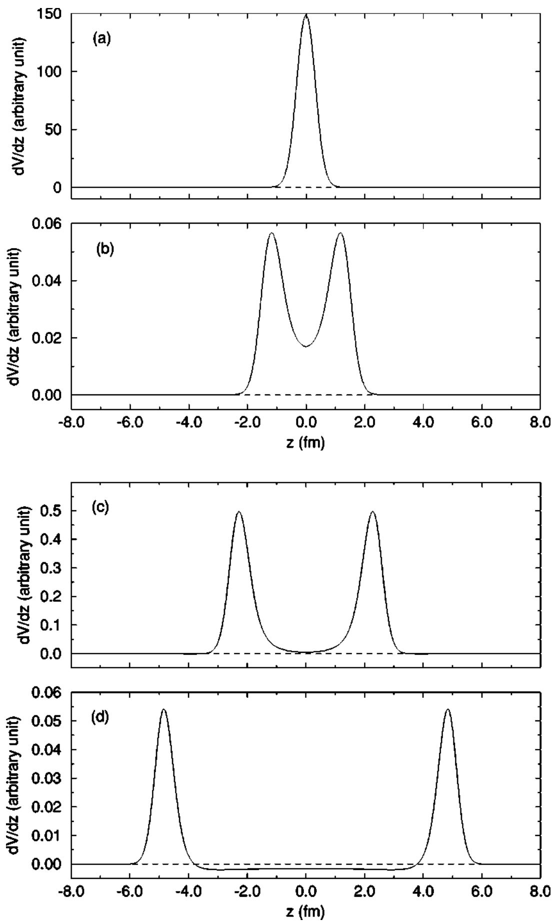

FIG. 2. Integrand in z of the scalar potential $V_C(d)$, for (a) $d=1$ fm, (b) $d=3$ fm, (c) $d=5$ fm, (d) $d=10$ fm; vertical axis: arbitrary unit.

ration must be two for all distances d; (2) the composite pion field must have the correct quantum numbers, being pseudoscalar, isovector and odd under G-parity; (3) the composite Lagrangian must be chiral symmetric, even under G-parity and invariant under the exchange of the two constituent Skyrmions.

The standard constructive approximate solution to the $B=2$ system is based on the PA, as discussed in the preceding section. In this approach, the composite pion field, obtained from Eqs. (10) and (16), is given by

$$\boldsymbol{P}_{pa}=\frac{1}{f_\pi}(\sigma_y \boldsymbol{\pi}_w+\sigma_w \boldsymbol{\pi}_y-\boldsymbol{\pi}_y\times\boldsymbol{\pi}_w), \tag{24}$$

where $\boldsymbol{\pi}_r$ is the pion field of the hedgehog with coordinate $\boldsymbol{r}$ and $\sigma_r\equiv f_\pi \cos F(\boldsymbol{r})$. The function

$$S_{pa}=\frac{1}{f_\pi}(\sigma_y\sigma_w-\boldsymbol{\pi}_y\cdot\boldsymbol{\pi}_w) \tag{25}$$

is the composite analogous of σ and satisfies $S_{pa}^2+\boldsymbol{P}_{pa}^2=f_\pi^2$.

The field $\boldsymbol{P}_{pa}$ has a rather serious drawback as a candidate for the pion field, namely that it contains an azimuthal term

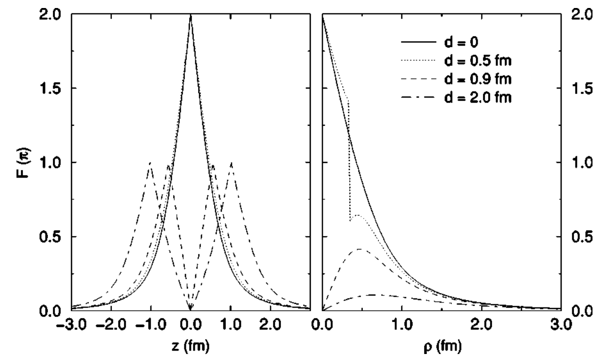

FIG. 3. Profile function F for the product *Ansatz*, in units of π, along the z (left) and ρ (right) axes, for various values of d.

which is both even under G-parity and antisymmetric under hedgehog exchange. Hence it does not have good pion quantum numbers, violating requirements (2) and (3) stated above.

This motivated us to try to understand whether this problem could be responsible for the absence of attraction found in the central potential. We considered several alternative possibilities, inspired in the PA. The basic idea is to propose a composite field $\boldsymbol{P}$, use it to define a function S by

$$S^2=f_\pi^2-\boldsymbol{P}^2, \tag{26}$$

construct the unitary field as

$$U=[S+i\boldsymbol{\tau}\cdot\boldsymbol{P}]/f_\pi, \tag{27}$$

and feed it into the Skyrme Lagrangian. We begin by describing briefly some unsuccessful attempts, in order to prevent readers from repeating them.

The simplest exchange-symmetric *Ansatz* would be the average $\boldsymbol{P}=(\boldsymbol{\pi}_y+\boldsymbol{\pi}_w)/2$. However, when $d=0$, one has $F_y=F_w=F_r$ and hence $\boldsymbol{P}=f_\pi \sin F_r \hat{\boldsymbol{r}}$ corresponds to a $B=1$ field, which must be disregarded.

This suggests that, in order to obtain $B=2$, it is mandatory to mix $\boldsymbol{\pi}$ and σ. In the case of the PA, which yields $B=2$ at all distances, we note that the chiral constraint between $\boldsymbol{P}_{pa}$ and S_{pa} allows one to write

$$U_{pa}=[S_{pa}+i\boldsymbol{\tau}\cdot\boldsymbol{P}_{pa}]/f_\pi\equiv e^{-i\boldsymbol{\tau}\cdot\boldsymbol{u}F_{pa}}, \tag{28}$$

where $\boldsymbol{u}$ is a unit vector, taken as pointing always away from the origin of the coordinate system, and F_{pa} is a profile function. In Fig. 3 we display the behavior of this angle along the axes z and ρ, for various values of the internucleon distance d. The solid lines correspond to the case $d=0$, which is spherically symmetric and it is possible to see that, along both directions, the chiral angle varies smoothly from 2π at the origin to 0 at infinity. In the case $d=0.5$ fm, shown in dotted lines, one notes that a discontinuity has appeared along the ρ axis. This discontinuity increases with distance and, at $d_{crit}=0.86$ fm, the chiral angle is such that $F_{pa}(\rho=0,z\to 0)=2\pi$ and $F_{pa}(\rho\to 0,z=0)=0$. Therefore, at this critical point, it is more natural to set $F_{pa}(0,0)=0$ and to work with two separate solutions, such as illustrated by the dashed and dot-dashed lines, corresponding to 0.9 and

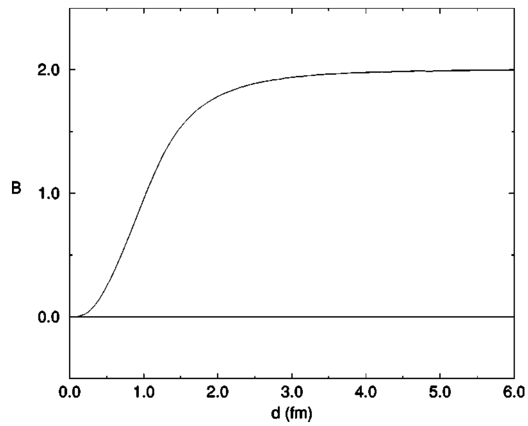

FIG. 4. Baryon number as a function of distance d (fm), for the *Ansatz* given by Eqs. (29) and (26).

2.0 fm, respectively. This suggests that, from d_{crit} onwards, each of the interacting Skyrmions acquires a considerable individuality.

The combination

$$\boldsymbol{P}=\frac{1}{f_\pi}(\sigma_y\boldsymbol{\pi}_w+\sigma_w\boldsymbol{\pi}_y) \tag{29}$$

is interesting, for it has an explicit physical meaning. As the function $\sigma(\boldsymbol{r})$ is associated with the quark condensate that surrounds the baryon labeled by $\boldsymbol{r}$, this field $\boldsymbol{P}$ represents each Skyrmion immersed in the vacuum distorted by the other one. The condition (26) allows one to determine S up to a sign. In the case $B=1$ the field σ changes sign when one goes from infinity to the origin and the same happens when $B=2$. The sign of S is also important and, in order to fix it, we note that the behaviors of Eqs. (24) and (29) along the z axis are identical, since the azimuthal component vanishes. We then forced the condition $S=S_{pa}$ along this axis. However, this *Ansatz*, based on Eq. (29), gives rise to a baryon number which varies with d, as shown in Fig. 4, and had to be abandoned.

This discussion illustrates the fact that it is not trivial to build an *Ansatz* with a good topology. We thus decided to adopt simultaneously the pion field as given by Eq. (29) and the function S_{pa} of the PA, Eq. (25), for its topology is automatically correct. With this option, the unitarity constraint reads

$$S^2+\mathbf{P}^2=f_\pi^2\eta^2, \tag{30}$$

where

$$\eta=\sqrt{1+[(\boldsymbol{\pi}_y\cdot\boldsymbol{\pi}_w)^2-\boldsymbol{\pi}_y^2\boldsymbol{\pi}_w^2]/f_\pi^4} \tag{31}$$

and the pion field becomes in fact $\boldsymbol{P}/\eta$. This form for the dynamical variable is the same as that proposed by Nyman and Riska, in their symmetrized product *Ansatz* [18]. This *Ansatz* has a topology similar to the PA, as illustrated in Fig. 3. The corresponding baryon number density is given in the Appendix and, in the classical case, yields $B=2$ for all distances when integrated over space, as shown in Fig. 5.

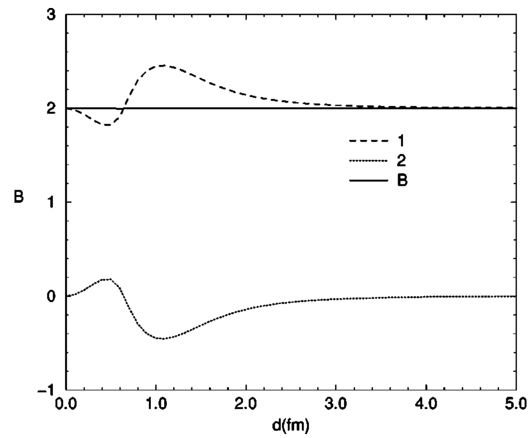

FIG. 5. Baryon number as a function of the separation distance (fm) for the symmetrized *Ansatz* (solid); separate contributions B_1 (dashed) and B_2 (dotted), as defined in Eq. (A1).

IV. RESULTS AND CONCLUSIONS

In order to derive the potential, we use the quantized fields of Eq. (11), obtained by rotating the constituent Skyrmions. This idea of rotating individual hedgehogs corresponds to an approximation and deserves some attention. The quantization of a hedgehog, as discussed in Sec. II, amounts to multiplying the classical field by the matrix D, which depends on three free parameters. In the case $B=1$, this procedure does not change the baryon current. This can be seen by writing the baryon density for quantized fields as

$$B^0=-\frac{1}{12\pi^2}\epsilon_{abc}\epsilon_{\alpha\beta\gamma}\frac{1}{\sigma}\partial_a D_{\alpha i}\pi_i\partial_b D_{\beta j}\pi_j\partial_c D_{\gamma k}\pi_k \tag{32}$$

and using the result

$$D_{\alpha a}D_{\beta b}=\tfrac{1}{3}\delta_{\alpha\beta}\delta_{ab}+\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}D_{\gamma c}\epsilon_{cab}+\text{isotensors} \tag{33}$$

in order to obtain

$$B^0=-\frac{1}{12\pi^2}\epsilon_{ijk}\epsilon_{abc}\frac{1}{\sigma}\partial_a\pi_i\partial_b\pi_j\partial_c\pi_k\,. \tag{34}$$

This shows that the baryon density is the same for both quantized and classical fields.

Analogously, in the case $B=2$, quantization would require a matrix $\overline{D}$, depending on six collective coordinates. However, the determination of this general matrix may prove to be very difficult and, in the spirit of the the sudden approximation, one normally uses $\overline{D}\approx D^{(y)}I^{(w)}+I^{(y)}D^{(w)}$, where I is an identity matrix and $D^{(y)},D^{(w)}$ are operators over the Skyrmions labeled by $\boldsymbol{y}$ and $\boldsymbol{w}$, respectively. The price one pays for this approximation is that it leads to a

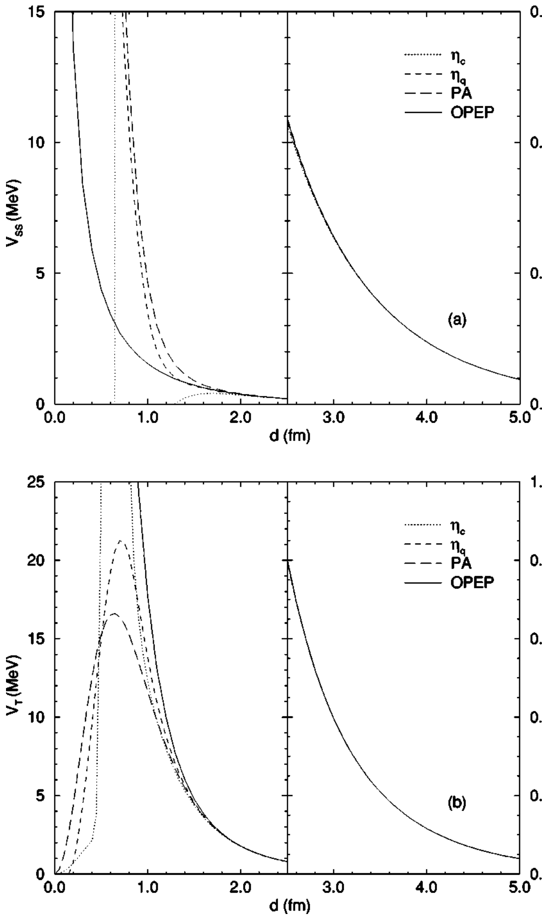

FIG. 6. Comparison among Skyrme model G-parity odd (a) spin-spin and (b) tensor potentials from symmetrized *Ansatz*, with classical (dotted) and quantized (dashed) versions of η, from product *Ansatz* (long dashed) and OPEP (solid).

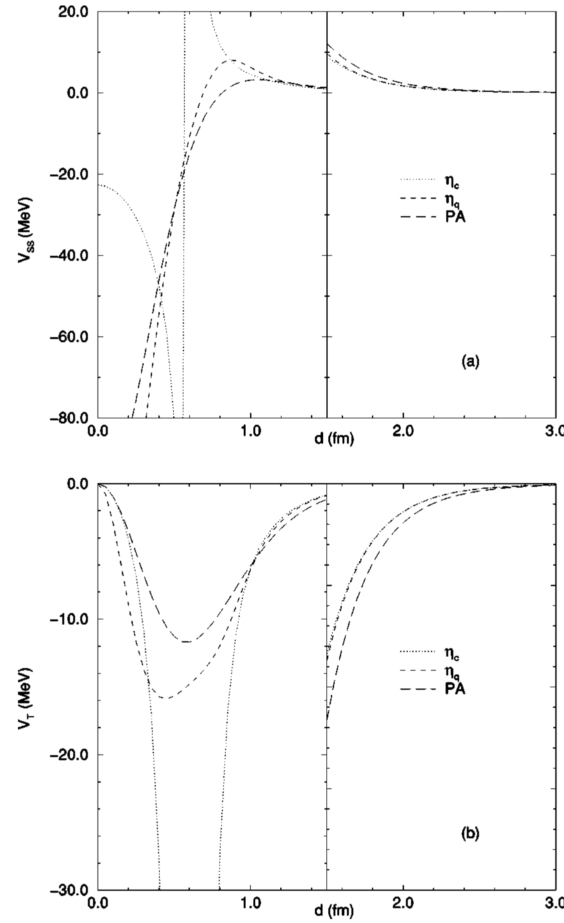

FIG. 7. G-parity even (a) spin-spin and (b) tensor potentials from symmetrized *Ansatz*, with classical (dotted) and quantized (dashed) versions of η, and from product *Ansatz* (long dashed).

quantized baryon number which depends on $D^{(y)}$ and $D^{(w)}$. This happens because the relation equivalent to Eq. (33) does not hold for the approximate matrix $\overline{D}$ and hence does not represent a major shortcoming for the symmetrized *Ansatz* (SA). Indeed, as pointed out by Sternheim and Kälbermann [19], this poses problems for short distances only.

A collective rotation of the pion field $\boldsymbol{P}$, as in the $B=1$ case, would leave S/η unmodified, as a classical function. However, this would also mean to treat the scalar product $\boldsymbol{\pi}_y \cdot \boldsymbol{\pi}_w$ as a classical quantity and would lead to serious contradictions, for the OPEP content of the isospin dependent channels relies on the quantum character of such a scalar product in Eq. (25).

Therefore, the individual rotation of each pion field is more consistent with a constructive approach, although not free of problems. In principle, every pion field $\boldsymbol{\pi}$ in the composite Skyrme Lagrangian should be quantized. When applying this prescription to the dynamical variable of the SA, one has to deal with the functions η^{-2} and η^{-4}, which depend on pion fields, coupled to operators D. The meaning of the quantized η^{-2} is that of a power series in D, which involves products of arbitrary numbers of these matrices and hence can only be handled by resorting to truncation. With this limitation in mind, we treat η^{-2} as a polynomial in $\boldsymbol{\pi}$ and thus its expectation value between two-nucleon states can be evaluated without ambiguities.

In order to test the implications of this assumption, in the sequence we present results with two versions of η, namely, a classical one,

$$\eta_c^{-2} = \{1-[1-(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})^2]s_y^2 s_w^2\}^{-1}, \tag{35}$$

and a quantized one, truncated at the first order in the D expansion, given by

$$\begin{aligned}\langle NN|\eta_q^{-2}|NN\rangle &= \langle NN|\{1-s_y^2 s_w^2 + D_{\alpha i}^{(y)} D_{\alpha j}^{(w)} D_{\beta k}^{(y)} D_{\beta \ell}^{(w)} \\ &\quad \times(\pi_y)_i(\pi_w)_j(\pi_y)_k(\pi_w)_\ell/f_\pi^4\}^{-1}|NN\rangle \\ &\approx \{1-\tfrac{2}{3}s_y^2 s_w^2\}^{-1}. \end{aligned}\tag{36}$$

Replacing the pion field of the symmetrized *Ansatz* into the interaction Lagrangian used to calculate the potential, one has

$$\mathcal{L}_{\mathrm{int}} = \mathcal{L}^S + D_{\alpha m}^{(y)} D_{\alpha n}^{(w)} \mathcal{L}_{mn}^V, \tag{37}$$

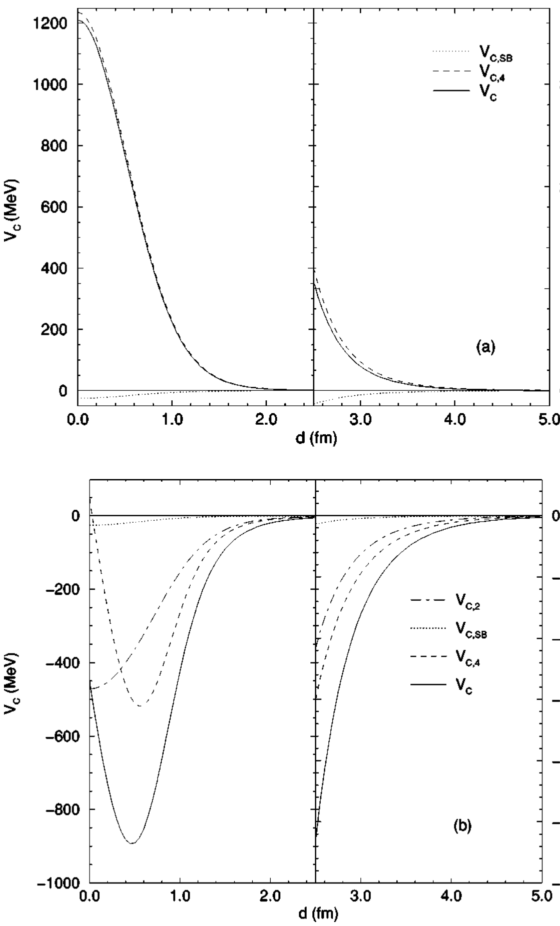

FIG. 8. Scalar potential in the Skyrme model: total result and components. Product *Ansatz* (a) and symmetrized *Ansatz* with $\eta=1$ (b).

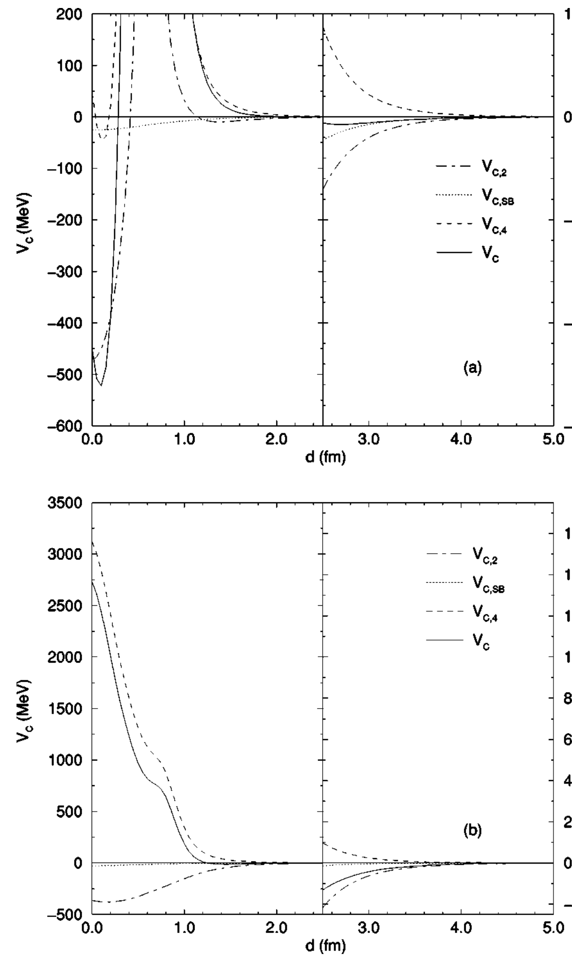

FIG. 9. Scalar potential in the Skyrme model with the symmetrized *Ansatz*: total result and components; (a) η_c and (b) η_q.

where the labels S and V stand, respectively, for isoscalar and isospin dependent parts of $\mathcal{L}_{\rm int}$. Using this result in Eq. (17), one gets

$$V(d)=V_C+\boldsymbol{\tau}^{(y)}\cdot\boldsymbol{\tau}^{(w)}[\boldsymbol{\sigma}^{(y)}\cdot\boldsymbol{\sigma}^{(w)}V_{SS}+S_{12}V_T], \tag{38}$$

where V_C, V_{SS}, and V_T are the usual central, spin-spin and tensor components. All terms receive both G-parity odd and even contributions.

The G-parity odd components of spin-spin and tensor terms of the potential are shown in Fig. 6, for the two possible choices of η, compared to the PA and pure OPEP results. One sees that all curves coincide for distances larger than 2 fm, indicating that all *Ansätze* reproduce asymptotically the OPEP. The results for the G-parity even terms are of minor importance here, as we are interested in the long range behavior of the potential, but they are included for completeness in Fig. 7. One should note that in the case of η_c, the potentials present a singularity at $d\sim 0.6$ fm, due to a zero of η_c.

Results for the scalar component V_C are presented in Figs. 8 and 9. In the former we display the behavior of the PA and the predictions from the SA with $\eta=1$, which is nonunitary and considered just for pedagogical purposes. Inspecting it one learns that the SA includes a contribution from $\mathcal{L}_2$, that was not present in the PA. Moreover the contribution from $\mathcal{L}_4$ is negative, and so is the net result for V_C.

The two valid options for the SA considered here, based on η_c and η_q, are given in Fig. 9. In both cases we observe that the unitarity constraint restores the repulsion due to $\mathcal{L}_4$, but in such a way that the net result is asymptotically attractive. On the other hand, the amount of overall attraction found in the SA depends on the specific quantization prescription adopted. At very large distances, the curves corresponding to η_c and η_q have the same geometry and yield, respectively, the following approximate values for the intensity of the potential: $K_c=14$ MeV and $K_q=57$ MeV. Comparing them with the empirical values in Table I, it is possible to see that predictions from the SA are qualitatively reasonable.

In summary, we have shown that the SA provides the correct baryon number for the two-nucleon system in the Skyrme model, as well as attractive central NN potentials. The correct quantitative feature is somewhere between the values obtained for the two versions of the normalization function η.

We conclude that it is indeed relevant to the central potential to eliminate the term with a wrong G parity from $\boldsymbol{P}_{pa}$. We expect that a deeper and more careful study of the quantization procedure will lead to a more accurate evaluation of the amount of attraction coming from the SA.

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APPENDIX: THE BARYON NUMBER

The explicit calculation of the zero component of the baryon current yields

$$B^0 = B_1^0 + B_2^0, \tag{A1}$$

where

$$B_1^0(\boldsymbol{r};d) = -\frac{1}{2\pi^2\eta^2}\left(c_y\frac{s_w}{w}+c_w\frac{s_y}{y}\right)^2 (F_y'+F_w'),$$

$$\begin{aligned}B_2^0(\boldsymbol{r};d) = &-\frac{1}{2\pi^2}\frac{1}{(c_yc_w-(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})s_ys_w)\eta^2}\left(c_y\frac{s_w}{w}+c_w\frac{s_y}{y}\right)\\ &\times\Bigg\{(1-(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})^2)\Bigg[c_yc_w\left(c_yF_y'-\frac{s_y}{y}\right)\left(c_wF_w'-\frac{s_w}{w}\right)\\ &-s_y^2s_w^2F_y'F_w'\Bigg]-\Bigg[\frac{s_ys_w}{\eta^2}J_y\Bigg(\left(c_y\frac{s_w}{w}+c_w\frac{s_y}{y}\right)\\ &\times(c_ws_y+(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})c_ys_w)+(1-(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})^2)s_yc_y\\ &\times\left(F_w'-c_w\frac{s_w}{w}\right)\Bigg)+(y\leftrightarrow w)\Bigg]\Bigg\},\end{aligned}$$

with

$$\begin{aligned}J_y = &((\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})^2-1)c_yF_y's_w+(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})(s_ys_w/w\\ &-(\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{w}})s_ws_y/y).\end{aligned}$$

The numerical integration of B_2^0 is tricky due to the presence of the function S in the denominator. Results are shown in Fig. 5, as functions of separation distance d. It shows that the SA presents the correct topology for the NN system.

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