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Size shrinking of deuterons in very dilute superfluid nuclear matter

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It is shown within the strong-coupling BCS approach that, starting from the zero-density limit of superfluid nuclear matter, with increasing density deuterons first shrink before they start expanding.

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The crossover from Bose-Einstein condensation (BEC) of bound fermion pairs in the low density limit to superconductivity or superfluidity for higher densities is a very actively pursued field of research, since, for instance, such phenomena may play a role in high T_c superconductors [1]. In a recent paper we have shown [2], using the strong-coupling BCS approach, that also in nuclear matter such a crossover may take place. Indeed deuterons are bound proton-neutron (p-n) pairs which may turn into p-n Cooper pairs at higher densities [2]. In fact the situation in superfluid nuclear systems with $\Delta/\epsilon_F \approx 1/10$, where Δ is an average gap value and ϵ_F the Fermi energy, rather resembles a strong coupling superconductor than the one of ordinary metals.

In this Brief Report we will continue our previous work [2] and study the size of the deuterons or p-n Cooper pairs as a function of density. It is expected on arguments of general grounds that with increasing density, starting from the zero-density limit, the deuterons first shrink before they expand [3]. Indeed once the deuterons come on average close enough so that they start feeling the Pauli principle with their immediate neighbors they can avoid the increasing repulsion in reducing their spacial extension. Of course this cannot go on forever and soon the deuterons will start overlapping, losing their binding, and increase in size. It is very interesting that such a general feature is already contained in the BCS approach to superfluidity [3]. Therefore we will present calculations of the coherence length of p-n pairing in the SD two-body coupled channels in symmetric nuclear matter based on the strong-coupling BCS approach. The interaction adopted in the gap equation is the Paris force projected onto the SD channel, which reproduces quite well the experimental phase shifts of p-n scattering as well as the deuteron binding energy. The single-particle energy $\epsilon(p)=p^2/2m+U(p)$ is calculated from the same Paris potential in the Brueckner-Hartree-Fock (BHF) approximation. The details of the procedure for solving the BCS equations are reported elsewhere [2]. The coherence length is defined by

$$\xi^2=\frac{\int d\vec{r}r^2|\psi(\vec{r})|^2}{\int d\vec{r}|\psi(\vec{r})|^2}=\frac{\int d\vec{p}|\nabla\tilde{\psi}(\vec{p})|^2}{\int d\vec{p}|\tilde{\psi}(\vec{p})|^2}, \qquad (1)$$

where $\psi(\vec{r})=\langle c^\dagger(\vec{r})c^\dagger(0)\rangle$ is the pairing function and $\tilde{\psi}(\vec{p})$ is the Fourier transform. Integration in momentum space is more suitable since the gap equation is solved in momentum space, giving $\Delta(\vec{p})$ and then

$$\tilde{\psi}(\vec{p})=\Delta(\vec{p})/\sqrt{[(\epsilon(p)-\mu)^2+\Delta(\vec{p})^2]}. \qquad (2)$$

In Fig. 1 the coherence length is plotted as a function of the density: the crosses correspond to using the free single-particle spectrum and the stars to using the BHF mean field. The values of ξ tend to coincide at low density where the mean field is negligible. At vanishing density ξ approaches the deuteron radius and then it decreases according to the general trend. The minimum at $\rho \simeq 0.036$ fm^{-3} amounts a deuteron radius of less than 2 fm and to an interparticle distance of about 3 fm. Above the minimum the coherence length rapidly rises in qualitative agreement with the weak-coupling prediction, i.e., $\xi \sim k_F/\Delta$.

The above-mentioned size shrinking can be traced back to the Pauli blocking effect. A simple explanation starts from the Schrödinger-like equation

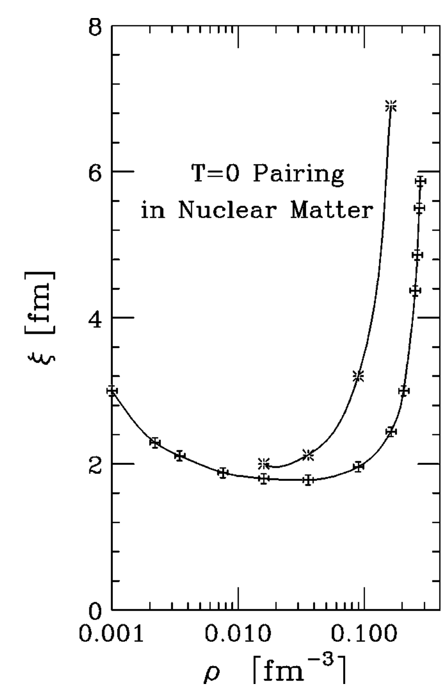

FIG. 1. Coherence length vs density in the BCS approximation.

$$(p^2/m-2\mu)\psi(p)=[1-2n(p)]\sum V(p,p')\psi(p'), \quad (3)$$

which is equivalent to the BCS equations [4,2]. At zero density this is the Schrödinger equation for the p-n system: the eigenfunction $\tilde{\psi}_0$ corresponding to the energy eigenvalue $2\mu_0=-2.2$ MeV is the deuteron ground state. At very low density one may treat the potential $U(p,p')=-2n(p)V(p,p')$ as a small perturbation and then estimate the energy correction for μ positive close to zero by

$$\delta E=\frac{\sum\sum \tilde{\psi}_0^*(p)U(p,p')\tilde{\psi}_0(p')}{\sum |\psi^0(p)|^2}$$

$$=\mu_0\frac{16}{\pi}\frac{p_F}{p_0}\left[1-\frac{p_0}{p_F}\arctan\left(\frac{p_F}{p_0}\right)\right],$$

where $p_0=\sqrt{(-2m\mu_0)}$. This energy correction makes the system more bound. One may also estimate from Eq. (4) the correction to the coherence length, which results in

$$\xi\approx\frac{\hbar}{\sqrt{2}p_0}\left(1-\frac{8}{3\pi}\frac{p_F^3}{p_0^3}\right). \quad (4)$$

It is worth noticing that this result is in agreement with the one of Ref. [3], where also a linear dependence of ξ on the density was found with zero-range force. In the derivation of Eq. (5) no assumption has been made on the force except that it gives a bound state at zero density.

At this point it may be appropriate to discuss under which circumstances one may find a Bose-condensed gas of deuterons or a transition from p-n Cooper pairs to a Bose-Einstein condensation of deuterons. In the first place one may think of the far tail of density of heavier $N\simeq Z$ nuclei like they are or will be produced in the new exotic nuclear beam facilities. In a region of densities $\rho/\rho_0\simeq\frac{1}{20}$ the radial distance from the center of heavier nuclei is such that deuterons with a rms of $\simeq 2$ fm (see Fig. 1) can be easily accommodated in a Bose-condensed state. This picture demands the validity of the local density approximation and to neglect quantum fluctuations. Both approximations are, of course, questionable for finite nuclei but we know by experience that always something remains in a more correct treatment of those very simplifying assumptions. In this respect it could be very interesting to trigger very peripheral nuclear reactions and to measure the yields of deuterons (or correlated p-n $T=0$ pairs) with respect to uncorrelated nucleons. Also in expanding nuclear matter as it is produced in the late stage of central collisions with energies of $E/A\sim\epsilon_F$ condensation phenomena in very low density nuclear matter could play a role. In this respect it should be noted that for densities where the chemical potential of the p-n pairs is negative, i.e., where there is Bose-Einstein condensation of deuterons, the influence of the Pauli principle due to additional neutrons (asymmetric case) should be minimal. This has been confirmed by recent numerical calculations [5].

In conclusion we have shown that in a very diluted superfluid gas of deuterons the deuterons as a function of density first shrink by $\sim$35% before they start expanding again. This relatively large effect is due to the Pauli principle which is fully respected in the BCS approach. The investigation has been performed at $T=0$. The extension to finite temperature and to asymmetric matter is on the way.

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